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On-chip quantum interference between the origins of a multi-photon state

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There has been broad interest in path identity in recent years due to its role as a foundation for numerous novel quantum information applications. Here, we experimentally demonstrate quantum coherent superposition of two different origins of a four-photon state, where multi-photon frustrated interference emerges from the quantum indistinguishability by path identity. The quantum state is created in four probabilistic photon-pair sources on one integrated silicon photonic chip, two combinations of which can create photon quadruplets. Coherent elimination and revival of the distributed four photons are fully controlled by tuning phases. The experiment gives rise to peculiar quantum interference of two possible ways to create photon quadruplets rather than interference of different intrinsic properties of photons. Besides many known potential applications, this kind of multi-photon nonlinear interference enables the possibility for various fundamental studies such as nonlocality with multiple spatially separated locations.

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1. INTRODUCTION

In 1994, Herzog *et al*. demonstrated a remarkable quantum interference effect [1]. Like induced coherence without induced emission [2], they overlapped the two paths of emerging photon pairs from two probabilistic photon-pair sources, in such a way that it cannot be distinguished—not even in principle—whether the pair has been created in the first or second source. As a result, the photons are in a coherent superposition of being created in either of the two sources. This generalization of superpositions of properties of photons to the mere origin of their creation has exciting consequences. They now tune a phase between the two crystals, and thereby manipulate the total amount of emerging photon pairs. This effect has been called frustrated quantum interference [3], also known as the quantum version of one type of nonlinear interference [4–6].

Frustrated pair creation has been the enabling principle behind numerous emerging quantum technologies. For example, it allows for a new form of infrared spectroscopy [7] or terahertz quantum sensing [8] by exploiting visible light for which cheap and efficient photon detectors exist. So while this phenomenon has been much

investigated at the two-photon level, a multi-photon generalization was not even proposed until 2017 [9,10]. There, taking inspiration from graph theory, it was discovered that the very particular alignment of paths in a four-crystal setup could lead to genuine four-photon interference that is not observable from two photons alone [10]. The effect can be understood by a coherent superposition of two origins of a four-photon state that interfere.

Here, we report the first experimental demonstration of this new multi-photon interference effect. In our four-photon experiment, the paths of the photons are overlapped in such a way that there are exactly two different possibilities of how each of the four detectors sees a single photon. Taking advantage of high-quality integrated optics at a silicon chip, we align the photon paths in such a way that it cannot be distinguished whether the pairs have been created in the first or second possibility. In that way, the origin of the four photons is in a coherent superposition, and we can observe constructive and destructive interference of four-photon systems that cannot be observed in the photon pairs themselves.

Our demonstration relies on a number of novelties compared to existing experiments. First, we extended the only other on-chip frustrated quantum interference experiment to multi-photon

[5], and the integrated pattern showed high scalabilty and would stimulate more new quantum optical experiments. Furthermore, we were able to demonstrate for the first time how two indistinguishable multipartite states beyond photon pairs can be generated experimentally using the concept of path identity [3]. This not only required the design of multiple sources of highly indistinguishable photon pairs, but also precise mode matching between the different layers of photon quadruple generation. Our work is an important part of path identity and lays the foundation for numerous novel applications such as quantum state generation, remote quantum metrology, and observation of new nonlocal multi-photon interference effects [3]. Among others, the quantum interference effect we demonstrate here is the core experimental resource for a large class of new quantum state generation protocols that range from heralded entangled state generation to concepts for quantum simulation [11,12].

2. PHYSICAL INTERPRETATION

Let us first look at the experiment by Herzog *et al.* [1], depicted in Fig. 1(a). Each of the two nonlinear crystals can probabilistically create a photon pair. If all properties of the photon pair are identical, there is no way to distinguish from which crystal a pair was created. In that case, the two-photon state is

$$|\psi\rangle = \frac{1}{\sqrt{2}}g\left(\overbrace{|a,b\rangle}^{\text{crystal II}} + \overbrace{e^{i\varphi}|a,b\rangle}^{\text{crystal I}}\right) = \frac{1}{\sqrt{2}}g\left(1 + e^{i\varphi}\right)|a,b\rangle, \tag{1}$$

where $|a,b\rangle$ stands for a photon in path a and one in path b, and g is related to the pair creation probability. For clarity, we ignore the vacuum and higher-order terms.

The probability to detect a photon pair in a and b is $P_{a,b} \approx g^2(1 + \cos(\varphi))$. With $\varphi = 0$, constructive interference increases the number of photon pairs by a factor of two compared to the number of pairs generated by a single crystal. With $\varphi = \pi$, the two possibilities for pair generation cancel each other. That means, even though the crystals would produce photons independently, no photon pair is created at all.

In our four-photon experiment, four photon-pair sources, I–IV, are used, as shown in Fig. 1(b). Their paths are overlapped in such a way that detectors A–D see photon quadruples only if sources I and
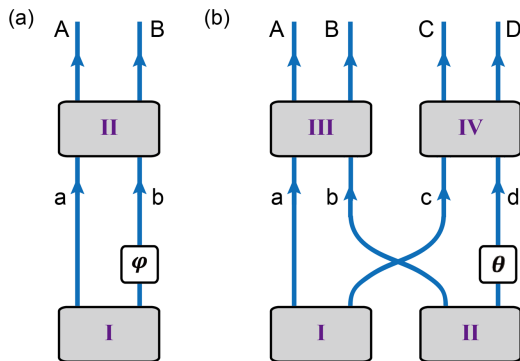

Fig. 1. Quantum interference by indistinguishable origins. (a) Quantum interference between the origins of a two-photon state. (b) Multi-photon generalization. For simplicity, the pump beams are not shown. In principle, the phases can also be added on the pump beams between crystals I, II and III, IV.

II or III and IV create a photon pair simultaneously. For small photon pair creation rate g, we can write the state as

$$|\psi\rangle = \frac{1}{2}g\left(\overbrace{|a,b\rangle}^{\text{crystal III}} + \overbrace{|c,d\rangle}^{\text{crystal IV}} + \overbrace{|a,c\rangle}^{\text{crystal I}} + \overbrace{e^{i\theta}|b,d\rangle}^{\text{crystal II}}\right)$$

$$+ \frac{1}{4}g^2\left(\underbrace{|a,b,c,d\rangle}_{\text{crystal III\&IV}} + \underbrace{e^{i\theta}|a,b,c,d\rangle}_{\text{crystal I\&II}}\right) + \dots$$

$$= \frac{1}{2}g\left(|a,b\rangle + |c,d\rangle + |a,c\rangle + e^{i\theta}|b,d\rangle\right)$$

$$+ \frac{1}{4}g^2\left(1 + e^{i\theta}\right)|a,b,c,d\rangle + \dots, \tag{2}$$

where $|a,b,c,d\rangle$ stands for a photon quadruple in detectors A–D. Again, by introducing a phase θ between the origins, we can observe constructive and destructive interference of the four-photon creation processes. For example, if the phase is $\theta = \pi$, no four-photon state can be observed in detectors A–D, while each of the individual crystals continues to create photon pairs.

3. EXPERIMENTAL SETUP AND RESULTS

High phase stability and mode matching among all the sources is quite important for our experiment; thus, employing bulk optics or fiber optics will be challenging. Integrated photonic devices enable localization and manipulation of photons at micro/nano scale, thus greatly improving stability and scalability [13]. We explore this multi-photon interference effect using an integrated silicon photonic chip, and the conceptual layout is shown in Fig. 2. Integrated silicon photonics has become a scalable platform for quantum information processing in recent years [14–18]. In the experiment, we pump four integrated photon-pair sources coherently and superpose different origins of the four-photon state to observe multi-photon frustrated interference. All phases are modulated by on-chip heaters.

Specifically, a 200 GHz bandwidth pulsed laser centred at 1550.11 nm acts as the pump. It is coupled into the chip using grating couplers and split coherently into two paths using a Mach–Zehnder interferometer configured to act as a 50:50 beam splitter (MZI1). The upper beam is split again at a 50:50 beam splitter (BS1) and acts as the pump for sources I and II, while the lower beam is delayed and subsequently pumps sources III and IV. Each photon-pair source (I–IV) is a 5 mm long single-mode spiral silicon waveguide. In the waveguides, two pump photons are annihilated, and signal–idler photon pairs are generated with the spontaneous four-wave mixing process [19]. In the experiment, we select signal and idler photons with respective central wavelengths of 1545.32 nm and 1554.94 nm and 100 GHz bandwidth (see Supplement 1, Fig. S3). After sources I and II, an unbalanced MZI (UMZI1) is used to filter out the pump beam. Subsequently, the signal and idler photons in both paths are then separated using another unbalanced MZI (UMZI2). In this way, we can achieve four photons in four paths as $|a,b\rangle$ and $|c,d\rangle$. In paths b and c, we place an MZI to act as a tunable beam splitter (MZI2). After MZI2, another pair of UMZI2s are used to combine signal and idler photons into one path. The lower pump (in the *delay line*
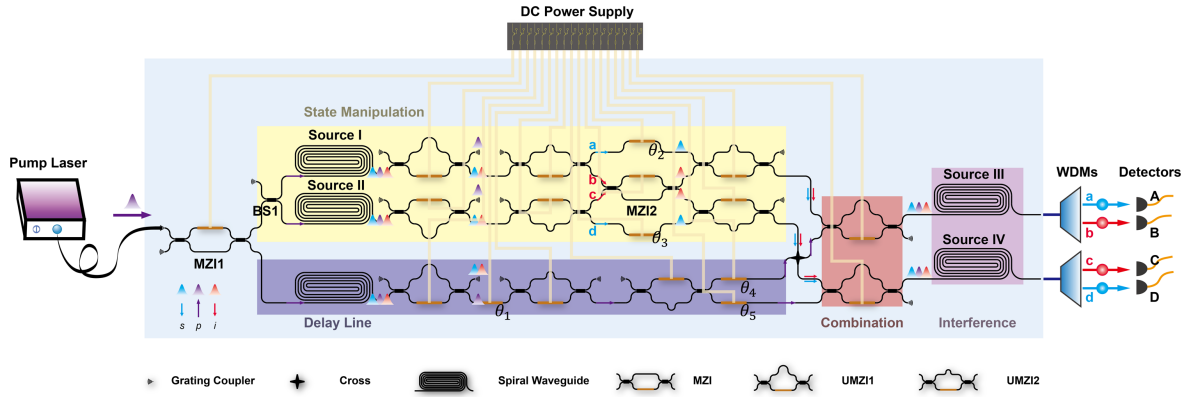
Fig. 2. Experimental setup. Four spiral waveguide sources produce photon quadruples (source I and source II can produce a four-photon state as well as sources III and IV). A tunable MZI (MZI2) is used to change from two-photon interference to four-photon interference. MZI, Mach–Zehnder interferometer; UMZI, unbalanced Mach–Zehnder interferometer; WDMs, wavelength-division multiplexers.

region) is combined with the photons created in sources I and II using two UMZI1s (in the *combination* region). The overlap is temporally controlled such that it enters into sources III and IV at the same time as the photon pairs created in sources I and II. Extra phases $\theta_1, \theta_2, \theta_3, \theta_4$, and θ_5 are introduced to observe the frustrated quantum interference effect. After photon pairs are coupled out from the chip, they are delivered into four single-photon detectors for detection and further time correlation analysis. More details about chip characterization are given in Supplement 1.

By setting MZI2 into the setting where the incoming photons remain in their paths, we demonstrate the two-photon frustrated interference process. The results are given in Supplement 1 and confirm the full controllability and high stability of the device. With the same filtering condition, we set MZI2 to swap the paths of the photons in b and c, resembling the configuration in Fig. 1(b). We get a two-photon state expressed as

$$|\psi\rangle_2 = \frac{1}{2}\left(\overbrace{e^{i\theta_2}|a, c\rangle}^{\text{Source I}} + \overbrace{e^{i\theta_3}|b, d\rangle}^{\text{Source II}} \right.$$
$$\left. + \overbrace{e^{2i(\theta_1+\theta_4)}|a, b\rangle}^{\text{Source III}} + \overbrace{e^{2i(\theta_1+\theta_5)}|c, d\rangle}^{\text{Source IV}} \right). \quad (3)$$

Correspondingly, the four-photon state is expressed as

$$|\psi\rangle_4 = \frac{1}{4}\left(\overbrace{e^{i(4\theta_1+2\theta_4+2\theta_5)}|a, b, c, d\rangle}^{\text{Source III\&IV}} + \overbrace{e^{i(\theta_2+\theta_3)}|a, b, c, d\rangle}^{\text{Source I\&II}} \right)$$
$$= \frac{1}{4}\left(e^{i(\theta_2+\theta_3)} + e^{i(4\theta_1+2\theta_4+2\theta_5)} \right)|a, b, c, d\rangle. \quad (4)$$

From these equations, if we change phases ($\theta_1, \theta_2, \theta_3, \theta_4$, and θ_5), four-photon coincidence counts will increase or decrease, while two-photon coincidence counts will remain unchanged. This is a multi-photonic generalization of two-photon frustrated quantum interference.

We first record four-photon coincidence counts when varying the phase θ_3, and the results are given in Fig. 3(a). This is the exact Fig. 1(b) scheme, that is, varying the phase on one photon. The pump power before the chip is set as 1.81 mW. Based on Eq. (4), the interference fringe can be fitted with $1 + V \sin[\pi(\varphi - \varphi_c)/T]$, where V is the fringe visibility, φ_c is the initial phase, and T is the oscillation period. The fringe visibility V is defined as $V = (d_{\max} - d_{\min})/(d_{\max} + d_{\min})$, where $d_{\max}$ and $d_{\min}$ are the maximum and minimum of the fitted data, respectively. The raw visibility of the four-photon coincidence fringe was estimated as 66.7% ± 3.1%, and the error bar was obtained from 500 Monte Carlo simulations assuming a Poissonian distribution of the measured counts (the same below). Two-photon coincidence counts with linear fitting are given in Fig. 3(b), and they show very small fluctuations. Besides phase θ_3, varying other phases on the silicon chip will also show a similar frustrated interference effect. We record four-photon coincidence counts when varying the phase θ_5 as an example, and the results are given in Fig. 3(c). At this point, we actually vary the phase of the pump laser of one nonlinear crystal. The fitting curve shows a two-fold oscillation frequency, and the raw visibility of the four-photon coincidence fringe was estimated as 70.7% ± 2.2%. Corresponding two-photon coincidence counts are given in Fig. 3(d), and they are basically the same as the results in Fig. 3(b).

To improve the visibility of four-photon interference results, we lower the pump power to 0.98 mW to reduce multi-photon noise. In this regime, the photon-pair generation rate of each source is estimated as $p = 0.01$, corresponding to $g = 0.1$. We record four-photon coincidence counts with an integral time of each point of 30 min when varying the phase θ_1, and the results are given in Fig. 4(a). At this point, we actually vary the phase of the pump laser of two nonlinear crystals (one quadruplet, sources III and IV). The fitting curve shows a four-fold oscillation frequency, and the raw visibility of the four-photon coincidence fringe was estimated as 78.3% ± 2.6%. In addition to multi-photon noise, the spectral purity of single photons and imbalance of the interference terms will also reduce four-photon interference visibility [5,20–23]. According to our experimental conditions, that is, a pump laser with 200 GHz bandwidth, signal–idler photons with 100 GHz bandwidth, and photon-pair generation rate of 0.01, the maximum achievable visibility is estimated to be 0.84. Further visibility
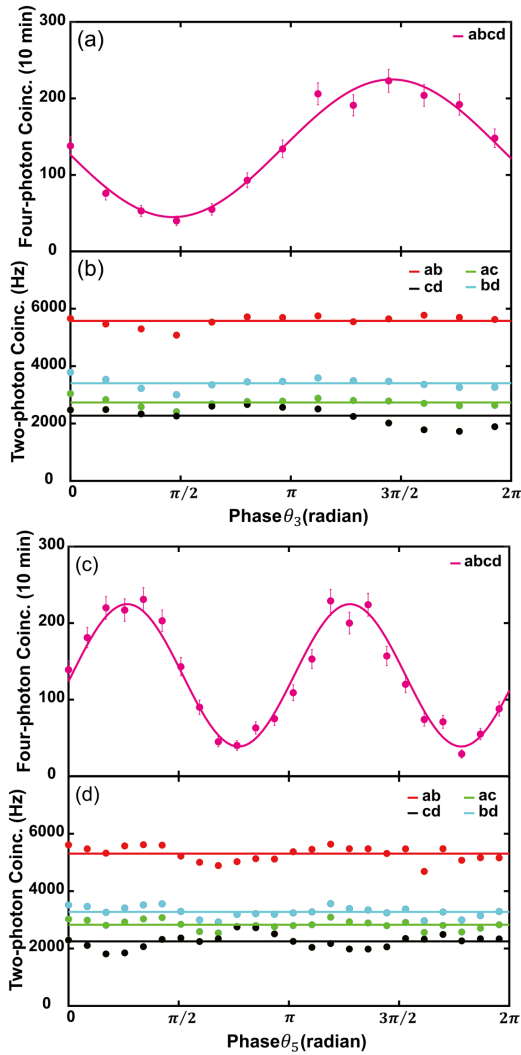
Fig. 3. Interference results with pump power 1.81 mW. Four-photon interference results (a) and two-photon fluctuation counts (b) when varying the phase θ_3. Four-photon interference results (c) and two-photon fluctuation counts (d) when varying the phase θ_5. Difference in two-photon counts is due to the imbalance in coupling and detection efficiency of different channels.

Fig. 4. Interference results with pump power 0.98 mW. Four-photon interference results (a) and two-photon fluctuation counts (b) when varying the phase θ_1.

4. DISCUSSION AND CONCLUSION

In contrast to the two-photon case, the multi-partite generalization of frustrated quantum interference demonstrated here has non-trivial consequences. This time not only the origin of the four-photon state, but also each origin consists of two locations that could be spatially separated. In this network, for example, source III could be in the main chip, and source IV in another chip. They can be easily separated by dozens of kilometers with a chip-to-chip setting linked with commercial fibers [26]. Because the pump laser and single photons all pass through the fiber, phases are still stabilized (see Supplement 1 for details). This separation will promote many exciting foundational experiments, for example, investigating more time delays of the interference effects [27].

To illustrate the uniqueness of this nonlocal quantum interference effect, let us imagine that the phase θ is close to crystal IV, and crystals III and IV are separated by a large distance. In that case, if we change the phase from zero to π, and observe a photon pair in C and D, then we know that there cannot be any photon pairs emitted in crystal III. Thereby, we can make statements about spatially separated and intrinsically random processes. This nonlocal interference resembles some properties of entanglement (where we can make statements about spatially separated, intrinsically random processes too). The crucial difference here is that we do not have superpositions of properties of photons, but between their origins. Observing this new type of nonlocal interference is a interesting goal for future quantum experiments [3]. Our experiment lays a stepping stone by demonstrating that this new quantum interference can indeed be observed in experiments.

Beyond the immediate physical interest, our work also suggests advances in the context of quantum technology applications. Based on our work, nonlinear interferometers with more photons can be constructed, and any application related to the two-photon version can be expanded to more photons. The significance of a multi-photon nonlinear interferometers goes far beyond just expanding the number of involved photons. Our experiment demonstrates coherently creating and overlapping highly distinguishable photon

improvement can be achieved by using photon-number resolving detectors and increasing the spectral purity of on-chip photon-pair sources with novel integrated parametric source designs [24,25].

In any case, only curve "cd" shows a slightly higher fluctuation (around 16.4% visibility with interference curve fitting), which is presumed to be caused by imperfect on-chip filtering that leads to frustrated two-photon quantum interference. However, the fluctuation of two-photon coincidence counts cannot explain the very large oscillation and pattern of four-photon interference. These results prove that we observe a totally new type of constructive and destructive multi-photon quantum interference, which cannot be understood by the behavior of local properties such as individual photon-pair productions (see Supplement 1 for details).
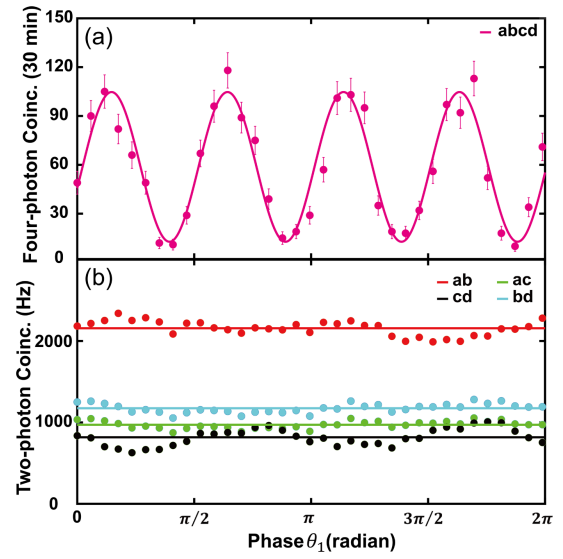
pairs from different sources. It is the basis of several new proposals such as special-purpose quantum computing schemes [10] and photonic quantum state generation [9,11,28]. By introducing the ability to amplify multi-photon states in a distributed, remote way, it could advance quantum technologies for quantum networks [29]. In fact, numerous proof-of-concept applications have used the idea of path identity to enhance quantum imaging, microscopy, and spectroscopy [3]. More advantages might be discovered in similar applications by exploiting the multi-photon interference effect realized here. More importantly, our work increases the capacity of integrated photonic circuits to process quantum information. The advantage in scalability of integrated photonic technologies enables hundreds or thousands of components to be integrated on the chip at the same time, which will greatly promote the realization of new quantum optics experiments and quantum technologies.

To conclude, we have experimentally observed constructive and destructive interference between two possible ways to create photon quadruplets on the silicon chip. This fundamentally multi-particle nonlinear quantum interference effect shows an intricate interconnection between quantum coherence, nonlocality, and entanglement, fueled by the concept of indistinguishability. We believe that our results are likely to spearhead and motivate follow-up studies related to the foundation of quantum mechanics and quantum technology applications.

Disclosures. The authors declare no conflicts of interest.

Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

Supplemental document. See Supplement 1 for supporting content.

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