COHERENCE LENGTH, COHERENCE WIDTH AND THE HELIUM ATOM MICROSCOPE

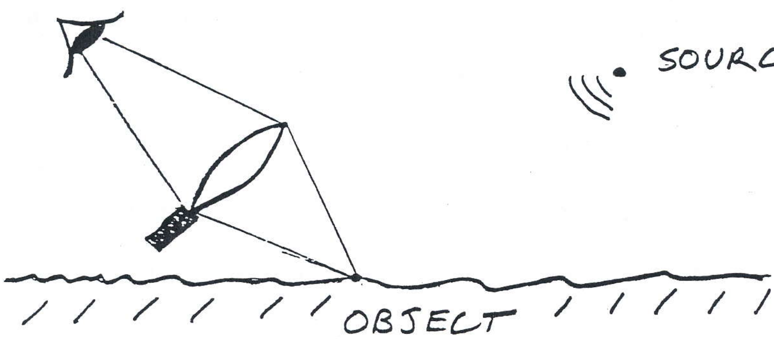
I start this discusion by quoting the first few sentences by H. M. Young on page 96 of his book "Optics and lasers" [1], to highlight a few points which can result in confusion with respect to the concepts of coherence:

" Until now, we have always assumed light to be completely *coherent*, in the sense that any interference experiment resulted in high quality interference fringes. In general, this is not the case, except with certain laser sources; the light from most sources is said to be *incoherent* or *partially coherent*. When conditions are such that light is incoherent, it is not possible to detect interference effects. A discussion of wave optics is incomplete without considering the conditions that must exist for an interference experiment to be performed successfully".

Athough there is nothing strictly wrong in the above discussion, I believe that to only discuss the concept of coherence with respect to the narrow range of experiments involving interference fringes, as is often done in text books, can lead to a misunderstanding of it's general relevance to wave experiments. I wish to therefore start by emphasising that the concept of coherence is applicable to <u>all</u> experiments involving waves, irrespective of whether interference fringes are generated or not. Secondly, I feel that to adopt the definition of an interference experiment as only one in which fringes are observed can also lead to difficulties in the understanding of coherence. The fact is that the results of <u>every</u> wave experiment are due to interference, again <u>irrespective</u> of whether fringes are present or not. Thus the effects of interference are always detected in an experiment, regardless of whether the illumination is coherent or incoherent. The quality of achieved experimental results however, depends upon the details of the wave interference and may indeed depend upon the coherence of the radiation field, but not necessarily so as will be later shown through example.

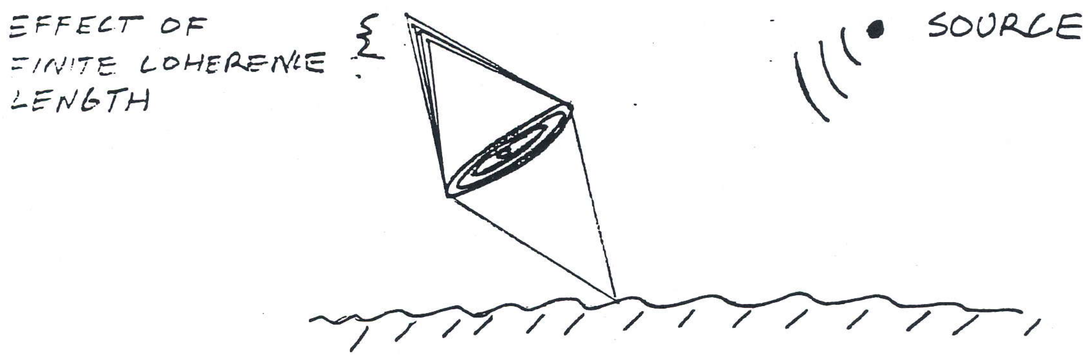
With the above remarks in mind, the important question to be answered then is: " What does the term coherence actually mean and what effect, if any, does it have on the quality of results in a given experiment ? ". Coherence effects are usually partitioned into two classifications, *temporal* and *spatial.* Spatial coherence (or coherence width) is defined to be a measure of the effects of finite band width of the radiation field on experiment. Temporal coherence (coherence length), on the other hand, is a measure of the effects of finite source size on the momentum resolution in a scattering experiment and depends also on the source to target distance. That is all all there is to it !! Coherence is just a convenient way to speak about the effects on experiment of finite source size and of finite beam band width. Nothing more ! The important point to note is this: If, for a given experiment, the results are independent of source size, then spatial coherence plays no role. If the results are independent of the degree of beam monochromaticity, then the temporal coherence of the beam plays is unimportant. Coherence is therefore not necessarily a prerequisite for a good experiment !! Here are some concrete examples:

EXAMPLE 1 : MAGNIFYING GLASS



Consider the case of an aberration free lens used to image a surface. A lens has the unique property that all imaging particles (photons in this case) leaving a particular point on the target are mapped onto a unique point in the image plane, irrespective of their momentum leaving the surface (assuming, of course, that they pass through the lens). Thus the position of the illuminating light source, or the finite solid angle it subtends at a point on the target (it's finite spatial extent), which together influence the angles at which scattered particles leave the target surface, do not affect image resolution. Futhermore, to the extent that the refractive index of glass and hence it's focal length is practically constant over the visible spectrum, the resolution of the magnifying glass is independent of the wavelength of the illuminating radiation. In other words, the resolution of the magnifying glass is independent of both the spatial and temporal coherence of the radiation field.

EXAMPLE 2 : THE FRESNEL ZONE PLATE

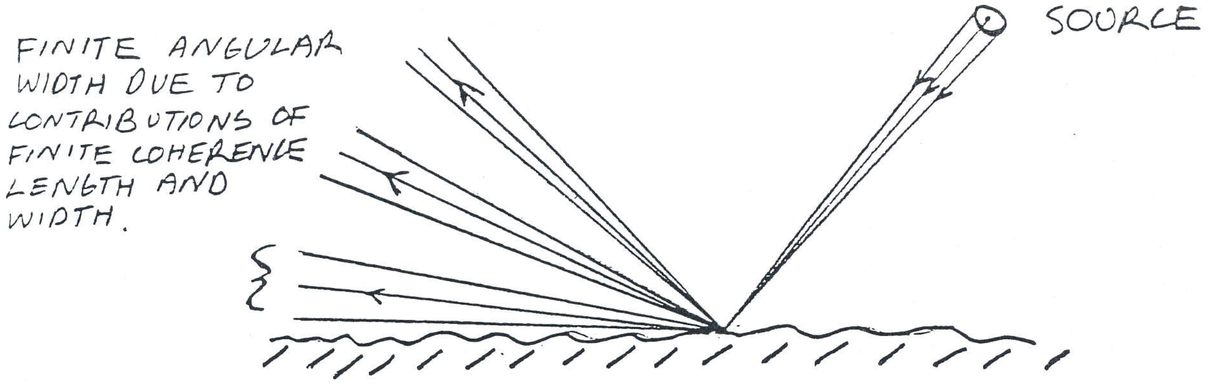


In this example, a Fresnel lens is used to image a surface. The arguments are essentially the same as above except that for this particular lens type, the focal length is strongly wavelength dependent. Thus the illuminating radiation must be highly monochromatic to ensure good image resolution. In other words, the resolution is independent of the spatial coherence of the illuminating radiation, but strongly dependent upon it's temporal coherence. Calculations [2] show that the required beam monochromaticity to obtain good focus with a Fresnel lens is given by the expression:

$$\frac{\lambda}{\delta\lambda} \simeq n \cdot m$$

where n is the number of rings and m the order of diffraction image.

EXAMPLE 3 : SURFACE DIFFRACTION EXPERIMENTS



In this example the target is illuminated by quasi-monochromatic radiation derived from a distant quasi-point source of radiation. Here both the finite bandwidth of the illuminating radiation and the finite source size conspire to smear out the diffraction pattern measured at a distant detector. Thus the quality of information obtained concerning the target surface, which is derived from the diffraction pattern of scattered radiation, is strongly dependent on both the spatial and temporal coherence of the illuminating radiation. The minimum spatial separation which can be resolved in the target, due to the combined momentum degrading effects of both finite source size and finite beam bandwidth, is often referred to in literature as the "transfer width" of a particular instrument.

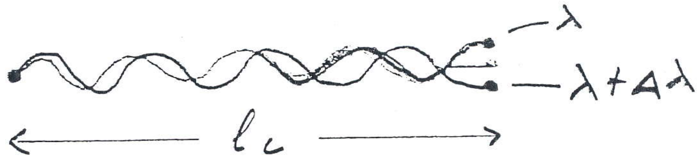
COHERENCE LENGTH - MATHEMATICAL BASIS

Consider a wavetrain of finite bandwidth. It can be represented as a superposition of harmonic waves of different wavelengths and different relative phases. Let λ and $\lambda + \delta\lambda$ represent the extreme wavelengths of it's frequency band. Consider first a point in space where these two components are in phase. As one proceeds away from this point along the wave propagation direction, then the two components become progressively more out of phase until after a distance l_c, defined as the *coherence length* , they are 180° out of phase with one another.

At this point:

$$m \lambda = l_c \qquad (1)$$

$$(m + 1/2)(\lambda + \delta\lambda) = l_c \qquad (2)$$

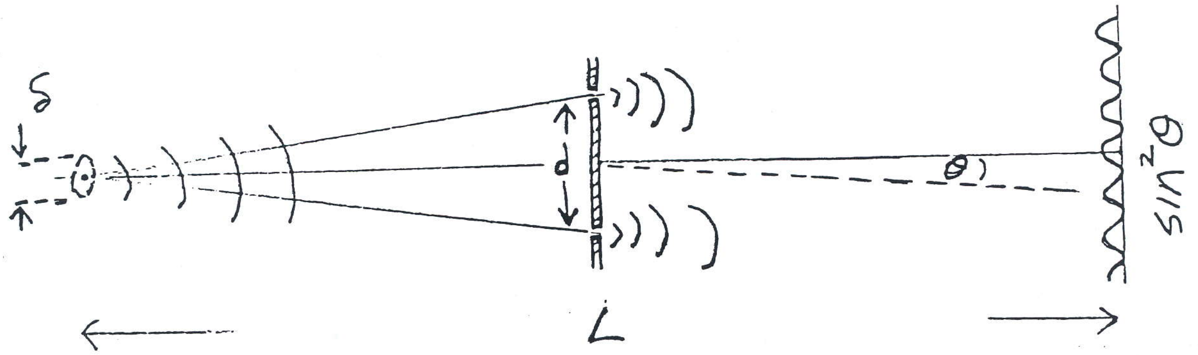


Combining (1) and (2) for the case that that $\delta\lambda \ll \lambda$, the coherence length l_c is given by the following expression:

$$l_c = \frac{\lambda^2}{2\,\delta\lambda}$$

COHERENCE WIDTH - MATHEMATICAL BASIS

Without going through a rigorous analysis of the problem. I will briefly describe how one can derive a formula for coherence width and what relevance it has to atom scattering experiments of the sort performed at the MPI, Göttingen. Consider the double slit experiment drawn below, where a point source is used to produce a series of interference fringes upon a distant screen.



The interference pattern at the image plane Σ is a $cos^2\theta$ function with maxima occuring at angles given by the expression:

$$m \lambda = d \sin\theta$$

Suppose the point source is now replaced by an extended source. Each infintesimal region of the extended source will produce it's own $cos^2\theta$ pattern shifted with respect to all others. The problem can then be viewed in two ways. The first, to ask what is the maximum source size δ permissible before interference fringes become completely unobservable. The second, which I shall now persue, is to ask what is the maximum permissible spatial separation between the slits, for a given source size, if the interference fringes are not to be completely smeared out (remembering that the fringes become more spread out as the slits become closer together). It can be shown [1] that the maximum permissible slit separation to satisfy this requirement is given by the formula:

$$d_c = \frac{0.16 \lambda L}{\delta}$$

So much for the double slit experiment, what relevance does this analysis have to helium scattering experiments ? Clearly, the results of the above discussion are the same if the slits are replaced by two infintesimal mirrors (superimposed upon an absorbing background) and the pattern observed at the source plane (neglecting the small region around the source). This then represents a simple model for a scattering experiment, where we seek to resolve two objects on a surface separated by a distance d. Of course atom scattering measurement are not normally performed at normal incidence as in this example and so the above expression has to be modified somewhat to account for this fact. Nevertheless, in this simple model, taking a source size $\delta = .5\ mm$, $\lambda = 2$ Å and a source to screen distance $L = .25\ m$, we obtain a spatial coherence $d_c = 160$ Å. This means that in this example, only structures on the target surface of spatial separation d less than 160 Å can be resolved due to the mixing of phase information resulting from the effect of finite source size.

SUMMARY

The concepts of temporal and spatial coherence are convenient ways of talking about the effect, on experiment, of finite source size and the finite bandwidth of the radiation field generated. That is all. The bearing these quantities have on the results of experiment, if any, is highly experiment dependent and must be assessed individually in each particular case.

REFERENCES

[1] M. Young. " Optics and Lasers ", Springer Series in Optical Sciences Volume **5**, Editor D. L. MacAdam, Springer-Verlag (1977) pgs. 96-105.

[2] G. Schmall, D. Rudolph, P. Guttmann and O. Christ, "Zone Plates for X-Ray Microscopy". Springer Series in Optical Sciences Vol. **43**. Editors G. Schmahl and D. Rudolph (1984) pg. 63.