Supplemental Document

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Microsphere kinematics from the polarization of tightly focused nonseparable light: supplement

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EXPERIMENTAL METHODS

An overview of our setup is shown in Fig. S1. The optical tweezer consisted of a pair of Leica 506195 HCX PL FLUOTAR 100x oil immersion objectives with NA 1.3. Samples were prepared by manually depositing a trace of silver conductive adhesive (SCA) on a BK7 glass substrate (thickness 80 µm, $n = 1.51$). This reflective layer was helpful for calibrating the axial location of the focal plane as well as for focusing the white light image. A drop of particle solution was deposited near the dry SCA. Placeholders of the same thickness were placed laterally on the substrate followed by a second substrate on top (all BK7). Figure S2 shows the sample geometry to scale.

The entire sample configuration was placed on a piezo-controlled precision stage (PI PZ 82E) and brought into contact with the objectives via immersion oil (Leica Type F, $n_e^{23} = 1.52$, $v_e = 46$), index-matched to the BK7 substrate.

In order to minimize spherical aberrations, pure 2,2'-thiodiethanol (TDE, $n_{\mathrm{TDE}} = 1.50$) [1] was used as the mounting medium. TDE has an optical index very close to that of BK7 ($n_{\mathrm{BK7}} = 1.51$).

A pair of cold mirrors placed above and below the optical tweezer allowed white light from a halogen lamp (Fiberoptic-Heim Linos LQ 1100) to form an image of the particles on a CCD camera (ImagingSource DMK 31BU03 with objective). These mirrors were removed during polarization measurements in order to eliminate unnecessary polarization disturbances. Additionally, identical pairs of turning mirrors in a periscope configuration were used to walk the beam into and out of the optical tweezer without shifting the polarization state.

Back focal plane images of the upper objective were taken with an InGaAs camera above the optical tweezer (Xenics XS–450), by folding away a mirror. This enabled beam characterization, effective–NA estimation, and precise beam alignment.

Fig. S3 shows the detection of a Stokes parameter. Overall, three non-polarizing beam splitters were used to split the beam from the tweezer into sub-beams with relative fractions 22%, 18%, 19%, and 19% of the initial power. The beam splitters had unequal transmission and reflection coefficients for *s*- and *p*-polarisation, but were all aligned in the same plane. The mismatch of *s*- and *p*-transmittance and reflectance was corrected by a tilted glass plate in each sub-beam (labeled "F" in Fig. S3). The overall polarization transformation occuring between optical tweezer and Stokes parameter detection due to unspecified phase shifts was reverted in each sub-beam using a "polarization gadget" consisting of three wave plates [2]. The total transmittance from tweezer to detectors was characterized for each sub-beam and factored into the responsivity of each detector. Large diodes ($\varnothing \geq 500$ µm) were found to be helpful in minimizing drifts, as was enclosing the experiment against air convection. s_0 was measured by direct detection while s_1, s_2, s_3 were measured by direct difference photodetection. Each differential photocurrent was amplified by a transimpedance amplifiying circuit inside each detector and recorded on an oscilloscope with 8 bit vertical resolution (LeCroy WaveSurfer 424).

Scanning measurements were performed by programming the piezo stage controller (PI E-710.3CD v7.040) to trace out a comb–like trajectory covering a rectangular area of the current transverse plane, as shown in Fig. S4. This was repeated in axial steps of 200 nm in order to cover the volume of interest. A single scan had a duration of 8.16 s, during which the Stokes parameters were measured continuously. After completion, the (x, y) coordinates were queried from the controller with a time resolution of 1 ms (Fig. S4). A regular grid was then constructed by linear tesselation inside the region of interest. The full map $\mathbf{s}(\mathbf{r})$ was built up by repeating this process, typically over 50 planes.

ALGORITHM

From a measurement $\mathbf{s}(t)$ of a free particle, the likelihood function

$$L(\mathbf{s}|\mathbf{r},t) = \frac{1}{(2\pi)^{4/2}} \prod_{i=0}^{3} \frac{1}{\sigma_i} \exp\left\{ \frac{-\left(s_i(t) - s_i(\mathbf{r})\right)^2}{2\sigma_i^2} \right\} \tag{S1}$$

was computed, where σ_i are the dark noise variances of the detectors, multiplied by a correction factor for tolerance of small experimental drifts. Using a constant prior distribution over a three-dimensional region

$$p(\mathbf{r}|t) = \sigma_{\text{prior}}^{-3} \prod_{k=\{x,y,z\}} \text{rect}\left(\frac{r_k'(t) - r_k(t_{-1})}{\sigma_{\text{prior}}} \right), \tag{S2}$$

where $r_k(t_{-1})$ are the Cartesian components of the position inferred at the previous timestep, and σ_{prior} an empirically chosen width, we computed the posterior probability distribution $P(\mathbf{r}|\mathbf{s},t)$ via Bayes's theorem

$$P(\mathbf{r}|\mathbf{s},t) = \mathcal{N}\, L(\mathbf{s}|\mathbf{r},t)\, p(\mathbf{r}), \tag{S3}$$

with the normalization factor $\mathcal{N} = \left[\int L(\mathbf{s}|\mathbf{r},t) p(\mathbf{r})\, d\mathbf{r}\right]^{-1}$, denoting the probability of the probe being located at position $\mathbf{r}$ conditional on having observed $\mathbf{s}(t)$. The maximum of this distribution was chosen as the new inferred position, and provided the center of the prior distribution for the next time step.

REFERENCES

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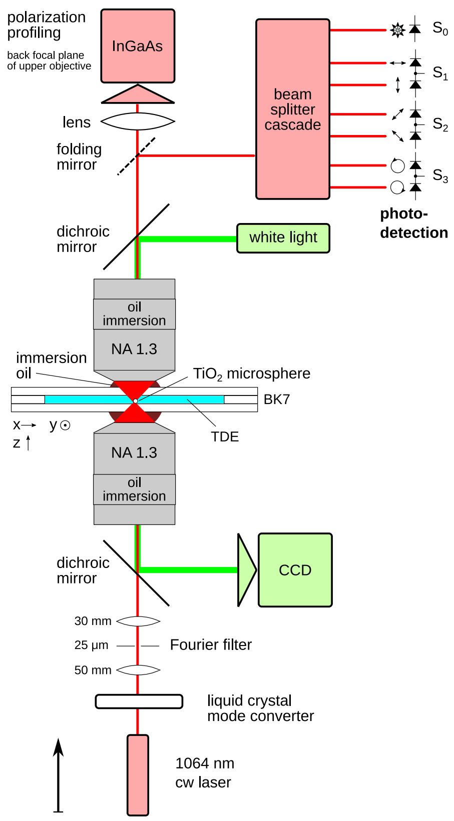

Fig. S1. Overview of experimental setup.

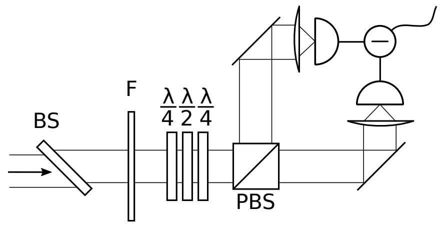

Fig. S3. Detail of the detection setup for one of the $s_{1,2,3}$ Stokes parameters. See text for description.

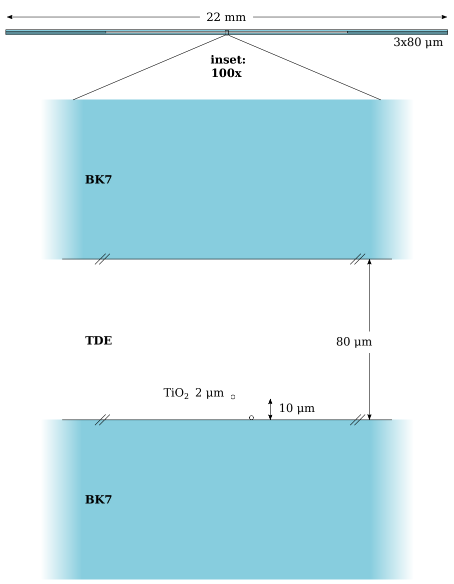

Fig. S2. Sample configuration (to scale).

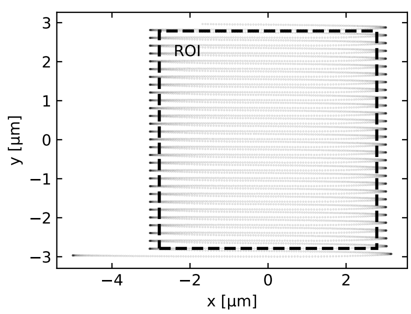

Fig. S4. Typical report table from a scan mapping x–y–position to time. The rectangle indicates the region of interest chosen for plotting.