

Interface to high-performance periodic coupled-cluster theory calculations with atom-centered, localized basis functions

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Summary

A major part of computational materials science and computational chemistry concerns calculations of total energy differences and electronic excitations of poly-atomic systems. Currently, the most prevalent method for such computations is density-functional theory (Hohenberg & Kohn, 1964) (DFT) based on the Kohn-Sham formalism (Kohn & Sham, 1965) (KS) together with one of the numerous exchange-correlation (xc) approximations (Civalleri et al., 2012; Sousa et al., 2007). The trade-off between comparably low computational cost and often reliably accurate results render it the dominant method in the field. Often, however, the accuracy of the xc approximations is not sufficient and uncertain, in particular when electronic correlations play a decisive role (Savin & Johnson, 2014; Zhang et al., 2016; Zhang & Grüneis, 2019).

Coupled-cluster (CC) methods (Čížek, 1966), while substantially more computationally expensive have proven to be significantly more accurate and reliable, at least for molecules and non-metallic solids. On these grounds, they allow for a systematic accuracy benchmark of other methods. Indeed, in molecular quantum chemistry the CC approach is considered the gold standard for a theoretical description of binding energies and electronic properties (Chan, 2019). It is typically used (at least) for benchmark studies. While the original CC methodology allows to calculate the groundstate total energy of a system, it is also possible to compute properties of excited states using the equation-of-motion formalism of CC (EOM-CC) with comparable accuracy and reliability (Stanton & Bartlett, 1993). The conventional CC approach is limited, however, to systems with about 50 electrons (Feller & Dixon, 2001; Gyevi-Nagy et al., 2019). By employing approximations, which exploit the locality of electronic correlation, materials with a couple of hundreds of electrons have been calculated via local natural orbital CC (LNO-CC) (Nagy & Kállay, 2019) and explicitly correlated pair natural orbital CC (PNO-CC-F12) (Ma & Werner, 2021). The extension of CC to periodic systems (Hirata et al., 2004) has been explored to a great degree by the research groups of [Andreas Grüneis](#) and [Garnet Chan](#).

In the work of [Andreas Grüneis et al.](#) the periodic formulation of CC was, for example, applied to study the adsorption behavior and surface chemistry of 2-dimensional materials including graphene (Al-Hamdani et al., 2017), boron-nitride (Brandenburg et al., 2019) and surfaces (Tsatsoulis et al., 2018). These studies showed that CC yields consistently accurate adsorption energies and reaction energetics. The work of the groups of [Garnet Chan](#) and [Timothy Berkelbach](#) addressed the electronic properties of 2- and 3-dimensional materials. By applying the periodic equation-of-motion (EOM) CC formalism, band paths, optical spectra and band gaps of various materials (diamond, silicon, nickel oxide and others) have been investigated (Gao et al., 2020; McClain et al., 2017; Wang & Berkelbach, 2020).

It is important to note that all the periodic CC calculations published so far utilized pseudopotentials (Kresse & Joubert, 1999) to largely disregard the effect of core electrons. One

notable distinction between the works of the two groups, is the type of basis set used. While the **CC4S** code of Andreas Grüneis et al. uses a plane-wave basis, the group of Garnet Chan perform their calculations using gaussian basis sets (McClain et al., 2017; Sun et al., 2018). Localized atom-centred basis sets like gaussian orbitals or numerical atomic orbitals (NAO) (Blum et al., 2009) can potentially decrease the computational cost. This is mostly due to the locality of the basis functions and their improved description of the atomic core region, which decreases the number of basis functions necessary for accurate computations of the system.

This paper describes a generalizable interface, called **CC-aims**, to the **Coupled-Cluster for solids** code (**CC4S**) developed by Andreas Grüneis et al. The interface is formulated in a general way and demonstrated here for the all-electron **FHI-aims** code (Blum et al., 2009). A generalization to other codes with atom-centered basis functions is straight forward. This interface expands the power of electronic-structure theory codes to a variety of correlated methods. These include Møller-Plesset perturbation theory to second order (MP2), coupled-cluster theory including single and double excitations (CCSD) and CCSD including the perturbative treatment of triple excitations (CCSD(T)). Implementations of EOM-CC for neutral (EE-EOM-CC) and charged (IP-EOM-CC/EA-EOM-CC) excitations are currently under development. **CC-aims** can be used directly by any software package which uses a localized basis set and employs a resolution-of-identity scheme (Ren et al., 2012) (RI). On the one hand, this includes local RI schemes, which expand products of atomic orbital basis function (AOs) pairs only using auxiliary basis functions which are localized on either of the atoms of the AOs. Primary examples for this family of localized schemes is the RI-LVL approach employed by **FHI-aims** (Ihrig et al., 2015), **ADF** (Förster & Visscher, 2020) and **ABACUS** (Lin et al., 2020) and the the pair-atomic RI (**PARI**) (Merlot et al., 2013). On the other hand, more conventional non-local schemes, which are predominantly used in molecular calculations, like **RI-V** (Whitten, 1973) and **RI-SVS** (Feyereisen et al., 1993) are recognized by **CC-aims** as well.

Statement of need

The only input quantity of **CC4S**, which is not typically calculated in quantum chemistry or electronic structure codes, but which is computed by **CC-aims**, is the Coulomb vertex (Hummel et al., 2017). The Coulomb vertex, a rank-3 tensor, constitutes a memory-saving, approximation of the rank-4 tensor of Coulomb integrals. The storage of Coulomb integrals is the major memory bottleneck in CC calculations, so that the utilization of the Coulomb vertex expands the scope of system sizes which can be calculated. While in the case of localized orbitals the herein presented **CC-aims** interface can be used, in the case of plane-wave basis sets a different approach to calculate the Coulomb vertex has to be taken, which is described in (Hummel et al., 2017). For Quantum chemistry programs employing a localized basis additionally the use of an RI scheme is needed or needs to be implemented. **CC-aims** allows software packages which either lack certain Quantum chemistry algorithms completely or which only offer insufficiently optimized implementations easy access to these methods. Interfaces like **CC-aims** will substantially accelerate the research done in areas, where DFT is too inaccurate or too unreliable, by allowing many electronic structure codes to participate in these investigations without the time-consuming effort of implementing correlated wave-function methods.

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References

- Al-Hamdani, Y. S., Rossi, M., Alfe, D., Tsatsoulis, T., Ramberger, B., Brandenburg, J. G., Zen, A., Kresse, G., Grüneis, A., Tkatchenko, A., & others. (2017). Properties of the water to boron nitride interaction: From zero to two dimensions with benchmark accuracy. *The Journal of Chemical Physics*, *147*(4), 044710. <https://doi.org/10.1063/1.4985878>
- Blum, V., Gehrke, R., Hanke, F., Havu, P., Havu, V., Ren, X., Reuter, K., & Scheffler, M. (2009). Ab initio molecular simulations with numeric atom-centered orbitals. *Computer Physics Communications*, *180*(11), 2175–2196. <https://doi.org/10.1016/j.cpc.2009.06.022>
- Brandenburg, J. G., Zen, A., Fitzner, M., Ramberger, B., Kresse, G., Tsatsoulis, T., Grüneis, A., Michaelides, A., & Alfè, D. (2019). Physisorption of water on graphene: Subchemical accuracy from many-body electronic structure methods. *The Journal of Physical Chemistry Letters*, *10*(3), 358–368. <https://doi.org/10.1021/acs.jpcllett.8b03679>
- Chan, B. (2019). The CUAGAU set of coupled-cluster reference data for small copper, silver, and gold compounds and assessment of DFT methods. *The Journal of Physical Chemistry A*, *123*(27), 5781–5788. <https://doi.org/10.1021/acs.jpca.9b03976>
- Civalleri, B., Presti, D., Dovesi, R., & Savin, A. (2012). On choosing the best density functional approximation. *Chem. Modell*, *9*, 168–185. <https://doi.org/10.1039/9781849734790-00168>
- Čížek, J. (1966). On the correlation problem in atomic and molecular systems. Calculation of wavefunction components in urself-type expansion using quantum-field theoretical methods. *The Journal of Chemical Physics*, *45*(11), 4256–4266.
- Feller, D., & Dixon, D. A. (2001). Extended benchmark studies of coupled cluster theory through triple excitations. *The Journal of Chemical Physics*, *115*(8), 3484–3496. <https://doi.org/10.1063/1.1388045>
- Feyereisen, M., Fitzgerald, G., & Komornicki, A. (1993). Use of approximate integrals in ab initio theory. An application in MP2 energy calculations. *Chemical Physics Letters*, *208*(5-6), 359–363. [https://doi.org/10.1016/0009-2614\(93\)87156-w](https://doi.org/10.1016/0009-2614(93)87156-w)
- Förster, A., & Visscher, L. (2020). Low-order scaling $g_0 w_0$ by pair atomic density fitting. *Journal of Chemical Theory and Computation*, *16*(12), 7381–7399.
- Gao, Y., Sun, Q., Jason, M. Y., Motta, M., McClain, J., White, A. F., Minnich, A. J., & Chan, G. K.-L. (2020). Electronic structure of bulk manganese oxide and nickel oxide from coupled cluster theory. *Physical Review B*, *101*(16), 165138. <https://doi.org/10.1103/physrevb.101.165138>
- Gyevi-Nagy, L., Kállay, M., & Nagy, P. R. (2019). Integral-direct and parallel implementation of the CCSD (t) method: Algorithmic developments and large-scale applications. *Journal of Chemical Theory and Computation*, *16*(1), 366–384. <https://doi.org/10.1021/acs.jctc.9b00957>
- Hirata, S., Podeszwa, R., Tobita, M., & Bartlett, R. J. (2004). Coupled-cluster singles and doubles for extended systems. *The Journal of Chemical Physics*, *120*(6), 2581–2592.
- Hohenberg, P., & Kohn, W. (1964). Inhomogeneous electron gas. *Physical Review*, *136*(3B), B864.
- Hummel, F., Tsatsoulis, T., & Grüneis, A. (2017). Low rank factorization of the coulomb integrals for periodic coupled cluster theory. *The Journal of Chemical Physics*, *146*(12), 124105. <https://doi.org/10.1063/1.4977994>
- Ihrig, A. C., Wieferink, J., Zhang, I. Y., Ropo, M., Ren, X., Rinke, P., Scheffler, M., & Blum, V. (2015). Accurate localized resolution of identity approach for linear-scaling hybrid density

- functionals and for many-body perturbation theory. *New Journal of Physics*, 17(9), 093020. <https://doi.org/10.1088/1367-2630/17/9/093020>
- Kohn, W., & Sham, L. J. (1965). Self-consistent equations including exchange and correlation effects. *Physical Review*, 140(4A), A1133.
- Kresse, G., & Joubert, D. (1999). From ultrasoft pseudopotentials to the projector augmented-wave method. *Physical Review b*, 59(3), 1758. <https://doi.org/10.1103/physrevb.59.1758>
- Lin, P., Ren, X., & He, L. (2020). Accuracy of localized resolution of the identity in periodic hybrid functional calculations with numerical atomic orbitals. *The Journal of Physical Chemistry Letters*, 11(8), 3082–3088. <https://doi.org/10.1021/acs.jpcllett.0c00481>
- Ma, Q., & Werner, H.-J. (2021). Scalable electron correlation methods. 8. Explicitly correlated open-shell coupled-cluster with pair natural orbitals PNO-RCCSD (t)-F12 and PNO-UCCSD (t)-F12. *Journal of Chemical Theory and Computation*, 17(2), 902–926. <https://doi.org/10.1021/acs.jctc.0c01129>
- McClain, J., Sun, Q., Chan, G. K.-L., & Berkelbach, T. C. (2017). Gaussian-based coupled-cluster theory for the ground-state and band structure of solids. *Journal of Chemical Theory and Computation*, 13(3), 1209–1218. <https://doi.org/10.1021/acs.jctc.7b00049>
- Merlot, P., Kjærgaard, T., Helgaker, T., Lindh, R., Aquilante, F., Reine, S., & Pedersen, T. B. (2013). Attractive electron–electron interactions within robust local fitting approximations. *Journal of Computational Chemistry*, 34(17), 1486–1496. <https://doi.org/10.1002/jcc.23284>
- Nagy, P. R., & Kállay, M. (2019). Approaching the basis set limit of CCSD (t) energies for large molecules with local natural orbital coupled-cluster methods. *Journal of Chemical Theory and Computation*, 15(10), 5275–5298. <https://doi.org/10.1021/acs.jctc.9b00511>
- Ren, X., Rinke, P., Blum, V., Wieferink, J., Tkatchenko, A., Sanfilippo, A., Reuter, K., & Scheffler, M. (2012). Resolution-of-identity approach to hartree–fock, hybrid density functionals, RPA, MP2 and GW with numeric atom-centered orbital basis functions. *New Journal of Physics*, 14(5), 053020. <https://doi.org/10.1088/1367-2630/14/5/053020>
- Savin, A., & Johnson, E. R. (2014). Judging density-functional approximations: Some pitfalls of statistics. *Density Functionals*, 81–95. https://doi.org/10.1007/128_2014_600
- Sousa, S. F., Fernandes, P. A., & Ramos, M. J. (2007). General performance of density functionals. *The Journal of Physical Chemistry A*, 111(42), 10439–10452.
- Stanton, J. F., & Bartlett, R. J. (1993). The equation of motion coupled-cluster method. A systematic biorthogonal approach to molecular excitation energies, transition probabilities, and excited state properties. *The Journal of Chemical Physics*, 98(9), 7029–7039. <https://doi.org/10.1063/1.464746>
- Sun, Q., Berkelbach, T. C., Blunt, N. S., Booth, G. H., Guo, S., Li, Z., Liu, J., McClain, J. D., Sayfutyarova, E. R., Sharma, S., & others. (2018). PySCF: The python-based simulations of chemistry framework. *Wiley Interdisciplinary Reviews: Computational Molecular Science*, 8(1), e1340.
- Tsatsoulis, T., Sakong, S., Groß, A., & Grüneis, A. (2018). Reaction energetics of hydrogen on si (100) surface: A periodic many-electron theory study. *The Journal of Chemical Physics*, 149(24), 244105. <https://doi.org/10.1063/1.5055706>
- Wang, X., & Berkelbach, T. C. (2020). Excitons in solids from periodic equation-of-motion coupled-cluster theory. *Journal of Chemical Theory and Computation*, 16(5), 3095–3103. <https://doi.org/10.1021/acs.jctc.0c00101>
- Whitten, J. L. (1973). Coulombic potential energy integrals and approximations. *The Journal of Chemical Physics*, 58(10), 4496–4501. <https://doi.org/10.1063/1.1679012>

- Zhang, I. Y., & Grüneis, A. (2019). Coupled cluster theory in materials science. *Frontiers in Materials*, 6, 123. <https://doi.org/10.3389/fmats.2019.00123>
- Zhang, I. Y., Rinke, P., Perdew, J. P., & Scheffler, M. (2016). Towards efficient orbital-dependent density functionals for weak and strong correlation. *Physical Review Letters*, 117(13), 133002. <https://doi.org/10.1103/physrevlett.117.133002>