Nonlinear global gyrokinetic stellarator simulations of GENE-3D with kinetic electrons

Felix Wilms[1], Alejandro Bañón Navarro[1], Gabriele Merlo[2],
Leonhard Leppin[1], Tobias Görler[1], Tilman Dannert[3], Florian Hindenlang[1], Frank Jenko[1]
[1] *Max Planck Institute for Plasma Physics, 85748 Garching, Germany*
[2] *Oden Institute for Computational Engineering and Sciences, Austin,Texas 78712, USA*
[3] *Max Planck Computing and Data Facility, 85748 Garching, Germany*

Abstract

Thanks to advances in code development and algorithmic performance optimisation, it is possible nowadays to perform global, gyrokinetic simulations in 3-D stellarator geometry. Such studies are of extreme importance, as only they can take into account effects like radially global $E \times B$ shear or global profile shapes, which play a key role in understanding plasma confinement [1]. While studies like [2–5] paved the way for understanding global ITG turbulence in stellarators, all results presented up to this point were provided by using an adiabatic electron model. However, kinetic electron effects can play a major role in order to understand global plasma performance in Wendelstein 7-X (W7-X) [6].

For this reason, GENE-3D [7] was recently upgraded to an electromagnetic stellarator turbulence code. In this contribution, we present the first application by comparing global, nonlinear simulations of ITG turbulence in Wendelstein 7-X using kinetic electrons with and without electromagnetic effects due to finite plasma-β.

Numerical model

GENE-3D is a Eularian code solving a gyrokinetic Vlasov-Maxwell system of equations in order to study global plasma turbulence in stellarators. It uses a δf-approach, in which the full distribution function F_σ of species σ is split into a local Maxwellian $F_{M,\sigma}$ and a first order perturbation $F_{1,\sigma}$. Under this assumption, the collisionless gyrokinetic equation, including electromagnetic effects through a parallel vector potential, can be cast in the form:

$$\begin{aligned}\frac{\partial F_{1,\sigma}}{\partial t} &= R_\sigma + q_\sigma v_{||}\frac{F_{M,\sigma}}{T_{0,\sigma}}\mathscr{G}\left\{E_{||}^{ind}\right\}\\ R_\sigma &= -\left[v_{||}\mathbf{b}_0 + (\mathbf{v}_{\nabla B}+\mathbf{v}_c)\right]\cdot\left(\nabla F_{1,\sigma} + \frac{q_\sigma F_{M,\sigma}}{T_{0,\sigma}}\nabla\mathscr{G}\{\phi_1\}\right) + \frac{\mu}{m_\sigma}\mathbf{b}_0\cdot\nabla B_0\frac{\partial F_{1,\sigma}}{\partial v_{||}}\\ &\quad -\mathbf{v}_\chi\cdot\left[\nabla\ln(n_{0,\sigma}) + \nabla\ln(T_{0,\sigma})\left(\frac{m_\sigma v_{||}^2/2+\mu B_0}{T_{0,\sigma}} - \frac{3}{2}\right)\right]F_{M,\sigma}\\ &\quad -\mathbf{v}_\chi\cdot\left(\nabla F_{1,\sigma} + \frac{q_\sigma F_{M,\sigma}}{T_{0,\sigma}}\nabla\mathscr{G}\{\phi_1\}\right)\end{aligned} \tag{1}$$

with the particle drifts defined as

$$\mathbf{v}_{\nabla B} \equiv \frac{\mu c}{q_\sigma B_0^2}\mathbf{B}_0\times\nabla B_0, \quad \mathbf{v}_c \equiv \frac{v_{||}^2}{\Omega_\sigma}(\nabla\times\mathbf{b}_0)_\perp, \quad \mathbf{v}_\chi \equiv \frac{c}{B_0^2}\mathbf{B}_0\times\nabla\mathscr{G}\left\{\phi_1 - \frac{v_{||}}{c}A_{1,||}\right\}. \tag{2}$$

Here, $\Omega_\sigma = (q_\sigma B_0)/(m_\sigma c)$ is the gyrofrequency of species σ with mass m_σ and charge q_σ, $\mathbf{b}_0 = \mathbf{B}_0/B_0$ is the unit vector in the direction of the equilibrium magnetic field $\mathbf{B}_0$ and c is the speed of light. Furthermore, the operator $\mathscr{G}\{\}$ denotes the push-forward gyroaverage operator, which is the counterpart to the pull-back gyroaverage operator $\mathscr{K}\{\}$.

In order to determine the electrostatic potential ϕ_1 and the parallel vector potential $A_{1,||}$, GENE-3D solves the quasi-neutrality equation

$$\sum_\sigma q_\sigma^2\int\left(\frac{F_{M,\sigma}}{T_{0,\sigma}}\phi_1 - \mathscr{K}\left\{\frac{F_{M,\sigma}}{T_{0,\sigma}}\mathscr{G}\{\phi_1\}\right\}\right)d^3v = \sum_\sigma q_\sigma\int\mathscr{K}\{F_{1,\sigma}\}\,d^3v \tag{3}$$

and the parallel component of Ampere's law

$$\nabla_\perp^2 A_{1,||} = -\frac{4\pi}{c}\sum_\sigma j_{1,||,\sigma} = -\frac{4\pi}{c}\sum_\sigma\int v_{||}\mathscr{K}\{F_{1,\sigma}\}\,d^3v. \tag{4}$$

Finally, the parallel inductive electric field $E_{||}^{ind} = -(1/c)\,\partial A_{1,||}/\partial t$ is determined by taking the derivative of Ampere's law with respect to time, resulting in the equation

$$\nabla_\perp^2 E_{||}^{ind} - \frac{4\pi}{c^2}\sum_\sigma q_\sigma^2\int v_{||}^2\mathscr{K}\left\{\frac{F_{M,\sigma}}{T_{0,\sigma}}\mathscr{G}\left\{E_{||}^{ind}\right\}\right\}d^3v = \frac{4\pi}{c^2}\sum_\sigma q_\sigma\int v_{||}\mathscr{K}\{R_\sigma\}\,d^3v. \tag{5}$$

Code Verificaiton

In order to ensure the correct implementation of the electromagnetic upgrade of GENE-3D, verification studies were performed against the global tokamak code GENE [8]. As a setup, the parameters described in [9] were used.

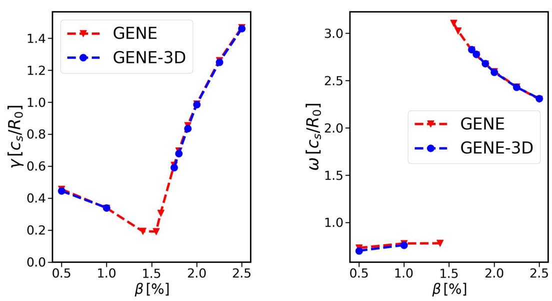

Figure 1: *Linear growth rates and mode frequencies as a function of the plasma-β; red shows results obtained by GENE, blue those from GENE-3D.*

Figure 1 shows an excellent agreement between both codes, for the linear growth rates as well as the mode frequencies, verifying a correct implementation of the electromagnetic terms in GENE-3D.

Global ITG simulations of Wendelstein 7-X

As the main focus of this work, global simulations of Wendelstein 7-X have been carried out in order to investigate the stabilisation of ITG turbulence through finite plasma-β effects. For this, a set of profiles, shown in figure 2 have been chosen in order to ensure a strong drive from the ion temperature gradient. The Standard configuration has been chosen for the investigation, where the specific geometry has been generated consistently with the previously mentioned profiles. A hydrogen plasma with realistic electron mass and a $\rho_s^*(x/a = 0.5) = 1/184$ was used. For the comparison, the first case uses an electron plasma-β of 10^{-4} at half radius as an electrostatic limit, whereas the electromagnetic case was chosen to have $\beta_e(x/a = 0.5) = 0.5\%$.

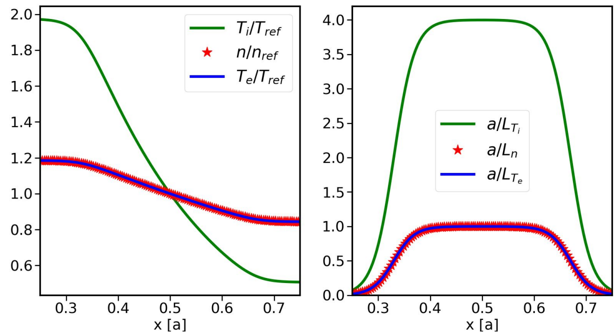

Figure 2: *Global density and temperature profiles used in the stellarator simulations, together with their gradients.*

In a first step, linear simulations of both scenarios were performed, globally with GENE-3D as well as local flux-tube simulations at $x/a = 0.5$, $\alpha = 0$ with GENE for comparison. Both codes show a reduction of the linear growth rates through electromagnetic effects, as can be seen in figure 3. However, the growth rates of the global simulations are lower in both cases, highlighting again the importance of global effects.

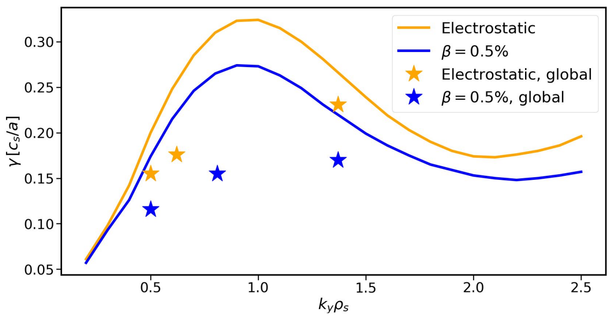

Figure 3: *Linear growth rates as a function of binormal wave number of electrostatic and electromagnetic W7-X scenarios; global simulation results are marked as stars.*

Nonlinear global simulations with GENE-3D convey the same message. Figure 4 shows the time traces of the volume-averaged heat fluxes, as well as the radial profiles of the electrostatic heat fluxes, averaged over a flux-surface and in time, for both scenarios. The prior reveals a negligible electromagnetic heat flux, even in the case using $\beta_e = 0.5\%$, as well as the ionic contribution dominating over that of the electrons, which indicates that both cases still represent ITG turbulence. The radial profiles show a reduction in turbulence of about 10% through the increase in β, with the peak value of the ion heat flux going from $(15.8 \pm 2.8)\,\mathrm{Q_{GB}}$ to $(14.3 \pm 1.3)\,\mathrm{Q_{GB}}$ and the electron flux decreasing from $(2.5 \pm 0.4)\,\mathrm{Q_{GB}}$ to $(2.3 \pm 0.3)\,\mathrm{Q_{GB}}$. Therefore, the nonlinear results are in line with the linear considerations before.

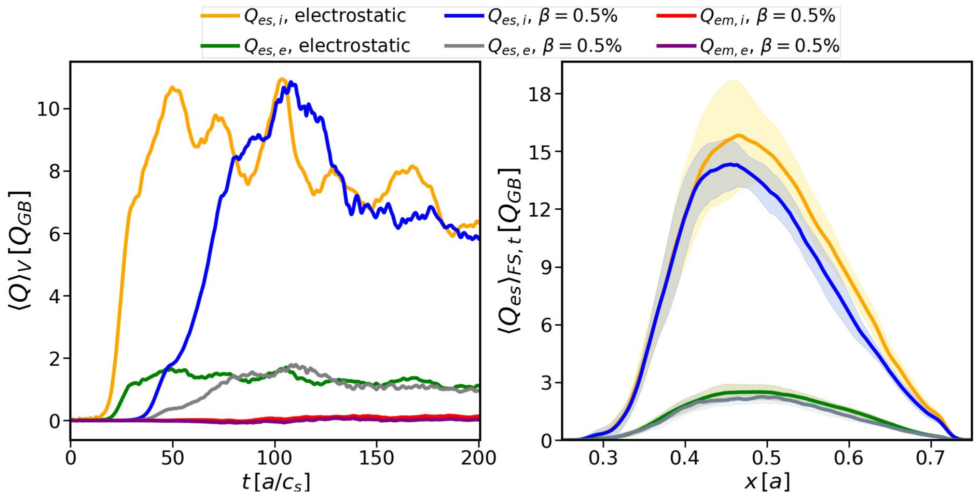

Figure 4: *Left: Volume-averaged time traces of nonlinear heat fluxes; right: radial profiles of flux.surface averaged electrostatic heat fluxes.*

Conclusion and outlook

The extension of GENE 3D to electromagnetic simualtions has been discussed. The underlying model equations where outlined, together with verification studies against the global tokamak code GENE. Finally, linear and nonlinear global simulations of Wendelstein 7-X with realistic electron mass were performed, showing a slight reduction of ITG turbulence through finite plasma-β.

In future works, the goal is to perform simulations with different model complexity of a realistic W7-X discharge in order to build towards a validation of gyrokinetic theory against experimental measurements.

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