Computer-Inspired Concept for High-Dimensional Multipartite Quantum Gates

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An open question in quantum optics is how to manipulate and control complex quantum states in an experimentally feasible way. Here we present concepts for transformations of high-dimensional multiphotonic quantum systems. The proposals rely on two new ideas: (i) a novel high-dimensional quantum nondemolition measurement, (ii) the encoding and decoding of the entire quantum transformation in an ancillary state for sharing the necessary quantum information between the involved parties. Many solutions can readily be performed in laboratories around the world and thereby we identify important pathways for experimental research in the near future. The concepts have been found using the computer algorithm MELVIN for designing computer-inspired quantum experiments. As opposed to the field of machine learning, here the *human* learns new scientific concepts by interpreting and analyzing the results presented by the machine. This demonstrates that computer algorithms can inspire new ideas in science, which has a widely unexplored potential that goes far beyond experimental quantum information science.

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One collective goal of quantum optics research is to find ways for controlling complex quantum systems, both for investigating fundamental questions of quantum mechanics and for potential applications in quantum technology [1,2].

The complexity of a quantum system increases with the number of parts involved and the number of dimensions of its individual parts. For single photonic quantum systems, it has been well known for 25 years how to perform arbitrary unitary transformations [3], which has since become a foundation for integrated photonics [4–7]. Also in other degrees of freedom of photons, single qudit quantum gates have been well understood—for example, using discretized time steps [8] or spatial modes of photons [9–12] and high-dimensional multi-degree-of-freedom operations on single photons [13].

Multiphoton operations are more intricate, as photons do not interact with each other. To overcome this difficulty and perform an effective interaction between two photons, ancillary states are used to herald probabilistic transformations such as controlled-NOT (CNOT) gates [14–16]. The quality of these transformations has immensely increased, enabling on-chip demonstrations of arbitrary two-dimensional two-photon gates, as well as theoretical concepts for arbitrary photonic qubit transformations [17]. Summing up, the special cases of multiphotonic qubit transformations and single-photonic arbitrary high-dimensional transformations are well understood. However, the general case of transformations of n photons in d dimensions is still open.

Here we show blueprints for experimental realizations of arbitrary multidimensional multiphotonic transformations. We use the orbital angular momentum (OAM) of photons as the high-dimensional degree of freedom, but the concepts can readily be generalized to other high-dimensional encoding systems [18]. The concept is based on encoding the essence of quantum transformations in an ancilla state, which mediates the necessary quantum information between the involved photons. This is made possible by a new quantum nondemolition measurement and the exploitation of a genuine high-dimensional interferometer. Several of our experimental proposals are feasible with state-of-the-art technology. Interestingly, the concept of encoding transformations in ancillary states is reminiscent of the first work on linear optics quantum computing by Knill, Laflamme, and Milburn [14] and by Gottesman and Chuang [19]. These researchers have shown that efficient quantum computation can be achieved with linear optics and access to a sufficiently large entangled quantum state.

Our concept for high-dimensional multiphotonic gates has many potential applications. It could enable the first proof-of-principle experiments toward high-dimensional quantum computation [20,21]. As an example, encoding qubit information in high-dimensional systems has significant advantages in robustness against errors [22,23] and can reduce quantum circuit complexities [24,25], and high-dimensional error-correction codes have shown advantages in terms of resources [26,27]. All of these examples require

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high-dimensional multiqudit gates, especially high-dimensional generalizations of the CNOT such as the CSUM gates, which we explain here.

High-dimensional multiphoton gates can be used for Bell-state or Greenberger–Horne–Zeilinger-state measurements, which are an essential requirement of quantum communication protocols. Therefore, the experimental setups proposed here can enable complex high-dimensional multiphotonic quantum teleportation and entanglement swapping and thereby form a fundamental element for high-dimensional quantum repeaters or high-dimensional quantum dense coding. Additionally, we introduce the first high-dimensional counterparts and extensions of quantum nondemolition measurements, which could find independent applications in quantum protocols.

From a conceptual point of view, the gates presented here are based on independent access to quantum states with different parities and can thus be generalized to other photonic degrees of freedom and potentially also to other quantum systems beyond quantum photonics.

Inspirations from computers.—We discovered the initial concepts presented here via computer-designed experiments using the algorithm MELVIN [28]. To do that, we had to formulate the question of effective photon-photon interaction in the most general way. The total search space of quantum optical setups here corresponds to roughly 10^{30}–10^{40} possibilities. Efficient exclusion principles allowed us to significantly reduce the number of calculations (details in the SM [29]). A total of roughly 150 000 CPU hours has then finally uncovered the seed of ideas we present in this manuscript. The human scientists in our team were subsequently able to understand computer-inspired ideas. As a consequence, we were able to exploit and generalize those ideas to many other high-dimensional and multipartite cases without the requirement of performing other expensive calculations. The potential of scientifically interpretable designs from computers that can inspire conceptual insights is widely unexplored. The idea is not restricted to the design of quantum experiments but could be applicable to many other areas of physics (see [30] for a concise overview of computer-inspired designs in physics).

High-dimensional control operations.—The simplest nontrivial case of a multiphotonic transformation is a two-dimensional CNOT gate. The four transformations are written as

$$\text{CNOT}|0,0\rangle = |0,0\rangle, \qquad \text{CNOT}|0,1\rangle = |0,1\rangle,$$
$$\text{CNOT}|1,0\rangle = |1,1\rangle, \qquad \text{CNOT}|1,1\rangle = |1,0\rangle. \qquad (1)$$

A more compact way is

$$\text{CNOT}|c,t\rangle = |c\rangle \hat{X}^c|t\rangle = |c,(c+t)\%2\rangle, \qquad (2)$$

where % stands for the modulo operation, and $\hat{X}$ stands for the Pauli-X operation (with $\hat{X}|0\rangle = |1\rangle, \hat{X}|1\rangle = |0\rangle$, or

more compactly, $\hat{X}|n\rangle = |(n+1)\%2\rangle$). Thus, one can think about the CNOT as an $\hat{X}$ operation applied c times on the target photon. Crucially, the CNOT operates coherently on superpositions of terms in Eq. (1), which distinguishes it from classical operations and enables its usage in quantum applications.

We generalize the concept to high-dimensional systems [25,31,32]. A high-dimensional generalization of the CNOT is a controlled-$\hat{X}$, $C\hat{X}$, which acts as

$$C\hat{X}|c,t\rangle = |c\rangle \hat{X}_d^c|t\rangle = |c,(c+t)\%d\rangle, \qquad (3)$$

with the high-dimensional Pauli-$\hat{X}$ gate acting as $\hat{X}|n\rangle = |(n+1)\%d\rangle$. Informally, we increase the value of the target photon by the value of the control photon (modulo d). This can also be considered as a CSUM gate. Generalizations of other quantum gates are controlled-controlled-$\hat{X}$ gates (CC$\hat{X}$, which generalizes the important three-qubit TOFFOLI gate) acting on two control photons and one target as $CC\hat{X}|c_1,c_2,t\rangle = |c_1,c_2\rangle \hat{X}_d^{c_1 \cdot c_2}|t\rangle = |c_1,c_2,(t+c_1 \cdot c_2)\%d\rangle$, and CPHASE acting as $\text{CPHASE}|c_1,c_2\rangle = w^{c_1 \cdot c_2}|c_1,c_2\rangle$ (where $\omega = e^{i2\pi/d}$, which are essential components in the generation of graph states that are used in measurement-based quantum computing [33]).

Experimental concepts.—Here we present the experimental concepts to perform high-dimensional multiphotonic transformations. We consider the OAM of photons as the discrete degree of freedom [34,35], which means that $|2\rangle_a$ stands for a single photon in path a with OAM value $\ell = 2$. However, our concept is general enough that it can be translated to any other discrete high-dimensional system, such as path encoding or time encoding, or potentially even beyond photons.

First, we explain how a well-studied two-input two-output optical element [36] can be used in a genuine three-dimensional way. This element is crucial for what comes next, a step by step construction of a three-dimensional generalization of a CNOT. We show how a three-photon ancilla state can effectively transform a three-dimensional target photon. Then we introduce a new experimental nondestructive measurement scheme to extract the quantum information of a three-dimensional control photon. The combination of these two structures leads to a full three-dimensional $C\hat{X}$ gate. Finally, we show experimental methods to generate other complex multiphotonic transformations.

Genuine high-dimensional two-input two-output element.—Conventionally, two-input two-output elements perform either the same action on all incoming modes (such as the beam splitter, which transmits or reflects an incoming photon purely by chance) or two different actions on two classes of modes (such as the polarizing beam splitter, which transmits horizontally polarized photons and reflects vertically polarized ones).
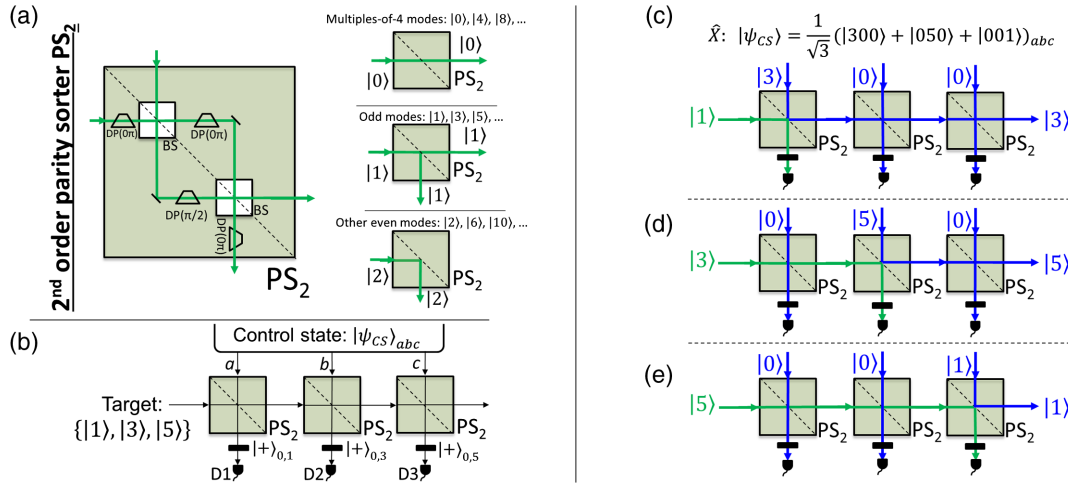
FIG. 1. An ancilla state-based setup for three-dimensional control gates $\hat{X}$, $\hat{X}^2$, and $\hat{X}^3$. (a) A genuinely three-dimensional action of a second-order parity sorter (PS_2). On the left is the physical implementation as an interferometer with Dove prisms that introduces a reflection and a mode-dependent phase of $\alpha = 0$ or $\alpha = \ell\pi/2$, introduced by Leach *et al.* [36]. Right, the abstract representation of the element. It can separate photons deterministically with mode $|0\rangle$ (transmitted) from photons with mode number $|2\rangle$ (reflected). Crucial for our requirements is that photons with odd mode numbers are randomly separated into two output paths, just as in a conventional beam splitter. Thus, the PS_2 is a two-input, two-output element, which can perform entirely different transformations on three orthogonal modes. (b) The state of a three-photon state $|\psi_{CS}\rangle$ determines the transformation of the target photon. The target photon is overlapped with the three ancilla photons at a PS_2 each. One output mode is connected to a detector, which heralds a correct transformation. If all detectors see a photon, the transformation was successful, and the output photon has the correct state. Importantly, PS_2 transmits modes $|0\rangle$ and probabilistically splits odd mode numbers. The state is encoded in the odd number space $\{|1\rangle, |3\rangle, |5\rangle\}$. The desired transformation can be achieved by adjusting the ancilla state. $|+\rangle_{0,n} = (|0\rangle + |n\rangle)/\sqrt{2}$ stands for projections into a two-dimensional subspace. (c)–(e) We show the $\hat{X}$ transformation in detail. The ancilla state for the $\hat{X}$ transformation is $|\psi_{CS}\rangle = (1/\sqrt{3})(|3,0,0\rangle + |0,5,0\rangle + |0,0,1\rangle)$, which is a three-photon W-state [37] (up to local transformations), and its generation has been discussed in [38,39]. If the target photon is $|1\rangle$, the only way to have all detectors see a photon (after the filter) is to have the photon in path a being $|3\rangle$ which leaves to the output port and the photons in path b and c both being in the state $|0\rangle$ and going to detectors D2 and D3. This happens with a probability of $p = 0.5^4$ because odd modes need to reflect in a specific way four times. The other two terms of $|\psi_{CS}\rangle$ will not be able to cause all three detectors, D1–D3, to click. The state $|0,5,0\rangle$ will not be able to create a click in detector D2, and $|0,0,1\rangle$ will not produce a click in either D1 or D3. Therefore, the pattern in the image is the only possible combination. Similar reasoning leads to the conclusions for the target photons being in $|3\rangle$ and $|5\rangle$. Exactly the same logic holds for the other two types of transformations: $\hat{X}^2$ with $|\psi_{CS}\rangle = (1/\sqrt{3})(|5,0,0\rangle + |0,1,0\rangle + |0,0,3\rangle)$ and $\hat{X}^3$ with $|\psi_{CS}\rangle = |0,0,0\rangle$. For equal transformation probability, the ancillary states need to be weighted, as shown in Fig. 3 and the Supplemental Material (SM) [29].

A key understanding is that a well-established experimental element can actually perform three different transformations [see Fig. 1(a)]. A second-order parity sorter will transmit modes $|0\rangle$, $|4\rangle$, and $|8\rangle$ and reflect $|2\rangle$, $|6\rangle$, and $|10\rangle$. Interestingly, for odd modes the PS_2 performs another transformation: it splits the wave function by chance into a reflected and transmitted beam (just as a beam splitter). This is important because, in this way, we can mix deterministic and probabilistic operations at a single element.

Controlling a photon by other photons.—Now we show how a three-dimensional photonic state can be controlled with three other photons. The correct transformation is heralded by the simultaneous clicks of three detectors, D1–D3 [see Fig. 1(b)]. The three photons from a controlling state $|\psi_{CS}\rangle$ are combined with a target photon at three PS_2 [explained in Fig. 1(a)]. In front of every detector is a mode filter, which projects the photons into the state

$(1/\sqrt{2})(|0\rangle + |1\rangle/|3\rangle/|5\rangle)$. A photon in the state $|0\rangle$ is deterministically transmitted to the detectors. An odd photon splits probabilistically at the PS_2. In order that all detectors fire simultaneously (and a photon at the output), the state $|\psi_{CS}\rangle$ is prepared in such a way that the target input photon has to go to one of the detectors and the output photon is replaced by one from $|\psi_{CS}\rangle$. In this way, only one photon configuration can lead to simultaneous detections in detectors D1–D3, which are shown in Fig. 1(b). All other configurations lead to at least one detector not firing or no photon leaving the setup. By changing the $|\psi_{CS}\rangle$, we can perform various other manipulations of the target photon. The three examples of $|\psi_{CS}\rangle$ in Fig. 1(c)–(e) will lead to a $\hat{X}$ transformation controlled solely by the three photons of $|\psi_{CS}\rangle$.

Quantum nondemolition measurement and a three-dimensional $C\hat{X}$.—Our goal is to generate, as a first example, a three-dimensional $C\hat{X}$ ($C\hat{X}|c,t\rangle = |c\rangle\hat{X}^c|t\rangle$)
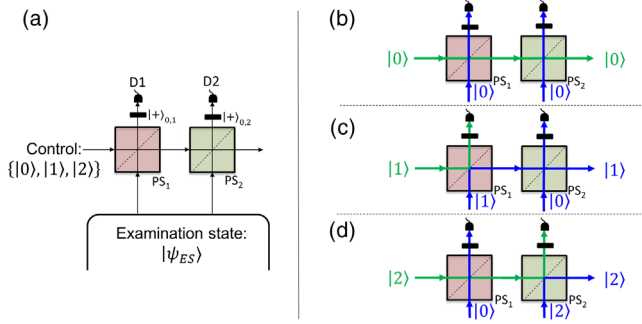
FIG. 2. High-dimensional quantum nondemolition measurement—extracting information about the control photon without destroying it. (a) We combine the control photon with a two-photon examination state $|\psi_{ES}\rangle$ at a PS_1 (which transmits even modes and reflects odd modes) and a PS_2 [as described in Fig. 1(a)]. (b)–(d) For each of the possible control states $|0\rangle$, $|1\rangle$, and $|2\rangle$, there is only one combination that produces clicks in the detectors D1 and D2. If both detectors register a photon, the output photon has the correct mode number. Thus, we can probe the control photon with three different ancilla states, and we learn about its state, heralded by a photon in both detectors.

using an ancilla state that mediates the information from the control photon to the target photon. Above we have already seen how $|\psi_{CS}\rangle$ can control a target photon. Of course, later $|\psi_{CS}\rangle$ will become part of the mediating ancilla state. The remaining question now is how to extract information from the control photon (in an examination state $|\psi_{ES}\rangle$) without destroying its quantum information. We employ a similar idea as before: using two ancilla photons and two detectors, the quantum information is correctly extracted if the two detectors fire simultaneously. In our example, Fig. 2, $|\psi_{ES}\rangle$ can be one out of three quantum states. Each of them makes detectors D1 and D2 click for a different input state.

For example, if $|\psi_{ES}\rangle = |0, 0\rangle$, the two detectors will click, and a photon will exit the setup only if the input photon was in the state $|0\rangle$. In all other cases, either no photon exits the setup (which can be considered as a loss) or not all detectors fire simultaneously. However, if they click, we know the photon was in the state $|0\rangle$ without destroying its quantum information.

By now, we can extract quantum information from the input state without destroying it, and we can control the target transformation using ancillary photons. Finally, we can combine these two ingredients. We use an entangled state that combines $|\psi_{ES}\rangle$ and $|\psi_{CS}\rangle$ into $|\psi_{Anc}\rangle$, as shown in Fig. 3(a). For example, if the input state is $|1\rangle$, only $\psi_{ES} = |1, 0\rangle$ can lead to clicks in detectors D1 and D2. Thus, $|\psi_{Anc}\rangle$ collapses into $|\psi_{Anc}\rangle_1 = (1/\sqrt{3})(|3, 0, 0\rangle + |0, 5, 0\rangle + |0, 0, 1\rangle)$. This state introduces an $\hat{X}$ transformation at the target photon, exactly as described in Fig. 1. Therefore, a click in detectors D1–D5 heralds a successful three-dimensional controlled-$\hat{X}$ transformation. The concept can be generalized to arbitrary d-dimensional controlled-$\hat{X}$ transformations, as we show in the SM [29]. In Fig. 3(b), we show a high-dimensional controlled-controlled-$\hat{X}$ (a generalization of the TOFFOLI gate), and in Fig. 3(c), we show a three-dimensional CPHASE.

The setups depend on post-selection and heralding of ancillary states. The gate success probability for a three-dimensional $C\hat{X}$ is equal for all modes, and we give the intuition for the case of $c = |0\rangle$. The transformation only works if the ancillary state collapses into $|00\rangle|000\rangle$ with a probability of $P_1 = 1/19$. Further, the target photon exits in the output mode, three PS_2 need to be passed, each with a probability of $p = 1/2$ for odd modes, leading to a total probability of $P_2 = 1/(2^3 \times 19) = 1/152$. Finally, each detector projects into a superposition of two modes, which
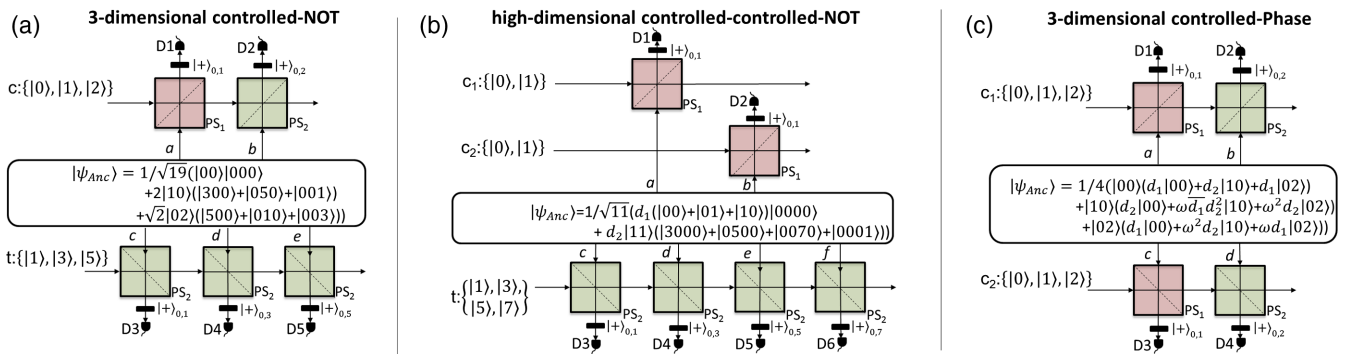

FIG. 3. Three high-dimensional multiphotonic quantum transformations. (a) A high-dimensional two-particle three-dimensional controlled-$\hat{X}_3$ gate, which can be described as $\hat{T}|c, t\rangle = |c\rangle\hat{X}^c|t\rangle = |c, (c + t)\%3\rangle$. It is a combination of the elements in Fig. 1 and Fig. 2, including correct weights to take mode-dependent success probabilities into account. If all detectors (D1–D5) click simultaneously, the transformation is successful and the two photons are emitted in the correct state. (b) A controlled-controlled-$\hat{X}_4$ quantum gate with two control photons being in a two-dimensional state while the target photon is in a four-dimensional state. The explicit transformation is $\hat{T}|c_1, c_2, t\rangle = |c_1, c_2\rangle\hat{X}^{c_1 \cdot c_2}|t\rangle = |c_1, c_2, (t + c_1 \cdot c_2)\%4\rangle$. (c) A three-dimensional controlled phase operation combines twice the method of Fig. 2. The transformation is $\hat{T}|c_1, c_2\rangle = \omega^{c_1 \cdot c_2}|c_1, c_2\rangle$. $d_1 = \exp(i\pi/4)$ and $d_2 = \sqrt{2}$ are normalization constants, and $\omega = \exp(i2\pi/3)$ is a root of unity.

would constitute another reduction of the probability by $P_{\text{triggers}} = 1/2^{\#(\text{triggers})}$. However, the triggers can be constructed by multioutcome measurements (projecting both into $(|0\rangle \pm |n\rangle)/\sqrt{2}$ in two separate detectors). The results from different detector-click patterns can then be propagated either by classical post-processing or active feed-forward methods. Thereby, the fundamental success probability of the three-dimensional $C\hat{X}$ gate in Fig. 3(a) is $P_{3dC\hat{X}} = 1/(2^3 \times 19) = 1/152$. In an analog way, one finds that the success probability for the $2-2-4$ dimensional $CC\hat{X}$ gate in Fig. 3(b) is $P_{CC\hat{X}} = 1/(2^4 \times 11) = 1/176$ and for the three-dimensional CPHASE in Fig. 3(c) is $P_{\text{CPhase}} = 1/16$.

General multiphotonic high-dimensional transformations.—We can apply the same idea to more complex transformations, such as controlled-controlled-$\hat{X}$ in Fig. 3(b) or a controlled-phase gate in Fig. 3(c). In both cases, we apply the same concepts as shown in Figs. 1 and 2. More general transformations, which are not simple control operations (an example is $\hat{T}|c_1, c_2\rangle = |(c_1 + c_2)\%d, (2c_1 + c_2)\%d\rangle$), can be generated in a very similar way (see the SM [29]).

One important remaining question is how to create the ancillary state experimentally. The last five years have seen a plethora of high-dimensional multiphotonic experiments [40–44], indicating that large classes of quantum states are accessible. Recently, *entanglement by path identity* has been proposed [38], which is a conceptually very efficient method to produce high-dimensional multiphotonic entanglement. A map from entangled states to graph theory allows one to analyze the generation of these states in a systematic way (see in particular [39,45] and the first experimental demonstration [46]). In addition, high-dimensional Bell-state measurements [47,48] can be used to entangle separated quantum states, and quantum teleportation is used to perform photon-number dependent measurements [40]. We show in the SM [29] how to create several of these states using well-known experimental concepts.

Conclusion and outlook.—We have presented general multiphoton high-dimensional transformations that rely solely on known experimental techniques. Among them are high-dimensional generalizations of the crucial CNOT, TOFFOLI, or CPHASE gates. Several of these concepts can readily be implemented in laboratories around the world.

The experimental configurations presented here are probabilistic and require a number of additional ancillary photons. This is expected, as the same holds true for the well-studied two-dimensional case. In order to improve the practicality of these setups, it would be interesting to study how (if possible) to reduce the number of ancillary photons and increase the success probability of the gates.

For experimental implementation, high stability of the experimental setups will be necessary. This can be done in bulk optics (such as shown here [47]) or by implementing these methods into integrated circuits [4,6,49], ideally with the possibility of having photon-pair sources on chip [50–53], to produce ancilla states in a stable manner. Therefore, it will be interesting to translate the concepts presented here to the path-encoding degree of freedom.

The concepts for these gates have been discovered using computer-designed quantum experiments, specifically a highly efficient version of the algorithm MELVIN [28]. Several other automated algorithms have been generated recently for the design of novel quantum-optical experiments [30,54–58]. Our result indicates the possibility that computers can be used in a widely unexplored way, namely to inspire human scientists.

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[29] See Supplemental Material at http://link.aps.org/ supplemental/10.1103/PhysRevLett.125.050501 for inspiration from the Computer; other examples for multi-photonic operations; examples for experimental design of ancillary states.

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