Mind the Gap: Acoustical Answers to Cosmological Concerns in First-Century B.C.E. China

Noa Hegesh, *Tel Aviv University and Max Planck Institute for the History of Science*

Abstract: From the mid-third century B.C.E., Chinese experts used the manipulation of sound as a technology to synchronize society with the cosmos. In the Western Han, the polymath Jing Fang 京房 (78–37 B.C.E.) detected an acoustical problem, known in the West as the Pythagorean comma, in the musical system. In the Chinese system, the comma is characterized by a minute but audible discrepancy, a gap between two pitches that should sound identical and that carry the same numerical representation. Jing Fang reduced the comma by designing a model of sixty pitches, superposed onto the calendar. This essay argues that his acoustical endeavors are inseparable from his ultimate goal: to track seasonal change accurately throughout the year and use it in weather prognostications. Sound was the tool he used to measure the yearly flux of *qi* and interpret what contemporary thinkers had termed the hidden realities and subtle transformations of the cosmos.

> ***This is the correspondence between sound and qi.***
> **—Jing Fang**

Noa Hegesh is a historian of sound, interested in musical thought and in sound as technology. Her research, focused on early and early medieval China, explores the ways people thought about and discussed sound and how they shaped and used tuning theories within and outside of musical performance, in such areas as calendrics, prognostication, and weights and measures. She earned a doctorate in East Asian Languages and Civilizations from the University of Pennsylvania. In September 2018 she joined the Max Planck Institute for the History of Science in Berlin as a postdoctoral fellow in Department III: Artefacts, Action, Knowledge and as part of the institute's research group "Epistemes of Modern Acoustics." With Dagmar Schäfer, she was awarded a two-year Exploration Grant (2021 and 2022) by the Einstein Center Chronoi for a new group project titled "Transience: Politics and Practices of Time in Early Medieval China (Fourth to Seventh Century)." For more details on this working group see the website: ec-chronoi.de/exploration/transience. Her column "The Sound of Weights and Measures" has been published in *Nature Physics*, with the aim of bringing the historical study of sound in early China to a wide, non-Sinological audience. In October 2021 she joined the Department of East Asian Studies at Tel Aviv University. The Department of East Asian Studies, Tel Aviv University, 69978 Tel Aviv, Israel; nhegesh@tauex.tau.ac.il.

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From around the mid-third century B.C.E., Chinese textual sources began discussing sound as a natural phenomenon that could be detected, measured, and expressed in numbers. By the first century B.C.E., authors of treatises had introduced sound as a measurement, with pitch as its unit—packaging the concept of sound together with other measurements such as length, weight, and volume. Sound measured cosmic *qi*, which by then referred to the stuff, or the vital force, from which all things are made. Offices as well as individual experts used it to offer models to assess and maintain synchronization between cosmos and man throughout the year. For this purpose, they manipulated sound and used it in various fields: mathematical astronomy, prognostication, ritual musical performance, and the standardization of weights and measures. The more accurate the measurement of a pitch and its timing in the yearly cycle, the better the analysis of the synchronicity between the human realm and processes in the cosmos. In this sense, we can inquire into the components that made up musical systems in general, to understand how those who used them thought about sound.

This essay examines the concept of sound and its manipulation by the Western Han expert Jing Fang 京房 (78–37 B.C.E.), along with his musical temperament model of the sixty tuning standards (*liushi lü* 六十律). The model appears in the "Treatise on Tuning Standards and Mathematical Astronomy" ("Lüli zhi" 律歷志) in the *History of the Later Han* (*Hou Han shu* 後漢書).[1] Jing Fang, a Han court official and specialist in prognostication and the interpretation of hexagrams, was also proficient in the traditional tuning system and the mathematical procedures behind it. He identified a problem in the mathematical cycle that generated the basic twelve pitches of the tuning system, which when expressed in numerical values are called the twelve tuning standards (*shier lü* 十二律).[2] The algorithm for generating tuning standards, called "Addition and Subtraction by a Third" (*sanfen sunyi* 三分損益), was in use, at the latest, from the mid-third century B.C.E. Generating twelve tuning standards and organizing them in ascending order created a chromatic scale of twelve untempered semitones.[3] Each of these twelve tuning standards could in turn serve as a foundation for constructing scales and modes of its own. These scales usually consisted of a five-note pattern (*wu sheng* 五聲), resembling today's major pentatonic scale (C–D–E–G–A).[4]

Simply stated, Jing Fang thought the calculation of the twelve tuning standards *should* produce a cycle that began and ended on the same numerical value—the fundamental tuning standard called Yellow Bell (*Huangzhong* 黃鍾), from which the remaining eleven pitches

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[1] "Treatise on Tuning Standards and Mathematical Astronomy" 律歷志, in *Hou Han shu* 後漢書 (Beijing: Zhonghua, 1974), pp. 2999–3024.

[2] I translate the character *lü* 律 as "tuning standard" (instead of "pitch," "pitch standard," or "tonometer") to convey its relative, rather than fixed, function. A *lü* denotes the conveyance of measured sound through numbers. When one *lü* is used in relation to another *lü* it creates a ratio, which can also be named a music interval. Only when the text ascribes a defined measure to the *lü* (referring to a length of a string or pipe) does it become a fixed pitch. But under the Chinese nomenclature, even with a definite length, the term *lü* could also refer to an entire scale that begins from a chosen *lü* pitch. Pitches within a scale are sometimes called *yin* 音. They can be relative or become fixed when referred to a specific scale, and the note names for the five degrees in the five-note scale are called *yin* or *sheng* 聲. I use "pitch" when it is certain that the discussion relates to the sonic characteristic or to a recognizable measurement. Additionally, I refer to "tuning" and not "temperament," much like the often-used "Pythagorean tuning" and not "Pythagorean temperament." This is an aesthetic choice. By "traditional tuning system" I mean the twelve tuning standards and the five-note scale system, which existed (with variants) by the Eastern Zhou and is the most prevalent system in Han and pre-Han textual sources.

[3] Early Chinese texts did not define or discuss semitone intervals, nor did they use terminology such as "intervals" or "octaves." There is one example, which comes from the excavated bell sets of the Marquess Yi of Zeng (fifth century B.C.E.), in which the prefix *bian* 弁 denotes the lowering of a pitch by a semitone. See Lothar von Falkenhausen, *Suspended Music: Chime-Bells in the Culture of Bronze Age China* (Berkeley: Univ. California Press, 1993), p. 285.

[4] Throughout this essay, my choice of C as the equivalent Western fundamental note is arbitrary and meant to facilitate understanding for the reader.

are then generated.[5] The existing model resulted in a discrepancy between the numerical value of the first and the thirteenth tuning standards. The thirteenth tuning standard, *Zhishi* 執始, commenced the next set of twelve tuning standards.[6] It should be clarified that in this model the thirteenth tuning standard is not the octave; rather, it is calculated so that it returns to the point of origin, the fundamental tuning standard. In this case, the second set of twelve tuning standards from *Zhishi* is meant to replicate the first set. If played on a pipe or string, the discrepancy between the Yellow Bell and *Zhishi* fundamentals is audible, sounding like a gap of nearly a quarter tone. In the West, this type of gap has been independently recognized and termed the Pythagorean comma. It comes as no surprise that Jing Fang engaged with the same problem in another culture. However, his framework for recognizing it was rooted in his own cultural circumstances, which included the cosmological concerns mentioned above—his use of sound in prognostication and the detection of *qi*.

The problem of the comma and returning to Yellow Bell was as conceptual as it was technical. The desire to construct closed cycles preoccupied contemporary thinkers working with the cosmic model, especially in the realm of mathematical astronomy.[7] For Jing Fang, this problem also applied to the realm of tuning, and he was the first to identify and confront it. By expanding the generation of tuning standards from twelve to sixty, Jing Fang greatly reduced the comma by the fifty-fourth tuning standard, *Seyu* 色育. The reduced comma is known in China today as the Jing Fang comma (*Jing Fang yin cha* 京房音差).[8]

In the minds of late Warring States and Han elites, musical performance, with its philosophical underpinnings, was one of several areas to which practices and ideas pertaining to sound belonged. This essay argues that Jing Fang utilized sound in a different way: to him, sound was a natural phenomenon, and tuning standards were the technology for assessing the state of cosmic *qi* throughout the year. He offered the court a better option for assessing the transition of *qi* through an experiment called "Observing *qi*," which employed his sixty tuning standards and included a model for distributing them across the days of the year. This model was probably at the heart of at least one of his prognostication methods and may have been the impetus for his recognition of the systematic comma problem in the existing model—and his attempt to solve it.

To position this essay in the literature, the first section discusses previous scholarship on temperament and cosmology in preimperial China. The second section introduces Jing Fang and frames his accomplishments in the Han court. The third section focuses on the structure and history of the "Treatise on Tuning Standards and Mathematical Astronomy" as it appears in the *History of the Later Han* and turns to analyze the concept of sound reflected in the text.

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[5] The original meaning and reasoning behind some of the names of the twelve tuning standards are still unknown, and so their English translations remain problematic. Nevertheless, I rely here on John Major's translation, most often used in secondary scholarship: John S. Major, "Celestial Cycles and Mathematical Harmonics in the 'Huainanzi,'" *Extrême-Orient Extrême-Occident*, 1994, *16*:121–134. On the meaning and grouping of tuning standard nomenclature see Lothar von Falkenhausen, "On the Early Development of Chinese Musical Theory: The Rise of Pitch-Standards," *Journal of the American Oriental Society*, 1992, *112*:433–439. The origins and naming of the additional forty-eight tuning standards added by Jing Fang also remain unclear. I do not provide English translations for them here.

[6] The name "*Zhishi*" can be translated as "governing the beginning," possibly indicating its function in opening the next set of twelve tuning standards. As the names of Jing Fang's tuning standards do not appear in other Han and pre-Qin textual sources, it is possible that he invented them as part of his model.

[7] Nathan Sivin, "Cosmos and Computation in Early Chinese Mathematical Astronomy," *T'oung Pao*, 1969, 55(1/3):1–73; and Liu Tseng-kuei 劉增貴, "Calendrical Computation Numbers and Han Dynasty Politics: A Study of Gu Yong's Three Troubles Theory," in *Chang'an 26 BCE: An Augustan Age in China*, ed. Michael Nylan and Griet Vankeerberghen (Seattle: Univ. Washington Press, 2015), pp. 293–322.

[8] Chen Yingshi 陳應時, "Wei 'Jing Fang Liushi Lü' shen bian" 為 "京房六十律" 申辯, *Yiyuan: Yinyue ban* 藝苑: 音樂版, 1986, pp. 6–13.

The fourth section explicates the model Jing Fang created and the comma within their cosmological context, focusing on the categorization of sound, the generation of tuning standards, and the distribution of sound across the calendar in groups and subgroups. The fifth section expounds on Jing Fang's choices and goals in terms of acoustics. The final section raises questions regarding the relatively unexplored topic of sound prognostication methods.

I. FRAMING SOUND AND COSMOLOGY IN THE LITERATURE

Pre-Qin and Han elite societies inquired into sound on multiple levels. Much Western scholarship has focused on the institutionalization of music, performance, and philosophy as aids to the cultivation and moral guidance of the ruler and society.[9] Scholars have also inquired into the theoretical and technical components of early Chinese musical systems and their relation to politics and cosmology. John Major, for example, analyzed two models that related the twelve tuning standards to the months of the calendar from the "Celestial Patterns" ("Tianwen" 天文) chapter in the *Huainanzi* 淮南子 (139 B.C.E.). Lothar von Falkenhausen's reconstruction of early Chinese musical systems and their components, especially in the case of the invaluable bell and chime sets excavated from the tomb of Marquess Yi of Zeng (*Zeng Hou Yi* 曾侯乙), has significantly enhanced our understanding of the complexity and diversity of early Chinese musical culture. Robert Bagley's analysis of the Zeng musical system and his theory regarding the development of bells as musical instruments has also greatly impacted the field. The contribution Jing Fang made to the realm of early acoustics has not gone unnoticed either. Ernest McClain and Ming Shui Hung have analyzed the mathematical calculation of the sixty tuning standards in great detail. They determined how Jing Fang handled the numbers in his list, including his choice to ignore certain remainders and balance this by correcting other results, as well as translating his string and pipe measurements. Fritz Kuttner discussed the sixty tuning standards in relation to the formulation of equal temperament in the sixteenth century by Zhu Zaiyu 朱載堉 (1536–1611). Research on this topic in Chinese is richer, including works by Wang Zichu 王子初 and, most notably, the musicologist Chen Yingshi 陳應時, who wrote extensively on Jing Fang and early Chinese musical temperament. Many of these contributions situate Jing Fang squarely within a historical context relating to acoustics, while leaving aside the question of how he thought about sound. This essay argues that Jing Fang's acoustical accomplishments are not independent of the context in which they are presented in his text. Dismissing their context is like illuminating only one corner of a large painting. In this sense, the essay complements the earlier thinking of the Japanese scholar Nobuo Horiike 堀池信夫.[10] Jing

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[9] A far-from-exhaustive list includes Kenneth J. DeWoskin, *A Song for One or Two: Music and the Concept of Art in Early China* (Ann Arbor: Univ. Michigan Center for Chinese Studies, 1982); Scott Cook, "Xun Zi on Ritual and Music," *Monumenta Serica*, 1997, *45*:1–38; Cook, "The 'Lüshi Chunqiu' and the Resolution of Philosophical Dissonance," *Harvard Journal of Asiatic Studies*, 2002, *62*:307–345 (Cook's essays are two in a series of articles on musical thought); Roel Sterckx, "Transforming the Beasts: Animals and Music in Early China," *T'oung Pao*, 2000, *86*(1/3):1–46; Erica Brindley, *Music, Cosmology, and the Politics of Harmony in Early China* (Albany: State Univ. New York Press, 2012); and Ingrid M. Furniss, *Music in Ancient China: An Archaeological and Art Historical Study of Strings, Winds, and Drums during the Eastern Zhou and Han Periods* (Amherst, N.Y.: Cambria, 2008). On the institutionalization of music in the Bureau of Music (*yue fu* 樂府) see Michael Loewe, "The Office of Music, c. 114 to 7 B.C.," *Bulletin of the School of Oriental and African Studies*, 1973, *36*:340–351, rpt. in Loewe, *Crisis and Conflict in Han China, 104 B.C. to A.D. 9* (London: Allen & Unwin, 1974), pp. 193–210; Helmut Wilhelm, "The Bureau of Music of Western Han," in *Society and History: Essays in Honor of Karl August Wittfogel*, ed. Gary L. Ulmen (The Hague: Mouton, 1978), pp. 123–135; and Kamatani Takeshi, "The Early Bureau of Music," *Acta Asiatica*, 1996, *70*:37–53.

[10] Major, "Celestial Cycles and Mathematical Harmonics in the 'Huainanzi'" (cit. n. 5); von Falkenhausen, *Suspended Music* (cit. n. 3); Robert Bagley, "Percussion," in *Music in the Age of Confucius*, ed. Jenny F. So (Washington, D.C.: Freer Gallery of Art and Arthur Sackler Gallery, Smithsonian Institution, 2000), pp. 35–64; Ernest G. McClain and Ming Shui Hung, "Chinese Cyclic Tunings in Late Antiquity," *Ethnomusicology*, 1979, *23*:205–224, esp. p. 207; Fritz Kuttner, "Prince Chu Tsai-Yü's Life

Fang's endeavor was situated deep within the cosmological model of the "Han Paradigm."[11] Several features of the text indicate this, among them the supremacy of patterns and numbers (*shu* 數), a preoccupation with temporal cycles, and the notion that *qi* fluctuates throughout the year and resonates with sound in the cosmos—all notions that the "Han Paradigm" employed.

Jing Fang clearly states, at least according to the information in the treatise, that his approach to sound was meant to facilitate weather prognostication. His biographical section in the *History of the Han* (*Han shu* 漢書) supports this claim.[12] He distributed his sixty tuning standards in accordance with the solar year in a way that leaves little doubt that they fulfilled uses beyond temperament. The use of terminology such as "*yin-yang*" and "*qi*" in relation to sound in his theory should not be dismissed as lip service but must be understood as part of the fundamental non-negotiables of his time. These presented sound as a detectable manifestation of *qi* attuned to the natural order of the cosmos and asserted that sound, like all things, had by then been divided into the two mutually generating categories of *yin* and *yang*—the dyadic principle that governed the cosmos and everything in it.[13] Moreover, in early imperial China, the accurate calculation and distribution of sound throughout the year was as critical a matter as accurately calculating and granting the seasons (*shou shi* 授時) via the astronomical system. According to this idea, measuring sound, like computing a calendar, could result in natural disasters and social unrest if miscalculated.[14]

II. SITUATING JING FANG

The presence of Jing Fang as one of the few figures in the Han histories characterized by a strong and detailed association between sound, prognostication, and cosmology makes him especially significant for our study of sound conceptualization in early imperial China. His approach to acoustics shows that sound manipulation had practical uses that went beyond music or relating the twelve pitches to the months of the year as part of a cosmic model.

Jing Fang 京房 (78–37 B.C.E.) was a specialist in the art of interpreting the sixty-four hexagrams in the *Classic of Changes* (*Yijing* 易經) and demonstrated outstanding skill in the mathematical calculations of the music system. His style name (*zi* 字) was Junming 君明, and his place of origin was Dunqiu 頓丘 County, then located on the western border of Dong Commandery

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and Work: A Re-evaluation of His Contribution to Equal Temperament Theory," *Ethnomusicology*, 1975, *19*:163–206, esp. p. 172; Wang Zichu 王子初, "Jing Fang he tade liushi lü" 京房和他的六十律, *ZhongguoYinyue* 中國音樂, 1984, 3:24–26; Chen Yingshi 陳應時, "Wei 'Jing Fang Liushi Lü' shen bian" 為 "京房六十律" 申辯 (cit. n. 8); and Horiike Nobuo 堀池信夫, "Kyō Bō no Rokujūritsu: Ryō Kan Keigaku no Tenkai to Ritsurekigaku" 京房の六十律—両漢経学の展開と律暦学, *Nihon Chugoku Gakkai Ho*, 1979, *31*:74–89.

[11] On the "Han Paradigm" see Christopher Cullen, *Heavenly Numbers* (Oxford: Oxford Univ. Press, 2017), p. 19.

[12] *Han shu* 漢書 (Beijing: Zhonghua, 1964), *juan* 75, pp. 3160–3167.

[13] Earlier examples of musical systems, such as the amalgamated system inscribed on the fifth-century B.C.E. bell and chime-stone sets unearthed in the tomb of the Marquess Yi of the state of Zeng, do not directly present such a division, and that is only one of many aspects in which they differ from the tuning standard sets we find in later sources. Two early examples of a clear set of six core tuning standards and six secondary ones exist. The first appears in the "Discourses of Zhou" ("Zhouyu" 周語) section of the *Discourses of the States* (*Guoyu* 國語), a text edited sometime after ca. 425 B.C.E., with some sections dating to the Warring States, where the six core tuning standards are termed *lü* 律 and a secondary set of six tuning standards is termed *jian* 間, meaning "interstitial." They are not related to *yin* and *yang* and are not treated as a set of twelve. The second appears in the "Spring Offices, [Domain of the] Patriarch of Ancestral Affairs" ("Chun Guan Zong Bo" 春官宗伯) chapter in the *Rites of Zhou* (*Zhouli* 周禮), a text produced no later than the second century B.C.E., where the six core tuning standards are termed *lü* 律 and are categorized as *yang*, while the secondary set is termed *tong* 同, meaning "to accompany," and is categorized as *yin*. The author(s) of the text defined them here as a set of twelve tuning standards (*shier lü*). This conceptual change, which fused these two sets, with a remaining implicit categorial subdivision into *lü* 律 and *lü* 呂, occurred sometime before the mid-third century B.C.E. See von Falkenhausen, *Suspended Music* (cit. n. 3), p. 308. For his discussion on the differences between the Zeng bell system and systems in received sources see pp. 289–290; on tuning standards in the *Zhouli* and *Guoyu* see pp. 296–299.

[14] Cullen, *Heavenly Numbers* (cit. n. 11), p. 30. See also Daniel Patrick Morgan, *Astral Science in Early China: Observation, Sagehood, and Society* (Cambridge: Cambridge Univ. Press, 2017), pp. 95–97.

(*Dong jun* 東郡), in present-day Henan. He served the Western Han court during the reign of Emperor Yuan 元 (r. 48–33 B.C.E.). With regard to musical temperament, Jing is known for expanding the traditional chromatic scale from its basic twelve tuning standards to sixty, within the span of a single octave, and distributing them across the days, rather than the months, of the year. His model allows us a new way of thinking about the link between sound and calendrics. Jing Fang also wrote about a large-scale zither-like tuning instrument he had designed and called the *zhun* 準, or equalizer, meant to facilitate the measurement and marking of these sixty pitches by converting them from numbers into audible sound. The *zhun* is said to have been lost soon after his time, and, sadly, the surviving description is only a general one.

The "Treatise on Tuning Standards and Mathematical Astronomy" presents these mathematical endeavors in acoustics in the context of an experiment called "Observing *qi*" (*hou qi* 候氣), which Jing Fang designed to predict the transition of cosmic *qi* throughout the year by detecting its correlation with sound. The experiment called for an isolated room (to avoid external effects), in which pitch pipes filled with ashes were planted in the earth. When *qi* corresponded with the correct pitch measure the ashes would scatter. For Jing Fang, this was analogous to measuring the sun's shadow using the gnomon:

> This birth of the five tones out of the *yin* and the *yang*, their division into the twelve *lü* and transformation into sixty, all are ways to record [the transition of *qi*] and model the classification of things. Heaven is modeled by the shadow, and Earth is modeled by the echo.[15] These are the pitch pipes. When the *yin* and the *yang* are harmonized, the shadow reaches [a certain length]. When the pitch pipes and the *qi* correspond, the ashes fly off.[16]

The validity of such an experiment today is of no interest here. The notion of the correlation between sound and *qi* was not unique to Jing Fang, and so the experiment provided an apparatus to exemplify it.[17]

These advances in tuning joined his achievements as a prognosticator of natural disasters who advised the Han emperor. The treatise also details his acoustic model and ties it directly to the yearly cycle and indirectly to a method of prognostication. It seems likely that Jing Fang utilized the already existing concept of sound as a measuring technology for *qi* in at least one of his prognostication schemes. These accomplishments begin to situate Jing Fang as a key figure in expanding the scope of utilizing sound in the first century B.C.E.

A final piece in this mosaic is Jing Fang's decision to change his surname from Li 李 to Jing 京, in accordance with a technique he used called "Inferring from the tuning standards" (*tui lü* 推律). In view of his background as an expert in the tuning system and in prognostication and interpretation, the rationale behind this process probably had to do with an association between his name, *qi*, and its manifestation through sound or its numerical representation. His biography in the *History*

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[15] The distinction between shadow and echo reflects the combination of sight and sound, which also tie into the sagacious sensorial qualities of "keenness of hearing and sight" (*congming* 聰明) often referred to in early Chinese textual sources.

[16] "Treatise on Tuning Standards and Mathematical Astronomy" 律歷志 (cit. n. 1). Unless otherwise indicated, all translations in this essay are my own (most of them from the "Treatise on Tuning Standards and Mathematical Astronomy" as it appears in the 1974 Zhonghua edition of the *History of the Later Han* [cit. n. 1]; other sources are indicated in the pertinent footnotes).

[17] Derk Bodde translated and analyzed the "Observing *qi*" experiment and its subsequent appearances in Chinese history. I will not repeat its details in full here. See Derk Bodde, "The Chinese Cosmic Magic Known as Watching for the Ethers" (1959), in Bodde, *Essays on Chinese Civilization*, ed. Charles Le Blanc and Dorothy Borei (Princeton, N.J.: Princeton Univ. Press, 1981), pp. 351–372. For a recent translation of the "Observing *qi*" section see Lisa Raphals, *Divination and Prediction in Early China and Ancient Greece* (Cambridge: Cambridge Univ. Press, 2013), pp. 170–172. It bears mention that different methods of observing *qi* also existed and were used in prognostication. See A. F. P. Hulsewé, "Watching the Vapours: An Ancient Chinese Technique of Prognostication," *Nachrichten der Gesellschaft für Natur- und Völkerkunde Ostasiens*, 1979, *125*:40–51.

of the Han does not detail the technique or the ideas behind it, but there are additional references to the verb "infer" (*tui*) in relation to the numerical value of tuning standards. This name change is intriguing because it points to another link between tuning standards and divination.[18] Was there a regular divination practice involving tuning standards and name changes, or is it mentioned precisely because it was uncommon? Little about Jing Fang seems to suggest common practices, and much of his uniqueness seems to lie in his ability to incorporate acoustics within larger divinatory and cosmological theories.[19]

Jing Fang was also part of the political milieu at the court of Emperor Yuan. He functioned in several official positions and in 45 B.C.E. was appointed Gentleman of the Palace (*Langzhong* 郎中), an honorary post at the imperial court. Alongside this, or as part of his expertise in interpreting the *Changes*, he was a prognosticator with a good record of realized forecasts, as well as a political figure with an agenda, known for strongly advocating meritocratic promotion. His political activities, which directly targeted powerful officials such as the eunuch Shi Xian 石顯 and Wu Lü 五鹿, destabilized his position at court, eventually leading to an accusation of slandering the emperor and his subsequent execution at the age of forty-one. His biography recounts several occasions when he counseled Emperor Yuan on identifying disloyal officials. Because of his excellent record in prognosticating natural disasters (*zaiyi* 災異), the emperor first consulted Jing regarding an uprising of the Western Qiang 西羌 people, which was followed by a solar eclipse. Later, Jing Fang approached the emperor, warning him once more about the quality of his officials. In both cases he used his stature as a prognosticator to advance his political concerns. He argued that the calamities and strange phenomena afflicting the realm were a result of the current political system, which preferred flatterers to men of talent. He was permitted to construct a method to examine the achievements of government officials and choose disciples who would carry it out after him. In his warnings, Jing Fang urged the emperor to heed strange natural phenomena and the state of society as signs that Shi Xian and his ilk were leading the dynasty to its demise.[20]

The author of Jing Fang's biography paints a picture in which it was this conflict with Shi Xian and Wu Lü that cost Jing his life. He was removed from the capital and appointed Governor of Wei Commandery (*Weidu taishou* 魏都太守). Increasingly fearful for his life, his admonitions against the deception and excess power of the high officials naturally grew even stronger. In several sealed memorials to the emperor, he added detailed predictions of severe weather and noted cosmological portents that could, unsurprisingly, only be lifted with his return to court. The last section of his biography tells that Shi Xian, afraid for his position, waited for Jing Fang's removal from the capital before charging him, together with Zhang Bo 張博, with colluding and slandering the government and the emperor. These accusations ended in the execution of both.

The background Jing Fang had in tuning computations seems to relate more strongly to his interest in prognostication than to ritual music performance.[21] His biography in the *History of the Han* and his description in the treatise tersely describe him as "fond of tuning computations (鍾律 bell standards) and proficient in tones and notes," or as having knowledge in the "Tones of the

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[18] A. F. P. Hulsewé suggested that apart from the possible cosmological benefits of this name change, Jing Fang perhaps "wanted to share the fame" of the other well-known interpreter of the *Changes* who bore the same name. While this is a plausible argument, taking the name of another expert hints at cloaking one's identity and could also have proven dangerous. See A. F. P. Hulsewé, "The Two Early Han 'I Ching' Specialists Called Ching Fang 京房," *T'oung Pao*, 1986, 72(1):161–162.

[19] Clues regarding the link between fixing family or clan names and blowing the pitch pipes to infer cosmic *qi* appear in the *Discussions in the White Tiger Hall* (*Bai Hu Tong* 白虎通) and in Wei Zhao's (韋昭 204–273 C.E.) third-century commentary on the "Discourses of Zhou" chapter in *Discourses of the States*.

[20] An abridged version of this account also appears in the chapter "Advice and Admonitions" ("Gui zhen" 規箴) in the fifth-century compilation *A New Account of the Tales of the World* (*Shi shuo xin yu* 世說新語).

[21] I deliberately refer to the tuning requirements of a ritual music setting, because ordinary popular musical performances probably did not adhere to strict requirements, and the act of tuning was not tied to cosmological implications in the latter setting.

five notes, and the numerical relations of the six tuning standards."[22] He is not presented as a music master. There is no mention of any musical performance, and the only instrument mentioned is the tuner designed for fixing pitches. His involvement with wider aspects of sound manipulation, quite separate from the sphere of music, is precisely what makes his contribution to what we would today term acoustics so unique.

III. THE CONCEPT OF SOUND IN THE "TREATISE ON TUNING STANDARDS AND MATHEMATICAL ASTRONOMY" IN THE *HISTORY OF THE LATER HAN*

On the one hand, Jing Fang was a known figure in the histories of the Han, especially when it came to his prognostication methods. On the other hand, with regard to acoustics, he remains somewhat elusive. There is no extant complete textual source by him on this subject, and so fully understanding his theory as a whole is a challenging task. The "Treatise on Tuning Standards and Mathematical Astronomy" was originally compiled by Cai Yong 蔡邕 (132–192 C.E.), who wrote a section on tuning standards, and Liu Hong 劉洪 (fl. 180 C.E.), who wrote a section on mathematical astronomy. The Jin dynasty historian Sima Biao 司馬彪 (ca. 240–ca. 306 C.E.) used this treatise in his own "Treatise on Tuning Standards and Mathematical Astronomy" in a work titled *Continued History of the Han* (*Xu Han shu* 續漢書). Finally, Sima Biao's treatise was incorporated into the *History of the Later Han*, because Fan Ye 范曄 (398–446 C.E.), the compiler of the rest of this history, was executed before he could complete the treatises (*zhi* 志) section of his work.[23] The *History of the Jin* (*Jin shu* 晉書) tells us that Cai Yong pursued the calculations of the sixty tuning standards, which had been lost by his time. It also tells us that contemporary acousticians (*yinjia* 音家) used this text but that the sixty tuning standards were not applied to musical performance (*liushi lü zhe wu shi yu yue* 六十律者無施於樂).[24] Jing Fang's theory of the sixty tuning standards somehow survived and made its way to us, filtered by at least three different compilers.

In the treatise, Cai Yong designates Jing Fang as the notable expert at the time of Emperor Yuan. He is introduced following a succinct chronological list of specialists in the music system during the reigns of the Western Han emperors. The list includes only three figures: Zhang Cang 張蒼, who served during the reign of the founding Emperor Gaozu; Emperor Wu 武帝 (r. 141–87 B.C.E.); and Liu Xin 劉歆 (53 B.C.E.–23 C.E.), who compiled his memorial on tuning standards and mathematical astronomy during the reign of the child Emperor Ping 平帝 (r. 1 B.C.E.–5 C.E.). This grand and important undertaking included over a hundred experts in music temperament. The text was later edited by Ban Gu 班固 (32–92 C.E.) and inserted as a treatise in the *History of the Han*.[25] The reader is then presented with an account of an event in which Emperor Yuan sends the Senior Tutor of the Heir Apparent Wei Xuancheng 韋玄成 (d. 36 B.C.E.) and the Advisory Counsellor Zhang 章 to ask Jing Fang questions at the Bureau of Music. The exact nature of these questions is unknown, but we hear Jing's voice for the first time in the treatise when Cai Yong quotes his reply. But then Cai Yong discloses that he proceeds with his own summary of Jing Fang's ideas, and so we lose Jing yet again.[26] The decision to dedicate the treatise

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[22] *Han shu* (cit. n. 12), *juan* 75, p. 3160. The six tuning standards (*liu lü* 六律) refer to the two sets of six *lü* each and to the scales potentially constructed from each of them using the five-note pattern. See the elaboration below.

[23] B. J. Mansvelt Beck, *The Treatises of Later Han: Their Author, Sources, Contents, and Place in Chinese Historiography* (Leiden: Brill, 1990), pp. 61, 1.

[24] *Jin shu* 晉書 (Beijing: Zhonghua shuju, 1974), p. 483.

[25] *Han shu* (cit. n. 12), *juan* 21, p. 955. Ban Gu credits Liu Xin's endeavor to Wang Mang 王莽, who usurped the throne in 9 C.E. Cai Yong purposely ignored this, it seems. See also Michael Loewe, "Ban Gu: Copyist, Creator, and Critic," *Bull. School Oriental African Stud.*, 2015, 78:342–343.

[26] Cai Yong shared aspects of Jing Fang's expertise in sound, astronomy, and prognostication. He probably also sympathized with Jing's revulsion at powerful eunuchs. Nevertheless, because Cai Yong expressed some views that do not accord with earlier views, his approach to Jing's material could diverge somewhat from its original intention.

to Jing Fang's theory seems to be an act of atonement, since Liu Xin did not so much as mention him in his treatise: "Fang spoke of the tuning standards in more detail than [Liu] Xin's memorial. His skill was applied by the Astrological officials. The Department of Observation used it. Most writings do not record it in detail. Therefore, [I] summarize his main ideas, to supplement the earlier records."

The tuning section of the treatise is subtitled "Tempering the *zhun* instrument and observing *qi*" (*lü zhun hou qi* 律準候氣). This is already a good indicator of the prime concern Cai Yong had for this section, which turns toward practical aspects of using sound in two ways: tempering a tuning instrument that could serve as a go-to standard for pitch measures related to the calendar cycle and utilizing pitch pipes in order to observe cosmic *qi* at specific moments throughout the year.[27]

All the information within the treatise, including the solution to acoustical problems proposed by Jing Fang, is geared toward the accomplishment of the author's goal: to give as much information as he can about these two processes and to position the discussion within its encompassing framework, that of numbers (*shu* 數). "*Shu*" is often translated as "numbers," but its meaning encompassed various forms of cyclical, repetitive events, especially those of an astrological and astronomical nature, which were interpreted through prognostication, mathematics, and numerology.[28] For this reason, Nathan Sivin translates "*shu*" as "regularities."[29] The acoustic theory of Jing Fang, his practice of observing *qi*, his prognostication according to natural phenomena, and his interpretation of the *Changes* are all intertwined within the concept of *shu*. The treatise opens with an excerpt from the *Zuo Tradition* (*Zuozhuan* 左傳),[30] which frames the entire discussion that follows in the context of *shu*: "What the ancients said about numbers was: 'phenomena are born, and then have counterparts.[31] Once they have counterparts, they sprout. Once they sprout, they have number.' That being so, when Heaven and earth had first taken shape, and men and things were already manifest, the calculation of numbers began."

Readers of the time would immediately have recognized this as a reference to the pitiful defeat and capture of the prince of Jin State and his men by the Qin army, after he attacked Qin while repeatedly ignoring counsel from his advisors and prognostications by his diviners. Han Jian defined *shu* as part of his retort to Lord Hui, in which he protests: "Could the former ruler's lapses in virtue have been reckoned in number? What good would it have done had he heeded this prognostication of the scribe Su?"[32] This retort uses *shu* to suggest that when a ruler's character is not virtuous, even the best prognostication can prove useless.

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[27] We cannot know whether this is Sima Biao's choice as compiler, Cai Yong's choice as treatise author, or Jing Fang's choice in whatever sources Cai Yong had at his disposal. What we can say is that this choice frames the text before us.

[28] Peng Yoke Ho, *Chinese Mathematical Astrology: Reaching out to the Stars* (London: Routledge Curzon, 2003), p. 6.

[29] See Sivin, "Cosmos and Computation in Early Chinese Mathematical Astronomy" (cit. n. 7), p. 62; and Nathan Sivin, *Granting the Seasons: The Chinese Astronomical Reform of 1280, with a Study of Its Many Dimensions and a Translation of Its Records* (New York: Springer, 2009), p. 372.

[30] The *Zuo Tradition* (fifth century B.C.E.) is one of the three commentaries to the *Annals Classic* (*Chunqiu* 春秋) and one of the most important surviving pre-Qin texts. The use of the *Zuo Tradition* in prognostication during the Western Han is beyond the scope of this essay.

[31] Edward H. Schafer translates "*xiang*" 象 as "counterpart," which is especially fitting in this case—despite his reliance on medieval rather than early texts—owing to its connotation of a "universe conceived as a resonant system of celestial and terrestrial correspondences." Edward H. Shafer, *Pacing the Void: T'ang Approaches to the Stars* (Warren, Conn.: Floating World, 2005), p. 5.

[32] I use the translation in Stephen W. Durrant, Wai-yee Li, and David Schaberg, *Zuo Tradition = Zuozhuan: Commentary on the "Spring and Autumn Annals"* (Seattle: Univ. Washington Press, 2016), p. 327.

Next, Cai Yong solidifies tuning standards as a measuring unit for sound. This had as much to do with measuring and calibrating high and low pitches as it did with the expression of sound in numerical values, which conveyed additional numerological significance:

> The records state that Da Rao created the ten heavenly stems and twelve earthly branches, and Li Shou created numbers.[33] When both (the sexagenary cycle and numbers) were established, they were applied to the gnomon and the management of the myriad affairs. One, ten, hundred, thousand, ten thousand are used in the same way. Tuning standards, degrees, weights and measures, and the calendar are among their other uses.
>
> Bodies are longer or shorter; they are measured in degrees. Objects can be many or few; they are encompassed by measures. Masses can be light or heavy; they are weighed by means of the steelyard. Pitch can be clear or turbid;[34] it is tuned[35] by the tuning standards. The motion of the Sun, Moon, and planets is recorded using calendric constants. After doing this, one can sum up the hidden realities and the subtle transformations.

Thinking of numbers and measurements as instruments to detect concealed, minute transient processes in the cosmos is crucial to understanding how Jing Fang thought about sound and how he was able to use the manipulation of sound for prognostication. Ignoring this would be tantamount to discussing Pythagorean tuning theory without addressing the concept of numbers in the Pythagorean construction of reality.

IV. THE COMMA AND THE SIXTY TUNING STANDARDS

Categorizing Sound

Jing Fang extended the mathematical calculations that generated the numbers of the basic twelve tuning standards in the Chinese musical system. He generated forty-eight additional tuning standards, each with its own name, number, and length of pipe and string, which are presented one after the other in a list, in ascending order. As noted above, McClain and Hung focused on the hard numbers for the sixty tuning standards. When attempting to repeat these calculations, it becomes clear that this was not a simple case of extending the existing twelve tuning standards to sixty. Instead, it was a matter of scrupulous calculations, manual corrections, and selective choices, aimed at solving the problem of circling back as close as possible to the initial number the calculations began at (more on this below) and at providing the numbers for the tuning standards (*lü* 律) section in integers only.[36] Nevertheless, because McClain and Hung focused on the astounding accomplishment in acoustics, they do not pay much attention to the context in which these numbers are presented and to the additional information in the very same list, such as the distribution of

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[33] Da Rao is the name of a mythological character said to be a minister of the Yellow Emperor. He is said to have created the sixty-year cycle. Li Shou, the mythological scribe (*shiguan* 史官) for the Yellow Emperor, is said to have created the counting of numbers.

[34] I have reservations about the translation of "*qing zhuo*" 清濁 as simply "high" and "low," and until a better definition is available I prefer to leave it as "clear" and "turbid" in the translation of early texts.

[35] I translate "*xie*" 協 here as "to tune," despite its more accurate translation as "to harmonize" (usually with a cosmological connotation), in order to avoid readers' potential misinterpretation of the word "harmonize" as a musical term.

[36] When reconstructing the calculation of the numbers for the tuning standards (not including their transcriptions into pipe lengths and string lengths), it becomes clear that as a rule, when dividing into three, Jing Fang intentionally ignored quotient remainders of 1/3 or 2/3, adding and decreasing only integers. Because of this, he also chose to correct the table manually in specific locations, probably to balance out the results. It is not possible simply to extend the *sanfen sunyi* method to sixty pitches and reach his numbers. I have done this work myself, but it is well explained in detail in McClain and Hung, "Chinese Cyclic Tunings in Late Antiquity" (cit. n. 10).

these tuning standards across the days of the year, the purpose behind this complex undertaking, or the concept of sound at the time of the Western Han.

In Chinese texts from around the third century B.C.E. onward tuning standards were viewed in accordance with the cosmological *yin-yang* dyad principle. Based on the complementary idea of mutual generation in which *yang* produced *yin* and vice versa, pitches are assumed to generate one another in the same way. The twelve tuning standards subdivided into two sets: six tuning standards categorized as *yang* called *lü* 律, and six others categorized as *yin* called *lü* 呂. A *yang* tuning standard generated a *yin* tuning standard, which further generated a *yang* tuning standard. For example: Yellow Bell, a *yang* tuning standard from the *lü* 律 category, mathematically generated Forest Bell (*Linzhong*), a *yin* tuning standard from the *lü* 呂 category. Forest Bell, in turn, generated Great Budding (*Taicou*), a *yang* tuning standard from the *lü* 律 category again. In this way, the first twelve tuning standards, sometimes referred to as *lülü* 律呂, fluctuate back and forth in a *yin-yang* cycle of mutual generation.

Jing Fang introduces the principles of the "Addition and Subtraction by a Third" algorithm, with its underlying principle of *yin-yang* mutual generation. He then humbly explains that he simply extended the calculations beyond the basic twelve tuning standards, adding and subtracting one third, until he reached sixty pitches.

> [When] up produces down, all threes produce twos. [When] down produces up, then all threes produce fours. When the *yang* goes downward it produces *yin*, and when *yin* goes upward it produces *yang*. When the cycle ends at Median Regulator (*Zhonglü* 中呂),[37] the twelve tuning standards are thus complete! When Median Regulator goes upward it produces [the tuning standard] *Zhishi*. When *Zhishi* goes downward it produces *Qumie* 去滅. Upward and downward they produce each other, culminating at *Nanshi* 南事. The sixty tuning standards are thus complete!

Temperament: Calculating Sound

Several texts dating from the mid–Warring States (403–221 B.C.E.) to the early imperial period discuss the basic requirements of the same music system Jing Fang used. The most valuable of these is the algorithm "Addition and Subtraction by a Third" mentioned above, which produced the numerical values of tuning standards. The earliest received source for this algorithm is probably the "Di Yuan" 地員 chapter of the *Guanzi*, although its dating is contested.[38] However, the relation between tuning standards and the calendar is also evident in excavated preimperial sources such as Scroll B of the Fangmatan daybooks (*rishu* 日書), dated to the mid- to late third century B.C.E., which also link the two with prognostication using the numerical value of tuning standards to predict auspicious and inauspicious days.[39]

The method for generating the twelve tuning standards uses intermittently changing ratios of 2:3 and 4:3. The practitioner chooses a number to represent the initial tuning standard. He then divides it by three and the quotient is added to or subtracted from the initial number. The result represents the next tuning standard. The ratio between the initial number and the following one can be translated into a musical interval. Reducing a given number by 1/3 results in 2/3

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[37] *Zhonglü* is the name of the twelfth generated tuning standard.

[38] Additional later sources include the *Annals of Master Lü*, *Records of the Grand Scribe* (*Shiji* 史記), *The Huainanzi*, and the "Treatise on Tuning Standards and Mathematical Astronomy" in the *History of the Han*.

[39] The excavated scrolls of Fangmatan (dated not earlier than 239 B.C.E.) were unearthed in 1986. See He Shuangquan 何双全, "Tianshui Fangmatan Qin jian zongshu" 天水放马滩秦简综述, *Wenwu*, 1989, 2:23–31, 102–103.

of its original value. The ratio between them is 3:2—the ratio of a musical interval known today as a fifth. Adding 1/3 of its value produces 4/3 of its value. The ratio between these numbers will be 4:3—the ratio of the interval known today as a fourth. Thus, the numerical value (and length) of the second pitch is 2/3 of the first pitch. That of the third pitch is 4/3 of the second. That of the fourth pitch is 2/3 of the third, and so on up to twelve pitches. It would seem that this is what Jing Fang refers to when he writes: "all threes produce twos, . . . all threes produce fours."[40] In the terms of modern Western music theory, a cycle of rising fifths and falling fourths generated an untempered, chromatic scale spanning a single octave.[41] In this tuning system, the thirteenth tuning standard, meant to begin the cycle again, disconcertingly did not have the same number as Yellow Bell. This is the problem Jing Fang tried to solve.

This cyclical system can be applied to fixed lengths of strings or pipes, but the ratios are only a mathematical expression of intervals.[42] In this way, *yang* pitches produce *yin* pitches and vice versa, generating the numbers and ratios of the first twelve tuning standards, ending on Median Regulator. Jing Fang continued the cycle beyond the twelfth pitch, to produce sixty pitches in the span of a single octave, concluding in a pitch called *Nanshi* (Southern Affairs).

The pitch generation scheme assigned the number 81 to the first tuning standard, Yellow Bell. The number 81 was significant in early Chinese cosmological numerology and was used in various areas. It was, for example, the largest number in the multiplication tables of the *Zhou bi suan jing*.[43] By the Han era, one of the major changes under the astronomical reform of Emperor Wu was the semantic change of the denominator or divisor (*rifa*日法) in measuring the mean synodic month from 940 days to 81. Christopher Cullen explains the underlying reason behind this number: "This is the fourth power of 3, the number representing the principle of *yang* 陽,which pertains to the heavens as opposed to earth, to male as opposed to female, and to light as opposed to darkness." Changing the divisor to 81 in this new Grand Inception Calendar (*Tai chu li* 太初曆), he argues, was an extension of the numerological associations with sound:

> The motive was not empirical, but was rather the wish to introduce into the system the cosmologically significant number 81, which . . . was not only the fourth power of the quintessentially yang number 3, but also represented the volume of the pitch pipe sounding the fundamental note of the scale, "Yellow Bell" *Huangzhong* 黃鐘. Since music was held to embody the same cosmic order as was exemplified in the heavens, it would have been highly satisfying to see the motions of sun and moon governed by this number.[44]

However, generating tuning standards starting at 81 was problematic because it allowed only the first five generated tuning standards to be expressed in integers:

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[40] If we use an image of a pipe (or string) to visualize this, the length of the pipe is divided into three, and 2/3 of its length is the measurement for the following pipe, which produces a pitch higher than the former pipe by an interval of a fifth. In the same manner, the length of the next pipe is divided into three, and 1/3 is added to it. The following pipe will measure 4/3 of the former and produce a pitch lower by an interval of a fourth. Thus, three (thirds) produce two (thirds), and three (thirds) produce four (thirds).

[41] In order to keep within an octave, the up and down cycle must break once by calculating a fourth downward twice in row. This happens between the sixth and seventh and between the seventh and eighth tuning standards. Additionally, to cycle back to the fundamental tuning standard, we must also calculate the thirteenth pitch as a fourth downward instead of a fifth upward.

[42] The same method does not apply to bells, since their construction is much more complex.

[43] Christopher Cullen, *Astronomy and Mathematics in Ancient China: The "Zhou Bi Suan Jing"* (Cambridge: Cambridge Univ. Press, 1996), p. 83. The composition date for the *Zhou bi suan jing* is unclear, owing to its composite nature and the "extremely varied degrees of interrelation" between its sections (p. 138).

[44] Christopher Cullen, *The Foundations of Celestial Reckoning: Three Ancient Chinese Astronomical Systems* (New York: Routledge, 2016), pp. 24–25, 25, 34.

1st		2nd		3rd		4th	
Yellow Bell *Huangzhong* 黃鍾		Forest Bell *Linzhong* 林鍾		Great Budding *Taicou* 太蔟		Southern Regulator *Nanlü* 南呂	
81	−1/3	**54**	+1/3	**72**	−1/3	**48**	+1/3
C		G		D		A	

5th		6th
Maiden Cleanliness *Guxian* 姑洗		Responsive Bell *Yingzhong* 應鍾
64	−1/3	**42.667**
E		B

Authors were aware of this problem and rounded the number for Responsive Bell upward or downward. The author of the Fangmatan daybooks, for example, used the number 43, while the author of the "Celestial Patterns" chapter in the *Huainanzi* used 42. To avoid fractional remainders, another scheme assigned Yellow Bell the irksome large number 177,147. This alternative number is divisible by 3 eleven times, allowing smoother calculations of the twelve tuning standards.[45] As far as we know, this large number first appears in the Fangmatan daybooks. If we calculate the twelve tuning standards starting from 81, the thirteenth tuning standard produces the number 80. The difference between 81 and 80 is the numerical visualization of a comma.[46] When using large numbers, and without the need to ignore remainders, it is the difference between 177,147 for the first tuning standard and 174,762 for the thirteenth.

Jing Fang justified the way he reached the sixty tuning standards by correlating his method of expanding the twelve tuning standards with the way the eight basic trigrams expand to sixty-four. This argumentation is meant to convince his audience that his use of sound and *qi* is in full accord with the hexagram studies of the *Classic of Changes*, which was his background: "This transformation of the twelve pitches reaching sixty is like the transformation of the eight trigrams reaching sixty-four. Fu Xi wrote the *Classic of Changes* and recorded the beginnings of the *yang qi*. From this came the method of the tuning standards." According to this text, the order of the sixty-four hexagrams in the *Classic of Changes*, beginning with pure *yang*, was the conscious precedent of the mythological Emperor Fu Xi for the transformation of twelve tuning standards into sixty.[47]

[45] Western scholarship has called the number series beginning from 81 "harmonic numbers," although in the Chinese texts they are simply called "numbers" (*shu*). The series beginning with 177,147 was referred to in Chinese texts as *da shu* 大數, translated as "large numbers." See Major, "Celestial Cycles and Mathematical Harmonics in the 'Huainanzi'" (cit. n. 5); and Marc Kalinowski, "Musique et harmonie calendaire à la fin des Royaumes Combattants: Les livres des jours de Fangmatan (239 Avant J.-C.)," *Etudes Chinoises*, 2011, 30:99–138.

[46] The comma 80/81, also known as a syntonic comma (21.506 cents), differs from the comma created by the large numbers (23.46 cents). In the Chinese case it is produced by rounding remainders upward or downward when calculating the twelve tuning standards from 81. When remainders are kept, the comma is the same as the one created by the large numbers (81/79.9097 = 23.46 cents). See the explanation of the comma, below.

[47] In the sixty-four hexagrams, unbroken lines signify *yang* and are represented by the number 9 (3 × 3). Broken lines signify *yin* and are represented by the number 6 (3 × 2). The first hexagram, *Qian* 乾, is composed entirely of *yang* lines. Its counterpart hexagram, *Kun* 坤, is composed entirely of *yin* lines.

Table 1. The seven-note scale starting at Yellow Bell as *gong*, showing its numerical expression using the algorithm. The duration of each tuning standard is a single day of the year.

Tuning Standard	Yellow Bell *Huangzhong* 黃鍾	Great Budding *Taicou* 太蔟	Maiden Cleanliness *Guxian* 姑洗	Luxuriant Guest *Ruibin* 蕤賓	Forest Bell *Linzhong* 林鍾	Southern Regulator *Nanlü* 南呂	Responsive Bell *Yingzhong* 應鍾
Note	*gong* C	*shang* D	*jue* E	*altered zhi* F#	*zhi* G	*yu* A	*altered gong* B
Number	177,147	157,464	139,968	124,416	118,098	104,976	93,312
Days	1	1	1	1	1	1	1

Distributing Sound across the Year: Groups and Subgroups

In addition to the sixty tuning standards, Jing Fang introduced a seven-note scale pattern, which he used to divide the year into seven groups, using each of the seven notes as a marker. The basic scale pattern, prevalent in early Chinese texts, is pentatonic (五聲 *wusheng*), consisting of the notes *gong* 宮, *shang* 商, *jue* 角, *zhi* 徵, and *yu* 羽. To these Jing Fang added two additional notes, which he treats as alterations (*bian* 變) of basic notes: altered *gong* 變宮 and altered *zhi* 變徵. In ascending order, the scale becomes (1) *gong*, (2) *shang*, (3) *jue*, (altered 4) altered *zhi*, (4) *zhi*, (5) *yu*, and (altered 1) altered *gong* (C–D–E–F#–G–A–B).[48]

Jing Fang does not introduce the scale solely as an acoustic product but links it with the winter solstice and argues that this is how one begins to identify sound and *qi* correspondence. His emphasis highlights the seven-note scale as one of the most important aspects of his sixty tuning standards (see Table 1):

> He established as the sound corresponding to the winter solstice the Yellow Bell [pitch] as the *gong* note, [then] Great Budding as *shang*, Maiden Cleanliness as *jue*, Forest Bell as *zhi*, Southern Regulator as *yu*, Responsive Bell as the altered *gong*, and Luxuriant Guest as the altered *zhi*. This is the beginning of sound-*qi* and is the correct ordering of the five tones.

Tuning standards and mathematical astronomy (*li* 歷) functioned as two overlapping, coexisting systems of cycles. One cycle followed the location of celestial bodies throughout the year, aided by the twenty-eight lodges, while the other traced and measured the *qi* that was also thought to reside there. Han elites did not regard sound-*qi* as a social construct in which man decides on a pitch and scale to associate with this or that month. Yellow Bell and its derived scale were simply the sound at the time of the winter solstice. Calculations, manipulations, and tools simply measured and traced an existing phenomenon. Jing Fang superimposes this seven-note scale onto the calendar and divides the yearly cycle into seven large sections. His overview of the system is general, yet clear once compared with the data in his list (see Figure 1). Each of the seven notes of the Yellow Bell scale shown in Table 1 extends across a single day and acts as a delineator. The remaining fifty-three

[48] The notes of the five-note scale are equivalent to the Western C–D–E–G–A (the choice of C is arbitrary). Using the *sanfen sunyi* algorithm, *gong* (C), generates the tuning standard for *zhi* (G). *Zhi* generates *shang* (D), then *yu* (A) and *jue* (E). The notes "altered *gong*" (B) and "altered *zhi*" (F#) result from generating two additional tuning standards to the scale by calculating a fifth upward from *jue*, followed by a fourth downward.

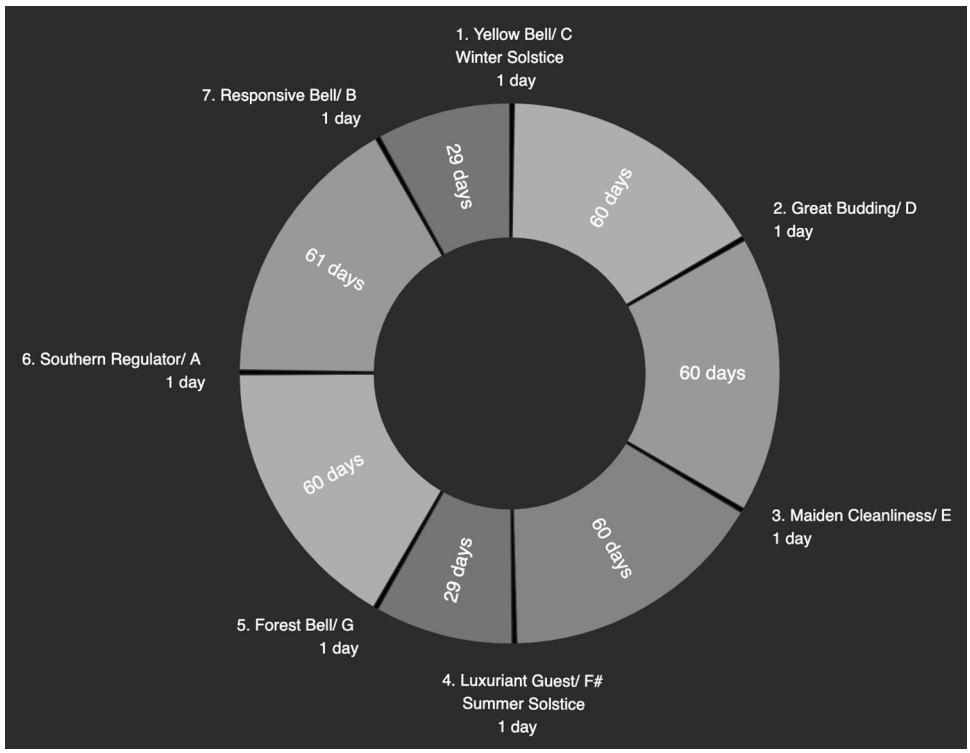

Figure 1. A single-day duration for each of the pitches in the seven-note scale, demarcating seven groups throughout the year. All figures generated by Calvin Yeh and Noa Hegesh.

tuning standards occupy the rest of the cycle within these divisions. At their designated times, each of the sixty tuning standards becomes *gong* and acts as a fundamental pitch:

> Each [of the seven notes] governs a single day. The remainder [of sounds] go through the cycle in sequence. The ones that correspond to [a single] day spontaneously [act as the] *gong* [degree of the scale], and *shang* and *zhi* then take over, according to their categories. When in the "Circulation of Rites" chapter it is said: "The five sounds, six tuning standards, and twelve pipes circle back to *gong*," it refers to this.
>
> In the distribution of the sixty tuning standards across the days, in which Yellow Bell begins on the winter solstice and repeats on the following winter solstice, the prognostication of *yin* and *yang*, cold and heat, wind and rain are created. With it, one sets in order all the tones and studies their highs and lows. As long as it is not the sounds of the grass and trees,[49] there is none that cannot harmonize with it. When in the "Documents of Yu" it is said: "Tuning standards harmonize sounds," it refers to this.

This overview of the model is intertwined with its technical purpose and moral justification. Jing Fang expresses a cosmological model of sound-*qi* based on two textual sources: the technical

[49] This probably refers to non-pitch-specific sounds, such as the rustling of leaves or gusts of wind. Jing Fang is interested in marking discernible sound in the form of pitches. "Sounds of the grass and trees" conveys sound clusters that are difficult to discern and do not belong to his discussion. This comment brings to mind the vivid opening image of the Heavenly Panpipes (*tian lai* 天籟) in the core chapter "On the Equality of Things" ("Qi wu lun" 齊物論) in the *Zhuangzi*.

principle of returning to the origin in the "Circulation of Rites" ("Li yun" 禮運) chapter in the *Record of Rites* (*Liji* 禮記)[50] and the moral function of tuning standards as unifiers and harmonizers of ritual music in the "Canon of Shun" chapter in the *Documents of Yu*.[51] The "Circulation of Rights" presents a larger cosmological and numerological model in which groups revolve and return to a source. It states that as the five phases (*wuxing* 五行), four seasons, and twelve months revolve back to a point of origin in time, the elements of the musical system—the five-note scale, the six tuning standards (shorthand for the set of twelve), and the twelve pitch pipes that resonate them—all return to *gong*. Jing Fang clarifies that by distributing the sixty tuning standards across 366 days of a year he aims to begin with the numerical value of Yellow Bell at the winter solstice and return to it on the next winter solstice, thus perfecting the cycle. This is the cosmological impetus behind the technical solution of "returning back to *gong*." In his comment that his model "gives rise to prognostication of *yin* and *yang*, cold and hot, and wind and rain," Jing Fang clarifies that his model facilitates the prognostication of weather, seasons, and fluctuations of *yin* and *yang*. Figure 1, based on data from Jing Fang's list, visualizes only the function of the seven-note Yellow Bell scale as division markers throughout the year, each pitch of the scale lasting a single day.

The distribution of days is detailed as part of the long list of tuning standards, presented one by one in ascending order. The information for each tuning standard contains its name and large number; the name of the following generated tuning standard; the *gong*, *shang*, and *zhi* degrees of its scale; duration in days; and conversion into pitch-pipe measurements and their augmentation into string length on the *zhun* tuner, the instrument Jing Fang constructed. The entries for the first two tuning standards are for Yellow Bell and *Seyu*. They immediately introduce the reduction of the comma, evident even without calculation, owing to the proximity of their numerical values:

> Yellow Bell. 177,147. Downward it produces Forest Bell. Yellow Bell is *gong*, Great Budding is *shang*, Forest Bell is *zhi*. One day. Pitch pipe: 9 *cun*. *Zhun*: 9 *Chi*.
>
> *Seyu*. 176,776. Downward it produces *Qiandai* 謙待. *Seyu* is *gong*, *Weizhi* 未知 is *shang*, *Qiandai* is *zhi*. Six days. Pitch pipe: slightly more than 8.98 *cun*. *Zhun*: 8.9 *Chi* and 15,973.

When examining the list, one may choose to rearrange the data for tuning standards and measurements according to various categories and examine them from the lowest to the highest, according to the order of their generation, duration of days, lengths, or different commas created along the way. The list Jing Fang produced is indeed a treasure chest of information.

The seven divisions are made up of groups of 61, 62, or 30 days. A closer look reveals that groups of 61 or 62 days subdivide into two groups of 30 or 31 days, demarcated by one of the remaining five tuning standards of the original twelve (see Figure 2). The tuning standards

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[50] The "Circulation of Rites" is a chapter in the *Record of Rites*, a compilation of ceremonial usages from the Zhou dynasty to the Han dynasty. Its date of compilation is debated among scholars, but it may have been compiled in the Western Han. For more details see Jeffrey K. Riegel, "Li chi," in *Early Chinese Texts: A Bibliographical Guide*, ed. Michael Loewe (Berkeley: Society for the Study of Early China and Institute of East Asian Studies, Univ. California, 1993), p. 293.

[51] This is a reference to the "Canon of Shun" ("Shun dian" 舜典) chapter in the *Documents of Yu* (*Yu shu* 虞書) within the *Venerated Documents* (*Shang shu* 尚書). Part of the virtuous actions of the Sage Ruler Shun involved regulating the seasons, months, and days of the calendar, standardizing the tuning standards, degrees, weights, and measures, and fixing the rituals. When reforming the state, Shun appointed the (mythological) musician Kui 夔 to be in charge of musical performance (*yue* 樂), to educate the young morally and create harmony between man and gods, saying: "Poetry expresses the will, song intones words, sound accords with intonation, and the tuning standards harmonize sound (詩言志, 歌永言, 聲依永, 律和聲)."

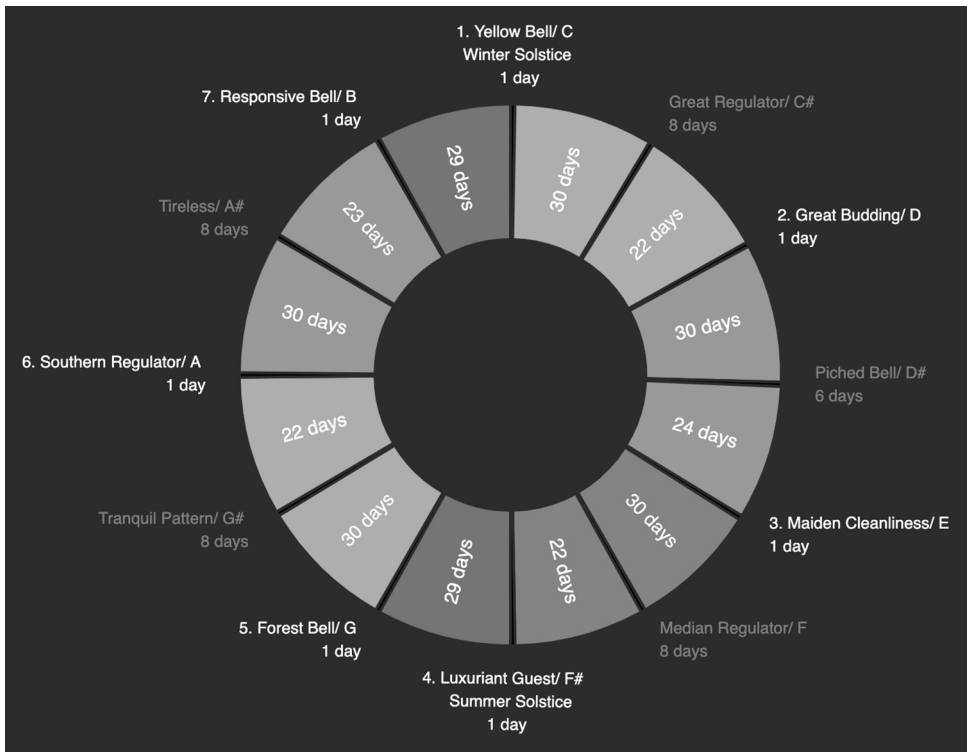

Figure 2. The remaining five pitches of the twelve tuning standards subdividing the seven groups, create an overall division of the year into twelve groups of 30 or 31 days.

marking the seven large groups and those marking the subgroups make up the basic chromatic set of twelve tuning standards, distributed unevenly across the year:

1. Yellow Bell, 61 days: Yellow Bell (1 + 30) + Great Regulator (30)
2. Great Budding, 61 days: Great Budding (1 + 30) + Pinched Bell (*Jiazhong* 夾鐘) (30)
3. Maiden Cleanliness, 61 days: Maiden Cleanliness (1 + 30) + Median Regulator (30)
4. Luxuriant Guest: 30 days (1 + 29)
5. Forest Bell, 61 days: Forest Bell (1 + 30) + Tranquil Pattern (*Yizi* 夷則) (30)
6. Southern Regulator, 62 days: Southern Regulator (1 + 30) + Tireless (*Wuyi* 無射) (31)
7. Responsive Bell: 30 days (1 + 29)

The duration in days assigned to each of the sixty tuning standards belonging to these subgroups spans anywhere between five and eight. For example: *Zhishi*, third on the list, lasts six days. *Kaishi* 開時, eighteenth on the list, lasts eight days. *Nanzhong* 南中, twenty-eighth on the list, lasts seven days. *Qiandai*, thirty-seventh on the list, lasts five days. The reason behind this method of distribution is still unknown.

Figure 3 visualizes the entire distribution of sound across the year, according to the information in the list. The first (innermost) cycle presents the seven demarcated groups of the seven-note scale (same as Figure 1). The second cycle records the duration of each of the sixty tuning standards in days. The third and fourth cycles contain the names of the sixty tuning standards. The fifth cycle shows how each tuning standard becomes a fundamental and lists the tuning standards in the *gong*, *shang*, and *zhi* degrees of its scale. In his exposition and in his list Jing Fang details only the *gong*,

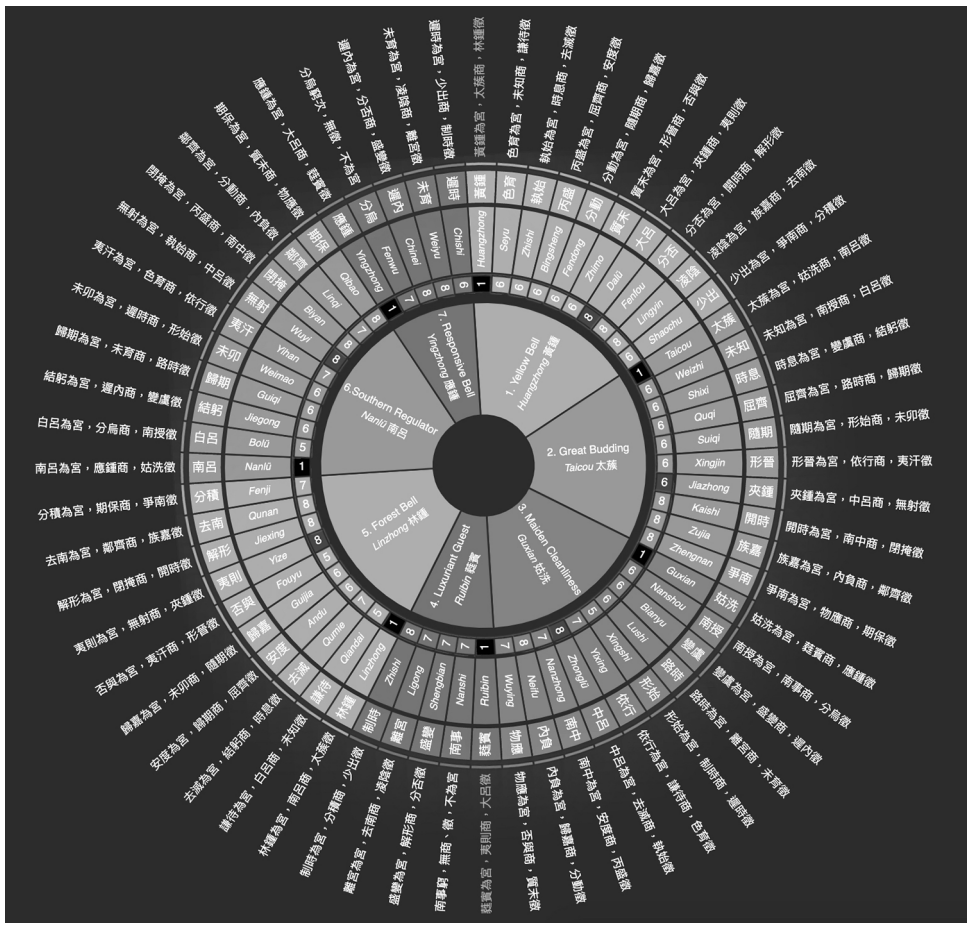

Figure 3. The distribution of the sixty tuning standards and scales across the days of the year. The black numbers on the interior segment (also black in the online version) indicate the seven-note scale as main group delineators, each lasting a single day (marked by the number 1). The intermediate gray numbers on the interior segment (green in the online version) indicate the remaining five tuning standards, which function as subgroup delineators (four lasting eight days and one lasting six days). The dark gray rays at the top and bottom of the exterior circle (yellow in the online version) mark the winter and summer solstices.

shang, and *zhi* degrees of the scale for each of the sixty pitches. They appear in ascending order (rather than order of generation: *gong–zhi–shang*), so we can surmise that he is referencing a scale, but there is no telling. It could reference a seven-note or a five-note scale; only the first, second, and fifth degrees (unlikely); or possibly options for modes (different ordering of the scale), if one calculates from the *shang* or *zhi* degrees.

On the one hand, the way Jing Fang distributed tuning standards across the yearly cycle resembles other contemporary and earlier models, which also superposed the distribution of the twelve tuning standards and the five-note scale over the twelve months of the year. Among these are the "Tones and Tuning Standards" ("Yin lü" 音律) chapter of the *Annals of Master Lü* (*Lüshi Chunqiu* 呂氏春秋), the "Celestial Patterns" ("Tian wen" 天文) chapter in the *Huainanzi*, which offers two such models, and the "Monthly Ordinances" ("Yueling" 月令) chapter in the *Record of Rites*. On

the other hand, the model Jing Fang proposed differs from others not only in its use of sixty tuning standards but in their distribution across days in addition to months and in their grouping scheme.

Each of the available models solidifies the link between sound and the solar year, seasonal change, and the transition of *qi*. The case of Jing Fang may provide a different schematic option, but these basic characteristics remain. One noticeable technical similarity is, of course, the position of Yellow Bell at winter solstice, but also that of Luxuriant Guest at the summer solstice.[52] The lunar months are of no concern to Jing Fang and do not appear in the text. Yet, despite the appealing division of the tuning standards into 30 and 31 days, they do not accord with any available model for the division of the yearly cycle into months. Neither do the subdivisions of the twelve tuning standards accord with the twenty-four or twelve solar terms (*jieqi* 節氣), which last fifteen or sixteen days. Their inner distribution does not lend itself to such further division. The tuning standards belonging to the very first subgroup beginning at Yellow Bell provide an example:

Yellow Bell	*Seyu*	*Zhishi*	*Bingcheng* 丙盛	*Fendong* 分動	*Zhimo* 質末	Total
1	6	6	6	6	6	31

Jing Fang based his model on principles used in other contemporary models, and I would suggest that these subdivisions are indeed somehow related to the solar months, yet equally they seem to be of his own making and require further explanation.

The philosophical ideas that underpin this theory likely lie in its tutorial purpose—it was to be used by technicians in the Department of Observation in the "Observing *qi*" experiment, as indicated later in the treatise:

> One cuts tubes to create pitch pipes; studies the sounds by blowing [into the pipes]; and arranges [them] according to the *qi* of things. This is the foundation of the Way. Because it was difficult for technicians to understand the subtleties of [the pipes'] sounds, it was hard to know what [relationships] they embodied, and their divisions were unclear, I made the *zhun* (Equalizing tuner) to use in its stead. Tones and sounds are profound and subtle. If they are gathered together, they can be explicated.

However, later generations who wished to reinstate his method were unable to apply the calculations to the tuner. Cai Yong tell of two such cases:

> In 84 C.E.,[53] the Expectant Observer of Tuning Standards Yin Rong petitioned the emperor, saying: "None of the officials knows how to tune the sixty tuning standards using the *zhun*. The Expectant Official Yan Chong taught the method of the *zhun* to his son Nan Xuan, and Xuan learned it thoroughly. I wish to summon Xuan to fill a vacancy among the educational officials, and put him in charge of tuning the musical instruments."

[52] One of the *Huainanzi* models presents a different option, a doubled-cycle model based on the twenty-four solar terms. In this model, Yellow Bell begins one cycle on the winter solstice and another on the summer solstice, while Luxuriant Guest falls on the spring equinox in the first cycle and on the fall equinox in the mirroring cycle. See further details in Major, "Celestial Cycles and Mathematical Harmonics in the 'Huainanzi'" (cit. n. 5).

[53] That is, in the reign of Emperor Zhang 章帝 of the Eastern Han (r. 75–88 C.E.).

> In 177 C.E., those in charge of the tuning standards convened at the Eastern Hall. The palace secretary to the Heir Apparent, Zhang Guang, and others inquired about the meaning of the *zhun*. Because Guang and the others did not know, they returned and reviewed the old archives. Thereupon they found this instrument. Although the design resembled that in Jing Fang's writings, they were unable to determine how tightly to adjust the strings or mark the tones.

These technicians needed little knowledge of the reasons behind the computations. What Jean-Claude Martzloff clarifies regarding the astronomical texts that were used in "particular domains" applies here as well: "In fact, all this is understandable, if one acknowledges that Chinese astronomical texts which have reached us were intended not for logicians but above all for technicians, mechanically carrying out repetitive, clearly defined tasks."[54]

The "Observing *qi*" experiment may have been used in the context of solstitial rituals and could have been applied to the tuning of instruments. The two solstices were thought to indicate the extreme states of *qi* and the commencement of nature's seasonal changes. The treatise provides an example in which an official able to execute the correct measurements of tuning standards could oversee the tuning of musical instruments for solstitial rituals. Astrologers and astronomers seem to have been the main utilizers of these calculations, and once they had ensured that the instruments were satisfactorily tuned the sounds could be deemed correct (*zheng* 正):

> Therefore, the Son of Heaven always arrived at the front hall at the summer and winter solstices. Those érudits who combine all Eight Capacities explained the Eight Timbres; listened to the scale (均) of the music; measured the shadow of the gnomon; timed the tuning (pitch pipes); assessed the ashes of the earth, and modeled the *yin* and the *yang*. On the winter solstice, the *yang qi* responds, then the pitch is clear, the shadow is at its longest, Yellow Bell passes through, and the weight of the earthly ashes is light. In the summer solstice the *yin qi* responds, then the pitch is turbid, the shadow is at its shortest, Luxuriant Guest passes through, and the earthly ashes are heavy. This takes place within the span of more or less five days. Each of those with the Eight Capacities reports by observing the conditions. The Director of Astrology seals it and petitions the emperor. If it corresponds, then there is harmony; if it does not, then [one should] divine.

The érudits (*shi* 士) of the eight capacities oversaw the correspondence between tuning and the solar calendar. They made sure that astronomical and musical instruments were tuned according to correct standards, and, as seen from the context of Jing Fang's theory, they not only measured the shadow of the gnomon but made sure the transition from one scale to the next was carried out at the right time and in accordance with the pitch-pipe observations. Their duties are reiterated in the "Treatise on Ritual and Etiquette" ("Liyi" 禮儀) in the *History of the Later Han* where the emperor summoned them three days prior to the solstices, when the gnomon shadow was at its longest or shortest, to examine astronomical instruments, assess the accuracy of the tuning of various musical instruments (among them the mouth organ 竽, bells, se zither, and chimes), and listen to the Yellow Bell scale. Even if these tuning methods were not applied to all musical performances, they might have applied to those during the solstices.[55]

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[54] Jean-Claude Martzloff, A *History of Chinese Mathematics*, 2nd ed. (New York: Springer, 2006), pp. 46–47.
[55] *Hou Han Shu* (cit. n. 1), pp. 3125–3126.

V. THE REDUCTION OF THE COMMA: A COSMOLOGICAL PROBLEM OF NUMBERS

Within the study of acoustics, a comma is a problem specifically related to tuning systems. Simply put, the comma identifies a gap between two pitches that should sound identical, but when calculated they differ in their numerical value and so would also differ in pitch when played together. Several types of commas have been detected throughout the history of temperament. For the purpose of this essay, I focus only on the comma that Jing Fang worked to reduce, known in the West as the Pythagorean comma (in the Western Han, it did not have a name). He defined it as a mathematical problem, and his goal for reducing the comma was to reach the end of the tuning cycle with a pitch as close as possible to the initial one. This meant getting as close as possible to the number of the Yellow Bell tuning standard: 177,147. The miniscule gap between the Yellow Bell and the pitch Jing Fang arrived at, *Seyu* 色育, is known in China as the Jing Fang comma (*Jing Fang yincha* 京房音差).[56] This complicated problem entailed several realizations evident from Jing Fang's list. First, when calculating the twelve tuning standards, the thirteenth pitch should be an exact octave from the first pitch, Yellow Bell, but it is not. The difference is audible (nearly a quarter tone), and the result creates a mathematical spiral rather than a cycle. Second, the yearly cycle must begin again on the winter solstice not only with an accurate Yellow Bell pitch but with the entire Yellow Bell seven-note scale.[57]

Pythagorean tuning uses musical intervals of fifths (a 3:2 ratio) and octaves (a 2:1 ratio) to create a scale. If we take a string and press it two thirds of the way up, when we pluck it we will hear the fifth (the interval between the pitches C and G). Pressing the same string at its halfway point produces the octave (C and C′). The Pythagorean method is close to the Chinese method, which uses fifths (and fourths) but does not directly discuss octaves. The mathematical problem with these systems is that after calculating twelve consecutive fifths, which produce the twelve semitones of the chromatic scale, the thirteenth pitch slightly overshoots the octave pitch and creates a comma—a gap.[58] In the Pythagorean method, the calculation of thirteen consecutive fifths should reach the same pitch as the calculation of seven octaves. In the Chinese system, the cycle of rising fifths and falling fourths should return to the initial pitch. However, in both cases, they do not.[59] To Jing Fang, apart from the acoustically audible problem, the comma is evidence of a broken cycle, where there should be a perfect one.

The sixty tuning standards are a near-perfect solution to the comma problem. They are an attempt to reduce the gap between the numerical value of the first tuning standard, Yellow Bell (177,147), and the thirteenth, *Zhishi* (174,762).[60] *Seyu*, the fifty-fourth generated tuning standard, is the pitch with the large number close enough to Yellow Bell that it is audibly indistinguishable from it. The comma was thus reduced to Jing Fang's satisfaction. In numbers, the ratio between Yellow Bell and *Seyu* is 177,147:176,776. This ratio (3.6 cents) is the Jing Fang comma.

But Jing Fang did not stop on the fifty-fourth tuning standard, instead continuing the cycle to sixty. It is likely that he did this in order to complete an entire run of the seven-note scale that

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[56] Wang Zichu 王子初, "Jing Fang he tade liushi lü" 京房和他的六十律 (cit. n. 10), p. 24.

[57] Chen Yingshi 陳應時, "Wei 'Jing Fang Liushi Lü' shen bian" 為"京房六十律"申辯 (cit. n. 8), p. 8.

[58] An equally tempered fifth equals 700 cents (100 × 7 semitones); a Pythagorean fifth is not adjusted and equals 701.955 cents (an exact 3:2 ratio). These extra 1.955 cents accumulate with the generation of each consecutive fifth.

[59] Seven octaves equal 8,400 cents (1200 × 7), while twelve Pythagorean fifths equal 8,423.46 cents (701.055 × 12). The nearly two additional cents carry over twelve times, eventually resulting in an audible difference of 23.46 cents (1.955 × 12), known in the West as a Pythagorean comma.

[60] Transcribing the ratio between these two tones into cents produces the comma. In order to do this, one inserts the ratio into the following logarithm, which translates ratios into cents: 1200*log (m/n)/log (2), where m/n is the ratio. Thus: 1200 log(177,147 ÷ 174.762) ÷ log(2) = 23.466614531.

Table 2. A comparison between the numerical representation of the Yellow Bell scale (first in the cycle) and the Seyu scale (last in the cycle).

Degree	Notes	Yellow Bell Scale	Yellow Bell Large Numbers	*Seyu* Scale	*Seyu* Large Numbers
1st	*gong* / C	Yellow Bell *Huangzhong* 黃鍾	177,147	*Seyu* 色育	176,776
2nd	*shang*/ D	Great Budding *Taicou* 太蔟	157,464	*Weizhi* 未知	157,134
3rd	*jue*/ E	Maiden Cleanliness *Guxian* 姑洗	139,968	*Nanshou* 南授	139,674
4th	*bian zhi*/ F#	Luxuriant Guest *Ruibin* 蕤賓	124,416	*Nanshi* 南事	124,154
5th	*zhi*/ G	Forest Bell *Linzhong* 林鍾	118,098	*Qiandai* 謙待	117,851
6th	*yu*/ A	Southern Regulator *Nanlü* 南呂	104,976	*Bailü* 白呂	104,756
7th	*bian gong*/ B	Responsive Bell *Yingzhong* 應鍾	93,312	*Fenwu* 分烏	93,116

begins from *Seyu* and parallels the Yellow Bell seven-note scale mentioned earlier.[61] Thus, there is a near match of a whole scale between the first and last seven pitches of the cycle (see Table 2).

None of the earlier texts that included the "Addition and Subtraction by a Third" algorithm calculated beyond the twelfth tuning standard, and so none raised any potential problems with the system, possibly because their originators were not aware that there was one. The problem of the comma could not be solved using the algorithm because fifths (3:2) are based on divisions of 3, while octaves (2:1) are based on divisions of 2. Since 3 and 2 do not have a common denominator, the "Addition and Subtraction by a Third" algorithm, or Pythagorean tuning, could never produce the desired result. In China, this problem was not solved until the fifteenth century when Zhu Zaiyu discovered tempered tuning, which divided the octave into twelve equal sections by using $\sqrt{2}$.[62]

Chen Yingshi argues that Jing Fang was aware of this problem, and so he ceased his calculation when he reached a number that answered the conditions as he saw fit. In the Liu-Song dynasty (420–479 C.E.), however, the experiment Jing Fang conducted was extended to 360 tuning standards, reducing the comma even further (from 3.63 to 1.845 cents). This goes to show that for Jing Fang stopping at sixty was a choice.[63] The treatise cites Jing Fang claiming

[61] Chen Yingshi 陳應時, "Wei 'Jing Fang Liushi Lü' shen bian" 為"京房六十律"申辯 (cit. n. 8).
[62] See Kuttner, "Prince Chu Tsai-Yü's Life and Work" (cit. n. 10).
[63] *Ibid.*

that his goal to cycle back to Yellow Bell (177,147) was inspired by the "Circulation of Rites" chapter. He does not explicate this idea further in the treatise, and the connection between his calculation and this quotation is not readily apparent. Jing Fang created a threefold cycle conjunction between notes, tuning standards, and the calendar. This mode of thought is reminiscent of the cyclical astronomical systems in the Qin and Han, which had the same intention.[64] Chen Yingshi's argument provides solid musicological and intellectual reasoning behind these last six pitches. Yellow Bell and *Seyu* provide the example of the reduction of the comma, and Jing continued because he was interested in calculating an entire seven-note scale that would match the original seven-note Yellow Bell scale (*yun* 均) as closely as possible.[65] Fritz Kuttner suggests that Jing Fang selected twelve pitches that came close to reaching the large numbers of the original twelve pitches (ranging from Yellow Bell to Median Regulator), but he ignores the seven-note scale and does not explain why Jing Fang continued beyond the fifty-fourth tuning standard to generate six additional pitches, reaching the sixtieth pitch, *Nanshi*.[66]

VI. THE USE OF SOUND IN PROGNOSTICATION

Scholars now know of several other cases in which tuning standards and divination worked hand in hand. Scroll B of the daybooks excavated in Fangmatan 放馬灘 (Gansu Province) date to the mid- to late third century B.C.E. and provide the earliest excavated example of the use of the algorithm for pitch generation. They also contain two methods to calculate the twelve tuning standards: using "harmonic numbers" beginning with 81 for Yellow Bell and using large numbers with 177,147 for Yellow Bell. The Fangmatan daybooks correlate the twelve standards with geographical locations and include numerous divination scenarios and instructions involving the five-note scale and the twelve tuning standards to determine auspicious and inauspicious days.[67] It is impossible to separate Jing Fang and his sixty tuning standards from his background in prognostication, methods of hexagram interpretation, and conceptualization of numbers. The Fangmatan daybooks, precedent of the numerical representation of sound in prognostication relating to time, cast Jing Fang's idea to use sound for his predictions in a new light and place him within the context of an existing phenomenon, although the evidence for it predates him by more than a hundred years.

Because of this connection, 1950s Chinese scholarship drowned his accomplishments in acoustics in accusations of mysticism (*shenmi zhuyi* 神秘主義) and superstition (*mixin* 迷信).[68] In response, Chen Yingshi insisted on divorcing Jing Fang's tuning method from any nonpositivist context and asserted that his method of calculating the sixty tuning standards was not an extension of his *Changes* interpretation method. However, declaring Jing Fang's accomplishments to be purely acoustic forces us to ignore too many factors. In order to understand his project, we must consider contemporary conceptualizations of sound. These include numbers (*shu*) as representing hidden cosmic processes; the idea of sound as a resonator for transient cosmic *qi* throughout the seasons of the year; and tuning standards as a measuring instrument for sound. We cannot ignore the possibility that in Jing Fang's view sound was a reliable tool for prognostication, especially in relation to the prediction of weather and natural disasters.

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[64] See Sivin, "Cosmos and Computation in Early Chinese Mathematical Astronomy" (cit. n. 7).

[65] Each of the numbers in the first seven-note scale (Yellow Bell to *Nanzhong*) is separated from its equivalent in the last seven pitches (*Seyu* to *Nanshi*) by the miniscule Jing Fang comma (3.62 cents).

[66] Kuttner, "Prince Chu Tsai-Yü's Life and Work" (cit. n. 10), p. 172.

[67] Donald J. Harper and Marc Kalinowski, eds., *Books of Fate and Popular Culture in Early China: The Daybook Manuscripts of the Warring States, Qin, and Han* (Leiden: Brill, 2017), pp. 468–475 (App. C).

[68] Yang Yinliu 楊蔭瀏, *Zhongguo gudai yinyue shigao* 中國古代音樂史稿 (Beijing: Renmin Yinyue 人民音乐, 1981), pp. 131–132.

Jing Fang's focus on day distribution and his framing of one-day markers and subdivisions share some characteristics of hexagram interpretation techniques, in particular a technique called the *qi* series (*gua qi* 卦氣), nestled under the large interpretation categories known as "Images and Numbers" (*xiangshu* 象數). This method was crafted by Meng Xi 孟喜 (fl. 47 B.C.E.), with whom Jing Fang is connected (albeit skeptically) in the "Biographies of Various Scholars" and through his two commentaries on Meng Xi's hexagram interpretation, which appear in the "Bibliographic Treatise" (Yiwen zhi 藝文志) in the *History of the Han*.[69] Meng Xi assigned the first four cardinal hexagrams to the solstices and equinoxes and their twenty-four lines to the twenty-four *qi* solar terms. He distributed the remaining sixty hexagrams across the days of the year so that each hexagram governs 6 7/80 days (*liu ri qi fen* 六日七分)[70] and their lines (*yao* 爻) govern the 365 ¼ days of the solar year. Twelve hexagrams govern the twelve lunar months, and their hexagram lines govern the seventy-two phases (*hou* 候).[71] The third-century commentator Meng Kang 孟康 annotated the section discussing Jing Fang's method of hexagram distribution according to the days of the year to predict catastrophes and weather in his biography in the *History of the Han*:

> This is a method of distributing hexagrams according to days. Each line governs one day, the sixty-four hexagrams become the three hundred and sixty days. The four surplus hexagrams—Zhen, Li, Dui, and Kan hexagrams—occupy the offices of the Regional Inspectors (*fangbo jiansi* 方伯監司). The days on which sacrificial offerings (*yong shi* 用事) for the two solstices and two equinoxes are conducted use the Zhan, Li, Dui, and Kan hexagrams. They are also the ruling *qi* of each [section] of the four seasons cycle. Each hexagram governs a season. The method of its occupation is observing the good and the bad for each [season], according to its days.[72]

Gua qi is relevant to the method Jing Fang constructed. Similarities exist in the distribution scheme, which utilizes the days of the year; in the use of the number 60, which fits well with Jing Fang's conceptual framework and mathematical results, whether fortuitously or by design; and in the declared connection to weather prognostication related to the identification of *qi* throughout the year. These similarities are general. The tuning standards theory, with its use of the seven-tone scale division and the twelve tuning standards, is possibly an amalgamation of some of these principles with an innovative model of the highest expertise in the intricacies of acoustics and its use within an existing system of sound-*qi* predictions. Further detailed research is required in order to link Meng Xi's method with this theory directly and to consider the various additional hexagram interpretation methods Jing Fang produced.

CONCLUSION

This essay shows that Jing Fang is a great example of nondichotomous expertise that merged realms of knowledge and operated freely on the foundational concepts and theories of the time. In this case: the contemporary emphasis on the relation between sound, the yearly cycle, and *qi*; the construction of perfect cycles that dominated the concepts of mathematical astronomy and the concept

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[69] On the prevalence of divination techniques across the entire bibliographic treatise of the *Han shu* and beyond see Lisa Raphals, "Divination in the *Han shu* Bibliographic Treatise," *Early China*, 2008–2009, 32:45–101. For a detailed study of Jing Fang's methods of hexagram interpretation see Guo Yu 郭彧, *Jing shi yi yuanliu* 京氏易源流 (Beijing: Huaxia 華夏出版社, 2007).

[70] I.e., 365 ¼ (days of the solar year) divided by 60 (hexagrams).

[71] On the phases (*hou* 候) see Sivin, *Granting the Seasons* (cit. n. 29), p. 81.

[72] *Han shu* (cit. n. 12), p. 3160.

of regularities, numerology, accuracy, and synchronicity; divination techniques to prognosticate catastrophes; and an established use of sound in divinatory techniques. All of these intertwined, each providing an aspect of the conceptual framework that led Jing Fang to solve a problem that is at the heart of what we now call acoustics.

Jing Fang's argumentation and theory about sound take for granted that it manifests *qi* emitted from the cosmos. This is not a negotiable idea or something that is treated as a social construct, which may argue for an association of sound with the calendar rather than as a physical phenomenon. Sound was thought to transition throughout the year, and early Chinese scholars were certain that they could track and measure it quite precisely. This assumption allowed Jing Fang to conduct his "Observing *qi*" experiment, reach his sixty tuning standards, and change his own name without questioning the very idea of cosmic sound and its manifestation in numbers. This is a scientific venture, a cosmological solution to an acoustical problem. Experts such as Jing Fang conceptualized sound as a tangible, useful component, a vibrating *qi* attuned to the natural order of the cosmos. Jing Fang's model of the sixty tuning standards exemplifies how tuning and sound were laden with cosmological meaning. Sound was conceptualized as a technology, and the goal of producing accurate pitches and calculations did not necessarily coincide with that of making music more beautiful or cultivating the listener. Its purpose was to obtain the keys to the cosmos.