Evidence of large recoil velocity from a black hole merger signal

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The final black hole left behind after a binary black hole merger can attain a recoil velocity, or a "kick", reaching values up to 5000 km/s. This phenomenon has important implications for gravitational wave astronomy, black hole formation scenarios, testing general relativity, and galaxy evolution. We consider the gravitational wave signal from the binary black hole merger GW200129_065458 (henceforth referred to as GW200129), which has been shown to exhibit strong evidence of orbital precession. Using numerical relativity surrogate models, we constrain the kick velocity of GW200129 to $v_f \sim 1542^{+747}_{-1098}$ km/s or $v_f \gtrsim 698$ km/s (one-sided limit), at 90% credibility. This marks the first identification of a large kick velocity for an individual gravitational wave event. Given the kick velocity of GW200129, we estimate that there is a less than 0.48% (7.7%) probability that the remnant black hole after the merger would be retained by globular (nuclear star) clusters. Finally, we show that kick effects are not expected to cause biases in ringdown tests of general relativity for this event, although this may change in the future with improved detectors.

Introduction.— When two black holes (BHs) orbit each other, they emit gravitational waves (GWs) which carry away energy and angular momentum. This causes the orbit to shrink in a runaway process that culminates in the merger of the BHs into a single remnant BH. At the same time, the GWs can also carry away linear momentum from the binary, shifting its center of mass in the opposite direction [1]. Most of the linear momentum is lost near the merger [2], resulting in a recoil or "kick" velocity imparted to the remnant BH.

Kicks are particularly striking for precessing binaries, in which the component BH spins are tilted with respect to the orbital angular momentum. For these systems, the spins interact with the orbital angular momentum as well as with each other, causing the orbital plane to precess [3]. Numerical relativity (NR) simulations revealed that the kick velocities for precessing binaries can reach values up to $\sim 5000~\rm km/s$ [4–6], large enough to be ejected from any host galaxy [7].

Kicks have important implications for BH astrophysics. Following a supermassive BH merger, the remnant BH can be displaced from the galactic center or ejected entirely [7], impacting the galaxy's evolution [8], fraction of galaxies with central supermassive BHs [9], and event rates [10] for the future LISA mission [11]. For stellar-mass BHs like those observed by LIGO [12] and Virgo [13], kicks can limit the formation of heavy BHs. BH masses greater than

 $\sim 65 M_{\odot}$ are disfavored by supernova simulations [14, 15], but have been seen in GW events [16–18]. This could be explained by second-generation mergers [19], in which one of the component BHs is itself a remnant from a previous merger, and is thus more massive than the original stellar-mass progenitors. However, if the kick from the first merger is large enough, the remnant BH would get ejected from its host galaxy and would not participate in another merger.

Unfortunately, observational evidence of large kicks has been elusive. While various candidates from electromagnetic observations have been identified, their nature is debated [20–28]. Similarly, observing kicks using GW signals has been challenging [29–34]. For example, Varma et al. [29] used accurate models based on NR simulations to show that kicks from precessing binaries can be reliably inferred with LIGO-Virgo operating at their design sensitivity. However, the GW events analyzed in Ref. [29], which only included signals in the first two LIGO-Virgo observing runs [35], were not loud enough to constrain the kick.

Since then, the LIGO-Virgo detectors have been further upgraded, and the GW data from the third observing run were released in two stages, O3a [17] and O3b [18]. Notably, O3a provided the first evidence for precession in the ensemble population of merging binaries [36], even though none of the individual GW events unambiguously exhibited precession [17]. Finally, in O3b, the binary BH merger GW200129 was identified as the first individual GW event showing strong evidence of precession [18, 37].

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Similarly, support for large kicks was identified in the ensemble population using the O3a data [38, 39], even though the individual events were not loud enough for an unambiguous kick inference [29, 40] (with the exception of GW190814 [41], which was found to have a *small* kick of $\lesssim 95$ km/s [34]).

In this Letter, we use the method developed in Ref. [29] to show that GW200129 has a large kick velocity (one-sided limit of $v_f \gtrsim 698$ km/s at 90% credibility). As an application of the kick constraint, we compute the retention probability for the remnant BH of GW200129 in various host environments, and discuss the implications for the formation of heavy stellar-mass BHs. Finally, we show that Doppler effects due to the kick on the remnant mass measurement are small for this event, and should not impact ringdown tests of general relativity (GR).

Methods.— We follow the procedure outlined in Ref. [29] to infer the kick from a GW signal. We begin by measuring the masses and spins of the binary's component BHs, employing Bayes' theorem [42]:

$$p(\lambda|d) \propto \mathcal{L}(d|\lambda) \pi(\lambda),$$
 (1)

where $p(\lambda|d)$ is the posterior probability distribution of the binary parameters λ given the observed data d, $\mathcal{L}(d|\lambda)$ is the likelihood of the data given λ , and $\pi(\lambda)$ is the prior probability distribution for λ . Under the assumption of Gaussian detector noise, the likelihood $\mathcal{L}(d|\lambda)$ can be evaluated for any λ using a gravitational waveform model and the observed data stream d [42]. A stochastic sampling algorithm is then used to draw posterior samples for λ from $p(\lambda|d)$. We use the Parallel Bilby [43] parameter estimation package with the dynesty [44] sampler.

For quasicircular binary BHs, the full set of binary parameters λ is 15 dimensional [18], and includes the masses and spins of the component BHs as well as extrinsic properties like the binary distance and orientation with respect to the observer. We are interested in a subset of these parameters, $\Lambda = \{q, \chi_1, \chi_2\}$ for inferring the kick. Here, index 1 (2) corresponds to the heavier (lighter) BH, with component masses $m_{1,2}$, component spins $\chi_{1,2}$, mass ratio $q = m_1/m_2 \ge 1$, and total mass $M = m_1 + m_2$. $\chi_{1,2}$ are dimensionless spins with magnitudes $\chi_{1,2} \leq 1$, and masses refer to the detector frame redshifted masses. In Eq. (1), we adopt the same prior choices as Ref. [45], with one difference: instead of a distance prior that is uniform in comoving volume (UniformComovingVolume [46]), we choose a prior that is uniform in comoving volume and source frame time (UniformSourceFrame [46]).

We employ the surrogate models NRSur7dq4 [47] and NRSur7dq4Remnant [47, 48] to infer the kick. These models are trained directly on 1528 precessing NR simulations and reproduce the simulations at a comparable accuracy [47]. NRSur7dq4 models the gravitational waveform, while NRSur7dq4Remnant models the mass m_f , spin χ_f , and kick velocity v_f of the remnant BH. These are currently the most accurate models in their regime of

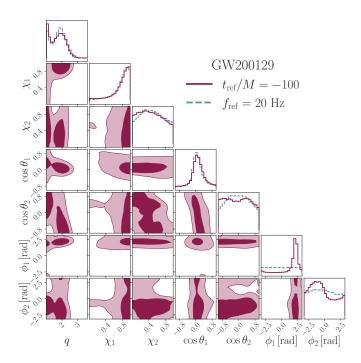


Figure 1. Constraints on the mass ratio and spins for GW200129, at reference time $t_{\rm ref}/M=-100$. The dark (light) regions represent the 50% (90%) credible bounds on joint 2D posteriors, while the diagonal plots show 1D marginalized posteriors. There is a preference for large χ_1 and $\cos\theta_1\sim 0$, meaning there is substantial spin in the orbital plane, which leads to precession. For comparison, we also show the 1D marginalized posteriors at $f_{\rm ref}=20\,{\rm Hz}$. The azimuthal spin angles (especially ϕ_1) are much better constrained at $t_{\rm ref}/M=-100$; this is critical for constraining the kick.

validity [47] (both models are trained on simulations with $q \le 4$ and $\chi_{1,2} \le 0.8$, but can be extrapolated to $q \le 6$ and $\chi_{1,2} \le 1$ [47]). NRSur7dq4 is necessary to accurately measure the spin vectors $\chi_{1,2}$, in particular, the spin orientations within the orbital plane [45], which have a strong influence on the kick [4]. Similarly, given the spin measurements, NRSur7dq4Remnant is necessary to accurately predict the kick velocity [29].

We first use NRSur7dq4 to obtain posterior samples for Λ . Following Ref. [45], we measure the spins at a fixed dimensionless reference time of $t_{\rm ref}/M=-100$ before the peak of the GW amplitude, rather than the traditional choice of a fixed GW frequency of $f_{\rm ref}=20\,{\rm Hz}$ [18]. Spin measurements at $t_{\rm ref}/M=-100$ are convenient for inferring the kick as the NRSur7dq4Remnant model is also trained at this reference time [47]. Finally, given the posterior samples for Λ , we evaluate NRSur7dq4Remnant to obtain the kick posterior. We also compute the effective prior distribution for the kick using prior samples for Λ as inputs for NRSur7dq4Remnant. The difference between the kick posterior and prior can be used to gauge how informative the data are about the kick [29].

GW200129 spin measurements.— GW200129 is the first GW event showing strong signs of precession [18,

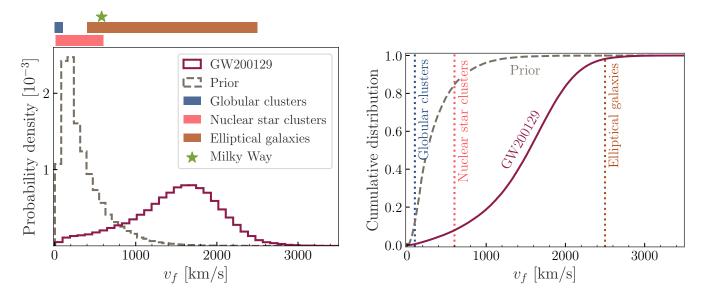


Figure 2. Left: Kick magnitude constraints for GW200129. We show the posterior and the effective prior, along with known ranges for the escape velocities for various types of host environments for comparison. There is a clear preference for large kicks in the posterior, with $v_f \gtrsim 698$ km/s at 90% credibility. Right: Cumulative distribution functions (CDFs) for the kick posterior and prior. The upper bounds of the escape velocity ranges from the left panel are shown as vertical dotted lines. The upper limit for retention probability of the merger remnant is given by the intersection of these lines with the posterior CDF.

37]. Figure 1 shows the posterior distribution for the binary parameters $\Lambda = \{q, \chi_1, \chi_2\}$ obtained using the NRSur7dq4 model at reference points $t_{\rm ref}/M = -100$ and $f_{\rm ref} = 20$ Hz; our constraints at $f_{\rm ref} = 20$ Hz are consistent with those of Ref. [37]. The spins are measured in a source frame defined at the given reference point: the z-axis lies along the instantaneous orbital angular momentum, the x-axis points along the line of separation from the lighter to the heavier BH, and the y-axis completes the right-handed triad. The spin vectors $\chi_{1,2}$ are decomposed into magnitudes $\chi_{1,2}$, tilts angles $\theta_{1,2}$ with respect to the z-axis, and azimuthal angles $\phi_{1,2}$ with respect to the x-axis. Due to precession, spins measurements vary between the two reference points but can be related by a rotation [45, 47].

For both reference points in Fig. 1, there is a clear preference for large orbital-plane spins for the heavier BH (large χ_1 and $\cos\theta_1 \sim 0$). Even though the spin of the lighter BH is not well measured, this is sufficient for precession. We stress that while precessing binaries tend to have larger kicks [4–6], precession does not necessarily imply a large kick, and it is important to directly compute the kick velocity as we do in the next section. In particular, the kick can vary from zero to ~ 5000 km/s just by changing the azimuthal spin angles, even for systems with large orbital-plane spins [4–6].

Next, the azimuthal angles (especially ϕ_1) in Fig. 1 are much better constrained at $t_{\rm ref}/M=-100$, while the other parameters do not change significantly. ¹ This feature is

Spins measured at $f_{\text{ref}} = 20 \,\text{Hz}$ can also be evolved consistently to $t_{\rm ref}/M = -100$ using NRSur7dq4 dynamics [47]. In fact, this procedure is internally applied by NRSur7dq4Remnant if the spins are specified at $f_{\rm ref} = 20 \,\mathrm{Hz}$ [47, 48]. Therefore, by construction, the kick posterior for individual GW events is independent of the reference point at which the spins are initially measured (modulo NRSur7dq4 spin evolution errors, which are small compared to the model errors [47, 49]). Therefore, for the purpose of this paper, the main benefit of the spin measurements at $t_{\rm ref}/M = -100$ is to illustrate why a successful kick constraint is possible in the first place. The supplement of Ref. [38] discusses other benefits, in particular, for constraining the ensemble population of spins and kicks. In the rest of the paper, we will use the spin measurements at $t_{\rm ref}/M = -100$.

GW200129 kick velocity.— Figure 2 shows our constraints on the kick magnitude v_f of GW200129, obtained

key for constraining the kick: even though the azimuthal angles are poorly constrained in the inspiral, they are well constrained near the merger [47] ($t_{\rm ref}/M=-100$ typically falls within $\sim 2-4$ GW cycles before the peak amplitude, independent of the binary parameters [47]). As the kick depends sensitively on the azimuthal angles near the merger [4], successfully measuring these angles at $t_{\rm ref}/M=-100$ is critical for constraining the kick.

¹ The posteriors for χ_1 , χ_2 , and q are expected to be consistent

between the two reference points (modulo parameter estimation uncertainty) as these parameters are independent of the reference point.

by evaluating NRSur7dq4Remnant on the NRSur7dq4 Λ posteriors at $t_{\rm ref}/M=-100$. In the left panel, we show the posterior and prior distributions for v_f , along with fiducial escape velocities for globular clusters [50], nuclear star clusters [50], giant elliptical galaxies [7] and Milky Way-like galaxies [51] for comparison. Unlike the events considered in Ref. [29], the v_f posterior is clearly distinguishable from the prior, and there is substantial information gain about the kick.

The kick magnitude is constrained to $v_f \sim 1542^{+747}_{-1098}$ km/s (median and 90% symmetric credible interval), or $v_f \gtrsim 698$ km/s (lower 10th percentile), making GW200129 the first GW event identified as having a large kick velocity. We note, however, that such large kick velocities are not surprising given previous constraints on the ensemble properties of merging binary BHs [38, 39]. For example, Fig. 3 of Ref. [38], which shows estimates of the ensemble kick distribution, includes nonneglibile support up to $v_f \sim 1500$ km/s.

The large kick of GW200129 raises the question of whether the remnant BH is ejected from its host environment. This has implications for the formation of heavy BHs through second-generation mergers in dense environments [19]. This formation channel is one possible way to explain observations of BHs with masses $\gtrsim 65 M_{\odot}$ [16–18], which fall within the mass gap expected due to the (pulsational) pair-instability supernova processes [14, 15]. To address this, we compute the retention probability for the remnant BH of GW200129 in globular clusters and nuclear star clusters, both of which host dense stellar environments where merger remnants can potentially interact with other BHs and form binaries.

The right panel of Fig. 2 shows the cumulative distribution functions (CDFs) for the v_f posterior and prior. As the posterior $\mathrm{CDF}(v_f)$ denotes the probability that the kick magnitude of GW200129 is below v_f , we take it to be the probability that the remnant BH is retained by a host environment with an escape velocity of v_f . The vertical dotted lines indicate the maximum escape velocity $v_{\rm esc}^{\rm max}$ for various host environments; $CDF(v_{esc}^{max})$ sets the upper limit on the retention probability for that host. In particular, assuming $v_{\rm esc}^{\rm max} = 100 \text{ km/s} (v_{\rm esc}^{\rm max} = 600)$ for globular (nuclear star) clusters there is a less than 0.48% (7.7%) probability that the remnant BH of GW200129 is retained by those hosts. This is consistent with Refs. [34, 38, 39], where globular clusters were already identified as an unlikely site for second generation mergers, even for more moderate kicks.

Remnant mass and Doppler shifts.— Our method provides predictions for both the magnitude and direction of the kick [29]. If the kick vector v_f has a significant component along (or opposite) the line-of-sight, the observed GW signal can be influenced by the kick. At leading order, the kick's effect can be described as a Doppler shift of the GW frequency [31]. However, as GR lacks any intrinsic length scales, a uniform increase in signal frequency is completely degenerate with a decrease in total mass M, and vice versa. Thus, if not explicitly accounted for, a

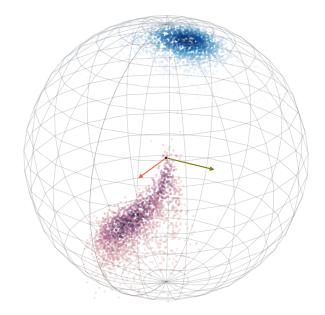


Figure 3. Posterior samples for the full kick vector v_f in the source frame at $t_{\rm ref}/M=-100$. Each purple marker indicates a kick posterior sample; an arrow drawn from the origin to the marker would show the kick vector v_f . The outer radius of the sphere corresponds to $v_f=2500$ km/s. The x-axis (orange) and y-axis (green) are shown as arrows near the origin; the x-y plane is orthogonal to the orbital angular momentum direction. The blue markers on the sphere show posterior samples for the line-of-sight direction to the observer. For both distributions, the color reflects posterior probability density. A rotating perspective of this plot can be seen at vijayvarma392.github.io/GW200129/#kick.

frequency shift due to a kick can bias mass measurements. In particular, because the kick is mostly imparted near the merger [2], the Doppler shift only affects the merger and ringdown part of the signal. This can lead to biases in the measurement of the remnant mass m_f [29, 52], and potentially impact tests of GR using the ringdown signal [53]. However, this effect is expected to be small for current detectors [29, 31].

In the following, we verify that the Doppler effect on the remnant mass measurement of GW200129 is indeed small. At leading order, the Doppler-shifted remnant mass is given by [31]:

$$m_f^{\rm DS} = m_f \left(1 + \boldsymbol{v}_f \cdot \hat{\boldsymbol{n}}/c \right),$$
 (2)

where c is the speed of light and $\hat{\boldsymbol{n}}$ is the unit vector pointing along the line-of-sight from the observer to the source. The line-of-sight direction is obtained from our inference setup, parameterized by (ι, ϕ) . ι is the inclination angle between the orbital angular momentum and the line-of-sight to the observer, and ϕ is the azimuthal angle to the observer in the orbital plane, both defined in the source frame at $t_{\rm ref}/M=-100$.

Figure 3 shows the posterior distributions for the full kick vector v_f (also defined in the source frame at

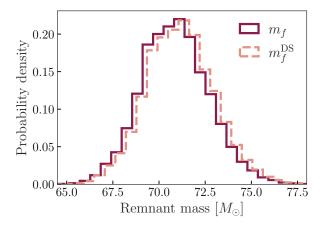


Figure 4. The remnant mass and the Doppler shifted remnant mass (estimated according to Eq. (2)) for GW200129. There is an overall redshift, as the kick direction in Fig. 3 is pointed (roughly) away from the observer. However, as these distributions are very close, we do not expect ringdown tests of GR to be impacted by the kick for this event.

 $t_{\rm ref}/M=-100)$ and the line-of-sight direction. We find that the kick and the line-of-sight are not very well (anti-)aligned; therefore, we do not expect significant Doppler shifts for this signal. Finally, Fig. 4 shows the posterior distributions for m_f (obtained from NRSur7dq4Remnant) and $m_f^{\rm DS}$ (computed using Eq. (2)) for GW200129. As expected, the difference between these distributions is very small, meaning that tests of GR should not be impacted by the Doppler effect for this event. As detector sensitivity improves, this may not be the case, however, and it may be necessary to explicitly account for this effect [29].

Conclusions.— We use NR surrogate models for the gravitational waveform and the remnant BH properties to infer the kick velocity for the binary BH merger GW200129. The kick magnitude is constrained to $v_f \sim 1542^{+747}_{-1098}$ km/s or $v_f \gtrsim 698$ km/s, at 90% credibility. Given the kick velocity, we estimate that there is at most a 0.48% (7.7%) probability that the remnant BH of GW200129 would be retained by globular (nuclear star) clusters. Finally, we show that the Doppler effect on the remnant mass is small compared to current measurement

uncertainty; therefore ringdown tests of GR are not expected to be significantly impacted by the kick for this event.

Observational evidence for kicks has far reaching implications for BH astrophysics. GW200129 is the first GW event identified as having a large kick velocity. Large kicks like this have been previously predicted based on the ensemble kick distribution of merging binary BHs [38, 39], and we can expect to see more such events as detector sensitivity improves. In particular, such observations can help resolve the mystery of the heavy BHs seen by LIGO-Virgo [16–18], by constraining the rate of second-generation mergers.

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