Supplementary Material: Collective narratives catalyse cooperation

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Evolutionary dynamics in an infinite population

In total, there are eight different strategies. The strategies can be enumerated by the generic profile $(a_1^\star, a_2^\star, u^\star)$ where $a_i^\star$ is the hunting strategy of the focal individual ($\star$) when in group consensus i and $u^\star$ is the preferred narrative of the focal individual (in the following the two narratives are called simply 1 and 2). The individuals in the tribe form a group of size G, and they need to reach a consensus on the group belief.

Group decision For the group to decide between, 1 or 2, we use a frequency-dependent process (for other methods of consensus narrative resolution, see below). The group thus choose to believe in 1 with probability,

$$f(k, u^\star) = \frac{k + \delta_{u^\star}}{G}. \qquad \text{(SI.1)}$$

and in 2 with probability $1 - f(k, u^\star)$. The number of individuals, besides the focal, who believe in 1 is denoted by k. The Kronecker delta $\delta_{u^\star}$ returns 1 if the focal individual prefers 1 (i.e. if $u^\star = 1$) and 0 otherwise.

Individual action. After the group consensus is reached, each individual in the group chooses the appropriate action. In 1, the focal individual is a hare hunter if $\delta_{a_1^\star}$ return 1 and a stag hunter otherwise. A similar reasoning works for 2 ($\delta_{a_2^\star}$).

Values of Hares and Stags. The value of hunting a Hare is denoted by P_H. The total probability that an individual gets a hare payoff is then denoted by

$$\Pi_H = P_H \sum_{k=0}^{G-1} \binom{G-1}{k} x_1^k (1 - x_1)^{G-1-k} (f(k, u^\star)\delta_{a_1^\star} + (1 - f(k, u^\star))\delta_{a_2^\star}) \qquad \text{(SI.2)}$$

where $x_1 = x_{HH1} + x_{HS1} + x_{SH1} + x_{SS1}$ the sum of the frequencies of individuals believe in 1 and thus $1 - x_1 = x_{HH2} + x_{HS2} + x_{SH2} + x_{SS2}$, the 2 believers. The value of a stag is given by P_S. The focal individual is a stag hunter according to the **Individual action** section. The group composition is a key determinant of the stag payoff since there is a minimum number of stag hunters necessary (M) for successful stag hunt. Besides the the focal individual, k individuals believe in 1 and $G - 1 - k$ in 2. However we need to sort how many of these individuals are stag hunters. The composition of the group is then denoted by,

$$P_{comp} = \sum_{\substack{l=0\\m=0\\n=0\\o=0}}^{k} \sum_{\substack{p=0\\q=0\\r=0\\s=0}}^{G-1-k} \binom{k}{l,m,n,o}\binom{G-1-k}{p,q,r,s} x_{HH1}^{l} x_{HS1}^{m} x_{SH1}^{n} x_{SS1}^{o} \times x_{HH2}^{p} x_{HS2}^{q} x_{SH2}^{r} x_{SS2}^{s} \chi(u^{\star})[Q(l,m,n,o,p,q,r,s)] \quad \text{(SI.3)}$$

where the Iverson bracket (Knuth, 1992) is used to test the statement $Q = (l+m+n+o=k) \wedge (p+q+r+s=G-1-k)$ with,

$$[Q(l,m,n,o,p,q,r,s)] = \begin{cases} 1, & \text{if } Q \text{ is true} \\ 0, & \text{otherwise.} \end{cases} \quad \text{(SI.4)}$$

The function $\chi(u^{\star})$ is a step function which (when $\chi(u^{\star}=1)$) ascertains if the focal individual prefers 1 and returns the function $\theta(1+n+o+r+s-M)$ (checking if the number of stag hunters meet the required threshold M). If $u^{\star}=2$ then the focal individual believes in 2 and $\chi(u^{\star})$ returns $\theta(1+m+o+q+s-M)$ (again checking if the number of stag hunters meets the required threshold M). Putting P_{comp} together with the rest of the probabilities we get the probability of successfully hunting a stag as,

$$\Pi_S = P_S \sum_{k=0}^{G-1} \binom{G-1}{k} P_{comp}[f(k,u^{\star})(1-\delta_{a_1^{\star}}) + (1-f(k,u^{\star}))(1-\delta_{a_2^{\star}})] \quad \text{(SI.5)}$$

The average payoff of an individual with strategy $(a_1^{\star}, a_2^{\star}, u^{\star})$ is then given simply by,

$$\pi_{(a_1^{\star},a_2^{\star},u^{\star})} = \Pi_H + \Pi_S \quad \text{(SI.6)}$$

The population dynamics can then be represented by the set of replicator equations (Hofbauer and Sigmund, 1998),

$$\dot{x}_i = x_i(\pi_i - \bar{\pi}) \quad \text{(SI.7)}$$

for each strategy i. There are eight possible strategies and hence the dynamics resides in a seven-dimensional simplex.

Dynamics between pure states The eight vertices of the simplex represent the pure strategies, homogeneous states where all individuals play the same strategy. We

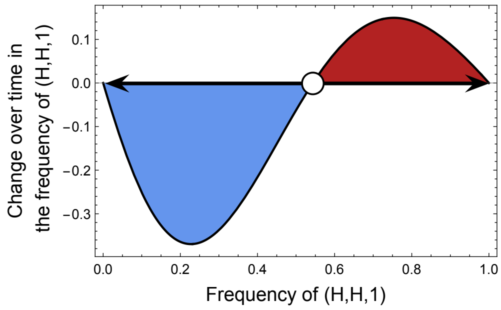

Figure SI.1: Dynamics on the edge between the pure strategies $(H, H, 1)$ and $(S, H, 1)$. If only a small fraction of the population plays the (H,H,1) strategy in a population predominantly composed of (S,H,1) individuals then the group form will consists mostly of stag hunters. While (S,H,1) individuals hunt stag it will be a stable strategy. If the number of (H,H,1) individuals is above the unstable threshold then stag hunting is not viable since the minimum number of stag hunters required for a successful hunt will not be present but the (H,H,1) individuals thrive. Parameters are: $G = 5, M = 4, P_S = 4; P_H = 1$.

study the dynamics between all the pairwise combinations of these pure states. Assume a population which can have only $(H, H, 1)$ and $(S, H, 1)$ individuals. For a group size $G = 5$ with a threshold number of stag hunters required for a successful hunt set at $M = 4$ a stag provide a payoff of 4 while a hare is worth 1. Using these values the average payoff of a $(H, H, 1)$ strategist is simply $\pi_{HH1} = (x_{HH1} + x_{SH1})^4$. The average payoff for a $(S, H, 1)$ player on the other hand is,

$$\pi_{\text{SH1}} = 4\left(4x_{\text{HH1}}x_{\text{SH1}}^3 + x_{\text{SH1}}^4\right) \tag{SI.8}$$

Plotting the replicator equation for a population of just these two types gives us Figure SI.1. In this manner we can describe the dynamics between all the eight vertices, as shown in the main text.

Evolutionary dynamics in finite populations

We assume a finite population of size N. From this population we choose individuals to form a group of size G. If the number of individuals with strategy j is given by i_j encapsulated in the vector $\mathbf{i}$, then the average payoff of a strategy $(a_1^\star, a_2^\star, u^\star)$ is given by,

$$\begin{aligned}\pi_{(a_1^\star,a_2^\star,u^\star)} &= \sum_{k=0}^{G-1}\Bigg(P_H\frac{\binom{i_1i_3i_5i_7}{k}\binom{i_2i_4i_6i_8}{G-1-k}}{\binom{N-1}{G-1}}(f(k,u^\star)\delta_{a_1^\star}+(1-f(k,u^\star))\delta_{a_2^\star}) \qquad \text{(SI.9)}\\ &\quad +P_SP_{comp}(k,u^\star,\mathbf{i})(f(k,u^\star)(1-\delta_{a_1^\star})+(1-f(k,u^\star))(1-\delta_{a_2^\star}))\Bigg).\end{aligned}$$

The composition of the group P_{comp} is reevaluated for finite populations as,

$$P_{comp} = \sum_{\substack{l=0\\m=0\\n=0\\o=0}}^{k}\sum_{\substack{p=0\\q=0\\r=0\\s=0}}^{G-1-k}\frac{\binom{i_1}{l}\binom{i_2}{p}\binom{i_3}{m}\binom{i_4}{q}\binom{i_5}{n}\binom{i_6}{r}\binom{i_7}{o}\binom{i_8}{s}}{\binom{N-1}{G-1}}\chi(u^\star)[Q(l,m,n,o,p,q,r,s)] \qquad \text{(SI.10)}$$

again with Q as defined in Eq. (SI.4). With this approach we can calculate the average payoff of each strategy when playing with another strategy. However for finite populations, we convert the payoff π_i of a strategy i to its fitness ψ_i via a mapping of the form $\psi_i = 1 + \omega\pi_i$ where i encompasses the strategies encoded by $(a_1^\star, a_2^\star, u^\star)$. Such a combination with ω allows us to tune the impact of the game on the fitness (Traulsen and Hauert, 2009). If ω the selection intensity is very low $\omega \to 0$ then the strategies are neutral with respect to each other. Evolutionary dynamics would then be a random walk between the eight strategies. On the other hand for $\omega \to 1$ the game completely determines the difference between the strategy fitness. All of this definitely assumes that the strategies do not go extinct, i.e. the mutation probability is non-zero $\mu > 0$.

Assuming small mutation rates $\mu \to 0$, the dynamics typically takes place between two strategies only. Hence a pairwise comparison of the fitnesses of the strategies proves to be instructive. The fitness of a strategy i playing against strategy j is given by $\psi_{i,j}$ This allows us to calculate the fixation probability of a single strategy i player in a population of $N-1$ strategy j players as,

$$\rho_{i,j} = \frac{1}{\sum_{k=1}^{N-1} \prod_{m=1}^{k} \frac{\psi_{j,i}}{\psi_{i,j}}} \tag{SI.11}$$

Collating the fixation probabilities between all pairwise combinations provides us with the following transition matrix $\mathbf{A}$,

$$\mathbf{A} = \begin{pmatrix} 1-\sum\text{column} & \rho_{1,2} & \rho_{1,3} & \rho_{1,4} & \rho_{1,5} & \rho_{1,6} & \rho_{1,7} & \rho_{1,8} \\ \rho_{2,1} & 1-\sum\text{column} & \rho_{2,3} & \rho_{2,4} & \rho_{2,5} & \rho_{2,6} & \rho_{2,7} & \rho_{2,8} \\ \rho_{3,1} & \rho_{3,2} & 1-\sum\text{column} & \rho_{3,4} & \rho_{3,5} & \rho_{3,6} & \rho_{3,7} & \rho_{3,8} \\ \rho_{4,1} & \rho_{4,2} & \rho_{4,3} & 1-\sum\text{column} & \rho_{4,5} & \rho_{4,6} & \rho_{4,7} & \rho_{4,8} \\ \rho_{5,1} & \rho_{5,2} & \rho_{5,3} & \rho_{5,4} & 1-\sum\text{column} & \rho_{5,6} & \rho_{5,7} & \rho_{5,8} \\ \rho_{6,1} & \rho_{6,2} & \rho_{6,3} & \rho_{6,4} & \rho_{6,5} & 1-\sum\text{column} & \rho_{6,7} & \rho_{6,8} \\ \rho_{7,1} & \rho_{7,2} & \rho_{7,3} & \rho_{7,4} & \rho_{7,5} & \rho_{7,6} & 1-\sum\text{column} & \rho_{7,8} \\ \rho_{8,1} & \rho_{8,2} & \rho_{8,3} & \rho_{8,4} & \rho_{8,5} & \rho_{8,6} & \rho_{8,7} & 1-\sum\text{column} \end{pmatrix} \tag{SI.12}$$

The normalised right eigenvector of $\mathbf{A}$ corresponding to the largest eigenvalue (which is 1) provides the stationary distribution of the system (Hauert et al., 2007). This analytical result is plotted as full lines in Figure SI.2 for a given choice of parameters.

Costly beliefs

If generating and believing in a new narrative, 2, is cognitively costly, the individuals who prefer 2 would pay a cognitive cost. As the cost increases indeed the new narrative will be harder to fix in the population. However the belief in the new narrative still acts as a catalyst Figure SI.5. It appears in a finite population by chance but spreads as it is still better to hunt stags than hares. However when everyone is hunting stags, the cognitive cost of maintaining the belief reduces the frequency of the believers. For increasing costs clearly the belief declines however the population is left transformed in a stag equilibrium.

Magnitude of payoffs

In finite populations the magnitude of the payoffs obtained from the interactions is crucial in determining the long term outcome of the strategy proportions in a populations.

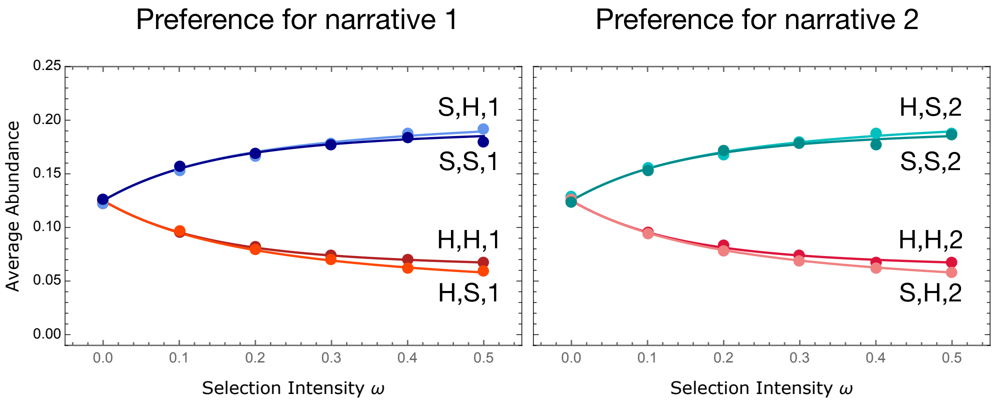

Figure SI.2: Average abundance in the long run. In the long run the strategies in the population stabilise at the proportions which can be calculated analytically (lines) and the results supported by individual based simulations (symbols). For a population of size 16, and a small mutation probability of $\mu = 10^{-3}$, the average abundance of the eight different strategies is denoted above for a variety of selection intensities (after 2×10^9 time-steps). The fitness of each type i is given by $\psi_i = 1 + \omega\pi_i$, where ω is the selection intensity. For $\omega = 0$ selection is neutral and all strategies exist in equal proportions ($1/8^{\text{th}} = 0.125$). As selection increases, we see the prevalence of the stag hunters in the population, irrespective of their belief. Parameters are $N = 16$, $G = 5$, $M = 4$, $P_S = 4$ and $P_H = 1$.

Numerous papers in evolutionary game dynamics have focused on the differences between the infinite and finite population differences (Taylor et al., 2004; Nowak et al., 2004). While a full analysis for our model is not considered here, it is available in the GitHub folder where the codes are deposited. The difference in the eventual outcome of the strategy distributions for different values of the stag and across selection intensities is denoted in Figure SI.6. For selection intensity $\omega = 1$ we show the corresponding matrix of pairwise fixation probabilities. For a population of size $N = 16$ the entries of the matrix represent the fixation probability of a single *row* strategy player in 15 individuals of *column* strategy players (normalised by the neutral fixation probability of $\rho_N = 1/16$).

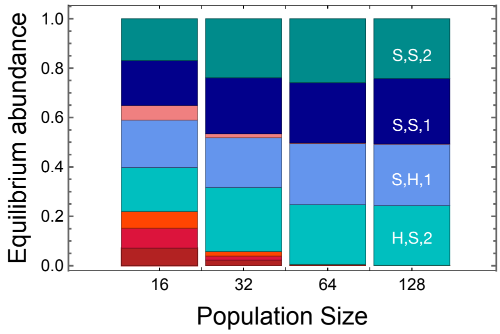

Figure SI.3: Equilibrium abundance across population sizes. For increasing population size, the effect of drift gets diluted and the deterministic equilibrium of the system emerges, which is composed of individuals hunting stags. Simulation parameters besides the changing population size are $\omega = 0.4$, $G = 5$, $M = 4$, $P_S = 4$ and $P_H = 1$ with $\mu = 10^{-3}$. The system status is reported after 5×10^6 time-steps.

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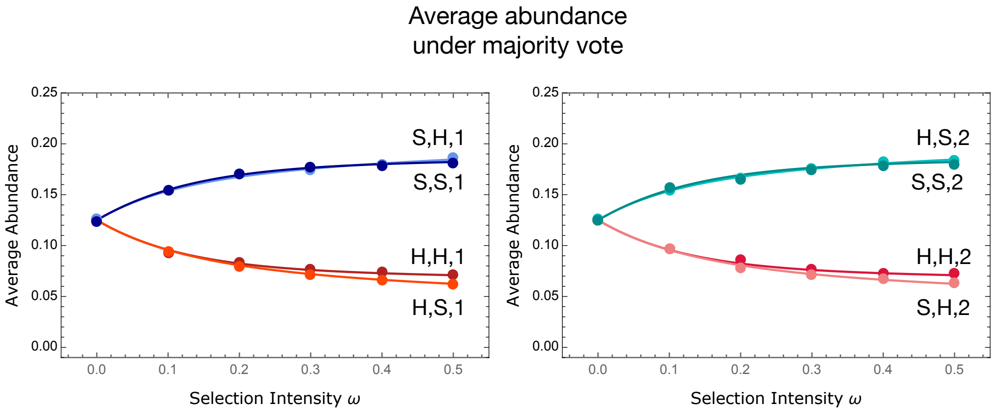

Figure SI.4: Average abundance in the long run under voting. In the long run the strategies in the population stabilise at the proportions which can be calculated analytically (lines) and the results supported by individual based simulations (symbols). For a population of size 16, and a small mutation probability of $\mu = 10^{-3}$, the average abundance of the eight different strategies is denoted above for a variety of selection intensities (after 2×10^9 time-steps). The fitness of each type i is given by $\psi_i = 1 + \omega\pi_i$, where ω is the selection intensity. For $\omega = 0$ selection is neutral and all strategies exist in equal proportions ($1/8^{\text{th}} = 0.125$). As selection increases, we see the prevalence of the stag hunters in the population, irrespective of their belief. Parameters are $N = 16$, $G = 5$, $M = 4$, $P_S = 4$ and $P_H = 1$.

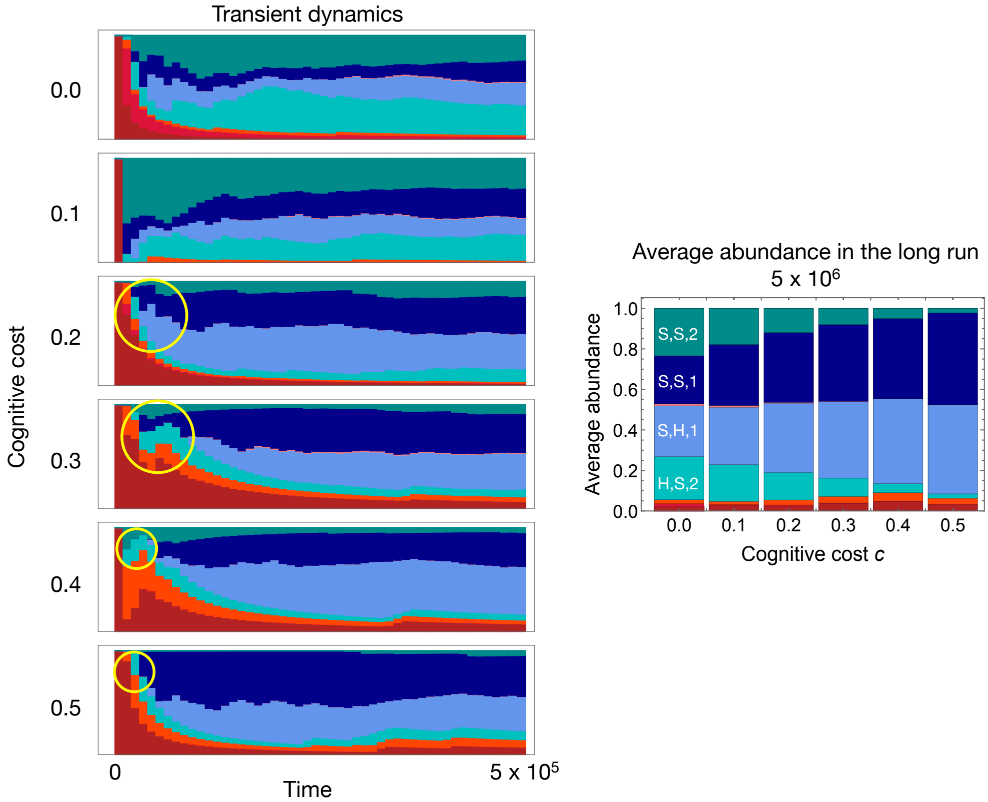

Figure SI.5: Costly beliefs. Even if the preference for the alternative belief accrues a cognitive cost, we show that it helps transform the population into a social group where everyone prefers to hunt stags. The alternative belief acts as a stepping stone (highlighted in the transients on the left by the circles), where the belief in 2 enables the spread of stag hunter who believe in 1. Thus acting as a true catalyst, the belief helps transform the population and then disappears. The dynamics of the eight strategies for different levels of cognitive costs is shown for a finite population of size 16, and a small mutation probability of $\mu = 10^{-3}$. The equilibrium average abundance of the eight different strategies, (which can also be calculated analytically) is shown in the right panel for a variety of cognitive costs (after 5×10^{6} time-steps). As selection increases, we see the prevalence of the stag hunters in the population, irrespective of their belief. Parameters are $N = 16$, $G = 5$, $M = 4$, $P_S = 4$ and $P_H = 1$. The selection intensity is set to $\omega = 0.5$.

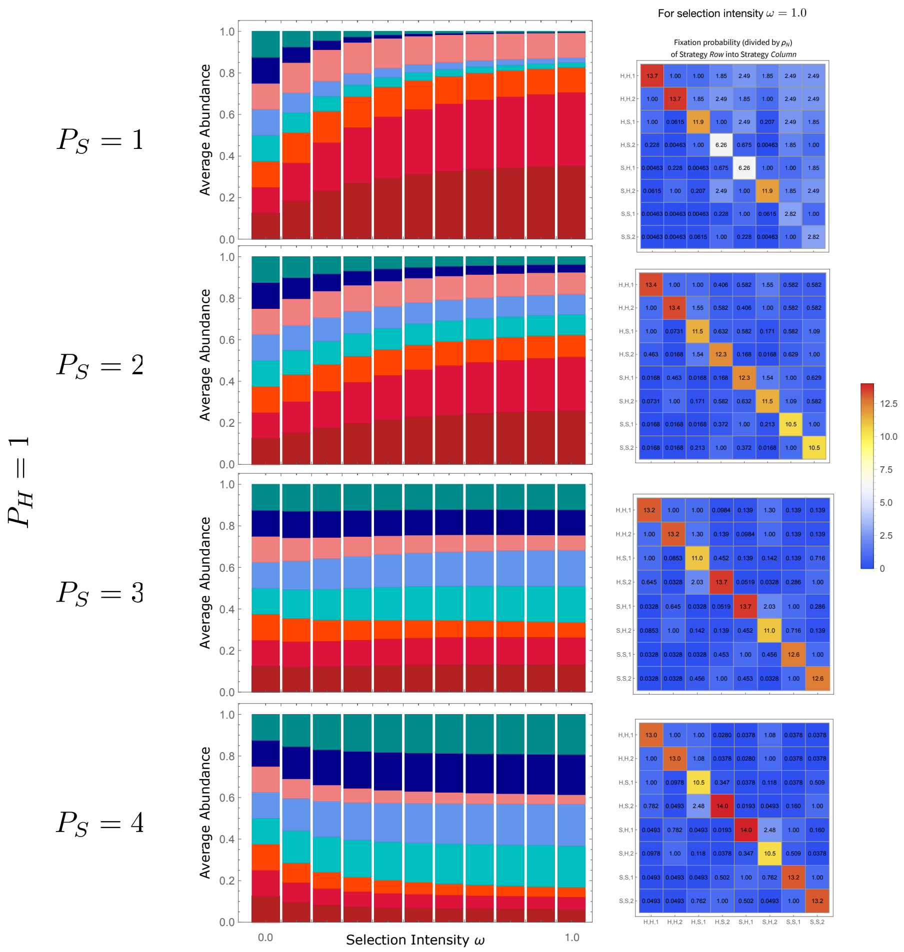

Figure SI.6: How much is a stag worth? Payoff magnitude in finite populations The column on the left shows the analytical result of the average abundance of strategies as a function of the selection intensity ω. When $\omega = 0$ all the strategies are identical and hence reach equal proportions in the long run. However as the selection intensity increases, the impact of the game is observed in the final distribution of the strategies. As we move down the rows, the stags get bigger. That is, the value of a stag increases from 1 (which is equal to P_H) to 4 (as used in the rest of the manuscript). For smaller values of P_S the gains of forming a successful hunting group are not larger than the hare payoffs but in fact also incur losses when forming groups with inadequate number of hunters. The results for large selection intensities can be understood by looking at the corresponding matrix of fixation probabilities as shown in the right column. From the values of the fixation probabilities (normalised by the neutral fixation probability of $1/N$) we see that the probability of staying in the hare equilibrium is larger for low P_S and moves to the stag equilibrium with increasing P_S. Parameters are $N = 16$, $G = 5$, $M = 4$, and $P_H = 1$.