COMPLEX

Knowledge Based Climate Mitigation Systems for a Low Carbon Economy

Review of existing literature on methodologies to model non-linearity, thresholds and irreversibility in high-impact climate change events in the presence of environmental tipping points

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1. Introduction

Nature is often expected to respond to gradual changes in a smooth way. However, studies of climate systems, lakes, coral reefs, oceans, forests and arid lands have shown that smooth change can be interrupted by sharp (or catastrophic) shifts to different regimes (Scheffer et al. 2001). 'Tipping points', 'thresholds', 'regime shifts', and 'irreversibility' — all are terms that describe the flip of a complex dynamical system from one steady state to another steady state, the so-called catastrophic shift. Catastrophic changes in the overall state of a system can derive from feedback mechanisms within it, and from linkages that are latent and often unrecognized. The change may be initiated by some exogenous shocks. Once set in motion, however, such changes can become explosive and afterwards will typically exhibit some form of hysteresis, such that recovery is much slower than the collapse. In extreme cases, the changes may be irreversible.

More specifically, climate change is a complex phenomenon plagued of uncertainties which are crucial for climate policy. Some of these uncertainties are related to the existence of non-linearities, thresholds, and irreversible events. The Stern report (Stern, 2007) points out three main non-linear changes and threshold effects from climate change. First, global warning will increase the chance of triggering abrupt and large-scale changes. Second, melting of polar ice sheets would accelerate sea level rise and eventually lead to substantial loss of land, affecting around 5% of the global population. Finally, global warming may induce sudden shifts in regional weather patterns with severe consequences. In this context, a critical issue is the definition of mitigation policies when the possibility for non-linearities, thresholds, and irreversible events is allowed. From the policy perspective, the existence of uncertainty about thresholds affects the timing and design of emissions abatement. In this sense, the uncertainty about the thresholds suggests that climate policy should be ''precautionary'' in the sense of favoring earlier and more intense intervention (Pindyck, 2007). However, due to this same uncertainty, it is not clear the level of precaution that should be adopted.

When it comes to assess the socio-economic impact of climate change and mitigation policies, current impact assessment models are unable to present non-linearities, thresholds and irreversibility or run catastrophe climate scenarios. Numerous studies have indicated that in the case of non-linear climate change impacts, optimal abatement increases substantially (Baranzini, et al., 2003, Gjerde, et al., 1998, Keller, et al., 2004, Kolstad, 1994, Mastrandrea, 2001, Tol, 2003, Yohe, 1996, Zickfield and Bruckner, 2003). The potential for non-linear and low-probability climate responses to anthropogenic greenhouse gas forcing, however, has received little attention in the climate change damage cost literature to date (Alley, et al., 2003, Higgins, et al., 2002, Tol, 2009, Wright and Erikson, 2003).

Besides climate change impacts irreversibility and non-linearitiesare also important characteristics of investment decisions, consumption preferences and production technologies. Most of the conventional economic models, however, are unable to take full account of biophysical and psycho-social characteristics of consumption and production (see Dasgupta and Heal, 1974).

This report presents a review of existing literature on the non-linearity, thresholds and irreversibility in the presence of environmental tipping points, their importance for policy measures and challenges for climate-energy-economic impact assessment modelling. Section 2 focuses on the catastrophe theory and critical transitions as well as complexity aspect of non-linearity in natural and socio-economic systems. It also provides a prototype modelling model of pollution management problems in the presence of environmental tipping points, threshold and irreversibility to illustrates the theoretical framework of the critical transition theory. Section 3 provides a review on non-linearities, thresholds and irreversibilities in economic literature and policy documents. Finally Section 4 reviews challenges for CEE impact modelling tools in the presence of abrupt non-linear responses focusing on three main modelling approaches in COMPLEX, Integrated Impact Assessment models including General Equilibrium models, system dynamics modelling, and agent-base modelling approaches.

2. Critical transition, non-linearities, thresholds and irreversibilities in natural and socio-economic systems

2.1 Definitions

Non-linearities:

Most coupled socio-environmental systems (SES) exhibit nonlinear behaviour (Liu et al. 2007). In general, this implies that small changes in independent variable(s) or in underlying micro behaviours may cause non-constant effects in a dependent variable or macro patterns and phenomena. In complex adaptive systems it is often the case that many interacting agents follow rules that produce complex nonlinear dynamics at macro-level (Axelrod 1997). Yet, even when processes at micro level or within subsystems are rather straightforward and linear, the interactions and feedbacks between them may cause non-linear responses resulting in a change in the trajectory of the system (Walker 2004). Cross-scale interactions and positive feedbacks among system elements, which lead to the emergence of nonlinear system responses, is a common feature of biological, physical and economic system (Peter et al 2004, Levin 1999, Carpenter et al 2011). Thus, nonlinear effects and their macro scale impacts stem from local processes, which shift from one state to another (Arthur 1999). A transition between alternate states often occurs when a threshold level of some control variables in a system is passed (Walker 2004), making such thresholds a common form of non-linearity (Liu et al.2007).

Emergent nonlinear system behaviour is characterized by discontinuities (Liu et al. 2007; Huggett 2005), what makes it difficult to predict systems behaviour on various spatial and temporal scales (Peter et al 2004). In coupled SES certain positive feedbacks within or between subsystems may trigger nonlinear increases in economic costs (Chapin et al 2000).

Thresholds:

Threshold is a critical value of independent variable where a system flips from one stable state to another (Muradian 2001; Wiens et al. 2002; Walker and Meyers 2004; Bennett et al. 2006; Kinzig et al. 2006). In resilience literature critical thresholds are called tipping point (Scheffer et al 2009). In mathematics thresholds are known as bifurcations (Andersen et al 2008). Bifurcation refers to a qualitative change in a steady state of an adaptive system at a faster time scale when a parameter on slower moving time scale goes through a critical value that causes the stable state on the fast time scale to become unstable (Brock 2004). While mathematics studies bifurcations as changes to cyclic or irregular transitions, an analysis of SES focuses primarily on a transition between two steady states (Andersen et al 2008). As reviewed by (Hugget, 2005) a threshold can be seen as a (1) a bifurcation point, (2) a boundary in space and time, (3) a critical value of independent variable, (4) a single point or a zone – where a “relatively rapid change” between alternate regimes occurs.

Crossing a threshold results in an abrupt shift of complex system from one regime to another, and may cause a cascade of thresholds crossed. Empirical research suggests that the positions of critical thresholds and chances of crossing them in one domain or scale dynamically react on the changes in other domains and scales creating a phenomenon of a moving threshold (Kinzig et al. 2006).

To some extent crossing a threshold may be used to identify a regime shift in SES. However, the data for identifying thresholds is often absent or incomplete (Huggett 2005).

Moreover, there might be a time lag between the system crossing a threshold and the reflection thereof in the domain-specific macro-measures of interest.

Irreversibility:

While complex adaptive SES are perpetually out-of-equilibrium going through marginal or non-marginal changes, some of those changes in system's states may be irreversible. According to Folke (2004) irreversibility is a consequence of changes in variables with long turnover times and a loss of SES potential and interactions between system elements, which are able to help the system to renew and reorganize back in a desired state. A transition, which is not reversible, is called hysteresis (Scheffer et. al. 2009). As Brock et al (2004) define it: a hysteresis is a change in a system state, which requires more efforts to shift it back to the previous desirable state compared to the original forcing that triggered the regime shift. For example, if a system flip was caused by a slow-moving variable going "up" and crossing a critical value, it needs to be forced "down" to a level quite far below the original value, which initiated this critical transition, to be able to "recover" the old state (Brock et al 2004).

The three terms – non-linearity, thresholds and irreversibility – are closely related to the notion of resilience. "Resilience reflects the degree to which a complex adaptive system is capable of self organization (versus lack of organization or organization forced by external factors) and the degree to which the system can build capacity for learning and adaptation." – a quote from (Adger et al 2005)

2.2 Complexity aspects in coupled non-linear CEE systems

2.2.1 Nonlinear dynamics in natural systems

Climate system

Non-linearities

It is no surprise that state-of-the-art Global Climate Models (GCMs) and their recent extensions – Earth System Models (EMS) – based on strongly non-linear equations of geophysical fluid dynamics often demonstrate strongly non-linear behaviour. It should be noted however that already very simple climate models, like one-dimensional energy-balance models (EBMs) that were by design unable to resolve atmospheric/ocean dynamics, often demonstrated prominent nonlinear behaviour, including the existence of possible multiple equilibria (Budyko, 1969; Sellers, 1969). And even simpler zero-dimensional models broadly used as climate modules of Integrated Assessment models to calculate global mean surface air temperature changes caused by anthropogenic GHG emissions, e.g. the model NICCS (Hooss et al., 2001), are usually nonlinear (CO2 concentration typically enters the r.h.s. of temperature dynamic equation under a logarithm, therefore the CO2 forcing is nonlinear etc.).

It should be noted in this respect that the entire modern theory of dynamic systems emerged from a seminal three-dimensional dynamic model of convection for which a strange attractor has been discovered by Lorenz (1963). Thus the modern theory of chaotic systems has its origin in geophysical fluid dynamics. In a recent review Ghil (2013) addresses two complementary approaches to modelling global climate system as a dynamic system – a deterministic nonlinear approach, which he calls "the Lorenz approach", and a stochastic linear approach, which he calls "the Hasselmann approach" (in the latter the "slow" climate variability is caused by random forcing by "fast" "weather" events (Hasselmann, 1976)). He argues that the unification of both approaches towards a stochastic nonlinear climate theory is necessary to address problems of climate variability and climate sensitivity to external forcing.

An analysis of ocean circulation – an important element of the global climate system – from the viewpoint of modern theory of dynamic systems started with a simple two-box model of the North Atlantic circulation, in which two steady-state flow regimes were revealed (Stommel, 1961). These ideas have been significantly advanced in recent years in the numeric bifurcation analysis of realistic ocean models, which are able to address features such as the bistable regimes of the Gulf Stream and Kuroshio, or the El Niño – Southern Oscillation (ENSO) phenomenon (see a comprehensive review in (Dijkstra, 2005)). Finding steady-state solutions of a hierarchy of ocean models of increasing complexity, accompanied by a numeric analysis of their stability and associated bifurcation diagrams in the model parameter space, is a promising area for the further development of more realistic climate models, since the values of many ocean model parameters, especially with respect to sub grid dynamic processes, are often not known with sufficient accuracy (Dijkstra, 2005).

Thresholds

Rial et al. (2004) relate the concept of thresholds in climate system to a question of the balance of amplifying (positive) and controlling (negative) feedbacks. They propose a metaphor of a net feedback, arguing that countless feedbacks in the climate system can be reduced to a net negative feedback and a net positive feedback, According to this metaphor, in unperturbed conditions the net negative climate-driving feedback of the Earth is slightly stronger than the net positive feedback, at least for small values of external/internal forcing. However if the forcing grows beyond the point at which the two competing feedbacks are balanced, then the explosive amplification produced by positive feedbacks leads to strong nonlinear effects. The point of balance between the two competing feedbacks defines a runaway threshold.

However, even below this critical runaway level, the negative impacts of human induced climate change can become so strong at some critical adaptation threshold that societies are no longer able to respond to the climate change impacts at an acceptable cost. Thus mitigation policies should be implemented such that this critical adaptation threshold is not exceeded. The inevitable uncertainty in the scientific and socioeconomic determination of the adaptation threshold has inevitably lead to discussions within the academic community. However, in the Copenhagen Accord (UNFCCC, 2010) this threshold, based on recommendations, among others, of Bruckner, et al (1999), was set at 2 degrees C. Jaeger and Jaeger (2011) provide an interesting overview of the history of emergence of 2C target, including a review of the criticism of this target. Whether the 2C threshold is well justified as a mitigation policy target or not, there is now increasing skepticism on the chances of retaining the global mean surface air temperature at or below this limit (Anderson and Bows, 2011; Peters et al., 2013). At the same time, some recent studies (Mann, 2009; Smith et al., 2009) have revised the climate change impacts associated with 2C temperature rise above the pre-industrial level towards higher severity levels. On this basis, Anderson and Bows (2011) suggest redefining the 2C limit as a threshold not between “acceptable” and “dangerous” climate change, but between “dangerous” and “extremely dangerous” climate change.

Irreversibilities

In IPCC AR5 WGI (2013, final draft) abrupt climate change is defined as a large-scale change in the climate system that takes place over a few decades or less, persists (or is anticipated to persist) for at least a few decades, and causes substantial disruptions in human and natural systems.

The IPCC AR5 WGI (2013, final draft) defines a perturbed state as irreversible on a given timescale if the recovery timescale from this state due to natural processes is significantly longer than the time it takes for the system to reach this perturbed state.

The concept of abrupt climate change is closely related to the concept of egrodicity (or supposed non-ergodicity) of the global climate system. The dynamic system is called ergodic if its behavior does not depend on its initial state and if its statistics as $t \to \infty$ does not depend on its initial position in phase space (Karol, 1988), If the climate system is ergodic, then for given external conditions there can be only one climate state; otherwise, dependent on the initial conditions, the climate system can move to different asymptotic regions in the phase space, so that multiple climate states may be possible for single given external conditions, the actual climate depending on the history of the system. Lorenz (1976) called ergodic climate systems “transitive” and non-ergodic climate systems “intransitive”; referring to paleoclimate evidence, he also proposed a hypothesis of “almost-intransitivity” of highly nonlinear climate systems. According to this hypothesis, the phase space of the global climate system is divided into separate regions. The trajectories of the climate system remain within a certain area over a finite time interval, but from time to time they move from one region to another. Lorenz (1976) provided an example of “almost-intransitive” dynamic system.

It should be noted that abrupt climate changes can be defined not only in the time domain (as above), but also in the frequency domain, where they can be manifested as changes in dominant oscillations or shifts in the phase between different climate signals (Lohmann, 2011). One example, suggested by modelling studies (e.g. Timmermann et al., 1999) is a possible change in El Niño events caused by global warming

Irreversible changes in the climate system may be triggered if the forcing exceeds certain thresholds. E.g. a recent modelling study by Robinson et al. (2012) suggests that an irreversible decay of the Greenland ice sheet (that would lead to 6 to 7 m sea level rise) could start when the global average temperature exceeds 1.5C above preindustrial level (with a range of 0.8 to 3.2degrees C); These estimates are substantially lower than those reported previously in IPCC AR4 Other model simulations suggest that not only the level of the forcing matters, but also its rate of its change (Stocker, 1999). For instance, simulations with a coupled climate model reported in (Stocker and Schmittner, 1997) produced irreversible changes of the Atlantic thermohaline circulation at CO2 concentration of 750 ppm if CO2 increases by 1% per year, while at a slower rate of only 0.5% per year the system withstands the increase of CO2 concentration up to the same level. The sensitivity to the rate of increase of the CO2 concentration is governed in this case by the rate increase of the heat uptake of the ocean.

Non-linearities in ecological systems

Ecosystems often do not respond smoothly to gradual change (Gunderson & Pritchard 2002), The nature and extent of the feedbacks can produce a change of direction of the trajectory of the system itself (Walker 2004). Biodiversity changes and ecosystem processes are likely to cause non-linearity, particularly when thresholds of ecosystem resilience are exceeded (Chapin et al 2000).

The most basic models of ecosystem dynamics in general and of population ecology in particular are substantially nonlinear. For instance, a model of population growth of organisms under limiting environmental constraints proposed by Verhulst in the 19th century (the logistic growth equation) has an S-curve as a solution, and the predator-prey Lotka-Volterra model (dated back to 1920s) describing the population dynamics in an ecosystem consisting of two interacting species is an interesting example of a conservative nonlinear system exhibiting periodic, strongly non-linear oscillations (Lorenz, 1993; de Vries, 2013).

2.2.2 Nonlinear dynamics in socio-economic systems

Economic systems, like many natural systems, are complex systems with non-linear dynamics, interactions and feedbacks loops. Among first studies of complexity in economic systems are Schumpeter and Hayek, and Simon. Indeed, with the recent explosion of interest in nonlinear dynamical systems, in mathematics as well as in applied sciences the fact that simple deterministic nonlinear systems exhibit bifurcation routes to chaos and strange attractors, with ‘random looking’ dynamical behaviour, has received much attention in economics (see for example Brock and Hommes, 1997 and 1998). The complex economic modelling paradigm was mainly developed by economists as well as natural scientists and computer scientists within multidisciplinary fields of research.

‘Tipping points’, ‘thresholds’, ‘regime shifts’, and ‘irreversibility’ — all are terms that describe the flip of a complex dynamical system from one steady state or equilibrium to another alternative state or equilibrium, the so-called catastrophic shift. Catastrophic changes in the overall state of a system can ultimately derive from how it is organized — from feedback mechanisms within it, and from linkages that are latent and often unrecognized. The change may be initiated by some exogenous shocks. Once set in motion, however, such changes can become explosive and afterwards will typically exhibit some form of hysteresis, such that recovery is much slower than the collapse. In extreme cases, the changes may be irreversible. For financial institutions, the Wall Street Crash of 1929 and the Great Depression are examples of such a shift.

In mathematical terms, this means that the system can undergo a catastrophic shift that is a small changes in certain parameter values of a nonlinear dynamical system can cause equilibria to appear or collide or disappear, or to become stable or unstable. This could lead abrupt and sudden changes of the behavior of the system or mathematically speaking can cause dramatic changes in geometrical qualitative structure of the system. It can be said that catastrophe theory is a special case of bifurcation theory, part of the study of nonlinear dynamical systems (cf. Kuznetsov, 2004). Bifurcation theory is widely argued to have been developed first by the great French mathematician, Henri Poincaré, as part of his qualitative analysis of systems of nonlinear differential equations (1880-1890). The other principal figure in this field was, Christopher Zeeman (1977), who was responsible for coining the term catastrophe theory.

The economic dynamics is mainly concerned with modelling fluctuations in economic and financial systems, such as prices, output growth, unemployment, interest and exchange rates (see for example Brock and Hommes, 1998). These dynamics can be also concerned with modelling coupled natural-socio-economic systems such as ecological-economic and climate-economic models (cf. Wagener, 2003; Moghayer and Wagener 2008). Regarding the main source of economic fluctuations there are two main conflicting views. The linear, stable view in which it is argued that these fluctuations are driven by random exogenous shocks to consumer preferences, technology, firm’s earning, dividend, etc. This view, which dates back 1930’s by Frisch, Slutsky, and Tinbergen, does not offer an economic explanation to those fluctuations, but rather exogenous forces to linear stable economic systems. In the nonlinear view, however, the economy may be unstable and even in the absence of external shocks, fluctuations in economic variables can arise.

Earlier studies of non-linearity in economic dynamics were conducted by Goodwin, Hicks, and Kaldor who developed non-linear, endogenous business cycle models in the 1940’s and 1950’s. However, this view was criticized especially with regards to the law of motion in these models that are considered ad-hoc and that the agent’s behavior was considered irrational. The latter was triggered by the rational expectations revolutions in 1960’s and 1970’s (cf. Muth (1961)). New classical economists developed alternatives within general equilibrium framework, characterized by optimizing behaviour of consumers and firms, market clearing for all goods in each period and all agents having rational expectations. An example of such models is Dynamic Stochastic General Equilibrium Models (Clarida et al., 1999), which is currently a dominating tool for policy analysis.

Zeeman (1974) was among the first application of critical transition theory or catastrophe theory in economics to study nonlinear economic dynamics. This paper models bubbles and crashes in stock markets. Debreu (1970) set the stage for doing so in regard to general equilibrium theory with his distinction between regular and critical economies, the latter containing equilibria that are singularities. Discontinuous structural transformations of general equilibria in response to slow and continuous variation of control variables can occur at such equilibria. Analysis of this possible phenomenon was carried out using catastrophe theory. Bonanno (1987) studied a model of monopoly in which there were non-monotonic marginal revenue curves due to market segmentation. Multiple equilibria can arise with smoothly shifting cost curves, which he analyzed using catastrophe theory. Beside these theoretical contribution, there have been a few empirical studies of catastrophic changes in economics. Fischer and Jammernegg (1986) was among few efforts to empirically estimate a catastrophe theory model in economics that was of a model of inflationary hysteresis involving a presumably shifting Phillips Curve.

Large numbers of economic models with critical transition can be seen in the urban and regional economics among them are Mees (1975), Wilson (1976), Dendrinos (1979), and Beckmann and Puu (1985). A good example in transport economics is Andersson (1986) that modeled “logistical revolutions” in interurban transportation and communications relations and patterns as a function of long run technological change using a fold catastrophe. In finance, Krugman’s (1984) of multiple equilibria in the demand for foreign currencies could rather easily be put into such a framework following along the lines of the Varian (1979) approach can be seen as application of catastrophe theory. Many models are now studied of multiple equilibria in foreign exchange rate models, with many of these taken very seriously given the numerous foreign exchange crises that have occurred in recent years.

Another important application of catastrophe theory or critical transition can be seen in the ecologic-economic systems focusing on the abrupt changes in biological populations and the state of ecosystems, including collapses to extinction as a result of interaction with human activities. Indeed, whenever human activity influences the state of an ecosystem, usually through some form of pollution that is a by-product of some kind of production activity, the difficult problem arises of assessing the relative interests of producers affecting the ecosystem and producers and consumers enjoying it.

In the next section, a recent research on the economics of lakes will be reviewed, presenting the results in critical transition and catastrophe theory and bifurcation form. The aim of this section is two-fold: first, to provide and illustration example of all the terms and definitions discussed in the previous sections; second, to provide a prototype example of a stylized non-linear coupled economic- environmental model that exhibits non-linearities, thresholds and irreversibility in the presence of environmental tipping points[1].

2.3. Critical transition, non-linearity, thresholds and irreversibility: an illustration modeling example

Nature is often expected to respond to gradual changes in a smooth way. However, studies of lakes, coral reefs, oceans, forests and arid lands have shown that smooth change can be interrupted by sharp (or catastrophic) shifts to different regimes (Scheffer et al. 2001; Carpenter 2003). One of the best-studied catastrophic shifts is the sudden loss of transparency and vegetation observed in shallow lakes, i.e. lakes with a depth less that 3 meters, as a result of human activities. Initially shallow lakes have clear water and a rich submerged vegetation. However, nutrient loading may change this. For instance nutrients arrive in the lake as a result of the use of artificial fertilizers on surrounding land; they are washed into the lake by rainfall.

Due to heavy use of fertilizers, at some point lakes might flip from a clear state to a turbid state that is caused by a dominance of phytoplankton. Lakes are hard to restore to the clear water state in the sense that the nutrient loads have to be reduced far below the level where the flip occurred before the lake returns to a clear state. In this case the lake is said to show *hysteresis*. In some cases the turbidity of the lake is even irreversible. The positive feedback through the effect on the submerged vegetation is one explanation for this *hysteresis effect*. The critical points at which the system flips (shifts) in a way that is not instantly reversible (or irreversible) are called *tipping points*.

The lake model that is used in this study gives a very simplified representation of these complex ecological feedback mechanisms that are active in a shallow lake. Indeed, the lake model in this study should be viewed as a metaphor for general ecological systems with

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[1] *The presented model and results are based on a recent research program carried out by Saeed Moghayer (authour of this report), and Florian Wagener in the Center for Nonlinear Dynamics in Economics and Finance at the University of Amsterdam (cf. Moghayer, 2012).*

tipping points, thresholds, non-linearities, and irreversibilities so that the analysis developed here will have a wider applicability (cf. Scheffer 2009).

2.3.1 Lake Dynamics

The dynamics of a lake, which was described above, can be modeled as a single non-linear difference equation

$$x_t = u_t + (1-\mathrm{b})x_{t-1} + h(x_{t-1})$$

Here x_t is the concentration of phosphorus, one of the main nutrients, in the lake. Artificial fertilisers containing phosphorus are used on the fields surrounding the lake. The phosphorus is washed into the lake by rainfall, yields a net inflow u_t of phosphorus. The parameter b denotes the sedimentation rate at which phosphorus leaves the water column and enters the sediment at the bottom of the lake. The last term models the internal production of phosphorus in the lake, e.g. through re-suspension of the sediment, and is assumed to be an S-shape function that has its inflection point at the point $x=1$:

$$h(\mathrm{x}) = \frac{x^q}{1+x^q}$$

The exponent q ,the *responsiveness* of the lake, is proportional to the steepness of h at $x=1$. Thus, steeper function h (resulting from higher q values) creates stronger *hysteresis*. For a constant pollution loading $u_t = u$ for all t, the fixed points of the lake are solutions of the equation

$$u = g(x) = b\,x - \frac{x^q}{1+x^q}$$

which is illustrated for $b=0.6$ and $q=2$ and $q=4$ in **Figure 1**.

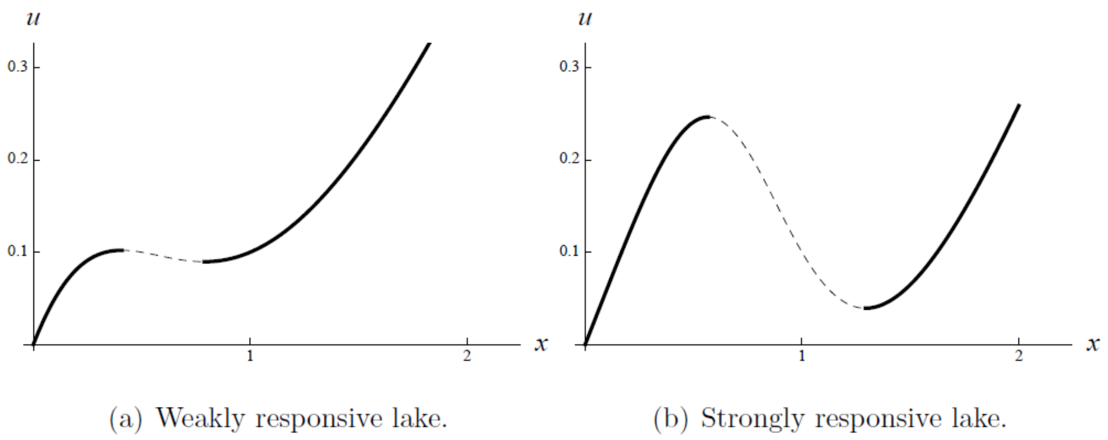


Figure 1: Location of fixed points for constant pollution streams $u_t = u$ for all t, plotted for $b=0.6$, and for (a) weakly ($q=2$) and (b) strongly responsive lakes ($q=4$). Indicated are stable (solid) and unstable fixed points (dashed).

For both values of q there is a range of u-values such that there are multiple steady states. However, the range is bigger for $q=4$ than for $q=2$. If the system starts in a low pollution steady state, and if u is then raised very slowly past the first critical value it switches to a high pollution steady state. A small subsequent decrement of u will not move the system back to the clean branch of steady states. For this, the pollution flow has to be lowered significantly, below the second critical value.

There is a value $b = b^*$ such that for $b < b^*$ the lake can be trapped in the high pollution steady state of phosphorus. This happens if the first flip, which occurs at $u = \bar{u}$, is irreversible. The critical value is $b \approx 0.57$ for $q = 4$ and $b = 0.5$ for $q = 2$ (see **Figure 2** for the case $q = 4$). In that case, only after a change in the value of b the lake can be restored to a clear state.

The sedimentation parameter is set the sequel of this section, it is assumed that the $b = 0.6$ so that the lake displays hysteresis but a flip to a low pollution steady state is reversible.

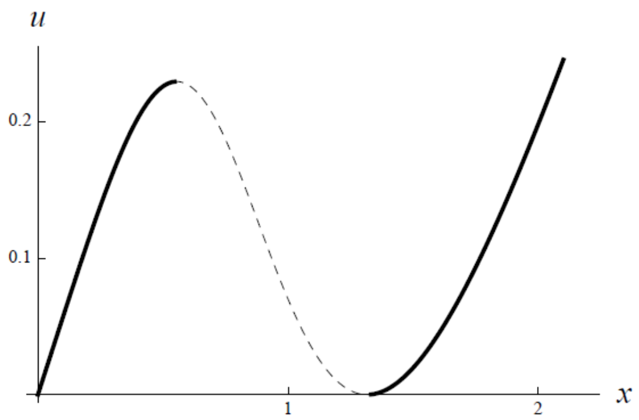


Figure 2: Irreversibility; location of fixed points for constant pollution streams $u_t = u$ for all t, plotted for $b = 0.57$ and $q = 4$. Indicated are stable (solid) and unstable fixed points (dashed).

2.3.2 Economics of lakes: optimal pollution management

In the ecological literature, management of shallow lakes is mostly interpreted as preventing the lake to flip or, if it flips, as restoring the lake in its original state. However, in this study the economics of the problem is analyzed in the sense of the trade-offs between the utility of the agricultural activities, which are responsible for the release of phosphorus, and the utility of a clear water lake (cf. Maler et. al., 2003). When the lake flips to a turbid state, the value of the ecological services of the lake decreases, but there is a high level of agricultural activities. It depends, of course, on the relative weight attached to these welfare components whether it is better to keep the lake clear or to use it as a waste dump.

Note that if it is better to keep the lake clear, it is very costly to let the lake flip first because of the *hysteresis*. The *complexity* of the lake optimal management problem derives from the *non-linear dynamics* of the lake that leads to a non-convex optimal control problem featuring several system parameters. The lake optimal management problems therefore have a rich structure that is the existence of *tipping points*. In such problems, depending on the values of these parameters, there may exist multiple steady states that are the long-run outcome of an optimal management policy. Also, the structure of optimal solutions may change if parameters are varied. In this study the bifurcation analysis developed Moghayer (2012) is used to classify the qualitative characteristics of the set of optimal solutions for different values of the model parameters.

In the lake pollution management problem, a social manager has to weigh the interest of the farmers that derive income from the use of artificial fertilizers against that of the lake users that suffer from pollution damage to the lake. Following Maler et. al. (2003), the social utility functional is modeled as

$$J = \sum_{1}^{\infty}\left(\log u_t - cx_{t-1}^2\right)e^{-\rho t}$$

Here c is the social preference parameter, and $\rho > 0$ the discount rate. The social manager tries to optimally manage the phosphorus pollution stream

$$\mathbf{u} = \{\mathrm{u}_t\}_{t=1}^{\infty}$$

that originates from the use of artificial fertilisers given that the concentration of x_t phosphorus in the lake follows the lake dynamic. The optimization problem is to maximize

$$J = \sum_{1}^{\infty}\left(\log u_t - cx_{t-1}^2\right)e^{-\rho t}$$

subject to the lake dynamic

$$x_t = u_t + (1-\mathrm{b})x_{t-1} + \frac{x^q}{1+x^q}$$

State space and control space are given as $\mathrm{X} = \mathrm{U} = (0, \infty)$ respectively.

The discrete Pontryagin function is

$$P = \log u - cx^2 + y\left(u + (1-\mathrm{b})\,\mathrm{x} + \frac{x^q}{1+x^q}\right),$$

where y is the co-state. The necessary condition $\frac{\partial P}{\partial u} = 0$ takes the form $0 = \frac{\partial P}{\partial u} = \frac{1}{u} + y$

Solving for u yields that $u = -1/y$. Substituting out u, the discrete Hamilton function is obtained as

$$\mathrm{H} = -\log(-\mathrm{y}) - \mathrm{cx}^2 + y\left((1-\mathrm{b})\,\mathrm{x} + \frac{x^q}{1+x^q}\right)$$

The necessary conditions read as

$$x_t = \frac{\partial H}{\partial y} = -\frac{1}{y_t} + (1-\mathrm{b})\,\mathrm{x}_{t-1} + \frac{x_{t-1}^q}{1+x_{t-1}^q}$$

$$e^{\rho} y_{t-1} = \frac{\partial H}{\partial x} = -2cx_{t-1} + y_t\left((1-\mathrm{b}) + \mathrm{q}\left(\frac{x_{t-1}^{q-1}}{1+x_{t-1}^q}\right)^{\wedge 2}\right)$$

Solving the second equation for y_t and substituting into the first yields the phase map, which
determines the state-costate dynamics by:

$$\mathrm{z}_t = (x_t, y_t) = \varphi(x_{t-1}, y_{t-1}) = \varphi(\mathrm{z}_{t-1})$$

where

$$\varphi(\mathrm{x},\mathrm{y})=\left(-\frac{g\,'(\mathrm{x})}{e^{\rho}y+2cx}+g(\mathrm{x}),\frac{e^{\rho}y+2cx}{g\,'(\mathrm{x})}\right)$$

with

$$g(\mathrm{x}):=(1-b)\,\mathrm{x}+\frac{x^{q}}{1+x^{q}}\ .$$

2.3.3 Solution structure and qualitative changes

In the rest of the section, the value of ρ is fixed to $\rho=0.03$. For $b=0.6$ and $q=4$, in **Figure 3**(f) fixed points and their stable and unstable manifolds are plotted for a range of values of the cost parameter c ; for all these values, the phase map has two saddle fixed points z_{-} and z_{+} .

Recall that the stable manifold of a fixed point $\bar{z}$, $W_{\bar{z}}^{s}$, is the set of all points whose forward iterates converge to $\bar{z}$:

$$W_{\bar{z}}^{s}=\{\mathrm{z}\in\mathbb{R}^{2}:\ \lim_{t\to+\infty}\ \varphi^{t}(\mathrm{z})=\bar{\mathrm{z}}\}$$

Analogously the unstable manifold of $\bar{z}$, $W_{\bar{z}}^{u}$, consists of the points backward asymptotic to $W_{\bar{z}}^{s}$:

$$W_{\bar{z}}^{u}=\{\mathrm{z}\in\mathbb{R}^{2}:\ \lim_{t\to-\infty}\ \varphi^{t}(\mathrm{z})=\bar{\mathrm{z}}\}$$

In Moghayer and Wagener (2008) it is shown that for every $x_{0}\in\mathbb{R}$, the problem to optimise

$$J=\sum_{1}^{\infty}\left(\log u_{t}-cx_{t-1}^{2}\right)e^{-\rho t}$$

subject to the lake dynamic

$$x_{t}=u_{t}+(1-\mathrm{b})x_{t-1}+\frac{x^{q}}{1+x^{q}}$$

has a solution. Moreover, the state-costate trajectory of such a solution is either on $W_{z_{+}}^{s}$ or $W_{z_{-}}^{s}$.

Figure 3(d) shows the bifurcation diagram of the lake system in the (b,c)-parameter space. The dashed curve represents saddle-node bifurcations, separating the region of values of the parameters for which the phase map has a fixed point from the region of multiple fixed points. Solid lines indicate indifference-attractor bifurcation curves, separating three parameter regions: *low pollution* region for which the clean steady state is globally optimal, (ii) the *high pollution* region for which the turbid steady state is globally optimal, and (iii) the *dependent on the initial state* region for which both the clean steady state and turbid steady state are locally optimal.

For the values of the physical parameters b and economic parameter c in the *unique equilibrium* region the phase map φ has a unique fixed point. This is a saddle, see Figure **Figure 3**(f) (a). The long run pollution level depends then on the values of the parameters c and b , changing within the region. If the pair (b,c) corresponds to a point of the *dependent on the initial state* region, the phase map φ has always two saddle fixed points characterized by respectively low pollution and high pollution (see **Figure 3**(c)). The clear state of the lake corresponds to a high level of water services and a low level of agricultural activities, whereas the turbid state corresponds to a high level of agricultural activities and a low level of water services. Depending on the initial pollution load, the social planner steers the lake to the clear or to the turbid steady state. If the pair (b, c) is in the *low pollution* region the optimal policy steers the lake to the clean steady state independently of the initial state of the lake; the clear state of the lake is globally optimal (see Figure **Figure 3**(a)). If (b, c) is in the *high pollution* region, see **Figure 3** (e & f), the optimal orbit lies on the stable manifold of the polluted equilibrium. Regardless of the initial state of the lake, the optimal policy steers the lake to the turbid state, that is the turbid steady state is globally optimal.

For a pair (b, c) in the *dependent on the initial state* region, there exist an indifference threshold, see **Figure 3**(c). If the initial state is below the threshold then the clean steady state is optimal, whereas if the initial state is above the threshold then the turbid steady state is optimal. Therefore, for a pair (b, c) in the *dependent on the initial state* region the lake is steered to the clear state only if it is initially not very polluted, otherwise it is steered to the turbid state. Note that at the *indifference threshold*, two different policies are radically different and non-equivalent, one corresponding to high agricultural activity, high pollution and convergence to the polluted steady state, whereas the other is characterized by lower pollution and convergence to clear steady state.

2.3.4 Conclusion

In this section, outcomes of the lake pollution problem and the long-term interest conflicts of the lake users have been presented, in the context of dynamic social planning. A characteristic feature of this problem, and of pollution problems in general, is the qualitative dichotomy in possible outcomes in the presence of tipping points: the lake (or the ecosystem, or the climate) ends up in either a clean or in a polluted state, both of which, if attained, is stablised by some kind of feedback mechanism.

This results to a qualitative aspect in socio-economic outcomes: the decision maker has to decide for or against production, for or against conserving the ecosystem. This qualitative distinction between the possible socially optimal outcomes enables us

to present the outcome of the analyses in the form of a bifurcation diagram presented in the last subsection, which gives a graphical overview of the qualitative characteristics of the solutions, depending on the parameters of the problem. The most critical region in these bifurcation diagrams is the "history dependent" region: in these cases, neglect by an actual decision maker that allows the ecosystem to flip can lead to large irrecoverable welfare losses.

The lake pollution problem is a prototype of a non-linear ecological-economic problem with multiple equilibria, thresholds, and irreversibility. Indeed, lake system, as mentioned in Scheffer (2009) is "a subtle twist on the Greek's view of our mind that is Mikos-Kosmos reflecting the entire world". It is also extensive enough to harbor many scales of complexity therefore served our purpose to present it as an illustration example that covers most of the definition, concepts and notions which were discussed in the previous sections of this report.

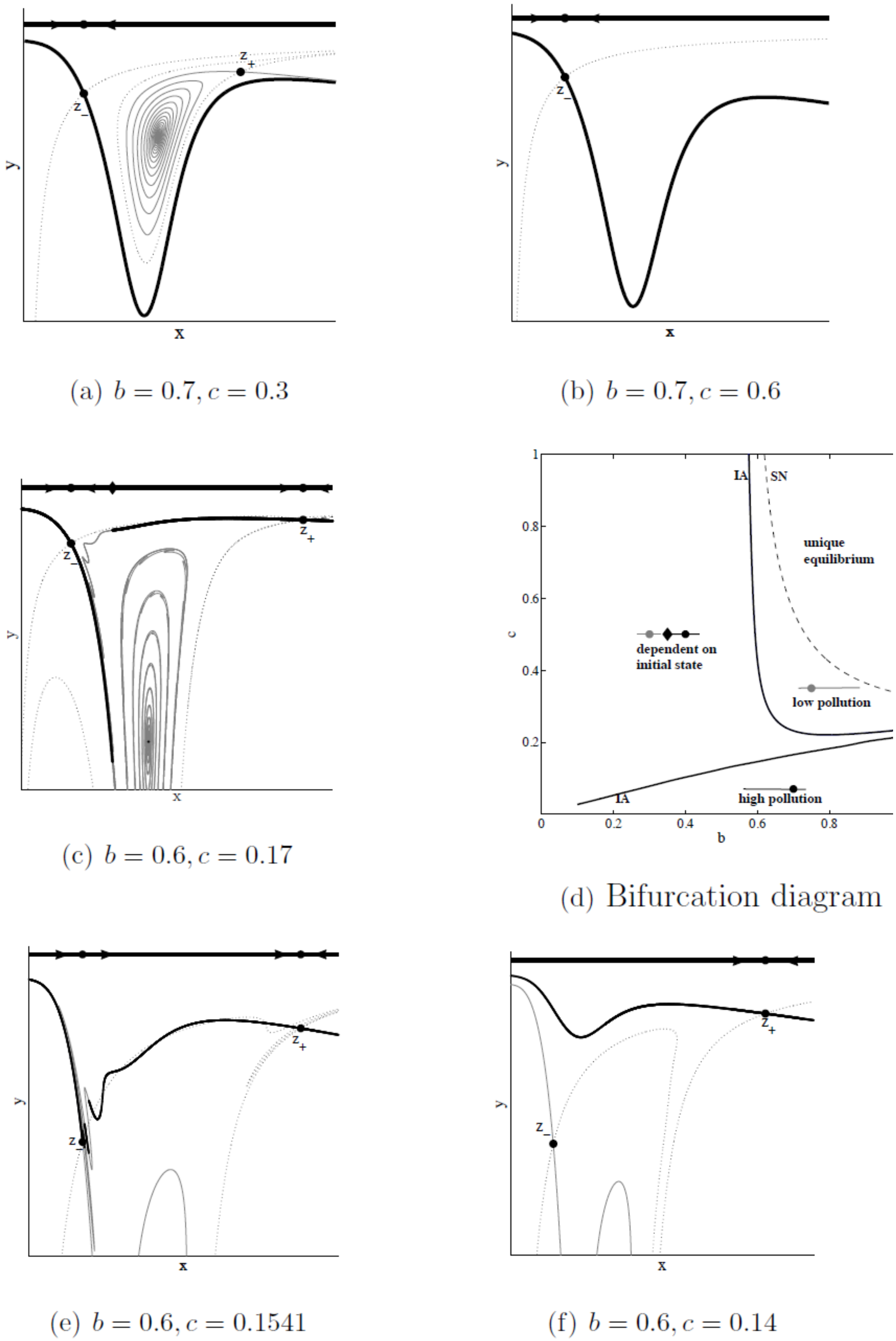


Figure 3: Plot (d) shows the bifurcation diagram of the lake system. The dashed curve represents saddle-node bifurcations, separating the region of values of the parameters for which the phase map has a fixed point from the region of multiple fixed points. Solid lines indicate indifference-attractor bifurcation curves Solid lines indicate stable manifolds, dotted lines unstable manifolds; optimal solutions are marked by thick lines; the vertical line through the indifference threshold is dashed. At the top of the graph, the optimal dynamics are indicated; attractors are marked by a circle, the indifference threshold by a diamond.

3. Climate mitigation policies in the presence of non-linearities, thresholds and irreversibilities

3.1 Non-linearities, thresholds and irreversibilities in economic literature and policy documents

3.1.1 Economics of climate change

Climate change is a complex phenomenon plagued of uncertainties which are crucial for climate policy. Some of these uncertainties are related to the existence of non-linearities, thresholds, and irreversible events. The Stern report (Stern, 2007) points out three main non-linear changes and threshold effects from climate change.

First, global warning will increase the chance of triggering abrupt and large-scale changes. These abrupt and large-scale changes could potentially destabilise regions, generating mass migrations and increasing regional conflicts. For example, the thermohaline circulation of the North Atlantic ocean, suggests the existence of thresholds, multiple equilibria, and other features that may result in episodes of rapid change (Stocker and Schmittner, 1997). While there is still uncertainty over the possible triggers for such changes, the latest science indicates that the risk is more serious than once thought. Figure 4 shows the consequences of climate change against increases in global mean temperature (°C) after 1990. Each column corresponds to a specific “reasons for concern’’ identified by the Third Assessment Report (TAR) of the IPCC (McCarthy et al., 2001) and represents additional outcomes associated with increasing global mean temperature. The color scheme represents progressively increasing levels of risk (Smith et al., 2009). The left hand side of the figure shows the risks as reported by the TAR while the right hand side reflects the update impacts associated to an increase in global mean derived from the Forth Assessment Report (AR4) of the IPCC (Parry et al., 2007).

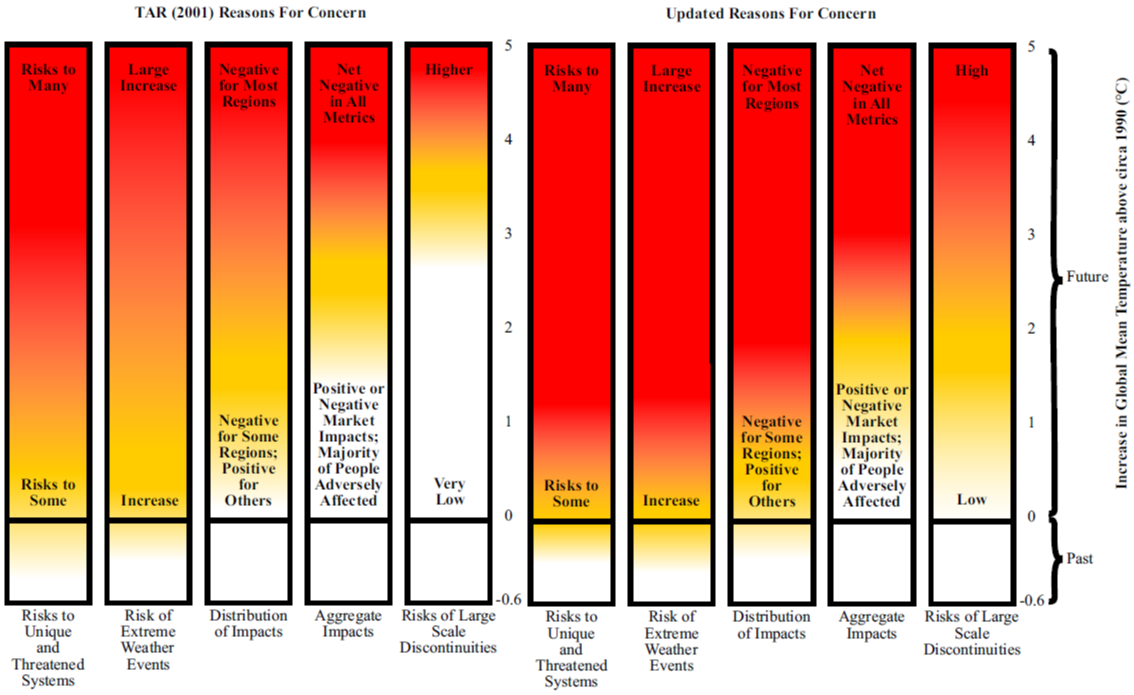


Figure 4: Risks from climate change, by reason for concern—2001 compared with updated data. Source: (Smith et al., 2009)

Second, melting of polar ice sheets would accelerate sea level rise and eventually lead to substantial loss of land, affecting around 5% of the global population. As temperatures rise, the world risks crossing a threshold level of warming beyond which melting or collapse of these polar ice sheets would be irreversible. This would commit the world to increases in sea level of around 5 to 12-m over coming centuries to millennia, much greater than from thermal expansion alone, and significantly accelerate the rate of increase. A substantial area of land and a large number of people would be put at risk from permanent inundation and coastal surges (Rahmstorf, 2007; Stern, 2007).

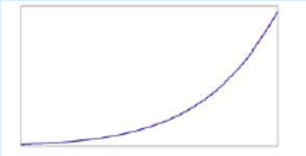
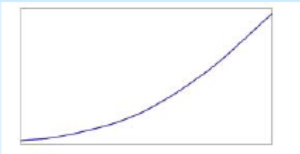
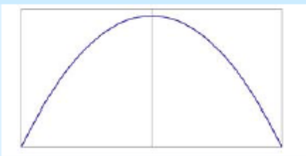
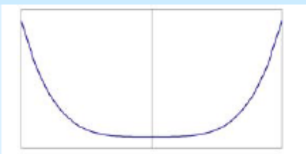
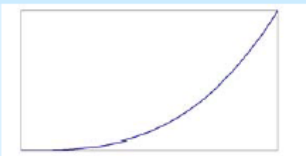
Finally, global warming may induce sudden shifts in regional weather patterns with severe consequences. The strongly non-linear nature and variability of some climatic systems, suggests that they may be particularly vulnerable to abrupt shifts. For example, recent evidence shows that an El Niño with strong warming in the central Pacific can cause the Indian monsoon to switch into a dry state, leading to severe droughts (Kumar et al., 2006). These types of shifts are temporary, but in the past, there is evidence that climate changes have caused such shifts to persist for many decades. If such abrupt shifts were replicated in the future, they could have severe impacts.

In this context, a critical issue is the definition of mitigation policies when the possibility for non-linearities, thresholds, and irreversible events is allowed. On the one hand, nonlinear phenomena characterize all aspects of global change dynamics: inputs and outputs are not proportional, change is often episodic and abrupt, rather than slow and gradual, and multiple equilibria are the norm (Rial et al., 2004). Accordingly, the damage caused by climate change does not increase linearly with the level of emissions/concentration of GHGs. Thus, damage functions tend to be highly nonlinear, with insignificant damages for low levels of pollution which turn to be severe or even catastrophic once some uncertain thresholds are reached. Similarly, the cost/benefits of climate mitigation may be very low for the abatement of low levels of emissions and extremely high for higher levels. However, as we will show in the following sections, most modelling approaches ignore these catastrophic events when assessing the cost of climate change. To some extent this is due to the fact that there is not enough direct quantitative evidence on the impacts at higher temperatures (Stern, 2007). For instance, (Hitz and Smith, 2004) found increasingly adverse impacts for several climate-sensitive sectors but were not able to determine if the increase was linear or not. Indeed, in most cases the shapes of the damage functions are unknown (see Tabel 1).

The issue of non-linearity is essential for the definition of optimal mitigation and adaptation strategies, and is especially relevant in the presence of a threshold or ‘‘tipping point’’ at which the impact climate change could become extremely severe, but we do not know where that point is (Pindyck, 2007). The implications of the existence of these critical thresholds have been widely addressed in climate science (Schellnhuber et al., 2006). Some climate researchers have argue that GHG emissions should be abated to avoid the high costs derived from exceeding climate thresholds (Rahmstorf, 1999), while others suggest that the trade-off between uncertain future climate damages and certain present costs for controlling emissions justify only low abatement levels (Tol, 1997). This discrepancy is close related to the parameter uncertainty about the threshold specific damages and the emissions level triggering a threshold (Keller et al., 2004).

Irreversibility is also a major problem for the design and implementation of climate policy. There are two major sources of irreversibility relevant for climate policy (Pindyck, 2007, 2000). On the one hand, emissions abatement policies usually impose sunk costs on society in the form of discrete investments expenditure flows. In either case, if future costs and benefits of the policy are uncertain, these sunk costs create an opportunity cost of adopting the policy, rather than waiting for more information about expected impacts. Consequently, traditional cost-benefit analysis would be biased toward policy action. On the other hand, environmental damage is often partly or totally irreversible. For example, GHGs accumulate in the atmosphere for long periods; thus, even if GHG emissions were drastically reduced, it would take many years to reduce the concentration levels in the atmosphere. Carbon dioxide is removed from the atmosphere by terrestrial vegetation or by the oceans, but this is a long process (Fisher, 2003). Further, many climate impacts such as the damage to ecosystems may be permanent. This means that adopting a policy now rather than waiting has a sunk benefit, that is a negative opportunity cost. This implies that traditional cost-benefit analysis will be biased against policy adoption (Pindyck, 2007).

Tabel 1: The types of relationship between rising damages and sectoral impacts. Source: (Stern, 2007).

Type of effect	Sector [location of source]	Proposed Functional Form		Basis
Climate system	Water [Chapter 1]	Exponential $y = e^x$		The Clausius-Clapeyron equation shows that the water holding capacity of air increases exponentially with temperature. This means that the water cycle will intensify, leading to more severe floods and droughts. There will also be more energy to drive storms and hurricanes.
	Extreme temperatures (threshold effects) [Chapter 1]	Convex curve (i.e. gradient increases with temperature)		Because of the shape of the normal distribution, a small increase in the mean dramatically increases the frequency of an extreme event.
Physical impacts	Agricultural production [Section 3.3]	Inverse parabolic ("hill function") $y = -x^2$		In cooler regions, low levels of warming may improve conditions for crop growth (extended growing season and new areas opened up for production), but further warming will have increasingly negative impacts as critical temperature thresholds are crossed more often. Tropical regions may already be past the peak. The shape and location of the curve depend on crop.
	Heat-related human mortality [Section 3.4]	U-shaped		Sharp increase in mortality once human temperature tolerances are exceeded (heatwaves and cold-snaps). Initially mortality will be reduced by warming in cold regions.
	Storm damage [Section 3.6]	Cubic $y = x^3$		Infrastructure damage increases as a cube of wind-speed
Human response	Costs of coastal protection [Section 3.5]	Parabolic $y = x^2$		Costs of sea-wall construction increase as a square of defence height

These two kinds of irreversibilities (sunk costs associated with an environmental regulation, and sunk benefits of avoided environmental degradation) interact with two kinds of uncertainty (over the future costs and benefits of reduced environmental degradation, and over the evolution of ecosystems) affecting optimal policy timing and design (Pindyck, 2000).

All these uncertainties are aggravated by the long time horizon of climatic change. The costs and, especially, the benefits from an environmental policy can extend for a hundred years or more. This long time horizon exacerbates the uncertainty derived from non-linearities, thresholds and irreversibilities.

From the policy perspective, the existence of uncertainty about thresholds affects the timing and design of emissions abatement. In this sense, the uncertainty about the thresholds suggests that climate policy should be ''precautionary'' in the sense of favouring earlier and more intense intervention (Pindyck, 2007). However, due to this same uncertainty, it is not clear the level of precaution that should be adopted.

3.1.2 Policy and governance literature

The issue of a non-linear transition from fossil-fuel-based to low-carbon economy, which goes through thresholds and exhibit irreversibilities, attracts the attention of policy-makers.

Firstly, policy discussion is structured around the fact that climate-energy-economy systems exhibit non-linear behavior with thresholds and irreversible pathways. Majority of climate mitigation policies worldwide are formulated using a “2 degrees Celsius” threshold as a target (McKinsey&Co 2009). It is widely accepted that if global warming is held below this threshold, humanity will avoid irreversible climate change with catastrophic non-linear impacts on all sectors of economy. In addition to the “2 degrees Celsius” threshold the World Bank (2012) considers a “4 degrees Celsius” point, which if crossed will lead to irreversible catastrophic consequences in 2060 or 2100.

The EU Roadmap 2050 (European Commission 2011) outlines an action plan to enable EU to achieve greenhouse gas reductions up to 80 to 95% agreed target by 2050 to avoid irreversible climate change. While policy discussions are often focused around non-linearities in climate system due to economic pressure, there is also a growing about non-linear changes in social and economic sphere systems due to changes in climate or our attitudes towards it. (Medhurst and Henry 2011) highlight that economic systems are constantly undergo changes. However, while some changes can be incremental and smooth, others are unpredictable sharp changes in trajectories driven either by external events or gradual changes and amplified by feedbacks. This non-linear dynamics, which has been also associated with long-term economic cycles such as Kondratieff waves, should be considered when exploring transitions to green economy. According to IHDP (2012) a successful transition to low-carbon economy is possible only when society moves “beyond incremental technological change toward system innovation”. Thus, business-as-usual with gradual improvement of energy technology and efficiency is not enough to achieve a green economy.

Due to path-dependency complex climate-energy-economy systems are difficult to change, i.e. they are likely to be irreversible when on a certain development trend (Liu et al. 2007). Thus, a transition to low-carbon economy is likely to undergo through a non-linear process of a system innovation and a paradigm shift (IHDP 2012). IHDP report highlights that a paradigm shift – a “change in the underlying social habits, beliefs, and assumptions that drive our behavior” – may break a traditional development path based on fossil-fuel-based economy. This systemic paradigm shift requires a deviation from the massive carbon-based production (energy supply side) as well as consumption (energy supply side).

Secondly, one of the most often discussed mechanisms that lead to non-linear responses in economic systems is emergence and diffusion of new low-carbon technologies. The Europe 2020 Strategy (European Commission 2010) sets up a goal to reduce GHG emissions by at least 20% compared to 1990 levels through an increase the share of renewable energy sources in the final energy consumption to 20% and through a 20% increase in energy efficiency. Thus, technological measures are at the core of the transformation to a low carbon economy. European Commission (2011) highlights that future modeling efforts should better represent penetration of low-carbon technologies and improvements in resource efficiency. These two may lead to non-linear impacts on economy and CO_2 footprints. Naturally, it is expected that break-through technological innovation occur spontaneously and lead to “unforeseeable structural change”. This rarely happens in a gradual way, which is used to justify marginal economic thinking in existing models.

Thirdly, another underlying mechanisms of non-linear response of socio-economic system frequently discussed in policy documents is behavioral change. In addition to supply-side effects driven by emergence and diffusion of low-carbon technologies, the demand-side effects also receive attention in policy documents. The implementation of the Energy Efficiency Directive of 2012 in EU relies on a change in consumer behavior, in particular in energy consumption practices (EEA 2013). Behavioral changes at household level are expected to bring about 6% (4 $GtCO_2$ per year) of the required GHG emission reduction by 2030 (McKinsey&Co 2009). Yet, as McKinsey report highlights, this is a very low bound of estimate, which inherits a lot of uncertainty and would be higher if considered before the implementation of technical measures. Behavioral changes that make a difference in terms of GHG emission reduction include cutting on travel, reduction of domestic heating and cooling, reducing appliance use and meat consumption (McKinsey&Co 2009). Yet, the impacts of major lifestyle changes have not been systematically quantified and are still beyond the capabilities of the quantitative modeling tools for climate mitigation (European Commission 2011).

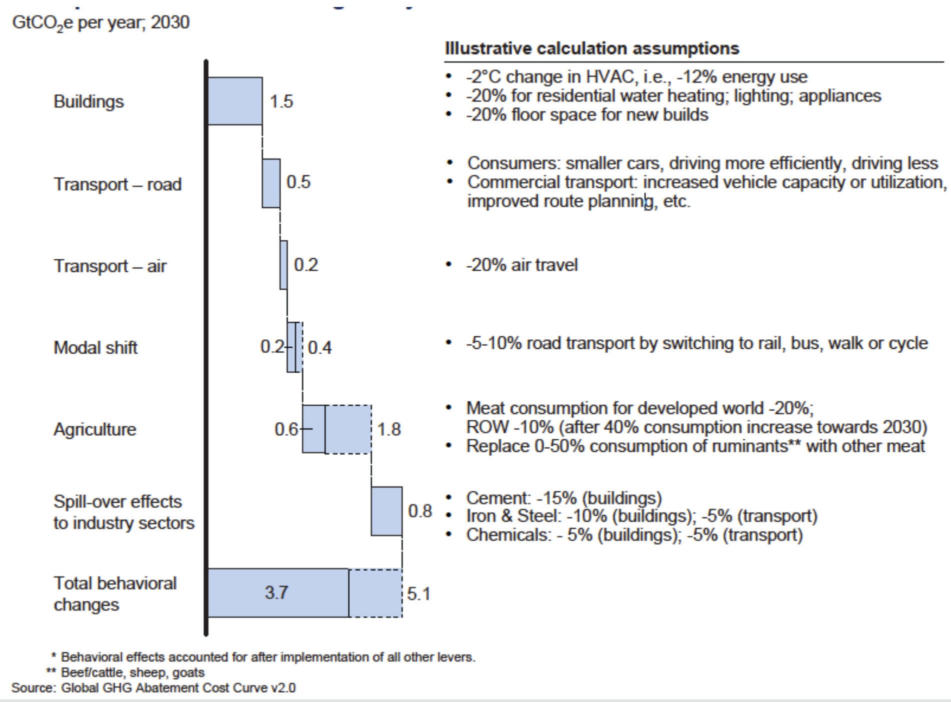


Figure 5: Impact of various forms of behavioral change on GHG emission reduction. Source (McKinsey&Co 2009).

Such behavioral changes at individual level are gradual. Yet, their impact on a societal level can experience social amplification effect, which is fueled by social interactions and shifting social norms regarding energy use, leading to thresholds and eventually non-linear transitions. A difference between a discrete technological change within one regime (carbon-intensive economies) and a *shift to a qualitatively new regime* (low-carbon economies) lays in acceptance of new norms by a *massive* range of actors and institutions (IHDP 2012). This said, practice shows that path-breaking regime shifts to a low-carbon economy start small and have roots at the local level (IHDP 2012). Thus, many incremental changes cumulatively may lead to a regime shift.

Faber and colleagues (2012) distinguish two types of behavioral change with respect to reducing GHG emissions: habitual actions and intended behavior. The former include frequently-repeated actions, i.e. habits. The latter is the outcomes of a planning process and conscious decision-making, e.g. a purchase of domestic appliances, and is usually a choice that does not occur on a daily/monthly or even annual basis. The impact of behavioral changes in routines as opposed to changes in one-time strategic choices on GHG emissions can be different. Figure 6 presents a quantification of the impact of the first category of behavioral change on GHG reduction.

Behavioral changes would require energy-awareness programs supported by the governments and potentially economic stimuli. The EEA (2013) argues that active engagement of a consumer would require a range of changes in energy markets, including its form of functioning and possible changes in their regulation. Persistent change in energy consuming behavior relies on long-term programs (educational, price, awareness, etc.) and should be studied in a dynamic way accounting for heterogeneity among households. As highlighted by EEA (2013) households and their behavioral change are not driven exclusively by economic reasoning. In contrast socio-demographic factors (age, education) as well as social norms, belief systems and cultural traits – which are changing over time – are prevalent. Quantification of the impact of a diffusion of 'green' beliefs and corresponding choices through society are vital.

Behavioural change	2020	2030	2050
1a. Buying and using an electric car	96-174	330-371	420-462
1b. Buying and using an plug-in hybrid	56-113	198-286	251-354
2. Buying and using a smaller car	80-96	74-88	71-84
3. Fuel efficient driving style	47	32	10
4. Teleworking	35-45	38-47	40-49
5. Virtual meetings	39	35	55
6a. Reduction of room temperature by 1°C	22	19	16
6b. Reduction of room temperature by 2°C	45	38	32
7. Optimised thermostat settings	11	10	9
8. Optimised ventilation behaviour	43	42	<<42
9. Shift to a vegetarian diet	266	270	271
10. Reduction of animal protein intake (one animal protein-free day per week)	50	50	50
11. Shift to a healthy diet	200	203	204

Note: The maximum realistic mitigation potential is defined as the reduction in GHG emissions achieved when the option is adopted by the largest number of actors possible, taking into account realistic and structural constraints, and where possible indirect effects and rebound effects.

Figure 6: Maximum mitigation potential for some categories of behavioral change for EU GHG emission mitigation targets, assuming all households adopt it (Mt CO2). Source: (Faber et al. 2012).

While considering behavioral change in a larger system of transition to low-carbon economy, Jevons Paradox (i.e. rebound effect) also requires attention (McKinsey&Co 2009; EEA 2013). Improvements in energy efficiency and emergence and diffusion of technological innovations may be bounced back by a behavioral response: as a unit of energy becomes cheaper and less harmful for the environment, a natural human response is to use more. This negative behavioral effect may reduce savings due to technical energy efficiency measures. Such feedbacks between technical and behavioral measures in coupled climate-energy-economy systems are likely to produce non-linear dynamics.

Lastly, another important factor, which is important when discussing potentially non-linear transitions to green economy, is impacts of transition for various economic sectors and their detailed representation in decision-support models. The feedbacks across sectors which may lead to non-linear economy pathways. The EU Roadmap 2050 (European Commission 2011) underlines that changes in technology may lead to structural changes, which require detailed specificities of sectors and their interactions. Moreover, as any structural change, a (non-linear) transition to green economy driven by emergence and diffusion of low-carbon technologies is expected to impose benefits and costs of transition. As with majority of economic cycles, e.g. Kondratieff waves, an economic system will experience short-term costs of transition (e.g. bankruptcies of carbon-intensive businesses, unemployment in corresponding sectors) in exchange for long-term benefits (slow down of climate change, healthier environment, green jobs, opportunities to develop in an energy-efficient way etc.) (Medhurst and Henry 2011). Costs and benefits will be unevenly distributes across sectors, demanding detailed representation of direct and indirect impacts of a transition to green economy. Some creative destruction as costs of transformation of energy use in various sectors, encourages path-dependence and attachment to the current fossil-fuel-based economy trend, which is difficult to reverse. Yet, a critical threshold on the expected future benefits and accumulating social commitment help to shift a system to a different trend (Medhurst and Henry 2011).

4. Challenges for CEE modeling in the presence of non-linear abrupt responses

4.1 General challenges in modeling for modeling non-linearities and resilience

In various disciplines regime shifts (Folke 2006; Biggs et al. 2009; Carpenter et al. 2011), critical transitions (Scheffer 2009), non-marginal changes (Stern 2008) are the terms, which are used to denote an abrupt structural change (Andersen et al. 2009). Such a non-linear systemic change may occur either due to a incremental change in some underlying variable(s) which gradually crosses a threshold, due to an external shocking event, or due to a combination of the two. While understanding of the nature of non-linear abrupt changes is essential for the proper estimate of cost and benefits of various policy actions, especially in the domain of climate change mitigation where impacts are intergenerational, the quantitative modeling of regime shifts in coupled CEE system is challenging. The literature on modeling coupled human-environment systems experiencing such non-linear dynamics identifies several critical issues (Filatova and Polhill 2012; Schlueter et al. 2012). They require a careful consideration when designing a software model, which is able to endogenously grow or capture non-linear responses of one of the subsystems or of a coupled system. On a model design stage it is vital to consider:

- the sources of regime shifts (endogenous or exogenous, originating in natural or social system, from a gradual change or a shocking event),
- the type of feedbacks between human and environmental systems (which could either amplify or absorb non-linear dynamics),
- methods for detecting and characterizing non-marginal change, a regime shift, and
- complexity aspects (thresholds, non-linearities and scales, including temporal scales to show if a phenomena is reversible or not).

The latter group is particularly relevant for this report. In what follows we review how various modeling approaches, which are most commonly used to design CEE models, treat the issues of non-linearity, thresholds and irreversibility. In particular, we look at Integrated Assessment Models (IAMs) including General Equilibrium Models, System Dynamics Models (SDs) and Agent-Based Models (ABMs).

4.2 Treatment of non-linearities in various modeling approaches

4.2.1 IAMs

Non-linear responses are strongly related with the feedbacks included in the modeling[2]; ultimately, all dynamics arise from the interaction of just two types of feedback loops, reinforcing (or positive) and balancing (or negative) loops. Among the high-resolution IA models the dominant approach has been the sequential (linear) representation from socioeconomic inputs to emission and climate impacts *without* considering feedbacks (Damage Function) to the “Human Activities” or “Ecosystem” modules (see Figure 7). In

[2] *In welfare optimization models, the inclusion of non-linearities is in close relationship with the discount rate used.*

these models (e.g. MiniCAM/GCAM, POLES, MESSAGE) feedbacks are usually restricted to the "Human Activities" module.

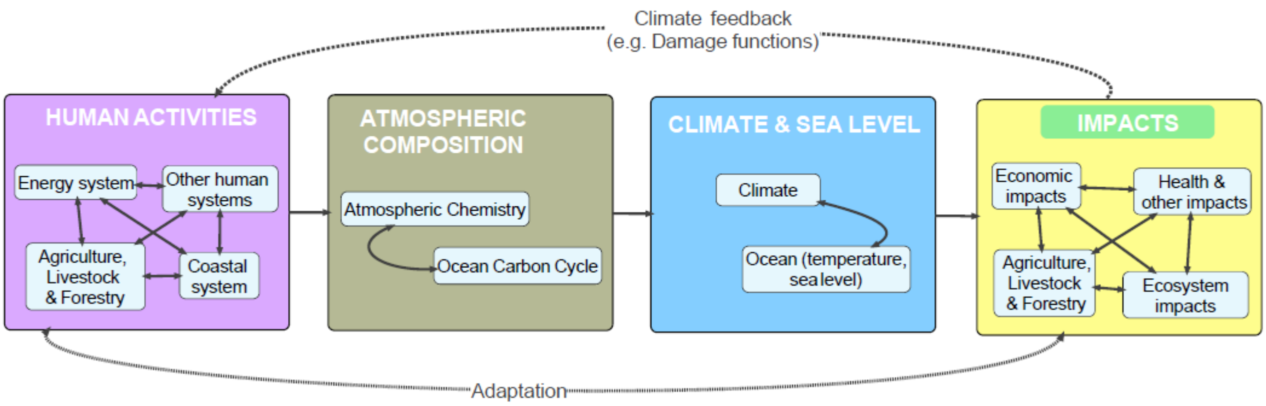


Figure 7: Sequential characterization of IAMs.

However, damage functions are implemented in some high-aggregated models such as DICE, FUND, PAGE, MERGE, etc. Damage functions have the form of non-linear equations mostly based on damage estimates related to doubling the CO_2 concentration from the pre-industrial level that are usually below the 2% of global GDP. The uncertainty on the damage functions currently used in IA models is extremely high (Arigoni and Markandya, 2009) and subject to concerns such as the degree of arbitrariness in the choice of parameters or the functional form which limit models' ability to portray discontinuities (Ackerman et al., 2009; Pindyck, 2013; Stanton et al., 2009; Stern, 2013).

Some models distinguish between economic impacts and non-economic impacts; only the former are included directly in the GDP (e.g. FUND, PAGE-09). However, many valuable goods and services (e.g. human health effects, losses of ecosystems and species) are not included in conventional national income, which suggests that usual damage functions may underestimate the damage costs of climate change. As an example, DICE, and a majority of its descendants, assumes that the exponent in the damage function is 2 –that is, damages are a quadratic function of temperature change: no damages exist at 0 ºC temperature increase, and damages equal to 1.8% of gross world output at 2.5 ºC (Nordhaus and Boyer, 2000; Nordhaus, 2008) (Figure 7). On the contrary, (Stanton et al., 2009) review of the literature uncovered no rationale, whether empirical or theoretical, for adopting a quadratic form for the damage function.[3] This is a key issue in IAM, since the results are significantly sensitive to this parameter (Dietz et al., 2007; Roughgarden and Schneider, 1999).

Feedbacks to the socioeconomic variables are not considered by IA models. For example, large scale population movement with likely associate conflict could happen at high levels of climate impact, being surely unreasonable to assume that we can be confident that this scale will be very small and invalidating, for example, the regional population exogenous projections (Stern, 2013).

Also, very few models explicitly assess the relationships between climate and ecosystem services, although modelers and policy makers have recognized that climate change problems have to be solved in harmony with other policy objectives such as economic development or environmental conservation (e.g., nonlinear impacts of temperature on crop yields (e.g. (Rosenzweig et al., 2013)).[4] In this sense, IMAGE and AIM can be considered among the most prominent models incorporating ecosystems services. These modes display a great spatial resolution in their ecosystem modules and have

[3] *This practice is endemic in IA models, especially in those that optimize welfare (e.g. DICE-family, MERGE, WITCH but also from other disciplines such as System Dynamics: ANEMI)PAGE2009 (Hope, 2011) uses a damage function calibrated to match DICE, but makes the exponent an uncertain (Monte Carlo) parameter.*

[4] *For an overview of IAM shortcomings in this field see (Calvin et al., 2013).*

participated in all the IPCC Assessments and in the Millennium Ecosystem Assessment (MEA, 2005). In the case of IMAGE 2.4 (Bouwman et al., 2006), it includes the Nitrogen cycle and a Biodiversity module as well as changes in climate (precipitation and temperature) impacting crop and grass yields. Also, the Carbon cycle model includes different climate feedback processes that modify Net Primary Productivity (NPP) and soil decomposition (and thus NEP) in each grid cell (0.5 by 0.5 degree resolution).[5] However, even in these models climate feedbacks to ecosystem services have a partial scope, they do not consider explicitly fundamental impact feedbacks related with the albedo-effect, the increase in climate extremes or sea-rise impact in coastal zones.

4.2.2 SDs

System dynamic models represent real-world applications of the formal mathematical theory of nonlinear dynamic systems, and thus, by definition, are designed to represent non-linearities. Coupled climate–socioeconomic system dynamics models applied to the study of the conomics of climate change include numerous non-linearities both in the economic modules (e.g. economic crises, bubbles on asset markets etc.) and in the climate modules (e.g. abrupt climate change). As example, a simple climate-socioeconomic system dynamic model by Kellie-Smith and Cox (2011) integrated for a very long term (from year 2000 to year 3000) generated under certain scenarios pronounced persistent low-frequency nonlinear oscillations of climate and macroeconomic variables.

Some system dynamic modelling studies suggest that the pronounced non-linearity of the real-word climate system (and supposedly even more pronounced non-linearity of real-world socioeconomic system) could surprisingly be beneficial for global mitigation policies. For instance, simulations with actor-based system-dynamic model MADIAM (Weber et al., 2005) revealed a strong non-linearity of the model towards properly designed mitigation strategies: revenues from a moderate carbon tax, when recirculated into the economy in the form of investments in endogenous carbon and energy efficiency improvement, had a more than linear impact in slowing down the global warming and accelerating the transition to a sustainable economy.

4.2.3 ABMs

ABMs are designed to model complex adaptive systems evolving along a non-linear path. Due to their technical ability to be implemented on a variety of spatial and temporal scales, they are naturally sited to be coupled with natural science models. In application to economics is it often realized either through technology diffusion on the supply side of a market or behavioral change on the demand side. Thus, ABMs have a high potential to simulate non-linear dynamics and responses in coupled CEE systems. Yet, it is not suited to model climatic systems, thus only non-linear response in socio-economic systems and energy markets can be considered. Abrupt shifts in climatic systems as a results of dynamics in energy and economy systems are never modeled with ABM.

While in other domains both technology diffusion and behavioral change have been studied, the applications of ABM to energy or climate mitigation, which also demonstrate non-linear response, are at their initial stage of development. A recent review of energy ABMs (Gerst et al. 2013) concludes that existing models can be divided into 4 groups: (1) ABMs focusing on technology diffusion in a single market with little or no feedback to macro-economy, (2) ABMs having a broader focus on the electricity market or overall energy use with little or no macroeconomic feedback, (3) ABMs of entire macro-economy of a country or the world at the costs of omitting technological detail and household behavior, (4) ABMs modeling interactions among countries with little or no feedback between domestic actors

[5] *Also, the IIASA Integrated Assessment Modeling Framework (including MESSAGE-MACRO model) includes some feedbacks in terms of changes in agricultural production (Tubiello and Fischer, 2007) or in the corresponding changing water needs for agricultural production (Fischer et al., 2007).*

and international policy. While all ABMs have some sort of non-linear functions or rule-based behavior on micro-level, here we focus on non-linear macro-dynamics of the emergent phenomena.

The ENGAGE ABM by Gerst and colleagues (2013) is the most developed ABM of CEE system to date, which also tries to connect across the 4 level of energy ABMs mentioned above. ENGAGE simulates heterogeneous firms and households while having an evolutionary representation of economic growth, energy technology, and international negotiations regarding climate change. It goes beyond conventional economic assumptions of many IAMs and CGE models such as homogenous households and firms, perfect information and are perfect rationality. Yet, it still represents an economy in a stylized manner (firms cover only two sectors – producers of capital and consumer goods). Households and firms are connected via labor and commodity and services markets. Energy enters as a cost factor in the production of goods and machines and is also consumed by households. Energy supply is represented by three energy technology firms ('carbon-heavy', 'carbon-light', and 'carbon-free') and one energy production firm. On the energy demand side households use a certain floor space and a certain number of appliances and cars, while good-producing firms use energy to run machinery, which can be replaced when its lifecycle is over. ENGAGE is applied to study the effect of domestic actors energy-related behavior on international and domestic climate policies, including carbon tax. Simulated energy technology market shares and energy intensity (ratio of annual energy use to real GDP) trajectories exhibit abrupt shifts. Emergent average household energy consumption and CO2 emissions also follow non-linear trends. This happens due to endogenous energy technology evolution, and is highly influenced by a policy scenario. For example, only when carbon tax is used as an investment in carbon-free R&D, economy a swift transition away from carbon-based-energy technologies. In this case low-carbon-energy fuels exponential economic growth by the end of the 21st century.

Chappin and Dijkema (2007) design an ABM of a decentralized System of Electricity Production Systems (SoEPS) in the Netherlands to explore the impact of CO_2 emission trading (CET) to in reducing CO_2 emissions. Their ABM shows that the impact of CET is small and visible only in a long time. However, authors admit that technological innovation among electricity producers, which is one of the crucial elements driving GHG emission reduction in the presence of CET, was not included in the model. Thus, this ABM is able to model only long-history of incremental innovation leading just to a smooth change. If diffusion of new technologies is implemented, this would imply a dramatic non-linear shift from fossil-fuel-based electricity production.

Castesana and Puliafito (2013) propose an ABM of endogenous economic growth studying the influence of population dynamics and growth of physical capital consumption on energy use and CO_2 emissions. This one-sector model operates on a global level and is partially parameterized with empirical data. A population of heterogeneous individuals goes through various life-stages potentially deciding to invest in human capital (education and development of technologies) that correspond to the investments in R&D at macro level. Agents make choices regarding their reproductive, economic and energy development driven by personal preferences and family influence. The trajectories of energy consumption and corresponding CO_2 emissions do not have linear correlation with smooth curves of population and GDP growth. The latter follow more volatile dynamic paths due to the fact that increase in energy consumption is partially offset by the improvements in technology. Moreover, authors highlight that agent-level factors may speed up or slow down a certain trajectories of energy use and CO_2 emissions, potentially amplifying non-linearities. In general a likely pathway towards a drop in anthropogenic carbon emissions is to encourage investments in human capital through education and low-carbon technologies.

Chappin and Afman (2013) developed an ABM of a consumer behavior regarding purchase of lamps. Their paper explores the nature and speed of possible transitions to low-electricity consumer lighting. This ABM explicitly model behavioral change on the demand side by assuming heterogeneous and dynamic preferences on lamps, which change with experience and through interactions via social network. As a result this ABM goes beyond simulating linear paths and is able to grow abrupt shifts to a non-conventional lightening technology under various policy scenarios. Authors highlight that complex market dynamics emerges as a result of interactions among consumers and bulb manufacturers, opinion exchange among consumers, and interactions between technologies. Non-linear transitions may not occur under specific assumptions about agents' heterogeneity and dynamics of individual perceptions.

Jackson (2010) designed an ABM to quantitatively evaluate electric utility energy efficiency and smart-grid programs. A forecast of annual electricity use and peak residential load over 15 years was simulated under an assumption of a residential customer growth rate of 1.2%. The results of a 'frozen' scenario (when equipment efficiencies and its utilization remain constant) show non-linear changes in annual electricity use (2.3% increase), while peak residential load changed almost linearly (1.3% increase). In contrast the 'baseline' scenario (smart grid 20% participation scenario) forecasts annual energy increases of 1.6% and annual peak load increases of 0.6%. In the 'smart grid 50% participation' scenario the peak annual growth is reduced to 0.2%. These non-linear response of the energy market driven by disaggregating the demand function into individual interacting consumers, which can be influenced by other agents leading to the dissemination of information on new technologies and utility programs. These complex dynamic is likely to be omitted when a traditional aggregated customer is used on the demand side.

The CITA ABM developed by Bravo and colleagues (Bravo et al. 2013) explores the relationships between household consumption (of food, transportation and energy) and the related GHG emissions under carbon tax and information campaign policies. CITA explores the behavioral change towards green alternatives or absence of such due to self-reinforcement and social influence, where heterogeneous preferences of agents for 3 domains are parameterized using Eurobarometer data. The effect of price policies on GHG emission reduction is moderate in the domains of transport and energy (3% and 5% respectively) and only in the food domain the effect is a non-linear significant reduction in the adoption of the brown leading to 17% GHG emission reduction. However, the policies aimed at behavioral change (changes in households preferences) lead to abrupt structural changes in emission reduction: in the transport domain declined by 15%, in the energy domain by 24%.

4.3 Treatment of thresholds in in various modeling approaches

4.3.1 IAMs

Stanton et al. (2009) IAM review finds that in only a few models damages are treated as discontinuous, with temperature thresholds at which damages show a major shift from lower temperatures. For example, DICE-2007 (Nordhaus, 2008) models catastrophe in the form of a specified (moderately large) loss of income, which is multiplied by a probability of occurrence (an increasing function of temperature), to produce an expected value of catastrophic losses. This expected value is combined with estimates of non-catastrophic losses to create the DICE damage function (i.e. it is included in the quadratic damage function discussed above). However, for much of Nordhaus's work using the DICE model the loss via the Damage Function at 5°C is *only* in the region of 5–10 percent GDP (see Figure 9). In much of Tol's work (see e.g. (Dietz et al., 2007)) on the FUND model damages at 5°C are still lower, around 1–2 percent of GDP (Figure 9).

In the PAGE-2009 model (Hope, 2011), the probability of a catastrophe increases as temperature rises above a specified temperature threshold (3 ºC above pre-industrial levels). For every 1°C rise in temperature beyond this, the chance of a large-scale discontinuity occurring rises by 20%, so that with modal values it is 20% if the temperature is 4°C above pre-industrial levels, 40% at 5°C, and so on. The threshold at which catastrophe first becomes possible, the rate at which the probability increases as temperature rises above the threshold, and the magnitude of the catastrophe when it occurs, are all Monte Carlo parameters with ranges of possible values. PAGE-2009 assumes that only one discontinuity occurs, and if it occurs it is permanent, aggregating long-term discontinuities, as ice-sheets loss, with short-term ones, such as monsoon disruption and thermohaline circulation. In fact, Nicholas Stern selected this model (PAGE-2002 version) for his Review "guided by our desire to analyse risks explicitly - this is one of the very few models that would allow that exercise" (Stern, 2006). However, still, climate feedbacks are poorly represented in this model in particular[6] and in climate IAMs in general (Whiteman et al., 2013).

However, as stated by (Stern, 2013) "*most reasonable modelers will accept that at higher temperatures the models go beyond their useful limits; Nordhaus suggests that we have insufficient evidence to extrapolate reliably beyond 3°C*". Since the climate science states that there are major risks of temperatures well above 3°C, the main concern thus lies in the incorrect extrapolation of these damage functions (Pindyck, 2013; Stanton et al., 2009; Stern, 2006). To illustrate this, whilst recognizing the wise cautionary advice of Nordhaus on making such extrapolations, (Ackerman et al., 2010) show that in a standard model, such as DICE-2007, temperature increases of up to 19°C might involve a loss in output of only 50 percent, against a baseline where the world is assumed to be many times richer by 2100 (**Figure 8**). This illustrates both the modest nature of damages and the perils of such extrapolation since such temperatures could even involve complete human extinction, indeed at much lower temperatures than that.

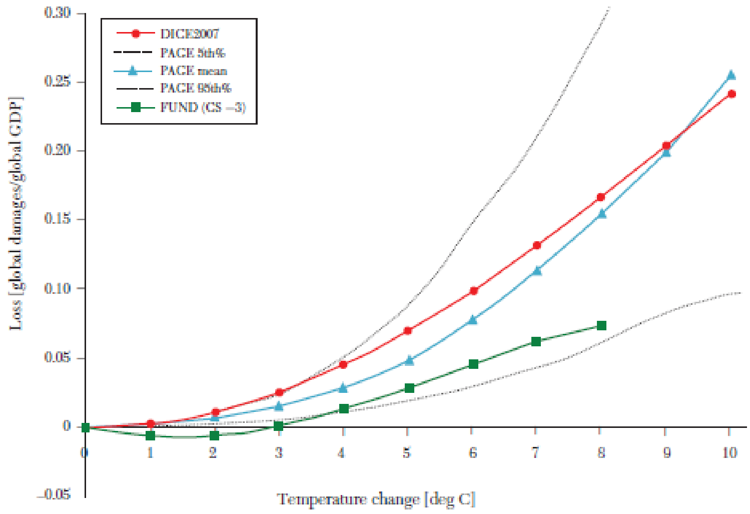


Figure 9: Annual Consumption Loss as a Fraction of Global GDP 2100 due to an increase in annual global temperature in the DICE, FUND and PAGE models. Source: (Stern, 2013)

The key point is the exogeneity of a key driver of growth combined with weak damages. With exogenous growth that is fairly high (say at 1 percent or more over a century or more) and modest damages, future generations are more or less assumed to be much better off (**Figure 10**). Exogenous growth of any long-term strength is challenged in the face of the scale of the disruption that could arise at these higher temperatures (e.g. potential large scale destruction of capital and infrastructure, mass migration, conflict) (Pindyck, 2013; Stern, 2013).

[6] *Better models are needed to incorporate feedbacks that are not included in PAGE09, such as linking the extent of Arctic ice to increases in Arctic mean temperature, global sea-level rise and ocean acidification," (Whiteman et al., 2013)*

Growth rate	With output loss				
	Yr 100	5%	10%	20%	50%
1%	270	256 (14)	243 (27)	216 (54)	135 (135)
2%	724	688 (36)	652 (72)	579 (145)	362 (362)
3%	1,922	1,826 (96)	1,730 (192)	1,538 (384)	961 (961)

Note: Table entries are output levels and losses are in parentheses (output in time zero = 100).

Figure 11: Output after a Century relative to now (base value = 100). Source: (Stern, 2013)

Some researchers have responded to the apparent absurdities of such weak damage functions by invoking higher order terms (see (Weitzman, 2012)), but the models still appear to suffer from the omission of the scale of damage that could arise from catastrophes, mass migration and serious conflict, most retain exogenous drivers of growth, and most have inherently narrow risk descriptions (Stern, 2013).

4.3.2 SDs

In the literature on coupled climate-economic SD models, there is relatively little discussion of threshold effects. Below we provide two interesting exceptions.

An interesting example is provided by Kellie-Smith and Cox (2011) for a highly stylized system dynamics model of a coupled global climate–socioeconomic system. With exogenous decarbonisation of the economy built into the model equations, projections of coupled climate-economic dynamics are computed for the 21st and 22nd century for two background economic growth rates: low (1% per year) and high (4% per year). For the low background economic growth rate, the global development is sustainable (a regime of “soft landing” at an equilibrium where the economy steadily grows at the decarbonisation rate). In contrast, for the high background economic growth rate, the global economy initially booms, but this is followed by an economic crash, and the resulting depression lasts for the entire 22nd century.

Another example of a threshold effect is the bifurcation of GDP losses caused by extreme weather and climate events simulated with the NEDyM model (Hallegatte et al., 2007): GDP losses increase sharply beyond a certain threshold value of the intensity and frequency of extremes.

4.3.3 ABMs

In the ABM literature in CEE domain thresholds are usually mentioned only with respect to the dynamics of socio-economic system and sometimes possible CO_2 emissions trajectories. Since ABMs are not directly used to model climatic systems (e.g. 2 degree Celsius threshold), there are no climate system thresholds considered directly. However, the latter may be used as a target for tested low-carbon policies entering ABM dynamics indirectly. The ABM examples below concern thresholds in energy-economy systems only.

The ABM of Chappin and Afman (2013) is driven by evolving preferences regarding low-cost electricity lams due to personal experiences and exchange of opinions with a social network. Yet, while agent’s perceptions evolve incrementally over time, the dramatic shift in market shares occur when an endogenous threshold value of adopters is reached. Changes in consumer preferences can be amplified or suppressed by changes in individual cost-effectiveness moving towards certain threshold values, e.g. when a decrease in electricity costs outweighs a jump in lamp purchasing costs.

When studying CEE system dynamics with their ABM Gerst and colleagues (2013) admit that the effect of the carbon tax on machine and goods consumptions, and consequently large-scale technological change, is dependent on how tax revenue is invested. While there is a linear relationship between firms’ R&D activity and economy-wide annual growth rates, the dynamic paths of market shares of various energy technologies under some policies (e.g. investing carbon tax into R&D) pass through certain threshold values. Various thresholds are also seen in aggregated energy intensities, which peak around year 2020. This is associated with lifecycle of machinery (20 years) and the fact that carbon tax is not high enough to trigger premature machine replacement.

Micro-level agent behavior is sometimes designed to exhibit thresholds. For example the CITA ABM assumes that consumer agents have two exogenously defined thresholds for need satisfaction and uncertainty (which impacts the forecasting ability regarding the consequences of agents' choices) with respect to food, transportation and energy consumption (Bravo et al. 2013). The threshold values were calibrated to match the empirical consumption trends. However, we are mainly interested in the thresholds in the response variables, i.e. macro-level dynamics. Such thresholds appear in the results of the CITA model under the scenario with households preferences change modeled as an information campaign to agents with low environmental preferences. Specifically, when the intensity of a policy reaches a certain value ($\sigma_3 > 0.5$) the brown consumption pattern disappears in all domains (food, transportation and energy use).

4.4 Treatment of irreversibility in various modeling approaches

4.4.1 IAMs

One of the most controversial conclusions to emerge from many of the first generation of climate IAMs was the perceived economic optimality of negligible near-term abatement of greenhouse gases. Typically, such studies were conducted using smoothly varying climate change scenarios or impact responses. Abrupt changes observed in the climatic record and documented in current models could substantially alter the stringency of economically optimal policies derived from IAMs. Such abrupt climatic changes—or consequent impacts—would be less foreseeable and provide less time to adapt, and thus would have far greater economic or environmental impacts than gradual warming (Mastrandrea and Schneider, 2001).

Despite critical uncertainties in the assessment of relationships such as climate sensitivity or damage functions (e.g. (Pindyck, 2013; Stern, 2013)), for the most part, IAMs adopt best guesses about likely outcomes (Ackerman et al., 2009; Kelly and Kolstad, 1998; Lomborg, 2010; Nordhaus, 2007; Tol, 2002; Webster et al., 2012). IPCC's focus in this issue has also being decisive: most visibly attention has been given to the communication of uncertainties by the natural scientists in the areas of climate science and impacts, and to a lesser extent, or at least very differently, by economic models and social scientists in the assessment of vulnerability, sources of greenhouse gas emissions, and adaptation and mitigation options (Pindyck, 2013; Stern, 2013; Swart et al., 2009).

Uncertainty, if incorporated at all, is usually analysed by running Monte Carlo simulations in which probability distributions are attached to one or more parameters. For example, the *Stern Review* (Stern, 2006), using the model PAGE-02, represents a step forward over the standard practice in this respect, employing a Monte Carlo analysis to estimate the effects of uncertainty in many climate parameters. As a result, the *Stern Review* finds a substantially greater benefit from mitigation than if it had simply used "best guesses".[7] Another recent applications are (Webster et al., 2012) with MIT-IGSM or (Cai et al., 2013), who developed a stochastic dynamic programming version of the DICE model. But these are rather exceptions: (Stanton et al., 2009) review did not identify any model assuming fat-tailed distributions that reliably samples the low probability tails, thus failing into providing an adequate representation of worst case extreme outcomes.

The probabilities of eventual warming of 4°C or more, on current emissions paths, may be of the order of 20–60% (e.g., (Rogelj et al., 2012; WEO, 2012)); thus, if the damage

[7] *Stern Review found that "without action, the overall costs of climate change will be equivalent to losing at least 5% of global gross domestic product (GDP) each year, now and forever." Including a wider range of risks and impacts could increase this to 20% of GDP or more, also indefinitely.*

functions are not included or calibrated to temperature increase until approximately 3 ºC (altogether with the common use of likely values instead of risk assessment), there is a wide range of possibilities currently outside the scope of the models. Therefore, it can be concluded that risk is understated in IAMs and models largely ignore the possibility of a catastrophic climate outcome (Ackerman et al., 2009; Pindyck, 2013; Stanton et al., 2009; Stern, 2013). (Lenton and Ciscar, 2013) review the limitations of the models and state that there is a "…huge gulf between natural scientists' understanding of climate thresholds or tipping points and economists' representations of climate catastrophes in IAMs." (Stern, 2013) summarizes: "the economic models add further underassessment of risk on top of the underassessment embodied in the science models, in particular because they generally assume exogenous drivers of growth, only modest damages from climate change and narrow distributions of risk".

4.4.2 SDs

The problem of abrupt/irreversible climate change is has not been extensively addressed in the existing literature on climate-economic SD models. Indeed, up to now most modeling exercises based on climate modules able to represent abrupt/irreversible climate dynamics or including discontinuous climate damage functions, have been performed within the utility maximization paradigm – a conventional wisdom of neoclassical economic growth theory. However, both of these climate modeling forms can be straightforwardly adopted in SD models. An interesting research agenda would therefore be to develop such system dynamic versions of traditional climate-socioeconomic models rooted in the utility maximization paradigm. These should then be able to provide a more realistic description of the impacts of abrupt/irreversible climate change and its interaction with the non-linear socio-economic system.

The problem of possible irreversible global change was originally addressed using system-dynamic modeling in the neighboring area of environmental and resource economics. An example that received extremely high visibility (and at the same time was severely criticized by many mainstream economists) is the "Limits to Growth" report and its follow-ups (Meadows et al., 1972, 1992, 2002). The authors argued, on the basis of simulations with the SD model World3, that maintaining the exponential growth of population, capital, resource use and pollution on a finite planet is unsustainable and will inevitably lead to an irreversible catastrophe, unless the timely correction measures are implemented at the global level.

4.4.3 ABMs

Many ABMs are characterized lock-in effects and strong path-dependency. Therefore the sequence of previous states constraint future states, and even gradual changes in behavior or technology may lead to irreversible changes in energy-economy system. As before irreversibility in climate systems is hardly ever considered in ABMs as they are not the best tools to simulate climatic systems.

The ABM of the carbon emission trading impact on shifting from carbon-intensive electricity production (Chappin and Dijkema 2007) suggests that as soon as investments in new technology are made, the switch from the old technology is irreversible. Various scenarios produced by the ENGAGE ABM by Gerst and colleagues (2013) all produce irreversible transitions to low-carbon economy. While depending on a policy, the transition can be swift or more gradual, the return back to carbon-intensive economy is unforeseeable.

The ABM of transition to low-electricity lightening (Chappin and Afman 2013) produces non-linear paths under various policies (banning, tax, subsidy). While this market system moves along transition pathways, this transition is irreversible. The shift to low-electricity lamps happen when either it becomes cost-efficient for consumers or when their dynamics preferences reach a certain level. There is no reverse dynamics modeled, also probably since it is unrealistic.

5. Discussions and conclusions

Occasionally, dramatic shifts occur in natural system as well social and economic systems. As reviewed in this report, the literature on critical transition theory suggests that such shifts can be associated to the existence of alternative stable state, thresholds and hysteresis in the system. For the management of such system and more specifically for the climate mitigation policies and measures, it implies a radically different view on policy options , and on the potential effects of global change on such systems. For instance, although the gradual changes in temperature might show little and proportional impact, once a threshold is reached and a flip occurred the large impact might be difficult or even impossible to reverse. Examples are the collapse of an overharvested population, ancient climatic transitions, and the collapse of Saharan vegetation. The critical transition in such systems can ultimately derive from how it is organized — and usually from feedback mechanisms, stabilizing or distabilising, within it.

In climate system, the critical transition is usually associated to the distabilising (positive) and stabilising (negative) feedbacks. For example, Rial et al. (2004)) proposes a metaphor of a net feedback. According to this metaphor, in unperturbed conditions the net negative climate-driving feedback of the Earth is slightly stronger than the net positive feedback, at least for small values of external/internal forcing. However if the forcing grows beyond the point at which the two competing feedbacks are balanced, then the explosive amplification produced by positive feedbacks leads to strong nonlinear effects. Even below this critical threshold, the negative impacts of human induced climate change can become so strong at some critical adaptation threshold that societies are no longer able to respond to the climate change impacts at an acceptable cost. Thus mitigation policies should be implemented such that this critical adaptation threshold is not exceeded.

Predicting such critical thresholds in a system and occurrence of catastrophic shift before they are reached is extremely difficult as the state of the system may show little change before the bifurcation points. However, recent attempts to assess whether alternative stable states and hence critical transitions are present in a system are now converging in different fields such as desertification, limnology, oceanography and climatology. These studies are now suggesting the existence of generic early-warning signals that may indicate for a wide class of systems if a critical threshold is approaching. The theoretical studies show that the dynamics of systems near a critical point have generic properties, regardless of differences in the details of each system. Therefore, sharp transitions in a range of complex systems are in fact related. In models, critical thresholds for such transitions correspond to 'catastrophic bifurcations'. Section 2.3 of this report reviews on of the prototype examples of such systems, the lake system, and analyse and classify the economic outcomes of such a shift.

Scheffer et al. (2009) reviews some of the generic early-warning indicators. The main indicator that is mentioned in the review is the so-called critical slowing down that might lead lead to three possible early-warning signals in the dynamics of a system approaching acritical threshold: slower recovery from perturbations, increased autocorrelation and increased variance in the resulting pattern of fluctuations. Although, these indicators are examined in some strong but stylized models, more work is needed to test the robustness of these signals. Also, detection of the patterns in real data is challenging and may lead to false results. In the Copenhagen Accord (UNFCCC, 2010) the critical threshold, based on recommendations, among others, of Bruckner, et al (1999), was set at 2 degrees C. Jaeger and Jaeger (2011) provide an interesting overview of the history of emergence of 2C target, including a review of the criticism of this target. Whether the 2C threshold is well justified as a mitigation policy target or not, there is now increasing skepticism on the chances of retaining the global mean surface air temperature at or below this limit (Anderson and Bows, 2011; Peters et al., 2013). At the same time, some recent studies (Mann, 2009; Smith et al., 2009) have revised the climate change impacts associated with 2C temperature rise above the pre-industrial level towards higher severity levels. On this basis, Anderson and Bows (2011) suggest redefining the 2C limit as a threshold not between “acceptable” and “dangerous” climate change, but between “dangerous” and “extremely dangerous” climate change.

In order to assess the economic impacts of climate change and the mitigation and adaptation related policies, the issue of non-linearity in the presence of tipping points is essential for the definition of optimal mitigation and adaptation strategies as the impact climate change could become extremely severe, however, there are a lot of uncertainities regarding the critical thresholds (Pindyck, 2007). Moreover, many climate impacts such as the damage to ecosystems may be irreversible. This means that adopting a policy now rather than waiting has a sunk benefit, that is a negative opportunity cost. This implies that traditional cost-benefit analysis will be biased against policy adoption (Pindyck, 2007).

While understanding of the nature of non-linear abrupt changes is essential for the proper estimate of cost and benefits of climate related policy actions, especially in the domain of climate change mitigation where impacts are intergenerational, the quantitative modeling of regime shifts in coupled CEE system and impact assessment models and tools is challenging. Current impact assessment models are not fully able to present non-linearities, thresholds and irreversibility or run catastrophe climate scenarios. Numerous studies have indicated that in the case of non-linear climate change impacts, optimal abatement increases substantially (Baranzini, et al., 2003, Gjerde, et al., 1998, Keller, et al., 2004, Kolstad, 1994, Mastrandrea, 2001, Tol, 2003, Yohe, 1996, Zickfield and Bruckner, 2003). The potential for non-linear and low-probability climate responses to anthropogenic greenhouse gas forcing, however, has received little attention in the climate change damage cost literature to date (Alley, et al., 2003, Higgins, et al., 2002, Tol, 2009, Wright and Erikson, 2003).

In this report we reviewed the shortcoming of various modeling approaches, which are most commonly used to design CEE models, treat the issues of non-linearity, thresholds and irreversibility. In particular, we look at Integrated Assessment Models (IAMs) including General Equilibrium Models, System Dynamics Models (SDs) and Agent-Based Models (ABMs).

As mentioned earlier, non-linear responses are strongly related with all dynamics arise from the interaction of just two types of feedback loops, distabilising (or positive) and stabilising (or negative) loops. Among the high-resolution IA models the dominant approach has been the sequential (linear) representation from socioeconomic inputs to emission and climate impacts without considering feedbacks (Damage Function) to the “Human Activities” or “Ecosystem” modules. In these models feedbacks are usually restricted to the “Human Activities” module. Moreover, Stanton et al. (2009) IAM review finds that in only a few models damages are treated as discontinuous, with temperature thresholds at which damages show a major shift from lower temperatures (see for example Nordhaus, 2008). In particular the review concludes that IAMs as well as GE models largely ignore the possibility of a catastrophic climate outcome (Ackerman et al., 2009; Pindyck, 2013; Stanton et al., 2009; Stern, 2013). (Lenton and Ciscar, 2013) review the limitations of the models and state that there is a “…huge gulf between natural scientists’ understanding of climate thresholds or tipping points and economists’ representations of climate catastrophes in IAMs.” (Stern, 2013) summarizes: “the economic models add further underassessment of risk on top of the underassessment embodied in the science models, in particular because they generally assume exogenous drivers of growth, only modest damages from climate change and narrow distributions of risk”.

Unlike GE modes and IAMs , ABMs have a high potential to simulate non-linear dynamics and responses in coupled CEE systems. Yet, it is not suited to model climatic systems, thus only non-linear response in socio-economic systems and energy markets can be considered. However, In the ABM literature in CEE domain thresholds are usually mentioned only with respect to the dynamics of socio-economic system and sometimes possible CO2 emissions trajectories. Since ABMs are not directly used to model climatic systems (e.g. 2 degree Celsius threshold), there are no climate system thresholds considered directly. Irreversibility, however, are addressed in ABMs. The ABM of the carbon emission trading impact on shifting from carbon-intensive electricity production (Chappin and

Dijkema 2007) suggests that as soon as investments in new technology are made, the switch from the old technology is irreversible. Various scenarios produced by the ENGAGE ABM by Gerst and colleagues (2013) all produce irreversible transitions to low-carbon economy. While depending on a policy, the transition can be swift or more gradual, the return back to carbon-intensive economy is unforeseeable.

System dynamic models represent real-world applications of the formal mathematical theory of nonlinear dynamic systems, and thus, by definition, are designed to represent non-linearities. Coupled climate–socioeconomic system dynamics models applied to the study of the economics of climate change include numerous non-linearities both in the economic modules (e.g. economic crises, bubbles on asset markets etc.) and in the climate modules (e.g. abrupt climate change). As example, a simple climate-socioeconomic system dynamic model by Kellie-Smith and Cox (2011) integrated for a very long term (from year 2000 to year 3000) generated under certain scenarios pronounced persistent low-frequency nonlinear oscillations of climate and macroeconomic variables. However, The problem of abrupt/irreversible climate change is has not been extensively addressed in the existing literature on climate-economic SD models. Indeed, up to now most modeling exercises based on climate modules able to represent abrupt/irreversible climate dynamics or including discontinuous climate damage functions, have been performed within the utility maximization paradigm – a conventional wisdom of neoclassical economic growth theory. However, both of these climate modeling forms can be straightforwardly adopted in SD models.

In order to tackle the aforementioned shortcomings of the current CEE impact models, the main goal of COMPLEX WP5 is to developed a system of models combining insights from different field of research such as critical transition and catastrophe theory, and IAMs, GEs, ABM, and SD modelling approaches with the emphasis on utilising the non-linear climate responses and regime-shifts of economic-ecological systems, modeling processes of diffusion and pervasive technical change and its implication, and representation of economic sectors with a significant potential for mitigation and resource efficiency. The system of model will be designed in such a way that it can serve as a so-called ‘fully integrated assessment model’ to evaluate mitigation policies, assessing the costs and inform policy makers in a more effective way. The next report will present the theoretical and conceptual framework for such a system.

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