

## Entanglement by Path Identity

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Quantum entanglement is one of the most prominent features of quantum mechanics and forms the basis of quantum information technologies. Here we present a novel method for the creation of quantum entanglement in multipartite and high-dimensional systems. The two ingredients are (i) superposition of photon pairs with different origins and (ii) aligning photons such that their paths are identical. We explain the experimentally feasible creation of various classes of multiphoton entanglement encoded in polarization as well as in high-dimensional Hilbert spaces—starting only from nonentangled photon pairs. For two photons, arbitrary high-dimensional entanglement can be created. The idea of generating entanglement by path identity could also apply to quantum entities other than photons. We discovered the technique by analyzing the output of a computer algorithm. This shows that computer designed quantum experiments can be inspirations for new techniques.

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In 1991 Zou, Wang, and Mandel reported an experiment where they induced coherence between two photonic beams without interacting with any of them [1,2]. They used two spontaneous parametric down-conversion crystals, where one photon pair is in a superposition of being created in crystal 1 and crystal 2—which can be described as  $|\psi\rangle = (1/\sqrt{2})(|a\rangle|b\rangle + |c\rangle|d\rangle)$ . The striking idea (originally proposed by Zhe-Yu Ou) was to overlap one of the paths from each crystal (Fig. 1), which can be written as  $|b\rangle = |d\rangle$ . This method removes the *which-crystal information* of the final photon in path  $d$ . In contrast to a quantum eraser, the information here is not erased by postselection. Instead, all photons arrive in the same output irrespectively in which crystal they are created. The resulting state can be written as  $|\psi\rangle = (1/\sqrt{2})(|a\rangle + |c\rangle)|d\rangle$ : one photon is in path  $d$ , and its partner photon is in a superposition of being in path  $a$  and  $c$ . There has been some follow-up work in recent years in the areas of quantum spectroscopy [3,4], quantum imaging [5,6], studies of complementarity [7–10], optical polarization [11], and in microwave superconducting cavities [12]. However, this striking idea has not been investigated in the context of quantum entanglement generation yet.

Here we show that by superposing photon pairs created in different crystals, and overlapping the photons' paths, one can generate very flexible experiments producing various types of entanglement, both in the multiphoton and the high-dimensional regime. We start by presenting different schemes to produce various multiphoton polarization-entangled states such as Greenberger-Horne-Zeilinger (GHZ) states [13] and  $W$  states [14,15], and contrast them with traditional methods of creating

these states [16]. The method is then generalized to high-dimensional multiphoton entangled states (such as a four-particle three-dimensional GHZ state), which so far can only be produced in a few special cases [17,18]. Furthermore, we present for the first time a method to create arbitrary high-dimensional two-photon entangled states, for example, arbitrary high-dimensional Bell states in orbital angular momentum (OAM) or frequency of photons.

*Multiphoton entanglement in polarization.*—First, we consider four-photon polarization entanglement [Fig. 2(a)].

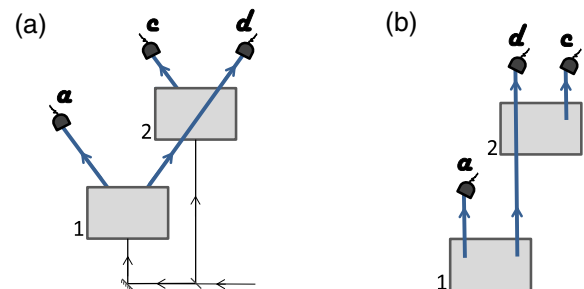


FIG. 1. (a) The simplest example which uses the overlapping modes has been discussed first in [1]. The two crystals (gray squares) can produce one pair of photons (blue lines), either in the first or in the second crystal with the pump beam depicted with black lines. If the two processes are coherent and the photons have the same frequency and polarization, it is not known in which crystal the photons are created. In that case, the resulting photon pair is in the state  $|\psi\rangle = (1/\sqrt{2})(|a\rangle + |c\rangle)|d\rangle$ . (b) A simple sketch of the same experiment. For simplicity, we will use this more abstract representation of physical experiments in the rest of the manuscript.

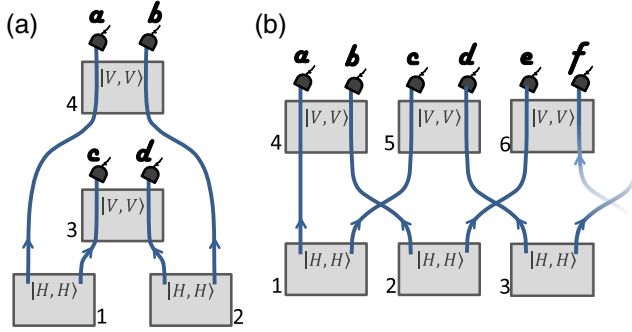


FIG. 2. Multiphoton entanglement with polarization. (a) In four crystals, two photon-pairs are produced. Crystals 1 and 2 produce horizontally polarized photons, while crystals 3 and 4 produce vertically polarized ones. Four-photon coincidences can only happen when crystals 1 and 2 fire together or when crystals 3 and 4 fire. This leads to a four-particle GHZ state  $|\psi\rangle = (1/\sqrt{2})(|H, H, H, H\rangle + |V, V, V, V\rangle)$ . (b) Entangled states with more numbers of particles can be created in an analogous way—here an  $n$ -photon GHZ state  $|\psi\rangle = (1/\sqrt{2})(|H, H, H, \dots\rangle + |V, V, V, \dots\rangle)$  is shown.

Crystals 1 and 2 can produce horizontally polarized pairs, while crystals 3 and 4 can produce vertically polarized ones. The crystals are pumped coherently, and the pump power is adjusted in such a way that we can neglect the cases where more than two pairs are created. The idea is that four-photon coincidences (i.e., one photon in each of the four paths) can only happen either when the two pairs come from crystals 1 and 2 or crystals 3 and 4. No other event produces four-photon coincidences. For example, if the pairs are produced in crystals 1 and 3, there will be two photons in path  $c$ , but none in path  $b$ . The resulting four-photon state can be written as (see Supplemental Material [19] for a detailed calculation)

$$|\psi\rangle = (|H_a, H_c\rangle + |H_b, H_d\rangle + |V_a, V_b\rangle + |V_c, V_d\rangle)^2, \\ |\psi\rangle \xrightarrow{\text{fourfold}} \frac{1}{\sqrt{2}}(|H_a, H_b, H_c, H_d\rangle + |V_a, V_b, V_c, V_d\rangle), \quad (1)$$

where  $H$  and  $V$  stand for horizontal and vertical polarization, respectively, and the subscript stands for the photon's path. The final result is a four-photon GHZ state. A realistic diagram of the experimental setup as well as discussion about requirements for temporal coherence and indistinguishability (applying the methods from [20] to the four-photon case) can be found in the Supplemental Material [19]. In an analogous way, by increasing the number of crystals and the pump power, entangled states with more photons can be created. In Fig. 2(b) the scheme for creating an  $n$ -photon GHZ state is shown.

In contrast to our new method, the traditional way of creating four-photon entangled states requires two crystals each producing a pair of polarization entangled photons. One photon from each crystal goes to a polarizing

beam splitter (PBS), which removes the which-crystal information. Triggering on events where all four detectors click, a four-particle GHZ  $|\psi\rangle = (1/\sqrt{2})(|H, H, H, H\rangle + |V, V, V, V\rangle)$  state is created [21]. With that traditional method, GHZ entanglement with eight photons [22,23] and very recently, up to 10 photons have been created [24,25].

The new scheme does not need entangled photons to start with. Furthermore, removal of the which-crystal information using a PBS is not necessary, as it has never been created in the first place. Our method does not use cascaded down-conversion, as it has been shown in recent letters producing multiphoton polarization entangled states [26,27]. In our examples, stimulated emission does not happen (which would introduce noise in the entangled state), because the input modes into the crystal are orthogonal to the output modes (for instance, having different polarization). In other cases, such as for the generation of the four photon  $W$  state (Fig. 3), stimulated emission can happen, but its contributions to the four-photon coincidences are negligible.

*Multiphoton entanglement in higher dimensions.*—The principle can be generalized to produce high-dimensional multiphoton entangled states [28,29]. High-dimensional entanglement has been investigated mainly in the two-photon case [30–33], with two recent exceptions which investigated three-dimensional entanglement with three photons [17], and the teleportation of 2 degrees of freedom of a single photon [34]. Figure 4(a) shows our proposal for an experiment creating a three-dimensional four-party GHZ state, starting from crystals which create separable photon pairs. There are three layers of two crystals—i.e., six

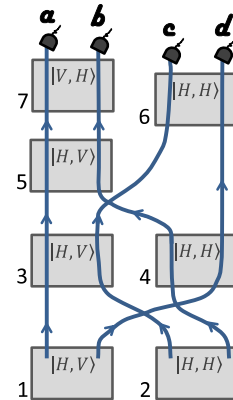


FIG. 3.  $W$  states, such as  $|\psi\rangle = \frac{1}{2}(|VHHH\rangle + |HVHH\rangle + |HHVH\rangle + |HHHV\rangle)$ , represent a different type of multiphoton entanglement. While the GHZ state is considered as the most nonclassical state, a  $W$  state is the most robust entangled state because the loss of one particle leaves an entangled state. Here, fourfolds can only happen if both crystals 1 and 2 produce a pair of photons, or crystals 3 and 4, or crystals 5 and 6, or 6 and 7. Interestingly, in this setup, one photon from crystal 1 can stimulate an emission in crystal 3. However, this will not lead to fourfold coincidences, as there is no photon in path  $b$ . Thus, this event can be neglected.

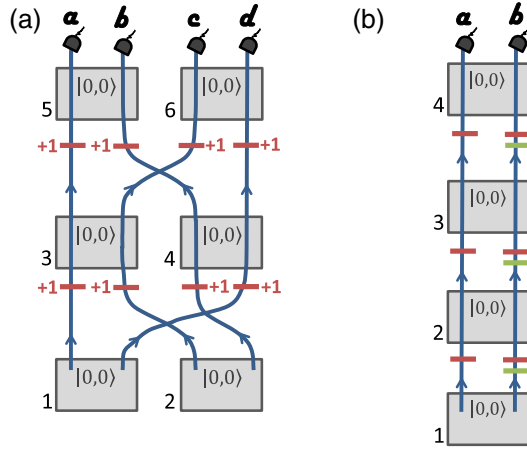


FIG. 4. Multiphoton entanglement with high-dimensional degrees of freedom (in this example: orbital angular momentum). (a) The setup produces two photon pairs. Four-photon coincidences can only occur when crystals 1 and 2 fire together, or crystals 3 and 4, or crystals 5 and 6. The produced photon pairs are all in the lowest mode (such as OAM = 0). After each layer of crystals, a hologram increases the OAM of the photons (depicted as a red line). After the third layer, photons from crystals 1 and 2 have an OAM = 2 and photons from the middle layer have an OAM = 1, which leads to a four-particle three-dimensional entangled GHZ state  $|\psi\rangle = (1/\sqrt{3})(|0, 0, 0, 0\rangle + |1, 1, 1, 1\rangle + |2, 2, 2, 2\rangle)$ . (b) The same technique can also be applied to two-particle states to produce general high-dimensional states. All crystals produce pairs of Gaussian photons. The red lines indicate OAM holograms, and the green lines indicate phase shifters. For example, if the holograms are all OAM = 1 and the phases are ignored, then the final output state is  $|\psi\rangle = \frac{1}{2}(|0, 0\rangle + |1, 1\rangle + |2, 2\rangle + |3, 3\rangle)$ . However, if one changes phases and the OAM in photon  $b$ , arbitrary four-dimensional states can be created—for example all 16 four-dimensional Bell states. By increasing the number of crystals, more dimensions can be added.

crystals that are pumped coherently, and photon pairs are created in two of them (because the pump power is set to such a level that higher-order emissions can be neglected).

Each photon from the first layer (crystals 1 and 2) passes through two mode shifters, which in total shifts its mode by +2. In the case of the OAM of photons [35], mode shifters are holograms which add one unit of OAM to the photon (an analogous method could be done with discrete frequency or time bins [36,37]). Photons from the second layer (crystals 3 and 4) pass through one mode shifter, while the photons created in the uppermost layer (crystals 5 and 6) stay in their initial mode. The resulting four-photon state can be described in the following way (see Supplemental Material [19] for details):

$$|\psi\rangle = (|2_a, 2_d\rangle + |2_b, 2_c\rangle + |1_a, 1_c\rangle + |1_b, 1_d\rangle + |0_a, 0_b\rangle + |0_c, 0_d\rangle)^2, \quad (2)$$

where 0, 1, and 2 stand for the mode number (such as the OAM of the photon), and the subscript denotes the photon's path. In the same way as before, by neglecting cases where more than two photon pairs are produced, one finds that a fourfold coincidence event in detectors  $a$ ,  $b$ ,  $c$ , and  $d$  can only be created either if crystals 1 and 2 fire together, or crystals 3 and 4, or crystals 5 and 6. This leads to

$$|\psi\rangle^{\text{fourfold}} \rightarrow \frac{1}{\sqrt{3}}(|2_a, 2_b, 2_c, 2_d\rangle + |1_a, 1_b, 1_c, 1_d\rangle + |0_a, 0_b, 0_c, 0_d\rangle). \quad (3)$$

Our idea can further be generalized to cases of more than four photons. As an example, a scheme for six photons entangled in five dimensions is shown in the Supplemental Material [19]. In general, adding additional columns (and increasing the pump power) increases the photon number  $n$ , while adding additional layers increases the dimensionality of entanglement  $d$ . That allows for the creation of arbitrary  $n$ -photon states entangled in  $d = n - 1$  dimensions (for even  $n$  and  $d = n$ , for odd  $n$  when one photon is used as a trigger). Furthermore, symmetries of the states can be exploited which leads to a vast number of available entangled states (such as asymmetrically entangled states, which exist only when  $n$  and  $d$  are both larger than two [28]). A detailed analysis of which states can be produced in this way is given in the Supplemental Material [19].

The efficiency,  $E$ , of state generation is the probability of getting a desired state from all  $n$ -fold photon terms. The GHZ state has an efficiency of  $E = \frac{d}{[(n-d)/2]^{n/2}}$ , other states have higher efficiencies (details in the Supplemental Material [19]). The efficiency of this method and the commonly used technique for polarization GHZ states are the same [24]. The expected efficiency of a three-dimensional three-photon GHZ state (the only high-dimensional GHZ state where the experimental implementation is known) with our new technique is significantly higher than the known technique [18].

*Two-photon arbitrary high-dimensional entanglement.*— Finally, we show that the same technique can be applied to generate arbitrary high-dimensional entangled two-photon states, starting again with only separable (nonentangled) photon pairs. As shown in Fig. 4(b), four crystals are set up in sequence (only one photon pair is produced) and their output modes are overlapped. Between each crystal, one adds arbitrary phase shifters and mode shifters. That allows for adjusting every individual term in the superposition independently. For example, with all phases set to  $\phi = (\pi/2)$  and all mode shifters being +1, the setup creates  $|\psi\rangle = \frac{1}{2}(|0, 0\rangle + i|1, 1\rangle - |2, 2\rangle - i|3, 3\rangle)$ . The dimension can be increased by increasing the number of layers (crystals); the minimum number of layers for creating a  $d$ -dimensional entangled state is  $d$ .



Traditional methods for producing high-dimensional entanglement exploit the entanglement produced directly in a crystal. Such methods can only produce very restricted type of states. Furthermore, those states are never maximally entangled and have low rates of production. Our technique overcomes these restrictions and can produce arbitrary high-dimensionally entangled two-photon states. We can also tune the amount of entanglement in the following ways: (i) by adjusting the pump laser power between different crystals, we can produce nonmaximally entangled pure states; (ii) by pumping the crystals with pumps that are not fully coherent to each other, we can produce entangled mixed states.

The number of photon pairs created does not depend on the number of crystals in the experimental setup. For example in Fig. 4(b), even though there are four crystals, only one photon pair is created. Therefore, the expected two-photon rate is of the same order as in a conventional single-crystal source. Moreover, our method requires only separable photon pairs to begin with. Therefore, for the OAM of photons, the production rates can be significantly higher than the rate achievable with a traditional method (where higher-dimensional entanglement is created directly in the crystal). This is because it is substantially easier to create photon pairs in zero-order (Gaussian spatial mode) than in higher-order modes.

Interestingly, the simplest special case of the technique presented here is a commonly used source of two-photon polarization entanglement. The so-called *cross-crystal source* uses two crystals, one after the other, where the first one can create a horizontally polarized photon pair, and the second one creates vertically polarized photon pairs [38]. Pumping both crystals at the same time and producing one pair of photons, one can create a  $|\psi\rangle = (1/\sqrt{2})(|H, H\rangle + |V, V\rangle)$  state. That technique can now be seen as a special case of a much broader technique to produce highly flexible high-dimensional multiphoton states in various degrees of freedom, by exploiting the superposition of photon-pair origins and overlapping of paths of photons.

To conclude, we investigated new types of photonic entanglement generation by combining two methods. First, photon pairs which originate from different crystals are coherently superposed. Second, we aligned photon paths to manipulate the structure of the entangled state. It allows for the generation of very general quantum entangled states for high-dimensional and multiphotonic systems. Our method can be favorably suited for photonic quantum computation schemes, particularly in miniaturized compact devices.

Topics for future research involve the following: exploring the relationship of generating entanglement by propagation, detection, and post selection [39], by using the indistinguishability [40–42], or by using linear optics [43]; treating the temporal coherence (such as investigated in [20] for two-photon states) in general for multiphoton

experiments; generalizing the creation of entanglement by path identity (or more generally, the identity of *some* degree of freedom) to other quantum entities, e.g., microwave superconducting cavities [12], atomic systems [44,45], trapped ions [46], or superconducting circuits [47].

Finally, we discovered this technique by analyzing the output of a computer algorithm which designs new quantum optical experiments [18]. From there, we generalized the idea (see Supplemental Material [19]). It shows that automated designs of quantum optical experiments by algorithms can not only produce specific quantum states or transformations, but they can also be a source of inspiration for new techniques—which can further be investigated by human scientists.

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- [1] L. J. Wang, X. Y. Zou, and L. Mandel, Induced coherence without induced emission, *Phys. Rev. A* **44**, 4614 (1991).
- [2] X. Y. Zou, L. J. Wang, and L. Mandel, Induced Coherence and Indistinguishability in Optical Interference, *Phys. Rev. Lett.* **67**, 318 (1991).
- [3] S. P. Kulik, G. A. Maslennikov, S. P. Merkulova, A. N. Penin, L. K. Radchenko, and V. N. Krasheninnikov, Two-photon interference in the presence of absorption, *J. Exp. Theor. Phys.* **98**, 31 (2004).
- [4] D. A. Kalashnikov, A. V. Paterova, S. P. Kulik, and L. A. Krivitsky, Infrared spectroscopy with visible light, *Nat. Photonics* **10**, 98 (2016).
- [5] G. B. Lemos, V. Borish, G. D. Cole, S. Ramelow, R. Lapkiewicz, and A. Zeilinger, Quantum imaging with undetected photons, *Nature (London)* **512**, 409 (2014).
- [6] M. Lahiri, R. Lapkiewicz, G. B. Lemos, and A. Zeilinger, Theory of quantum imaging with undetected photons, *Phys. Rev. A* **92**, 013832 (2015).
- [7] T. J. Herzog, J. G. Rarity, H. Weinfurter, and A. Zeilinger, Frustrated Two-Photon Creation via Interference, *Phys. Rev. Lett.* **72**, 629 (1994).
- [8] T. J. Herzog, P. G. Kwiat, H. Weinfurter, and A. Zeilinger, Complementarity and the Quantum Eraser, *Phys. Rev. Lett.* **75**, 3034 (1995).
- [9] A. Heuer, R. Menzel, and P. W. Milonni, Induced Coherence, Vacuum Fields, and Complementarity in Biphoton Generation, *Phys. Rev. Lett.* **114**, 053601 (2015).
- [10] A. Heuer, R. Menzel, and P. W. Milonni, Complementarity in biphoton generation with stimulated or induced coherence, *Phys. Rev. A* **92**, 033834 (2015).

- [11] M. Lahiri, A. Hochrainer, R. Lapkiewicz, G. B. Lemos, and A. Zeilinger, Partial polarization by quantum distinguishability, [arXiv:1510.04192](https://arxiv.org/abs/1510.04192).
- [12] P. Lähteenmäki, G. S. Paraoanu, J. Hassel, and P. J. Hakonen, Coherence and multimode correlations from vacuum fluctuations in a microwave superconducting cavity, *Nat. Commun.* **7**, 12548 (2016).
- [13] D. M. Greenberger, M. A. Horne, and A. Zeilinger, Going beyond bell's theorem, *Bell's Theorem, Quantum Theory and Conceptions of the Universe* (Springer International, Berlin, 1989) p. 69.
- [14] A. Zeilinger, M. A. Horne, and D. M. Greenberger, Higher-Order Quantum Entanglement, in *Proceedings "Squeezed States and Quantum Uncertainty", College Park*, edited by D. Han, Y. S. Kim, and W. W. Zachary, NASA Conference Publication No. 3135 (National Aeronautics and Space Administration, College Park, 1992).
- [15] M. Bourennane, M. Eibl, C. Kurtsiefer, S. Gaertner, H. Weinfurter, O. Gühne, P. Hyllus, D. Bruß, M. Lewenstein, and A. Sanpera, Experimental Detection of Multipartite Entanglement using Witness Operators, *Phys. Rev. Lett.* **92**, 087902 (2004).
- [16] J.-W. Pan, Z.-B. Chen, C.-Y. Lu, H. Weinfurter, A. Zeilinger, and M. Żukowski, Multiphoton entanglement and interferometry, *Rev. Mod. Phys.* **84**, 777 (2012).
- [17] M. Malik, M. Erhard, M. Huber, M. Krenn, R. Fickler, and A. Zeilinger, Multi-photon entanglement in high dimensions, *Nat. Photonics* **10**, 248 (2016).
- [18] M. Krenn, M. Malik, R. Fickler, R. Lapkiewicz, and A. Zeilinger, Automated Search for New Quantum Experiments, *Phys. Rev. Lett.* **116**, 090405 (2016).
- [19] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.118.080401> for a realistic diagram and coherence time requirements for the four-photon polarization entanglement setup; detailed calculations of the quantum states and effects of misalignments; constructions of general experiments; investigation of efficiencies; and information how the computer-inspired technique was found.
- [20] A. K. Jha, M. N. O'Sullivan, K. W. C. Chan, and R. W. Boyd, Temporal coherence and indistinguishability in two-photon interference effects, *Phys. Rev. A* **77**, 021801 (2008).
- [21] J.-W. Pan, M. Daniell, S. Gasparoni, G. Weihs, and A. Zeilinger, Experimental Demonstration of Four-Photon Entanglement and High-Fidelity Teleportation, *Phys. Rev. Lett.* **86**, 4435 (2001).
- [22] X.-C. Yao, T.-X. Wang, P. Xu, H. Lu, G.-S. Pan, X.-H. Bao, C.-Z. Peng, C.-Y. Lu, Y.-A. Chen, and J.-W. Pan, Observation of eight-photon entanglement, *Nat. Photonics* **6**, 225 (2012).
- [23] Y.-F. Huang, B.-H. Liu, L. Peng, Y.-H. Li, L. Li, C.-F. Li, and G.-C. Guo, Experimental generation of an eight-photon greenberger-horne-zeilinger state, *Nat. Commun.* **2**, 546 (2011).
- [24] X.-L. Wang, L.-K. Chen, W. Li, H.-L. Huang, C. Liu, C. Chen, Y.-H. Luo, Z.-E. Su, D. Wu, Z.-D. Li *et al.*, Experimental Ten-Photon Entanglement, *Phys. Rev. Lett.* **117**, 210502 (2016).
- [25] L.-K. Chen, Z.-D. Li, X.-C. Yao, M. Huang, W. Li, H. Lu, X. Yuan, Y.-B. Zhang, X. Jiang, C.-Z. Peng *et al.*, Observation of ten-photon entanglement using thin  $3 \times 6$  crystals, *Optica* **4**, 77 (2017).
- [26] H. Hübel, D. R. Hamel, A. Fedrizzi, S. Ramelow, K. J. Resch, and T. Jennewein, Direct generation of photon triplets using cascaded photon-pair sources, *Nature (London)* **466**, 601 (2010).
- [27] D. R. Hamel, L. K. Shalm, H. Hübel, A. J. Miller, F. Marsili, V. B. Verma, R. P. Mirin, S. W. Nam, K. J. Resch, and T. Jennewein, Direct generation of three-photon polarization entanglement, *Nat. Photonics* **8**, 801 (2014).
- [28] M. Huber and J. I. de Vicente, Structure of Multidimensional Entanglement in Multipartite Systems, *Phys. Rev. Lett.* **110**, 030501 (2013).
- [29] J. Lawrence, Rotational covariance and greenberger-horne-zeilinger theorems for three or more particles of any dimension, *Phys. Rev. A* **89**, 012105 (2014).
- [30] A. C. Dada, J. Leach, G. S. Buller, M. J. Padgett, and E. Andersson, Experimental high-dimensional two-photon entanglement and violations of generalized bell inequalities, *Nat. Phys.* **7**, 677 (2011).
- [31] J. Romero, D. Giovannini, S. Franke-Arnold, S. M. Barnett, and M. J. Padgett, Increasing the dimension in high-dimensional two-photon orbital angular momentum entanglement, *Phys. Rev. A* **86**, 012334 (2012).
- [32] M. Krenn, M. Huber, R. Fickler, R. Lapkiewicz, S. Ramelow, and A. Zeilinger, Generation and confirmation of a  $(100 \times 100)$ -dimensional entangled quantum system, *Proc. Natl. Acad. Sci. U.S.A.* **111**, 6243 (2014).
- [33] Y. Zhang, F. S. Roux, T. Konrad, M. Agnew, J. Leach, and A. Forbes, Engineering two-photon high-dimensional states through quantum interference, *Sci. Adv.* **2**, e1501165 (2016).
- [34] X.-L. Wang, X.-D. Cai, Z.-E. Su, M.-C. Chen, D. Wu, L. Li, N.-L. Liu, C.-Y. Lu, and J.-W. Pan, Quantum teleportation of multiple degrees of freedom of a single photon, *Nature (London)* **518**, 516 (2015).
- [35] M. Krenn, M. Malik, M. Erhard, and A. Zeilinger, Orbital angular momentum of photons and the entanglement of laguerre-gaussian modes, *Phil. Trans. R. Soc. A* **375**, 20150442 (2017).
- [36] D. Richart, Y. Fischer, and H. Weinfurter, Experimental implementation of higher dimensional time-energy entanglement, *Appl. Phys. B* **106**, 543 (2012).
- [37] A. Martin, T. Guerreiro, A. Tiranov, S. Désignolle, F. Fröwis, N. Brunner, M. Huber, and N. Gisin, Quantifying photonic high-dimensional entanglement, [arXiv:1701.03269](https://arxiv.org/abs/1701.03269).
- [38] P. G. Kwiat, E. Waks, A. G. White, I. Appelbaum, and P. H. Eberhard, Ultrabright source of polarization-entangled photons, *Phys. Rev. A* **60**, R773 (1999).
- [39] C. Thiel, J. Von Zanthier, Thierry Bastin, E. Solano, and G. S. Agarwal, Generation of Symmetric Dicke States of Remote Qubits with Linear Optics, *Phys. Rev. Lett.* **99**, 193602 (2007).
- [40] N. Killoran, M. Cramer, and M. B. Plenio, Extracting Entanglement from Identical Particles, *Phys. Rev. Lett.* **112**, 150501 (2014).
- [41] T. Sasaki, T. Ichikawa, and I. Tsutsui, Entanglement of indistinguishable particles, *Phys. Rev. A* **83**, 012113 (2011).

- [42] M. C. Tichy, F. Mintert, and A. Buchleitner, Limits to multipartite entanglement generation with bosons and fermions, *Phys. Rev. A* **87**, 022319 (2013).
- [43] P. Migdał, J. Rodríguez-Laguna, M. Oszmaniec, and M. Lewenstein, Multiphoton states related via linear optics, *Phys. Rev. A* **89**, 062329 (2014).
- [44] M. Keller, M. Kotyrba, F. Leupold, M. Singh, M. Ebner, and A. Zeilinger, Bose-einstein condensate of metastable helium for quantum correlation experiments, *Phys. Rev. A* **90**, 063607 (2014).
- [45] R. Lopes, A. Imanaliev, A. Aspect, M. Cheneau, D. Boiron, and C. I. Westbrook, Atomic hong-ou-mandel experiment, *Nature (London)* **520**, 66 (2015).
- [46] R. Blatt and D. Wineland, Entangled states of trapped atomic ions, *Nature (London)* **453**, 1008 (2008).
- [47] R. Barends, J. Kelly, A. Megrant, A. Veitia, D. Sank, E. Jeffrey, T. C. White, J. Mutus, A. G. Fowler, B. Campbell *et al.*, Superconducting quantum circuits at the surface code threshold for fault tolerance, *Nature (London)* **508**, 500 (2014).