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From the Quadrivium to Modern Science[1]

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Abstract: The ultimate objective of this work is to demonstrate that it is possible to reconsider the emergence of modern science as a process of disintegration of the quadrivium, which was considered a stable scheme for the organization of knowledge. The argument considers the quadrivium according to the Boethian systematization that was used to organize the curricula of the late medieval universities. This argument follows the development of each of its disciplines and illustrates the practical turn they underwent. The period between the thirteenth and seventeenth centuries is explored, and shows that during this period, the quadrivium potentially included a fifth autonomous discipline, calendric. The article concludes by describing epistemological considerations to the mechanisms of disintegration of knowledge structures.

Keywords: Boethius; quadrivium; calendric; knowledge structure; practical turn

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Introduction

The historical phenomenon often referred to as "the emergence of modern science," situated temporally in the early modern period and spatially in Europe, is usually associated with singular, exceptional events often removed from their context. These may encompass specific empirical demonstrations but, in reality, they mostly coincide with the publication of specific scientific works. Most famous in this respect is Nicolaus Copernicus' *De revolutionibus orbium coelestium,* published in 1543. Names such as Copernicus, Galileo Galilei, Isaac Newton belong to the shared global narratives regarding the birth of modern science.

The present work, instead, invites the reader to reconsider their perspective by looking at the ways in which scientific knowledge was organized, and investigating whether a general transformation of the conceptual, organizational schemes of knowledge can be used to explain the same phenomenon. This perspective therefore shifts the focus away from historical singularities toward an overarching view of the accepted and dominant scientific knowledge. To accomplish this, the article will focus exclusively on the mathematical sciences—and therefore on the quadrivium—and on the period ranging from the foundation of universities until the mid-seventeenth century. Given the long-term scope and wide spectrum of subjects, only the general trends and some paradigmatic but brief examples will be mentioned, while references to historical sources and literature for further reading will be added in the footnotes.

The ultimate objective is to demonstrate that it is possible to reconsider the emergence of modern science as a process of desegregation and disintegration of established structures of knowledge organization. In the following section, an overview of the quadrivium, its disciplines, and their evolution will be briefly illustrated, while the epistemic mechanisms will be disentangled in the final section.

The quadrivium and its evolution

During the late Middle Ages, a political and territorial identity began to take shape in Europe at the same time as the centralization of power was moving toward the Roman Church.[2] In this context, a new educational institution—the university—emerged. Universities were often founded in conjunction with or closely related to preexisting schools, mostly of a religious nature such as those of the mendicant friars.[3] First in Bologna and Paris, universities

[2] Robert I. Moore, *The First European Revolution. c. 970-1215* (Malden, MA: Blackwell Publishing, 2000).

[3] Michèle M. Mulchahey, "The Dominican Studium System and the Universities of Europe in the Thirteenth Century. A Relationship Redefined,» in *Manuels, Programmes de cours et techniques d'enseignement dans les universités médiévales*, ed. Jacqueline Hamesse (Louvain-la-Neuve: Institut d'Études Médiévales de l'Université Catholique de Louvain, 1994), 277-324. Hastings Rashdall, *The*

were founded in an increasing number of cities. From the outset, their scholars and pupils implemented and enriched the existing mobility network of the religious orders throughout Europe,[4] a phenomenon that peaked during the early modern period. This network, which expanded exponentially due to the circulation of hundreds of thousands of printed books from the second half of the fifteenth century,[5] ensured that knowledge was shared and debated, and that its organization, and therefore the way it was taught in the educational institutions, became increasingly homogeneous.[6]

The entry point of the university curricula was represented by the faculty of liberal arts, where the mathematical disciplines were taught.[7] Everyone who had access to higher education had to first become acquainted with the quadrivial disciplines: arithmetic, music, geometry, and astronomy.[8] The organizational scheme of the quadrivium has very ancient origins, but the systematization that most influenced the curriculum, as implemented by the universities, is the one employed in late antiquity by Boethius.[9] Boethius wrote a text for each of the four disciplines and these were conceived from the outset for pedagogic purposes. The texts were interrelated and mutually dependent because, as Ann E. Moyer clearly explains in her seminal paper, the quadrivium as an organizational knowledge structure and, therefore, as an organization of the curriculum, was based on a fundamentally moral and ethical conception of the world according to which the mathematical understanding of the world and its harmony was a way of contemplating God and creation.[10]

Universities of Europe in the Middle Ages, 3 vols. (Oxford: Oxford University Press, [1895] 1997).

4 Clifford H. Lawrence, *The Friars: The Impact of the Mendicant Orders on Medieval Society*, 2 ed. (London: Bloomsbury Academic, 2013).

5 Angela Nuovo, *The Book Trade in the Italian Renaissance* (Leiden: Brill, 2013).

6 For the process of homogenization of knowledge during the early modern period, see Matteo Valleriani et al., "The Emergence of Epistemic Communities in the *Sphaera* Corpus: Mechanisms of Knowledge Evolution," *Journal of Historical Network Research* 3 (2019): 50–91, https://doi.org/10.25517/jhnr.v3i1.63; Matteo Valleriani, "Prolegomena to the Study of Early Modern Commentators on Johannes de Sacrobosco's *Tractatus de sphaera*," in De sphaera *of Johannes de Sacrobosco in the Early Modern Period: The Authors of the Commentaries*, ed. Matteo Valleriani (Cham: Springer Nature, 2020), 1-23, https://doi.org/10.1007/978-3-030-30833-9_1; Maryam Zamani et al., "Evolution and Transformation of Early Modern Cosmological Knowledge: A Network Study," *Scientific Reports* 10 (2020): 19822 (https://doi.org/10.1038/s41598-020-76916-3).

7 Paul F. Grendler, "The *Sphaera* in the Jesuit Education," in *Publishing Sacrobosco's* De sphaera *in Early Modern Europe. Modes of Material and Scientific Exchange*, ed. Matteo Valleriani and Andrea Ottone (Cham: Springer Nature, 2022), 369-407, https://doi.org/10.1007/978-3-030-86600-6_11.

8 John North, „Das Quadrivium," in *Geschichte der Universität in Europa*. Band I: Mittelalter, ed. Walter Rüegg (München: Beck, 1993), 303–20.

9 Noel Harold Kaylor and Philip Edward Phillips, *A Companion to Boethius in the Middle Ages* (Leiden: Brill, 2012).

10 Ann E. Moyer, "The Quadrivium and the Decline of Boethian Influence," in *A Companion to Boethius in the Middle Ages*, ed. Noel Harold Kaylor and Philip Edward Phillips (Leiden: Brill, 2012), 479-517.

Although the texts are mutually dependent, their content is logically ordered and therefore displays a hierarchy that reveals itself in two ways. The first is related to the gnoseological process and responds to the question of what one needs to learn first in order to understand the other aspects. The second is related to the moral and theological meaning of the quadrivium and concerns one specific subject: the number. If the world and all its parts operate in harmony and this harmony can be understood mathematically, then numbers provide the key to accessing harmony and the contemplation of the creator; this was the very first subject that students were confronted with.

In the following section, each of the four original mathematical disciplines, as described by Boethius, will be explained, along with the evolution of their content and function during the period considered here. A fifth discipline, calendric, will also be described and explained, with justification given as to why this discipline should not be excluded when considering the quadrivium of the late Middle Ages. To analyze the content of the individual disciplines, the scientific texts used for the classes will be taken into consideration. A further rationale behind this approach is that classes were historically defined by the books that were read by both teachers and students. A music class at the University of Leipzig at the beginning of the sixteenth century, for example, was assigned to read *Musica Muris* by Johannes de Muris, the author of this textbook.[11]

Arithmetic

The first class was devoted to the study of Boethius' *De institutione arithmetica*,[12] which begins with the definition of the number and presents, in the first of its two sections, what nowadays would be called the theory of numbers. Boethian arithmetic does not correspond to the current content of the homonymous discipline. The second part is a lengthy contribution to the theory of proportions. It is through this subject that Boethian arithmetic can be found across various expressions of culture in the late Medieval world, such as literature or even architecture, for instance, when architectural plans of churches and cathedrals reflect the most relevant of such proportions, as in the case of St. Michaelis Church.[13]

In the thirteenth century, however, classes in arithmetic had already expanded their subjects with students working, for instance, on texts that were called *Algorismus*. Originally written in

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[11] Matteo Valleriani and Nana Citron, "Conrad Tockler's Research Agenda," in De sphaera *of Johannes de Sacrobosco in the Early Modern Period: The Authors of the Commentaries*, ed. Matteo Valleriani (Cham: Springer Nature, 2020), 111-36, https://doi.org/10.1007/978-3-030-30833-9_5.

[12] Boethius and Gottfried Friedlein, *De institutione arithmetica libri duo. De institutione musica libri quinque* (Leipzig: Teubner, 1867).

[13] Michael Masi, *Boethian Number Theory. A Translation of the «De institutione Arithmetica»* (Amsterdam: Rodopi, 1983), 23-48.

the frame of Islamicate science by Muhammad ibn Musa al-Khwarizmi in the ninth century, they reached the Latin world through Cordoba in the eleventh century. A content-reduced form of these texts became a widely-disseminated textbook at the universities and lecturers, such as the first known lecturer for mathematics at the University of Paris, Johannes de Sacrobosco, wrote their own adaptations. In particular, the textbook *Algorismus* did not include algebra but still offered students the opportunity to expand in two different directions. The first and most profound was represented by the Hindu-Arabic numerical system that pupils became acquainted while performing arithmetical operations. The second direction was the subject of correlation between the decimal and the hexadecimal systems. As the latter was used in astronomy, this knowledge enabled the use of arithmetical computation within astronomical studies.[14]

A further development of arithmetic is the one usually associated with the Abacus schools. This was practical arithmetic that was not taught at the universities but as a mathematical discipline that still belonged to the general connection of quadrivium. It included the content of the *Algorismus* but expanded more into algebra and logistics. A fundamental difference, moreover, was represented by the fact that such practical mathematics took a problem-solving approach and the problems discussed were related to a variety of professions that ranged from architecture to banking.[15]

Music

From the original Boethian perspective, the approach to music was conceived to be fundamentally mathematical. Similar to what is today referred to as harmony, in music class students learned a particular group of proportions, namely those fitting to the Pythagorean monochord. The world's harmony was thought to be expressed in numerical form and distilled into a blueprint of musical harmony: the so-called Pythagorean scale.[16] The scale, originating from the famous legend of Pythagoras passing a blacksmith's shop, indicates that

[14] Nadia Ambrosetti, "Algorithmic in the 12th Century: The *Carmen de Algorismo* by Alexander de Villa Dei," in *History and Philosophy of Computing*, ed. Fabio Gadducci and Mirko Tavosanis (Cham: Springer International Publishing, 2016), 71-86.

[15] Jens Høyrup, *The World of the* Abbaco (Basel: Birkhäuser Verlag, forthcoming). I would like to express my gratitude to Jens Høyrup for sharing with me the manuscript of this book.

[16] For an introduction to the "Pythagorean-Platonic doctrine of world harmony," see James Haar, Musica Mundana. *Variations on a Pythagorean Theme* (Cambridge, Massachusetts: Harvard University, 1960), 1-79; Ann E. Moyer, Musica Scientia. *Musical Scholarship in the Italian Renaissance* (Ithaca: Cornell University, 1992). See also Francesco Pelosi, "Eight Singing Sirens: Heavenly Harmonies in Plato and the Neoplatonists," in *Sing Aloud Harmonious Spheres. Renaissance Conceptions of Cosmic Harmony*, ed. Jacomien Prins and Maude Vanhaelen (New York London: Routledge, 2018), 15-30. For a detailed, yet clear description of the Pythagorean scale, see Moyer, Musica Scientia. *Musical Scholarship in the Italian Renaissance*, 14-7.

musical proportions are mathematical in nature. The Boethian tradition was supported by the preservation and transmission of his work, *De institutione musica*.[17] Following this text, music was subdivided into three subjects: a) *musica instrumentalis*, the more practical aspects of music, b) *musica humana*, which allows for the conjunction of body and soul, and c) *musica mundana*, which correlates harmonic proportions with those resulting from the distances of the planets from the Earth and their speeds. The latter was the culmination of the subject as it enabled a mathematical contemplation of the world through its relationship to cosmology and astronomy.

In the late Middle Ages, however, the structural stability of the Boethian conception of quadrivium had already been weakened by the reception of Aristotle's *De coelo*, translated by Gerard of Cremona and William of Moerbecke, and by the commentaries of Averroes and Simplicius. Most pertinently, Aristoteles' work (Book 2, Part 9) contained two fundamental criticisms of the principal idea of the *musica mundana*. According to Aristoteles' doctrine, a sound could only be generated by means of a collision of two objects, a phenomenon that could not occur in the heavens, however, where the movements of the planets do not encounter friction. Secondly, if *per absurdum* the existence of such a sound were to occur, following Aristoteles, then the noise generated would have been loud enough to destroy the cosmos itself.[18]

The teaching of *musica mundana* as well as the idea of universal harmony survived but were increasingly relegated to niche subjects. Influential scholars such as Johannes de Muris, Jacques Lefèvre d'Étaples, Franchinus Gaffurius, and Heinrich Glareanus kept the tradition alive but never tried to contend with its mathematical apparatus. A comparison between the dissemination of their and Boethius' early modern printed works on the subject, with those related to astronomy, including Johannes de Sacrobosco' *Tractatus de sphaera*, clearly shows that the subject was not in demand.[19]

Music was instead increasingly associated with singing, as the late medieval emergence of polyphonic music demonstrates. Singing supported the ongoing process of shaping European identity and shifted the attention to practical music. During the sixteenth century, the focus

[17] Boethius, "De institutione musica libri quinque," in *De institutione arithmetica libri duo. De institutione musica libri quinque*, ed. Gottfried Friedlein (Leipzig: Teubner, 1867).

[18] Gabriela Ilnitchi, «*Musica Mundana*, Aristotelian Natural Philosophy and Ptolemaic Astronomy," *Early Music History*, no. 21 (2002): 37-74, on 44-50.

[19] Matteo Valleriani and Andrea Ottone, eds., *Publishing Sacrobosco's* De sphaera *in Early Modern Europe. Modes of Material and Scientific Exchange* (Cham: Springer Nature, 2022) https://doi.org/10.1007/978-3-030-86600-6.

shifted to the art of tuning, and musical theory took on a new form, based on practice, as the famous polemic between Gioseffo Zarlino and Vincenzo Galileo paradigmatically shows.[20]

Geometry

Knowledge of geometry was mostly associated with classes dedicated to reading either the first three or six books of Euclid's *Elements*, similar to what later students in artists' workshops, for instance, would also learn.[21] Alternatively, Euclid's *De quantitatibus datis* and Jordanus de Nemore's *De triangulis* were also often read in class. In the frame of the Boethian conceptualization of the quadrivium, geometry was required to understand celestial mechanics, for instance by those (few) pupils who moved on to study Ptolemy's *Almagest* or, more commonly, by (more) students who then learned how to calculate the positions of the planets by reading the texts titled *Theoricae planetarum*.[22] More practically, geometry helped to understand the operations related to astronomical instruments such as the astrolabe and the quadrant.

With regards to geometry, however, thirteenth-century universities had already expanded the number of subjects that students could learn. Statics—such as that based on the reading of Jordanus de Nemore's *De ponderibus*—and optics, in particular, belonged to the same framework, namely disciplines that were only partially relevant to the study of astronomy. Optics was a genuinely innovative discipline of the late Middle Ages as it increasingly diverged from the physiology of the eye and of sight. Fundamental works were, for example, those of the Franciscans Robert Grosseteste, Roger Bacon, and John Peckham. Obviously, the important work of Vitello (Witelo) must be included as well.[23]

As was the case with arithmetic and music, starting from the late Middle Ages and, more decisively during the early modern period, new forms of geometry emerged that were not directly connected to the dissemination of Euclid's *Elements* and were not taught at the universities. Instead, these forms of geometry could be learned by actively working on (large) building sites, particularly when engaging in the design process.[24] Finally, geometry became the fundamental *trait d'union* between natural philosophy and practical mechanics (such as

[20] Pietro Greco, *The Tuscan Artist* (Cham: Springer Nature, 2018).

[21] Matteo Valleriani, *Galileo Engineer*, Boston Studies in the Philosophy of Science (Dordrecht: Springer, 2010), 3-20.

[22] Michela Malpangotto, Theoricae novae planetarum Georgii Peurbachii *dans l'histoire de l'astronomie* (Paris: CNRS Edition, 2021).

[23] North, "Das Quadrivium."

[24] Wolfgang Lefèvre, "Architectural Knowledge," in *The Structures of Practical Knowledge*, ed. Matteo Valleriani (Dordrecht: Springer Nature, 2017), 247-70.

machine-building), a combination from which theoretical mechanics emerged and developed, particularly during the sixteenth century.[25]

Astronomy

Astronomy had been the queen of the mathematical disciplines since antiquity, but it is unclear whether thirteenth-century students and teachers considered it more relevant than the other quadrivial subjects. Historians of science certainly devoted more time to the reconstruction of its development.

At the universities, astronomy classes began with a subject that we would now call cosmology, as the initial aim was to qualitatively learn the composition of the cosmos and the movements of the celestial bodies. Compiled in the antiquity were Aristotle's *De coelo* and Ptolemy's *Almagest*. Both works, however, were read later in the curriculum. In preparation for these, there was several works, mostly entitled *De sphaera*, which describe the *machina mundi*. Among those most frequently used was the ninth-century *Elements* of Al-Farghani, the late medieval *Almagestum abbreviatum*, Grossteste's *Sphaera*, and Sacrobosco's *Sphaera*, which became the most widely used work up until the beginning of the seventeenth century, long after Copernicus's work was published. As mentioned, the class would continue to use the *Theorica planetarum*. Several versions of this text were compiled over the centuries until the *Theorica nova* by Georg von Peuerbach (compiled in 1454 and published by his student Regiomontanus in 1472) replaced all previous ones. To fill the gap between the *Sphaera* treatises and the *Theorica*, the students had at their disposal the tables against whose background calculations were executed: first the Toledan tables and then the Alphonsine.[26] In general, as Olaf Pedersen stated in 1975, a rather stable *corpus astronomicum* was built during the late medieval period. Treatises dedicated to the use and function of specific astronomic instruments or to the astronomy of the eighth sphere belonged, among others, to such a corpus.[27]

According to the original Boethian conception of the quadrivium, astronomy and music, and especially *musica mundana*, were supposed to offer a means for achieving the ultimate ethical mission of the quadrivium itself, but the thirteenth-century quadrivium was already diverging from such a mission, in part due to the crisis of the *musica mundana*. However, during this time, the teachings of astronomy became increasingly relevant for other reasons. In this case

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[25] Jürgen Renn, Rivka Feldhay, Matteo Valleriani, Matthias Schemmel, eds., *Emergence and Expansion of Pre-Classical Mechanics* (Dordrecht: Springer, 2018).
[26] North, "Das Quadrivium."
[27] Olaf Pedersen, "The Corpus Astronomicum and the Traditions of Mediaeval Latin Astronomy," *Copernicana*, no. 13 (1975): 57-96.

too, these reasons seem to show a practical turn as they concerned astrology and medicine, and in later centuries, cosmography and nautical astronomy.

The increasing relevance of astrology from the late medieval period until well into the sixteenth century is no mystery.[28] With the diffusion of printing technology, new genres related to practical science arose, especially in the German-speaking territories. Among the most relevant new genres were the extremely successful Almanacs and the *Practica*.[29] These were calendric-based astrological prognostications that concerned a variety of subjects, from individual everyday life to great political or military continental disruptions.

Largely due to a revival of Galen's theory of critical days,[30] astrological medicine became a fundamental scientific and cultural component of European society. As soon as sickness occurred, physicians were required to know the positions of the planets in order to be able to deliver a suitable prognosis. They were therefore very accomplished in using the *Theorica* and its volvelles to make precise calculations backwards in time.

Increased mobility in the thirteenth century and the phenomenon of exploratory travel starting in the fifteenth century finally led to the emergence of completely new astronomy-related disciplines. Nautical astronomy, with its new mathematical workflow, new mathematical instruments, and the requirement of planet-wide, precise observations, was indispensable for the early modern navigators.[31] From the new clusters of knowledge that were rooted in this new activity, cosmography emerged. Known today as geography, but entirely novel in the early modern period, the mapping of the Earth required the projection of celestial geometry—the geometry of the cosmos—onto the Earth's surface to create a spherical metric to precisely draw the contours of the distribution of land and water.

Calendric

The most striking divergence from the original Boethian conception of the quadrivium in its implementation in the frame of the university curriculum was due to the invasive presence

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[28] Darrel H. Rutkin, Sapientia Astrologica*: Astrology, Magic and Natural Knowledge, ca. 1250-1800* (Cham: Springer Nature, 2019).

[29] Richard L. Kremer, "Incunable Almanacs and *Practica* as Practical Knowledge Produced in Trading Zones," in *The Structures of Practical Knowledge,* ed. Matteo Valleriani (Dordrecht: Springer, 2017), 333-69.

[30] Glen M. Cooper, "Numbers, Prognosis, and Healing: Galen on Medical Theory," *Journal of the Washington Academy of Sciences* 90, no. 2 (2004): 45-60.

[31] Henrique Leitão, ed. Sphaera Mundi*: A Ciência na Aula de Esfera. Manuscriptos científicos do Colégio de Santo Antão nas colecções da BNP* (Lisboa: Biblioteca Nacional de Portugal, 2008); "Um Mundo Novo e uma Nova Ciência," in *360º Ciência Descoberta*, Catálogo da Exposição, ed. Henrique Leitão (Lisboa: Fundação Calouste Gulbenkian, 2013), 16-39.

of research and study concerned with time-reckoning systems. Most relevant among them regarding teaching was the *computus ecclesiasticus*, namely, the set of rules used to calculate Easter Day and therefore to set all movable feasts of the Christian liturgic calendar.[32]

The teaching of calendric, for instance in relation to the other quadrivial disciplines is not discussed by the historical actors and, apart from a few exceptions, is not even mentioned in the statutes of the universities. Historians therefore tend to unify this discipline with others, sometimes arithmetic (especially *Algorismus*) and sometimes astronomy (especially the use of the quadrant), as both are needed to accomplish the calendric computations. However, a short overview of major catalogs and repositories (e.g., Jordanus's database[33]) suffices to discover that the number of preserved manuscripts dedicated to the *computus ecclesiasticus* is comparable to, if not higher than, those concerned with the treatises dedicated to the *Sphaera*, including Sacrobosco's, the most widespread of them all. It is difficult to say at this stage how exactly the teaching of *computus* was organized, but it is undeniable that it must have played an integral role for the students and the teachers.

Knowledge on *computus ecclesiasticus* had been systematized during the eighth century by the Venerable Bede.[34] Mistakes in the calendars were well known but also accepted. The increased relevance of such disciplines and this subject in particular during the late Middle Ages was probably due to the formation of a European identity and increased mobility, as previously mentioned. The uniformity of the liturgic calendar all over Europe through joint effort illustrated the scope of the central bodies of the Church from the twelfth century. At the *Concilio Laudanensi* in 1109, through the *Decretum Gratiani* in 1150 and especially by means of the *Liber extra* enacted by Gregorius IX between 1232–1234, medieval culture had set the basis for a full liturgical homogenization, which was finally politically realized at the Council of Trent between 1545 and 1563 and, mathematically, with the Calendar Reform in 1582.

During the thirteenth century, the difficulties, made evident by continuous inaccurate previsions of Easter Day, remained insurmountable. The number of problems was high and difficult to discern but the most basic of them was due to the incommensurability between the Lunar and Solar calendars, an integration needed to calculate Easter Day. The solution of the Calendar Reform was to achieve a well-working Solar calendar "over which" any other system of time-reckoning could be set.

32 C. Philipp E. Nothaft, *Scandalous Error. Calendar Reform and Calendrical Astronomy in Medieval Europe* (Oxford, United Kingdom: Oxford University Press, 2018).

33 *Jordanus. An International Catalogue of Medieval Scientific Manuscripts*. https://ptolemaeus.badw.de/jordanus/start (Accessed February 20, 2022).

34 Faith Wallis, *Bede: The Reckoning of Time* (Liverpool: Liverpool University Press, 1999).

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Against this background, one could hypothesize that, during the late medieval period when this difficulty was known, the gradual uniformity of the Liturgic calendar occurred through a process of knowledge dissemination that attempted to distribute the same knowledge everywhere, despite its inaccuracies. The universities and schools, such as those of the Franciscans, apparently took over this task. As Léopold Delisle noted already in 1907, the last page of the thirteenth-century manuscript Lat 7475, preserved at the French National Library in Paris, was compiled in 1371 and shows the permission (and the prohibition to further transfer) of both the Pope (Gregory XI) and the King (Charles V) to send this manuscript to the then new Dominican convent of Troyes upon their request for books in order to learn how to calculate Easter Day.[35] Apart from the subject of geometry, the whole manuscript contains texts that cover all other quadrivial subjects. One could surmise that, during the late medieval period, the new mission of the quadrivium was the achievement of the liturgic synchronicity of Europe.

Enrichment and oblivion: the disintegration of the quadrivium

In the period considered, all original quadrivial disciplines experienced a remarkable evolution and transformation, primarily due to a clear practical turn.[36] Arithmetic moved from number theory to the so-called merchants' arithmetic. Music moved from the Pythagorean monochord and its cosmological applications to practical music and its theory. Geometry, primarily devoted to the investigations of the cosmos, was finally applied to mechanics. The goal of contemplation of cosmological harmony, associated with the discipline of astronomy, transformed into various goals relating to the more practical fields of astrology, medicine, calendric, navigation, and cartography. These disciplines which emerged from this development did not exhibit any mutual dependence or any shared mission. The distinct disciplines changed through a process of enrichment and oblivion. New subjects replaced older ones, though at different magnitudes and along different temporal scales. Consequently, the quadrivium and its foundations receded. Its overall concept and function were, however, neither discussed nor debated. The quadrivium as an organizational scheme of knowledge remained ever-present until it was simply forgotten as priorities changed and modern science emerged.

[35] Léopold Delisle, *Recherches sur la librairie de Charles V* (Paris: H. Champion, Libraire-Èditeur, 1907).
[36] Matteo Valleriani, ed., *The Structures of Practical Knowledge* (Dordrecht: Springer, 2017).

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