Department of Physics and Astronomy

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Master thesis

in Physics

submitted by

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born in Saarlouis

2022

Quantum Field Theory and Phenomenology in 5D Warped Space-Time: Gauge-Higgs Grand Unification

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at the

Max-Planck-Institut für Kernphysik

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Quantenfeldtheorie und Phänomenologie in gekrümmter 5D Raumzeit: Große Eich-Higgs Vereinheitlichung

Theorien mit zusätzlichen Raumzeit Dimensionen, z.B. Randall–Sundrum (RS) Modelle, bieten interessant Konzepte, um ungelöste Fragen des Standardmodelles der Teilchenphysik zu beantworten. Zu diesen zählen das Hierarchieproblem, die Unterschiede in den Fermion Massen und die große Vereinheitlichung der Grundkräfte. Darüber hinaus kann das Hierarchieproblem auch dadurch gelöst werden, dass man das Higgs Feld in die extra dimensionale Komponente eines fünfdimensionalen Eichfeldes einbettet. Dies wird als Eich–Higgs Vereinheitlichung bezeichnet und kann auch mit der großen Vereinheitlichung der Grundkräfte zu einer großen Eich–Higgs vereinheitlichen Theorie (GHGUT) kombiniert werden. In [1] wurde eine phänomenologisch valide GHGUT vorgestellt, welche auch in dieser Arbeit untersucht wird, aber viele Resultate haben darüber hinaus Validität. Um die quantitative Vereinheitlichung der Eichkopplungen zu untersuchen, werden im ersten Teil dieser Arbeit verschiedene Renormalisierungsmethoden in RS Modellen aus der Literatur untersucht und miteinander verglichen. Die jeweiligen Vor- und Nachteile werden herausgearbeitet und auf $SU(6)$ GHGUT angewandt. Im zweiten Teil werden dann weitere phänomenologische Aspekte der Flavor Physik sowie Präzisionsmessungen untersucht und mit konventionellen RS Modellen verglichen. Zu diesen Aspekten gehören die Erklärung der Hierarchien in den Fermion Massen, die Größe von Flavor verändernden neutralen Strömen sowie die elektroschwachen Präzisionsparameter.

Quantum Field Theory and Phenomenology in 5D Warped Space-Time: Gauge-Higgs Grand Unification

Theories with extra dimensions, e.g. warped extra dimensions like Randall-Sundrum (RS) models, are an interesting direction addressing unresolved questions in the Standard Model. These include the hierarchy between the Planck- and electroweak scale, the flavor puzzle, the unification of forces and many more. In fact, it is possible to solve the hierarchy problem by incorporating the Higgs field in the extra component of a five-dimensional gauge field. This is known as Gauge-Higgs Unification and can even be combined with Grand Unified Theories to form a Gauge-Higgs Grand Unified Theory (GHGUT). Recently [1] proposed a viable model for GHGUT, which will also be investigated in this thesis, but many results apply to general RS theories. To study the quantitative unification of the SM gauge couplings in these models, the first part of this thesis focuses on the renormalization of GHGUTs and general RS-models. Different regularization techniques proposed in the literature are introduced, their respective shortcomings and subtleties discussed and then applied to $SU(6)$ GHGUTs. In the second part, aspects of flavor phenomenology and precision physics in the above model are investigated, pointing out interesting differences compared to conventional RS models. These include the generation of hierarchies of the SM fermion masses, the size of flavor changing neutral currents and constraints from electroweak precision parameters

Contents

Part I

Introduction: Standard Model and Aspects of Grand Unification

The Standard Model (SM) of particle physics is currently the best description of nature at the subatomic scale. With the discovery of the Higgs boson at the Large Hadron Collider (LHC) in 2012, all of its content has been confirmed experimentally. Although, its predictions have been tested and verified countless times, the SM is far from being a complete explanation of all observations. Several of the shortcomings of the SM can be addressed in extensions of the SM called Randall-Sundrum (RS) models, as will be explained in detail later. RS models extend the usual four dimensional (4D) space by a finite warped extra dimension. While offering a solution to the gauge hierarchy problem, RS models also allow for multiple forms of unification. First, gauge fields in five dimensions (5D) have an extra component, which can incorporate the Higgs boson. In fact, because of the finite extra dimension this Higgs can get a potential dynamically, which triggers electroweak symmetry breaking (EWSB). This is known as Gauge-Higgs Unification (GHU). Second, RS models allow for the reduction of gauge symmetries by choosing appropriate boundary conditions (BC) on the endpoints of the extra dimension. This opens up the possibility to study Grand Unified Theories (GUT) in this context, which unify the three SM interactions into one gauge field. In fact it is possible to combine both GHU and GUTs in to one Gauge-Higgs Grand Unified Theory (GHGUT). Here all three SM interactions, as well as the Higgs boson, are described by one single gauge field.

One focus of this thesis is to study quantitatively if the unification of the SM gauge couplings can be achieved in such models. To do this, it is necessary to analyze how gauge couplings in RS models evolve with the energy scale, a consequence of renormalization. There exist several methods for this in the literature, but no clear consensus on the validity of them has been achieved so far. Therefore, in this work we compare these methods with each other, discuss their advantages and disadvantages, as well as certain flaws, and point out important differences. This should give a comprehensive view of the state-of-the-art renormalization in RS models. After crystallizing out one method, which is suitable to study high scale unification, we apply it to incarnations of GHGUTs based on the group $SU(6)$. We will be able to test unification in several incarnations of $SU(6)$ GHGUTs and give a postdiction for the weak mixing angle.

Another focus of this thesis are phenomenological aspects of GHGUTs. It is shown how a minimal fermion embedding in $SU(6)$ allows one to describe the flavor hierarchies. Apart from the masses, also the hierarchies in the Cabibbo-Kobayashi-Maskawa (CKM) matrix can naturally be achieved in these kinds of models. Furthermore, we will see that there exists an analogous mechanisms to the SM Glashow-Iliopoulos-Maiani (GIM) mechanism suppressing flavor changing neutral currents (FCNCs). Comparing the predictions of the process $\mu \to e\gamma$ from GHGUTs with current measurements will allow us to put stringent constraints on the viable parameter space of GHGUTs. Additionally, we will derive the constraints coming from electroweak precision tests (EWPT) in detail and compare it with the standard RS scenario, where the Higgs is localized on the IR brane.

This thesis is organized as follows. The current part serves as a general introduction and, as we explore extensions of the SM, we will start with a short review on the SM itself. There we introduce the necessary concepts and the relevant notation and afterwards discuss several of its shortcomings to motivate RS theories. We also review important aspects of renormalization and GUTs. Part II gives an overview of RS models. We will derive the 5D action of scalar, fermion and gauge fields and connect them explicitly to 4D fields via a Kaluza-Klein (KK) decomposition as well as holography. Apart from the KK profiles also the propagators and vertices for scalars, fermions and gauge fields are given for all possible BCs. The part closes with an introduction to GHU. Part III presents the concepts of GUTs in RS and their extensions to GHGUT models. Starting with Part IV the results which have been achieved during this master thesis are presented. First, to quantify unification in GHGUTs, detailed studies of renormalization techniques in RS models are performed and afterwards, applied to make predictions in $SU(6)$ GHGUTs. This is followed in Part V by a study of novel features of flavor aspects and precision tests in GHGUTs. Finally, Part VI summarizes the results and gives a brief outlook.

1 Standard Model

This chapter gives a short summary of the Standard Model (SM) of Particle Physics [2, 3, 4, 5] and the corresponding notations used in this thesis. More detailed overviews are available in the literature, see e.g. [6, 7, 8, 9, 10, 11, 12, 13], which are also the main sources of this summary.

The SM describes very successfully three out of the four known interactions and all particles which constitute the observed matter. These three interactions are the electromagnetic force as well the strong and weak nuclear forces. Luckily, the forth force, gravity, is on the elementary level much weaker than the other interactions so the SM provides a good explanation of the behaviour of fundamental particles. Thus a plethora of experiments are in agreement with the predictions of the SM [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24].

Mathematically, the SM is modelled by quantum field theories (QFT), which combine the developments of Special Relativity and Quantum Mechanics. A QFT consists of fields $\Phi(x)$ which are functions of the space-time coordinate x and particles are viewed as excitations in these fields. To classify these fields, transformations and their corresponding symmetries play an important role. One differentiates between external symmetries, i.e. space-time symmetries, and internal symmetries. The space-time symmetry transformations of the SM are the ones of Special Relativity: Poincaré transformations [10]. The infinitesimal space-time interval $\mathrm{d}s$ in a frame described by the coordinates x^μ is given by[1]

$$\mathrm{d}s = \eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu\,. \tag{1.1}$$

[1] Throughout this thesis we use the metric convention $(\eta_{\mu\nu}) = \mathrm{diag}(1, -1, -1, -1)$ and sum over repeated indices. Furthermore we work in natural units ($\hbar = c = 1$).

The set of linear transformations

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu} \,, \tag{1.2}$$

which leave the Minkowski metric invariant form the group of homogeneous Lorentz transformations $O(1,3)$. We only need the subgroup of proper orhochronous Lorentz transformation $SO(1,3)^{+}$, which can be obtained by a continuous change of parameters from the identity. Now, fields in QFTs have to transform under representations of $SO(1,3)^{+}$ [10]. As is common in representation theory we focus on irreducible representation and look at the ones with the lowest dimension. There exists one representation of $SO(1,3)^{+}$ with dimension one, the trivial representation. The corresponding (real or complex) fields $\phi(x)$ are Lorentz scalars and are thus called scalar fields. For $SO(1,3)^{+}$, it turns out that there are two two dimensional irreducible representations. They are both complex and referred to as spinoral representations, one describes a left-handed (LH) Weyl fermion $\psi_{L,\alpha}$ the other a right-handed (RH) Weyl fermion $\psi_{R,\alpha}$. Often one encounters a reducible representation composed of these two: the Dirac fermion $\psi = \begin{pmatrix}\psi_L & \psi_R\end{pmatrix}^T$. The fundamental representation of $SO(1,3)^{+}$ is real and gives rise to 4D vector fields A_μ. There are of course more representations of $SO(1,3)^{+}$, but these are the ones used in the SM.[2]

Next we focus on internal symmetries, which come in two forms, global and local symmetries [6]. The transformations of global symmetries are the same at each point x in space-time, whereas local symmetry transformations do depend on x. We focus in the following case on local symmetries, also called gauge symmetries, the case for global symmetries is analogous with the dependence on space-time dropped. There are many groups G which can be used as internal symmetries but we focus on the Lie groups $SU(N)$ and $U(1)$. Again quantum fields have to be representations under these, with the trivial, fundamental and adjoint representation used for the SM. The specific representations of a field are also referred to as its quantum numbers and fields, which do not transform trivially, are said to be charged under that specific gauge group. For a gauge symmetry it is necessary to include a vector field in the adjoint representation and this field is then called a gauge field. In this group space scalar/fermion fields, transforming in the fundamental representation, can be seen as vectors and gauge fields as matrices. Each group has an interaction strength g associated to it and for $SU(N)$ the transformations are best characterized in terms of the generators T^a of the corresponding Lie algebra. Thus the transformation read [6]

$$\phi(x) \to \phi(x) \qquad \text{(singlet)}\,, \tag{1.3}$$

$$\phi(x) \to e^{i\alpha^a(x)T^a}\phi(x) \qquad \text{(fundamental)}\,, \tag{1.4}$$

$$A_\mu(x) \to \left(e^{i\alpha^a(x)T^a}\right)\left(A_\mu(x) + \frac{i}{g}\partial_\mu\right)\left(e^{i\alpha^a(x)T^a}\right)^\dagger \qquad \text{(adjoint)}\,, \tag{1.5}$$

[2]These representations also determine the spin of the corresponding particles. Scalars are spin-0, fermions spin-1/2, vector bosons spin-1 particles. Note that the term *fermion* usually describes any particle with half-integer spin. In this thesis we are only interested in spin-1/2 fermions.

with similar relations for other fields/charges. For a $U(1)$ symmetry one can replace in the above transformation rules the generators T^a with a number, e.g. Q. Interestingly, Q, also called charge, can be chosen for each field differently. This is often depicted by an index $U(1)_Q$ on the group. A QFT can then be constructed by choosing a set of fields and specifying the corresponding representations.

The group G_{SM} of the SM consists of the (direct) product of three gauge symmetries

$$G_{\mathrm{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y \,. \tag{1.6}$$

$SU(3)_c$ describes the strong nuclear force, with the index c standing for color, the charge of the strong force, and the corresponding QFT is dubbed Quantum Chromodynamics (QCD). The combination $SU(2)_L \times U(1)_Y$ is called the electroweak (EW) force. Later in the text it is shown that the electromagnetic and weak nuclear forces emerge from this gauge group. Y is the (weak) hypercharge and the index L refers to the fact that $SU(2)_L$ only couples to LH Weyl fermions.

In the SM all matter fields are Weyl fermion fields and one needs one gauge field for each gauge group of G_{SM}. As we will explain in more detail below, the SM contains additionally one single complex scalar field, the Higgs doublet, to give the elementary particles their mass. The matter fields can be grouped into three different generations (or families), which will only differ by their masses. It is useful to arrange corresponding fields between generations into a three component vector. These vectors are in the SM the up-type flavors $u = (u, c, t)^T$, down-type flavors $d = (d, s, b)^T$, charged leptons $e = (e, \mu, \tau)^T$ and neutral leptons (neutrinos) $\nu = (\nu_e, \nu_\mu, \nu_\tau)^T$. Each of those will have a LH and RH field except for neutrinos which only have LH fields. In the "classical" SM there are no RH neutrinos. While the RH fields are chosen as singlets under $SU(2)_L$, the LH fields are in the fundamental representation which combines different types of fermions, the quark doublet $q_L = (u_L, d_L)^T$ and the lepton doublet $l_L = (\nu_L, e_L)^T$. The $SU(3)_c$ representations do not differentiate between LH and RH fermions, but between the quarks u, d and the leptons e, ν. Only the quarks are charged under $SU(3)_c$ while the leptons are singlets. Each fermion has a non-zero $U(1)_Y$ charge Y, called hypercharge.

Moving on, the gauge fields are denoted G, W, B for the gauge groups $SU(3)_c$, $SU(2)_L$, $U(1)_Y$ respectively. They are in the adjoint representation of their respective group and are not charged under the others. The last ingredient of the SM is a complex scalar field H, the Higgs doublet, which is charged under $SU(2)_L$ and $U(1)_Y$. Its role is to break the EW symmetry, thereby giving mass to the weak force carriers. This will be explained in more detail later in this text, but for now we summarize the SM fields and their quantum numbers in Table 1.1.

The central object of a QFT is the (classical) action S. From it correlation functions and observables can be calculated in a systematic way, see e.g. [6]. The action S can be build up in the following way: First construct operators out of the fields of the QFT and their derivatives. The sum of all operators gives the Lagrange density $\mathcal{L}$. Finally the integral of $\mathcal{L}$ over all of space-time is defined as the action

spin	field	representation $(SU(3)_c, SU(2)_L)_{U(1)_Y}$
1/2	q_L	$(\mathbf{3},\mathbf{2})_{1/6}$
	l_L	$(\mathbf{1},\mathbf{2})_{-1/2}$
	u_R	$(\mathbf{3},\mathbf{1})_{2/3}$
	d_R	$(\mathbf{3},\mathbf{1})_{-1/3}$
	e_R	$(\mathbf{1},\mathbf{1})_{-1}$
1	B	$(\mathbf{1},\mathbf{1})_{0}$
	W	$(\mathbf{1},\mathbf{3})_{0}$
	G	$(\mathbf{8},\mathbf{1})_{0}$
0	H	$(\mathbf{1},\mathbf{2})_{1/2}$

Table 1.1: Representations of SM fields under the SM gauge symmetries

S. For example two of these operators for a scalar field ϕ could be $(\partial_\mu \phi)(\partial^\mu \phi), \phi^2$. In general terms linear in the field can be set to zero by a suitable field redefinition and an overall constant is unobservable (in a theory without gravity). Therefore one often encounters operators similar to the above quadratic in the fields. The quadratic ones with derivatives of the fields are referred to as kinetic operators and the ones without derivatives mass terms. Operators with three or more fields, or combinations of different fields are considered interactions.

The action of the SM has several useful properties, e.g. unitarity, stability and causality [6]. Furthermore, S and therefore the combined operators should be invariant under all external and internal symmetries (up to a total divergence). One can now write down the most general Lagrange density consisting of all operators allowed by the above principles. It turns out there are in general infinitely many of them and one can systematically write them down if one orders them according to their mass dimension D. The mass dimension of an operator is the sum of the mass dimension of its fields and derivatives. Derivatives have a mass dimension $[\partial] = 1$ and from the fact that the actions S should be dimensionless in natural units the mass dimensions of the fields can be derived. In general scalar and gauge fields have mass dimensions $[\phi] = \left[A_\mu\right] = 1$ and fermions $[\psi] = 3/2$. For any fixed mass dimension D there are then only a finite number of operators. It is useful to split them into two parts, the set of operators whose mass dimension is less then four and the one which is greater than four

$$\mathcal{L} = \mathcal{L}_{D\leq 4} + \mathcal{L}_{D>4}\,. \tag{1.7}$$

In Chapter 3 the consequences of these operators for the so called renormalization of the theory are laid out further, but it turns out that only theories, where all operators have a mass dimension $D \leq 4$ are renormalizable, and theories containing higher dimensional operators are non-renormalizable. The SM is definied to be a renormalizable theory and in the following we write down the finite number of operators invariant under the local gauge symmetry G_{SM}

$$\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_{\mathrm{ferm}} + \mathcal{L}_{\mathrm{G,W,B}} + \mathcal{L}_{\mathrm{Higgs}} + \mathcal{L}_{\mathrm{Yukawa}}\,. \tag{1.8}$$

The first term in (1.8) contains the kinetic terms for the fermions involving the derivative operator ∂_μ. To ensure gauge invariance of this term a coupling to the gauge bosons must be added. This can conveniently be done in all terms by replacing the ordinary derivative with the covariant derivative

$$\partial_\mu \to D_\mu = \partial_\mu - i g_s t^a G^a_\mu - i g T^i W^i_\mu - i g' Y B_\mu \,. \tag{1.9}$$

If the field onto which the derivative acts is in the singlet representation we can set the corresponding generator to zero. For the fundamental representations of $SU(3)_c, SU(2)_L$ we take $t^a = \frac{\lambda^a}{2}, T^i = \frac{\sigma^i}{2}$ respectively. Here λ^a are the eight Gell-Mann matrices, σ^a are the three Pauli matrices (see Appendix A for both) and Y is the hypercharge of the field in question. The values of the corresponding coupling constants[3] g_s, g, g' change with the renormalization scale μ (see Chapter 3) and are given in the $\overline{MS}$-scheme at $\mu = m_Z$ by [14]

$$g_s(m_Z) \approx 1.22, \qquad g(m_Z) \approx 0.65, \qquad g'(m_Z) \approx 0.36 \,. \tag{1.10}$$

Furthermore, it is convenient to place the irreducible 1/2-representations into one reducible representation, the Dirac representation, and then recover the Weyl-fermions via projectors[4] $\Psi_{L,R} = P_{L,R}\Psi, P_{L,R} = \frac{1}{2}(1 \mp i\gamma^5)$. Then the fermionic Lagrangian reads [6]

$$\mathcal{L}_{\text{ferm}} = \bar{q}_L i \not{D} q_L + \bar{u}_R i \not{D} u_R + \bar{d}_R i \not{D} d_R + \bar{l}_L i \not{D} l_L + \bar{e}_R i \not{D} e_R \,. \tag{1.11}$$

Here $\bar{\Psi} = \Psi^\dagger \gamma^0$ and $\not{D} = \gamma^\mu D_\mu$. γ are the Dirac matrices satisfying $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ and their explicit forms, as well as that of γ^5, are given in Appendix A. Note that this Lagrangian does not mix left- and right-handed fields i.e. no mass term is present. This means that this part has an additional symmetry known as chiral symmetry [8]. Later we will show how a mass term is generated via the Higgs mechanism. Moving on now, the Lagrangian which includes the kinetic terms for the gauge fields consists of three parts

$$\mathcal{L}_{\text{G,W,B}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{FP}} \,. \tag{1.12}$$

The first term in (1.12), the Yang-Mills Lagrangian, is constructed from the field-strength tensors for each gauge field

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a - \frac{1}{4} W^i_{\mu\nu} W^{\mu\nu}_i - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \,. \tag{1.13}$$

These field-strength tensors can be constructed in general from $\left[D_\mu, D_\nu\right]$ [6] and read

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu \,, \tag{1.14}$$
$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g \epsilon^{ijk} W^j_\mu W^k_\nu \,, \tag{1.15}$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \,. \tag{1.16}$$

[3] Later we will also use the notation $g_3 = g_s$ $g_2 = g$ and $g_1 = g'$.

[4] Note that we use the convention given by (A.1) in Appendix A, which differs slightly from the usual 4D convention to better describe the 5D aspects of later chapters

Here f^{abc}, ϵ^{ijk} are the structure constants of $SU(3)$ and $SU(2)$ respectively. They specify the structure of the corresponding Lie algebras via

$$\left[t^a, t^b\right] = i f^{abc} t^c \,, \tag{1.17}$$

$$\left[T^i, T^j\right] = i\epsilon^{ijk} T^k \,. \tag{1.18}$$

Note that they are also the matrix components of the generators in the adjoint representation $(t^b_G)^{ac} = i f^{abc}$, $(T^j_G)^{ik} = i\epsilon^{ijk}$.

To properly quantize the theory one has to add a gauge-fixing Lagrangian $\mathcal{L}_{\text{gf}}$ and Faddeev-Popov ghosts $\mathcal{L}_{\text{FP}}$ [25]. Note that because of gauge symmetry no mass term for the gauge bosons is written down, but experimentally one finds massive gauge fields in EW interactions. Masses can only be generated consistently if one breaks the EW symmetry $SU(2)_L \times U(1)_Y$ spontaneously. In the SM this is done by the Higgs mechanism [26, 27, 28]. The Lagrangian for the SM-Higgs reads [2, 3, 4]

$$\mathcal{L}_{\text{Higgs}} = \left(D_\mu H\right)^\dagger \left(D^\mu H\right) - V(H) \,. \tag{1.19}$$

Apart from the kinetic terms (including the covariant derivative to guarantee gauge invariance) the SM-Higgs contains a potential $V(H)$. Note that this Lagrangian is invariant under $SU(2)_L \times U(1)_Y$ (also under $SU(3)_c$). Now for certain forms of the potential the value for the Higgs field in the vacuum can be non zero. In this case gauge invariance allows us to write the vacuum expectation value (VEV) of the field as

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \,. \tag{1.20}$$

Thus the vacuum is no longer invariant under the full $SU(2)_L \times U(1)_Y$ symmetry but only under a subgroup $U(1)_{\text{EM}}$. Since the symmetry is not broken by the Lagrangian, but by theVEV, this process is in general referred to as spontaneous symmetry breaking (SSB) and in the case of the EW symmetry as electroweak symmetry breaking (EWSB). $SU(2)_L \times U(1)_Y$ contains four generators but the VEV is only invariant under one linear combination, namely

$$Q = T^3 + Y \,, \tag{1.21}$$

and we will identify this operator with the electric charge of a particle and the remaining symmetry $U(1)_{\text{EM}}$ as the electromagnetic symmetry. To study the consequences of EWSB let us parameterize the full Higgs field by fluctuations around the VEV via four real fields $\varphi^i, i = 1, 2, 3$ and h

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} -i\sqrt{2}\varphi^+(x) \\ v + h(x) + i\varphi^3(x) \end{pmatrix} \,, \tag{1.22}$$

where $\varphi^\pm = \frac{1}{\sqrt{2}}\left(\varphi^1 \mp i\varphi^2\right)$. The field $h(x)$ parametrizes massive fluctuations around the VEV and in the SM it corresponds to the famous Higgs particle, which was discovered at the LHC in 2012 [29, 30]. According to the Goldstone theorem [31, 32] the remaining three generators correspond to three Goldstone bosons which are unphysical and can formally be removed from the theory thorough a suitable gauge transformation (unitary gauge). These are the fields $\varphi^{\pm}, \varphi^3$. Of course their degrees of freedom are not lost, they reappear as the the longuitudinal polarizations of massive EW bosons. Therefore it is sometimes said that the Goldstone bosons are "eaten" by the gauge bosons. The masses follow from the interaction of the gauge fields with the VEV, which can be seen explicitly by plugging (1.22) into (1.19)

$$\mathcal{L} \supseteq \frac{g^2 v^2}{8}\left((W^1_\mu)^2 + (W^2_\mu)^2\right) + \frac{v^2}{8}\left(gW^3_\mu - g' B_\mu\right)^2 . \tag{1.23}$$

To diagonalize the mass terms we rotate the fields in accordance to the remaining symmetry generator (1.21) and with canonically normalized kinetic terms [6]

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_w & -s_w \\ s_w & c_w \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} , \qquad W^{\pm}_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu \mp i W^2_\mu\right) . \tag{1.24}$$

Here the sine s_w and cosine c_w of the weak mixing angle θ_w are defined via [6]

$$s_w \equiv \sin(\theta_w) = \frac{g'}{\sqrt{g^2 + g'^2}} , \qquad c_w \equiv \cos(\theta_w) = \frac{g}{\sqrt{g^2 + g'^2}} . \tag{1.25}$$

This results in the mass terms

$$\mathcal{L} \supseteq m_W^2 W^+_\mu W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu , \tag{1.26}$$

with the masses for the W- and Z-bosons [6]

$$m_W^2 = \frac{g^2 v^2}{4} , \qquad m_Z^2 = \frac{(g^2 + g'^2)v^2}{4} . \tag{1.27}$$

The field A_μ remains massless and corresponds to the generator Q, thus can be identified with the photon field. The electric charges of the fields $Z, W^{\pm}$ are $q = 0, \pm 1$ respectively. Expressing the covariant derivative in terms of these fields we find [6]

$$\begin{aligned} D_\mu = \partial_\mu - i g_s t^a G^a_\mu - ieQA_\mu - i\frac{g}{\cos(\theta_w)}\left(T^3 - \sin^2(\theta_w) Q\right) Z_\mu \\ - i\frac{g}{\sqrt{2}}\left(T^+ W^+_\mu + T^- W^-_\mu\right) , \end{aligned} \tag{1.28}$$

where the electromagnetic coupling is identified by $e = g\sin(\theta_w)$ and we have defined $T^{\pm} = \frac{1}{\sqrt{2}}(T^+ \pm iT^-)$. At low energies the interaction with the W-bosons reproduce the Fermi theory with the Fermi coupling constant $G_F = \frac{g^2}{4\sqrt{2} m_W^2}$.

What remains to look at is the potential $V(H)$. In the SM it is given by

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 . \tag{1.29}$$

Here values $\mu^2 > 0, \lambda > 0$ allow for a non trivial minimum $v^2 = \frac{\mu^2}{\lambda}$. Putting the parametrization (1.22) into the potential one can determine the mass of the Higgs boson $m_h^2 = 2\mu^2 = \frac{\lambda}{2}v^2$.

Last but not least the Higgs can also give masses to the fermions through Yukawa interactions. For the coupling between fermions and gauge bosons gauge invariance implies that the coupling for corresponding fermions across families is the same. This restriction does not apply to the Yukawa couplings which can be matrices in flavor space. It is is the only part of the Lagrangian which distinguishes between different generations. The corresponding Lagrangian is given by [8]

$$\mathcal{L}_{\text{Yukawa}} = -y_u^{ij}\bar{q}_L^i H^c u_R^j - y_d^{ij}\bar{q}_L^i H d_R^j - y_e^{ij}\bar{l}_L^i H e_R^j + \text{h.c.}\,, \tag{1.30}$$

where $H^c = i\sigma^2 H^*$ and h.c. denotes the hermitian conjugate. After EWSB the terms above result in mass matrices $m_f^{ij} = \frac{v^2}{\sqrt{2}}y_f^{ij}$ for the fermion fields

$$\mathcal{L} \supseteq -m_u^{ij}\bar{u}_L^i u_R^j - m_d^{ij}\bar{d}_L^i d_R^j - m_e^{ij}\bar{e}_L^i e_R^j + \text{h.c.}\,, \tag{1.31}$$

which needs to be diagonalized to obtain the physical fields. Note that there is no mass matrix for neutrino fields, they will remain massless in the "classical" SM. Matrices can always be diagonalized by a bi-unitary rotation

$$\text{diag}(m_{f1}, m_{f2}, m_{f3}) = U_{f,L}^\dagger m_f U_{f,R}\,. \tag{1.32}$$

This diagonalization can be achieved for all three mass matrices by rotating the fields via

$$\begin{aligned} u_L &\to U_{u,L}u_L\,, \qquad & u_R &\to U_{u,R}u_R\,, \\ d_L &\to U_{d,L}d_L\,, \qquad & d_R &\to U_{d,R}d_R\,, \\ e_L &\to U_{e,L}e_L\,, \qquad & e_R &\to U_{e,R}e_R\,. \end{aligned} \tag{1.33}$$

For simplicity we will denote the fields after the rotation (also called the mass or physical basis) by the same letters as the original fields (also called the flavor basis). It will also be useful to rotate the neutrinos by the same amount as the electron

$$\nu_L \to U_{e,L}\nu_L\,. \tag{1.34}$$

The kinetic part as well as the coupling to photons, gluons and Z-bosons are completely universal in flavor space and thus are not effected by these unitary transformations. For the $W^\pm$-bosons the presence of the generators $T^\pm$ results in mixing between the different $SU(2)_L$–doublets. The fermionic field redefinitions then lead to flavour changing currents in the quark sector via the charged W-fields [6]

$$\mathcal{L} \supseteq \frac{g}{\sqrt{2}}\bar{u}_L^i \not{W}^+ d_L^i + \frac{g}{\sqrt{2}}\bar{\nu}_L^i \not{W}^+ e_L^i + \text{h.c.} \to \frac{g}{\sqrt{2}}(V_{\text{CKM}})^{ij}\bar{u}_L^i \not{W}^+ d_L^j + \frac{g}{\sqrt{2}}\bar{\nu}_L^i \not{W}^+ e_L^i + \text{h.c.}\,, \tag{1.35}$$

with the Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$V_{\mathrm{CKM}} \equiv U_{u,L}^{\dagger} U_{d,L} \,. \tag{1.36}$$

Since the SM-neutrinos are massless the definition (1.34) eliminates a similar term in the lepton sector. But in extensions of the SM one can give the neutrinos a mass and a similar matrix, the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix V_{PMNS}, can be constructed. The CKM matrix has four independent parameters, three mixing angles and one phase. The phase parameterizes the amount of CP-violation in the weak sector.

It turns out that the measured values for the CKM exhibit some hierarchies depending on the generation. Explicitly, one finds [14]

$$V_{\mathrm{CKM}} \sim \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97370 & 0.2245 & 0.00382 \\ 0.221 & 0.987 & 0.0410 \\ 0.0080 & 0.0388 & 1.013 \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} , \tag{1.37}$$

where we parametrized the hierarchies by the value λ and so far there is no agreed upon explanation for this structure. Furthermore, the entries of the PMNS matrix have no such hierarchies.

Since the CKM matrix only appears in the interaction of the charged W bosons, there are no FCNCs at tree level. This changes at loop level, where the exchange of multiple W-bosons, e.g. in so called box diagrams, can give rise to such FCNCs. The observed suppression of such currents is due to the Glashow-Iliopoulos-Maiani (GIM) mechanism [33], which is a direct consequence of the unitarity of the CKM matrix and for equal quark masses the cancellation would be exact.

As mentioned in the beginning, with these ingredients the SM describes successfully the particles and interactions around the EW scale. This concludes the short review of the SM and in the next section we study some of the remaining open questions of the SM.

2 Problems and Open Questions: The Standard Model as an Effective Field Theory

Although the SM very successfully describes nature on a subatomic level, there are several observations the SM does not explain and topics for which further inside would be desirable. Here we give an overview of some of the (theoretical) aspects, which suggests that there is physics beyond the SM.

First, one might imagine that there are reasons for the values of the 18 free parameters of the SM. Currently, they have to be determined from experiment, but deeper a understanding of their origins would be preferred. 13 out of the 18 parameters are the Yukawa couplings of the fermions with the Higgs doublet, which give the fermion masses and the CKM matrix. The flavor puzzle is the inability of the SM to explain the values of these parameters in the flavor sector and, since the differences between the arising fermion masses are relatively large, this becomes even more puzzling. Similarly, there are three independent gauge couplings in the SM. As we will see in Chapter 3 these three couplings have approximately the same value at high energies, which hints to the possibility that the three corresponding interactions might arise out of one single underlying one. This goes under the name of Grand Unified Theories (GUT) and will be further explored in Chapter 3. The last two parameters describe the Higgs potential, which triggers EWSB. There is currently no more fundamental reason for its form. Furthermore, its tachyonic mass parameter is the only dimensionfull parameter of the SM. If one considers other fundamental scales of the universe (see below), there is no apparent reason for it being around the EW scale. Explicitly, this might introduce a hierarchy problem, which concerns large differences between scales in a QFT and will be explained in more details below.

One goal of research could thus be to find successor theories which explain the origin of these SM parameters. This can be motivated by other deficits of the SM. There are several topics, already scratched in the chapter above which the SM does not explain. Most prominently, gravitational interactions are not included in the SM. They don't play a role at low energies, but it is believed that around an energy of $M_{\rm Pl} = \sqrt{\frac{\hbar c}{G_N}} \sim 10^{19}\,{\rm GeV}/c^2$, the Planck scale, the effect of gravity must be included. So far there is no established theory which completely includes the theory of gravity (General Relativity) and the SM.

Furthermore, the effect of neutrino oscillations has been observed [34], which implies that neutrinos have a (small) mass, but the "classical" SM describes them as massless particles. The precise nature of these neutrino masses is unclear.

Some readers are also already aware of the fact, that some of the allowed operators in the gauge sector have been omitted. The ones missing are of the form $\theta \frac{g_s^2}{64\pi^2} \tilde{F}_{\mu\nu} F^{\mu\nu}$, with the dual field strength tensor $\tilde{F}_{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$. It turns out that for $SU(2)_L$ and $U(1)_Y$ such a term has no observable effect [6], but for $SU(3)_c$ this is not the case. As this operator would induce large CP violating effects in the strong sector, which have not been observed, the corresponding operator coefficient $\theta_{\rm QCD}$ has to be very small or zero. As this breaks with the principle of writing down all allowed operators, this is dubbed the strong-CP problem [35].

There are also certain observations of cosmology which can not be explained by the particle content provided by the SM. There a new kind of matter, called dark matter, is proposed to explain the observed structures in space [36, 37, 38, 39]. There is no candidate for this in the SM. Similarly, there is an unknown effect, called dark energy, which drives the accelerated expansion of the universe [40, 41].

From cosmology one can also deduce that there is a certain baryon asymmetry in the universe, we see more particles than corresponding antiparticles, see e.g. [42, 43]. To explain this asymmetry three conditions, the Sakharov conditions [44], must be satisfied. Although these three conditions are qualitatively fulfilled in the SM, quantitatively their corresponding effects are to small to explain the observed baryon asymmetry.

Furthermore, there are also some experimental results from precision measurements, which are in slight tension with the SM. For example there are some anomalies in the B-sector, see e.g. [45], and discrepancies between the theoretical and measured values of the muon anomalous magnetic dipole moment $(g-2)_\mu$ [46].

These considerations suggest that the SM is not the end of the story and there should exist beyond the Standard Model (BSM) physics. This does not mean that the SM is invalid, it still describes the experiments of the last decades very well, rather it should be seen as a low energy effective field theory (EFT) of an underlying more fundamental theory. See e.g. [47, 48] for comprehensive reviews. In such set-ups the low energy EFT can be obtained from the more fundamental theory by integrating out the degrees of freedom above a scale Λ. The EFT is then valid up to this cut-off Λ. This procedure then in general produces also non-renormalizable operators. It is again useful to order these operators in terms of their mass dimension. Without any knowledge of the underlying theory the size of the coefficients of these operators is determined by the cut-off scale Λ. If the mass dimension of the operator $\mathcal{O}$ is D the coefficient scales like Λ^{4-D} in 4 dimensions. The Lagrangian can then be written as

$$\mathcal{L} = \mathcal{L}_{D\leq 4} + \sum_i \frac{c_i}{\Lambda}\mathcal{O}^{(i)}_{D=5} + \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}^{(i)}_{D=6} + \dots\,, \tag{2.1}$$

with the dimensionless Wilson coefficients c_i. The operators with $D \leq 4$ grow (or stay the same) with increasing Λ, whereas the operators with $D > 4$ are suppressed by more and more powers of Λ. For a large value of Λ this means that at low energies the renormalizable operators ($D \leq 4$) are sufficient to describe these processes. This could explain why the SM only needs renormalizable operators to describe the observed experiments with good accuracy. From these we can also conclude that Λ is larger than the EW scale, but it could be as large as $\Lambda \sim M_{\mathrm{Pl}}$. For the following discussion we assume a value of $\Lambda \sim M_{\mathrm{Pl}}$, but any large value of Λ leads to the same considerations. Note that the SM has only dimensionless parameters, except for the parameter μ in the Higgs-Sector. This parameter is of the order of the EW scale which is measured to be $v = 246\,\mathrm{GeV}$. Now the theory contains two scales v and M_{Pl} which are separated by 16 orders of magnitude and there seems to be no relation between them. This is the first part of the so called (gauge) hierarchy problem (HP): why are these two scales so different? The second part of the problem considers the EFT description above. From an EFT perspective the parameter μ is to be expected to be of the order of M_{Pl} and this would result in a similarly large Higgs mass. Even if this mass is small (or zero) to begin with, radiative corrections would lead to an $\mathcal{O}(M_{\mathrm{Pl}})$ Higgs mass. Note that this does not apply to fermions or gauge bosons. Their masses are protected from radiative corrections by chiral and gauge symmetry respectively. The SM-Higgs does not have such a protection mechanism. The hierarchy problem might hint to the possibility that new physics effects emerge at the TeV scale, as this can cut-off the divergencies at this relatively low scale, thereby solving the problem.

Having stated some of the problems of the SM we turn in the next section to some of the proposed solutions to these, with an emphasis on the above mentioned (gauge) hierarchy problem.

2.1 Solutions to the (Gauge) Hierarchy Problem

In this section we present directions which explore BSM physics to try to solve the above mentioned deficits of the SM. We focus primarily on the (gauge) hierarchy problem.

One possibility is to entitle the Higgs with a new symmetry to protect it from large radiative corrections. This is the idea of **Supersymmetry (SUSY)**, see e.g. [49, 50]. Here every particle is related by a SUSY to a supersymmetric counterpart, bosons are related to fermions and fermions to bosons. Thus the Higgs can inherit a stable mass from its supersymmetric partner, the Higgsino, because its fermionic mass is protected by chiral symmetry. The so called Minimal Supersymmetric Standard Model (MSSM) [51] was the most promising theory in the past, but the particles it predicts around the EW scale have not been found so far.

Instead of introducing a new symmetry, the dynamical evolution in the early universe could also lead to a hierarchically small weak scale. This is the idea of Cosmological Relaxation [52], where an axion-like field ϕ is introduced. Allowing for soft symmetry breaking of the axion shift symmetry, one can couple ϕ to the Higgs, without introducing large radiative corrections. The VEV of ϕ can then start from (large) positive values and decrease during the evolution of the early universe. Once the VEV of ϕ crosses to negative values the Higgs mass will be tachyonic, thereby triggering EWSB. In turn the effect of EWSB then stops a further decrease in the VEV of ϕ fixing the Higgs mass to its current value.

Another possibility are **technicolor theories** [53, 54]. Here the problem with the Higgs boson mass are circumvented by using a different mechanism than the Higgs mechanism to generate the masses of W- and Z-bosons. The mass is still produced by "eating" Goldstone bosons of a broken symmetry, but this breaking should be induced by a new sector of strongly interacting fermions. A motivation for this can be found in QCD. The up- and down-quarks have an approximate global $SU(2)_L \times SU(2)_R$ symmetry. Due to confinement at energies below $\Lambda_{\rm QCD} \approx 200\,\mathrm{MeV}$ the theory builds a quark condensate of fermion bilinears. This state breaks the $SU(2)_L \times SU(2)_R$ down to its diagonal subgroup $SU(2)_V$. Due to Goldstones theorem the three broken generators correspond to three Goldstone bosons, the pions. Note that in QCD, because the symmetry is only approximate, they are not truly massless, but their mass is lower then the typical scale $\Lambda_{\rm QCD}$ of the theory.

Technicolor mimics this by introducing so called techni-fermions, which interact by new strong technicolor force. Here the breaking scale should be around the EW scale and the corresponding Goldstone modes are absorbed by the EW bosons like in the SM case. But the simplest models of technicolor are not in accordance with EWPT [55] and don't explain the scalar particle discovered in 2012.

Composite Higgs theories [56, 57, 58, 59, 60] can be seen as interpolating between technicolor theories and the SM Higgs mechanism. On the one hand this Higgs is a pseudo Nambu-Goldstone boson (pNGB) of a global symmetry, like in Little Higgs theories [61, 62, 63], and on the other hand it is a composite particle of a new strongly interacting sector. Similar to the pion in QCD, it would not receive radiative corrections above its compositness scale and be naturally lighter than the confining scale. In such scenarios, like the Minimal Composite Higgs Model (MCHM) [64, 65], the Higgs doublet would be massless at tree level, but because of the explicit breaking of the global symmetry, loops of SM particles can generate a Higgs potential. In turn this Higgs potential can break the EW symmetry via the usual Higgs mechanism. Different from technicolor models, where no separation of scales exists, the EW scale v is dynamically generated and can be smaller than breaking scale f of the global symmetry breaking. The degree of misalignment $\xi = \frac{v^2}{f^2}$ is then the usual suppression factor for corrections to SM observables. The limit, $f \to \infty$ with fixed v, then decouples all other resonances and would produce the SM Higgs mechanism. In general, natural models predict top partners around the TeV scale, which have not been observed so far. There are many possible strong sectors which can lead to a light Higgs bosons but interestingly some of them can be related to the next class of theories by holography.

Via the AdS/CFT correspondence 4D Composite Higgs theories have a **five dimensional (5D)** dual interpretation. The ability of AdS_5 to solve the HP was shown by Randall and Sundrum [66] and phenomenological models of that kind are now known as Randall-Sundrum (RS) models. These models furnish the main part of this thesis and we will explain in detail in Chapter 4 how they solve the HP through their geometry. Another extra dimensional model, which solves the HP geometrically, are Arkani-Hamed-Dimopoulos-Dvali (ADD) models [67, 68]. In ADD only gravity can propagate into the extra dimensions and due to this dilution the gravitational force appears much weaker (the Planck-mass much larger) then the other interactions. The real cut-off of the SM could then be much lower.

Before we explore RS theories further in the next part, we first remain in 4D and review the process of renormalization and regularization and introduce GUTs.

3 Renormalization of Quantum Field Theories and Grand Unified Theories

Since this work studies the regularization and renormalization of QFTs in RS-models this chapter is devoted to explain some of the concepts of regularization and renormalization in more detail.

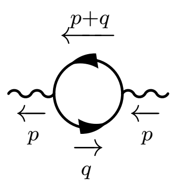
After writing down the classical action S in Chapter 1, there are several methods to get a fully fledged QFT. Two of the most well known are the Operator Formalism and the Path Integral Formalism [6]. Both allow one to define correlation functions between quantum fields starting from the classical action. These are the main quantities in a QFT, since they allow to calculate the so called S-matrix via the Lehmann-Symanzik-Zimmermann (LSZ) reduction formula [69], which in turn can be related to cross sections and decay rates. Unfortunately, one cannot calculate the correlation functions exactly in most cases. In these situations one can then try to expand these functions in a perturbative expansion in a small parameter, like a coupling constant, to get an approximate result. Every term in this expansion has a similar structure and one can use so called Feynman diagrams as a mnemonic device to write them down systematically. To construct Feynman diagrams one needs expressions for propagators of the fields involved and vertices connecting them. These can be derived from the classical action and for a warped 5D theory we do this in Chapter 8. At a higher order in perturbation theory one also needs to calculate loops of propagators. In general, this causes problems, which will be illustrated by the following toy example. We take a single Dirac fermion ψ coupled to an abelian $U(1)$ gauge symmetry

$$S = \int \mathrm{d}^4x \left[-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \bar{\psi} \left(i \left(\not{\partial} - i g_0 \not{A} \right) - m \right) \psi \right] . \tag{3.1}$$

From this a correlation function, which determines the coupling between the fermion and the gauge field, can be calculated perturbatively. At first order, or tree-level, one finds

$$g^{-2} = g_0^{-2} . \tag{3.2}$$

Thus the coupling is just given by the Lagrangian parameter g_0. At the next order one encounters a loop of fermions entailing the following integral [8]



$$\propto \int \frac{\mathrm{d}^4q}{(2\pi)^4} \frac{\mathrm{Tr}\big(\gamma^\mu(\not{q} + \not{p} + m)\gamma^\nu(\not{q} + m)\big)}{(q^2 - m^2)((q+p)^2 - m^2)} . \tag{3.3}$$

This is a divergent integral. To deal with this divergence one can use the method of regularization. Different regularization procedures exist in the literature, for example cut-off regularization, dimensional regularization or Pauli-Villars regularization.

In each case a regularization parameter and possibly a regularization scale is introduced. Via a suitable limit of the regularization parameter the full integral above is recovered. Here we will use dimensional regularization, which changes the dimension of Minkowski space from 4 to $4-2\epsilon$. To keep the mass dimension of the integral fixed one introduces the regularization scale μ. For certain ϵ the integral then converges and the result can be analytically continued around $\epsilon = 0$. This gives something like the following for the coupling, which will now depend on the momentum p of the external propagators,

$$g^{-2}(p^2) = g_0^{-2} + \frac{b}{16\pi^2}\left(\frac{1}{\epsilon} - \log\left(\frac{p^2}{\mu^2}\right) + \text{const.}\right) , \tag{3.4}$$

where b is a numerical coefficient. In the limit $\epsilon \to 0$ this would still give a divergent result. A solution to this is to renormalize the theory. This divergence gets replaced by using a renormalized coupling $g_{\text{ren.}}(\mu_0)$ at a scale μ_0 defined in the following way

$$g_0^{-2} = g_{\text{ren.}}^{-2}(\mu_0) + \frac{b}{16\pi^2}\left(-\frac{1}{\epsilon} - \log\left(\frac{\mu^2}{\mu_0^2}\right) + \text{const.}\right) . \tag{3.5}$$

For which the coupling results in

$$g^{-2}(p^2) = g_{\text{ren.}}^{-2}(\mu_0) - \frac{b}{16\pi^2}\log\left(\frac{p^2}{\mu_0^2}\right) + \text{const.} . \tag{3.6}$$

Now all dependence on the regularization parameter and scale have dropped out and we can send $\epsilon \to 0$. There is a freedom in which constant one absorbs in the renormalized coupling $g_{\text{ren.}}(\mu_0)$, which is the so called renormaliaztion scheme. Popular choices are the MS-scheme in which the constant is zero or the $\overline{MS}$-scheme in which often appearing constants like the Euler-Mascheroni contant γ_E and $\log(4\pi)$ are absorbed, too. From (3.6) it follows that different constants correspond to different values μ_0. For a fixed scale μ_0 the value of $g_{\text{ren.}}^{-2}(\mu_0)$ has to be measured in an experiment e.g. by measuring the coupling $g^{-2}(p^2 = \mu_0^2)$. The chosen scale μ_0 and thus the renormalization scheme are arbitrary and different choices for them should not change the physics. Therefore $g_{\text{ren.}}^{-2}(\mu_0)$ changes in a specific way with different μ_0. This is captured by the renormalization group equation (RGE)

$$\mu_0\frac{\partial}{\partial\mu_0}g^{-2}(p^2) = 0 \implies \mu_0\frac{\partial}{\partial\mu_0}g_{\text{ren.}}(\mu_0) = \beta(g_{\text{ren.}}) = \frac{b}{16\pi^2}g_{\text{ren.}}^3 . \tag{3.7}$$

The sign of the *beta*–function β gives interesting insights into the dynamics in the UV and IR. For $\beta > 0$ the coupling grows for higher energies and can even diverge at some large scale, giving rise to the so called Landau pole. This is the case for the $U(1)_Y$ coupling in the SM, but the predictivity of the perturbative result ends way before that scale. For $\beta < 0$ the coupling becomes stronger in the IR. This can lead to confinement, where only composite states can be observed, and this is exactly what happens for QCD. Furthermore, for high energies the coupling goes to zero, which allows one to treat the quarks as free particles in these composite states for scattering experiments. Note also that the EW gauge group is broken at low energies so there will be no confinement of the weak force.

It turns out that additional divergences appear at higher loop order and in different correlation functions. Many of them can be dealt with by defining a set of renormalized Lagrangian parameters. If only a finite set of parameters is needed for all possible divergences, the theory is called renormalizable, otherwise non-renormalizable. To estimate if a theory is renormalizable one can look of the mass dimension D of the operators of the Lagrangian. Through quantum corrections operators can produce other operators, but operators with $D \leq 4$ produce only operators with $D \leq 4$. On the other hand operators with $D > 4$ will in general produce operators of arbitrary high mass dimension. These are then an infinite set of parameters which need to be renormalized. Thus only theories, where the operators have mass dimension $D \leq 4$, are renormalizable, otherwise they are non-renormalizable. An EFT is in general not renormalizable, but if one restricts oneself to a specific order in the EFT cut-off, a consistent renormalization can be achieved.

Since the SM is per definition a renormalizable theory it only contains operators with mass dimension four or less. As discussed above, one consequence of renormalization is that the three gauge couplings of the SM depend on the energy scale. Defining[1] $\alpha_i \equiv \frac{g_i^2}{4\pi}$ this results in the following energy dependence [7]

$$\alpha_3^{-1}(\mu) = \alpha_3^{-1}(m_Z) - \frac{b_3}{2\pi}\log\left(\frac{\mu}{m_Z}\right), \qquad b_3 = -7\,, \tag{3.8}$$

$$\alpha_2^{-1}(\mu) = \alpha_2^{-1}(m_Z) - \frac{b_2}{2\pi}\log\left(\frac{\mu}{m_Z}\right), \qquad b_2 = -\frac{19}{6}\,, \tag{3.9}$$

$$\alpha_1^{-1}(\mu) = \alpha_1^{-1}(m_Z) - \frac{b_1}{2\pi}\log\left(\frac{\mu}{m_Z}\right), \qquad b_1 = \frac{41}{10}\,, \tag{3.10}$$

where we used the starting value $\mu_0 = m_Z$. This evolution is depicted in Figure 3.1, where one can see that around an energy scale of $M_{\text{GUT}} \sim 10^{14}$–$10^{16}$ TeV the couplings approximately meet. This could be a hint, that they have some common origin and this is the starting point of GUTs [70, 71, 72]. The idea is to start with a larger group and to recover the SM group via SSB. The minimal group which can lead to the G_{SM} is $SU(5)$. Since $SU(6)$ GHGUTs, which will be studied in this thesis, share many features with $SU(5)$ GUTs, we will take a closer look at the original Georgi–Glashow $SU(5)$ model[70], but many aspects of the following discussion also apply to different GUTs, e.g. GUTs based on $SO(10)$ or E_6 [14]. The three SM gauge fields can be combined in the adjoint representation **24** of $SU(5)$.

[1] for g_1 we include an extra factor $\frac{5}{3}$ which will become clear from the discussion about unification below.

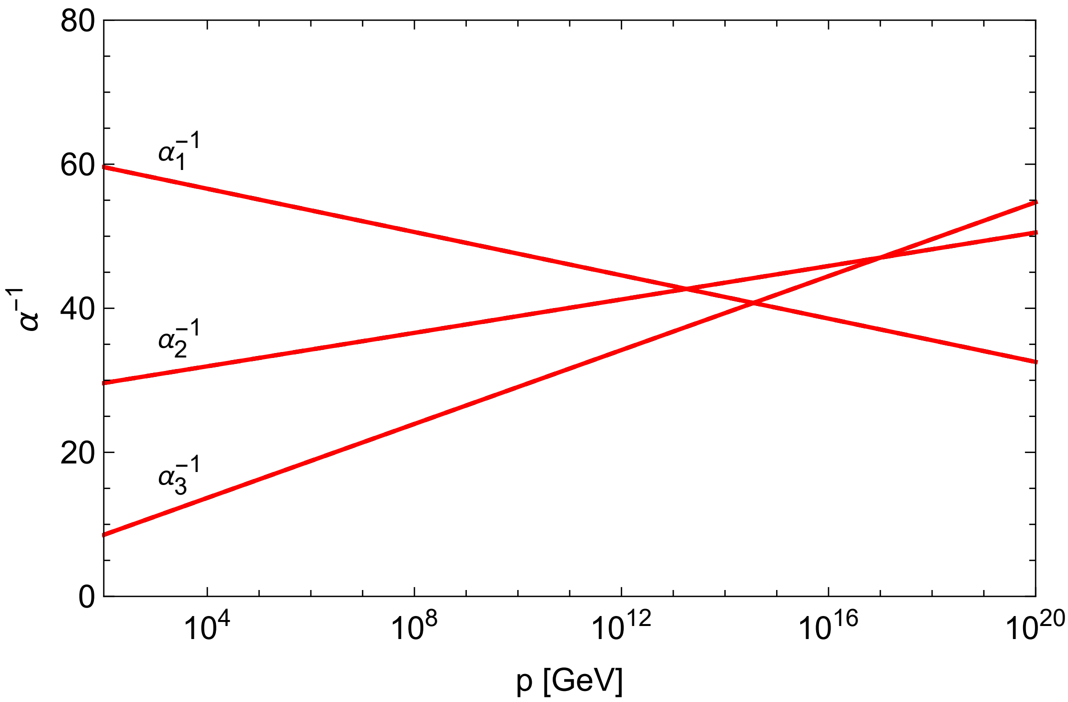


Figure 3.1: Running of the three SM couplings $\alpha_3, \alpha_2, \alpha_1$ at one-loop according to (3.8)–(3.10). The initial values are given by (1.10).

Symbolically we can write [7]

$$A^a_\mu T^a = \left(\begin{array}{c|c} SU(3)_c & X/Y \\ \hline X/Y & SU(2)_L \end{array} \right) , \tag{3.11}$$

where $SU(3)_c$ and $SU(2)_L$ correspond to the upper left 3×3 and lower right 2×2 blocks, respectively. We can also identify the hypercharge Y of $U(1)_Y$ with one of the completely diagonal generators, $T^{24} = c\,\mathrm{diag}(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, +\frac{1}{2}, +\frac{1}{2})$ with $c^2 = 3/5$, via the rescaling

$$Y = \sqrt{\frac{5}{3}}\, T^{24} \,. \tag{3.12}$$

As can be seen by the off-diagonal elements in (3.11) there are additional gauge bosons, called X/Y bosons, which can be viewed as one complex field with the SM quantum numbers $(\mathbf{3}, \mathbf{2})_{-5/6}$.

To get $G_{\rm SM}$ at low energies $SU(5)$ can be spontaneously broken via the Higgs mechanism by including a new scalar Φ in the $\mathbf{24}$ of $SU(5)$, which gets a VEV $\langle\Phi\rangle = V\,\mathrm{diag}(-\frac{1}{3},-\frac{1}{3},-\frac{1}{3},+\frac{1}{2},+\frac{1}{2})$ [7]. For the unification of gauge couplings we need $V \sim M_{\rm GUT}$ and thus the X/Y bosons will acquire a mass around this scale. Interestingly, interactions with the X/Y bosons can violate baryon and lepton number conservation leading to e.g. proton decay. As proton decay has not been observed so far, the lifetime of the proton is at least $\tau_p/\mathrm{BR}(p \to e^+\pi^0) > 1.67\times 10^{34}\,\mathrm{yr}$ [73]. Independent of the model this means for the mass M_X of the X/Y bosons: $M_X \gtrsim 10^{16}\,\mathrm{GeV}$.

Moving on to the fermion fields one can observe that one generation of quarks and leptons exactly fills the $\mathbf{5}^*$ and $\mathbf{10}$ representations of $SU(5)$

$$\begin{aligned}
&\mathbf{5}^* \to d_L^c(\mathbf{3}^*,\mathbf{1})_{+1/3} \oplus l_L(\mathbf{1},\mathbf{2})_{-1/2}\,,\\
&\mathbf{10} \to u_L^c(\mathbf{3}^*,\mathbf{1})_{-2/3} \oplus q_L(\mathbf{3},\mathbf{2})_{+1/6} \oplus e_L^c(\mathbf{1},\mathbf{1})_{+1}\,.
\end{aligned} \tag{3.13}$$

Here the RH SM fields are expressed by their charge conjugates[2], such that all fields are LH, for example $u_L^c \equiv (u^c)_L = (u_R)^c$.

This structure does not only explain why quarks interact via $SU(3)_c$ and leptons not, but also why $SU(2)_L$ couples only to one chirality, in this case LH fields. Moreover, the quantization of the hypercharge Y can be explained. Whereas in the SM these can a priory be any real numbers, the observed fractional charges follow directly from the hypercharge matrix given in (3.12) acting on the above fermion fields.

To include the Higgs doublet H, to break the EW symmetry, one is forced to include further scalar states to fill a complete $SU(5)$ representations. For example the minimal choice of a $\mathbf{5}^*$ also includes a scalar triplet φ. The Yukawa interaction between the full $\mathbf{5}^*$ and the fermions has two notable consequences. First, since the quarks and leptons are combined in the multiplets above there will be relations between the SM masses

$$m_b = m_\tau\,, \qquad m_s = m_\mu\,, \qquad m_d = m_e\,. \tag{3.14}$$

Even if renormalization effects are considered there are no such relations between the measured particle masses[3] [74, 75]. Second, the scalar triplet will couple quarks and leptons leading to proton decay. The corresponding constraint on the triplet mass $M_\varphi > 1.0\times 10^{12}\,\mathrm{GeV}$ [14] is weaker than on M_X since the first generation Yukawa couplings are much smaller. The mass of this triplet as well as the Higgs mass follow from the scalar potential, which includes also the GUT-scalar Φ [7]

$$\begin{aligned}
V(\phi,\Phi) = &-\frac{1}{2}m_\Phi^2\,\mathrm{Tr}\big(\Phi^2\big) + \frac{1}{4}\lambda_1\,\mathrm{Tr}\big(\Phi^4\big) + \frac{1}{4}\lambda_2\,\mathrm{Tr}\big(\Phi^2\big)^2\\
&+ m_\phi^2\phi^\dagger\phi + \frac{1}{4}\kappa_1(\phi^\dagger\phi)^2 - \frac{1}{2}\kappa_2\phi^\dagger\Phi^2\phi\,.
\end{aligned} \tag{3.15}$$

[2] Charge conjugation for a Dirac fermion ψ is defined via $\psi^c \equiv i\gamma^2\psi^*$

[3] To resolve this issue one can for example take a more complicated Higgs sector or one needs to take into account higher dimensional operators [7]

Here ϕ is the $\mathbf{5}^*$ and for the individual masses one finds

$$m_H^2 = m_\phi^2 - \frac{1}{8}\kappa_2 V^2\,, \tag{3.16}$$

$$m_\varphi^2 = m_\phi^2 - \frac{1}{18}\kappa_2 V^2\,. \tag{3.17}$$

Thus both masses are expected to be at the order $V \sim M_{\mathrm{GUT}}$, which can be seen as a concrete manifestation of the hierarchy problem. Again this also applies to radiative corrections. Even if the hierarchy problem is solved, there remains an additional problem for the tree level parameters. As we have seen above the triplet mass must be large and thus a small Higgs mass can only be achieved if m_ϕ^2 and $\frac{1}{8}\kappa_2 V^2$ cancel to at least sixteen significant digits. This is known as the doublet-triplet splitting problem [76, 77]. Although this cancellation is technically possible a deeper reason for this would be desirable.

Having a complete field content we can return on a more quantitative analysis of the unification of the gauge couplings. Since we started with one group there is only one gauge coupling and the SM gauge couplings are all equal at tree–level

$$g = g_3 = g_2 = \sqrt{\frac{5}{3}}g_1\,. \tag{3.18}$$

Note that we included an extra factor $\sqrt{5/3}$ for the coupling g_1 of the hypercharge group $U(1)_Y$ since we rescaled the $SU(5)$ generator T^{24} in (3.12) by this factor to get the hypercharge Y. Via renormalization these start to depend on the energy scale and, if the masses of the X/Y bosons and that of the triplet are of the order of M_{GUT}, the low energy particle spectrum will be that of the SM. This implies that the evolution of the couplings will be described by the same coefficients as in (3.8)–(3.10)

$$\alpha_3^{-1}(\mu) = \alpha^{-1}(M_{\mathrm{GUT}}) - \frac{b_3}{2\pi}\log\left(\frac{\mu}{M_{\mathrm{GUT}}}\right), \qquad b_3 = -7\,, \tag{3.19}$$

$$\alpha_2^{-1}(\mu) = \alpha^{-1}(M_{\mathrm{GUT}}) - \frac{b_2}{2\pi}\log\left(\frac{\mu}{M_{\mathrm{GUT}}}\right), \qquad b_2 = -\frac{19}{6}\,, \tag{3.20}$$

$$\alpha_1^{-1}(\mu) = \alpha^{-1}(M_{\mathrm{GUT}}) - \frac{b_1}{2\pi}\log\left(\frac{\mu}{M_{\mathrm{GUT}}}\right), \qquad b_1 = \frac{41}{6}\,. \tag{3.21}$$

In this model the three gauge couplings unify per definition, they have the same value $\alpha_{1,2,3}^{-1}(M_{\mathrm{GUT}}) = \alpha^{-1}(M_{\mathrm{GUT}}) \equiv \left(\frac{g^2}{4\pi}\right)^{-1}$ at some scale M_{GUT}. Of course this contradicts Figure 3.1, where the same RGE coefficients are used together with the measured low energy gauge coupling. To estimate how good/bad the level of unification is one can look at the following: Given the values for $\alpha^{-1}(M_{\mathrm{GUT}})$ and M_{GUT} one can postdict the three measured low energy gauge couplings. This means one can use two gauge couplings to determine $\alpha^{-1}(M_{\mathrm{GUT}})$ and M_{GUT} and postdict the third. In practice one often uses the the fine–structure constant $1/\alpha(m_Z) = 127.91$

and the strong coupling $\alpha_3(m_Z) = 0.1187$ to give a value for $\sin^2(\theta_W)$ [7]. Doing this for the Georgi–Glashow $SU(5)$ results in $\sin^2(\theta_W)(m_Z) = 0.207$. Comparing this with the value $\sin^2(\theta_W)(m_Z) = 0.23120$ from measurement, this is off by about 10% [7]. Note that at there might be threshold effects at scales where heavy particles start to contribute, which might improve the unification. In general these effects are too small for the Georgi–Glashow $SU(5)$ and the models we present in Chapter 13.

Of particular interest in GUT scenarios are supersymmetric extensions since on the one hand they solve the hierarchy problem and on the other hand improve the numerical unification of couplings. But SUSY GUTs still suffer from the doublet-triplet splitting problem and upcoming improvements on the proton decay rate will push the masses of the X/Y bosons above the ordinary SUSY GUT scale $M_{\text{GUT}}^{\text{SUSY}} \sim 2 \times 10^{16}\,\text{GeV}$ [14].

Another important quantity to look at when discussing running couplings and GUT groups are the differences of the coupling constants. They will also depend on the energy scale, and the difference between the runnings of different couplings is referred to as differential running. Looking at the individual contributions from gauge bosons, fermions and the Higgs doublet to the running one finds that the fermion contribution of all three gauge couplings is the same thus dropping out of the differential running. Geometrically this means all slopes in Figure 3.1 change by a common value, which does not influence the energy scale at which the graphs intersect and thus does not change if unification occurs or not. That the SM fermions do not contribute to the differential running is not that surprisingly when viewed from a GUT perspective: We saw in (3.13) that the SM fields form complete $SU(5)$ multiplets. In general it is true that there is no contribution to the differential running from complete $SU(5)$ multiplets and this will have interesting consequences on the models we discuss in Chapter 13.

In the course of this work we will see how GHGUTs share many features with ordinary 4D GUTs, while avoiding some of their problems. In particular, the necessary methods to study high scale unification are investigated and applied to $SU(6)$ GHGUTs to see if the SM unification can be improved.

Part II

Theory of Randall-Sundrum Models

This part introduces the key concepts of of RS-models, inspired by some excellent reviews of extra dimension [78, 79, 80, 81, 82, 83, 84, 85, 86]. First it is discussed how these models solve the hierarchy problem by introducing an extra space dimension to the usual four space-time dimensions. This is followed by reviewing the description of higher dimensional fields in this context as well as their relation to four dimensional fields. The propagators and Feynman rules of five dimensional theories are discussed and the concept of Gauge-Higgs Unification (GHU) is introduced.

4 RS-Metric and the Solution of the Hierarchy Problem

Already in 1921 T. Kaluza and O. Klein considered the idea of extra spatial dimensions. Their proposal was to unify the gravitational and electromagnetic forces in a five dimensional space-time [87, 88]. After the discovery of the other fundamental interactions this suggestion lost its relevance, but since then extra dimensions have been an interesting concept for theoretical physics. String Theory, as one of the candidates to unify all known interactions, requires at least six additional dimensions. Although it is believed that the effects of these extra dimensions from string theory are not visible at current collider energies they inspired further ideas in that direction. Arkani-Hamed-Dimopoulos-Dvali (ADD) [67, 68] proposed models with flat extra dimensions in which gravity could propagate into these, but the other interactions are confined to our usual four dimensional space time. This then explains the apparent weakness of gravity and the apparent large Planck scale. Later Lisa Randall and Raman Sundrum thought about models with one additional warped extra dimension [66]. These models are now called Randall-Sundrum (RS) models and are the main focus of this thesis. RS showed that a slice of 5D anti-de-Sitter space, AdS_5, which is a dynamical solution to the Einstein equations in 5D, can solve the HP. Mathematically this space can be described by the usual 4D Minkowski space together with an finite extra dimension spanning the interval[1]. There are two useful parametrizations of the metric [89, 90]

$$\mathrm{d}s^2 = \frac{1}{k^2 z^2}\left(\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}z^2\right), \quad \text{or} \quad \mathrm{d}s^2 = e^{-2kr|\phi|}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - r^2 \mathrm{d}\phi^2 \,, \tag{4.1}$$

where $z \in [\frac{1}{k}, \frac{1}{T}]$ or $\phi \in [0, \pi]$ describe the extra dimensional coordinates and the coordinates of the usal Minkowski space are labeled by x^μ, $\mu = 0, 1, 2, 3$. The scale k can be seen as the curvature of the AdS_5 space and $L \equiv kr\pi$ is the size of the extra dimension. We call the first the conformally flat metric and the second one the

[1] Such an interval can be constructed from an extended space by a process called orbifolding, see e.g. [82]

non-factorizable metric of RS. The two descriptions are related by the coordinate transformation [90]

$$z = \frac{e^{kr|\phi|}}{k} \,. \tag{4.2}$$

In particular, from this the relation $T = ke^{-L}$ follows, which also illustrates an important feature of the change of scales along the extra dimension. Given a scale M_0 at $\phi = 0$ there exists a corresponding scale M_π at $\phi = \pi$ given by [90]

$$M_\pi = M_0 e^{-L} \,. \tag{4.3}$$

Now, even if all fundamental parameters of the theory are of the order of the Planck scale ($k, 1/r, M_0 \sim \mathcal{O}(M_{\text{Planck}})$) the exponential factor in (4.3) can can lead to a large suppression of M_π. For example, if $L \approx 37$ the scale M_π is of order $T \sim \mathcal{O}(\text{TeV})$.[2] If one can associate the EW scale v with the scale T on the brane at $\phi = \pi$ this might provide a solution to the HP. One also refers to the brane at $\phi = 0$ as the UV-brane (or Planck-brane) and to the brane at $\phi = \pi$ as the IR-brane (or TeV-brane) as they are associated with the scales $k \sim \mathcal{O}(M_{\text{Planck}})$ and $T \sim \mathcal{O}(\text{TeV})$ respectively. By confining the SM fields to the IR-brane RS showed that it is indeed possible to get a correct Higgs VEV in these models, but it was soon realized that having the fields propagate in the full 5D bulk can also generate particle masses at the EW scale [92, 93, 94, 95].

In the next chapters it is described how TeV masses can be generated by bulk fields, starting with an overview of how one should properly treat bulk fields in AdS_5.

5 Action and Boundary Conditions

This chapter describes how QFTs in AdS_5 work. The concepts are almost the same as in the familiar 4D case, but special care has to be given to the non-flat metric and the finiteness of the extra dimension. As in 4D the fields are classified according to their representations of the space-time symmetry. Even though the space is warped, locally it behaves like a 5D Minowski space, so the representations can be chosen with respect to a 5D Lorentz symmetry. The trivial, spinoral and vector-like representations are similarly called scalar, fermion and vector/gauge fields respectively. These will be studied in turn in the following, starting from the simplest field, the scalar field.

[2] This small hierarchy between the scales k and $1/r$ can in fact be naturally generated, for example by the Goldberger-Wise stabilization mechanism [91].

5.1 5D Scalars

Considering for simplicity only the space-time symmetries without any internal symmetries, the invariant operators for a real scalar field ϕ can easily be written down. Looking only at bulk terms quadratic in the field ϕ one can write down the most general action as [79]

$$S = \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \sqrt{G} \left(\frac{1}{2} G^{MN} \left(\partial_M \phi\right) \left(\partial_N \phi\right) - \frac{1}{2} m^2 \phi^2 \right), \tag{5.1}$$

where G^{MN} is the AdS_5 metric and G its determinant. The capital letters M, N run from $0, 1, 2, 3, 5$, where the first four components correspond to the usual Minkowski space indices $\mu = 0, 1, 2, 3$ and the last one stands for the extra fifth dimension. From now on we will always use the conformally flat metric. With this the action can be simiplified to

$$S = \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \frac{1}{2k^3 z^3} \left(\eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - (\partial_5 \phi)(\partial_5 \phi) - \frac{1}{k^2 z^2} m^2 \phi^2 \right). \tag{5.2}$$

To be a proper QFT the variation of this classical action with respect to the field ϕ has to vanish. Explicitly this reads

$$\begin{aligned} \delta S &= \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \frac{1}{k^3 z^3} \left(\eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \delta\phi) - (\partial_5 \phi)(\partial_5 \delta\phi) - \frac{1}{k^2 z^2} m^2 \phi \delta\phi \right) \\ &= \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \frac{1}{k^3 z^3} \left(-\partial^2 \phi + k^3 z^3 \partial_5 \left(\frac{1}{k^3 z^3} \partial_5 \phi \right) - \frac{1}{k^2 z^2} m^2 \phi \right) \delta\phi \\ &\quad + \int \mathrm{d}^4x \left[\frac{1}{k^3 z^3} \left(-(\partial_5 \phi)(\delta\phi) \right) \right] \Bigg|_{1/k}^{1/T}, \end{aligned} \tag{5.3}$$

where we used partial integration in the last line. Requiring the action to vanish under a general variation $\delta\phi$ implies that both terms have to vanish separately. The first one is the bulk equation of motion (EoM), which is a generalization of the Klein-Gordon equation. The second term arises because of the finite extra dimension and will give boundary conditions (BC) on the fields ϕ. Note that such a term in principle arises also in 4D, but one generally assumes that the fields and their derivatives vanish at infinity. Looking at each term individually we find the EoM for a bulk scalar [79]

$$\left(-\partial^2 + k^3 z^3 \partial_5 \left(\frac{1}{k^3 z^3} \partial_5 \right) - \frac{1}{k^2 z^2} m^2 \right) \phi(x, z) = 0\,. \tag{5.4}$$

The solution to this equation will be discussed at a later point. The BCs at $z = \frac{1}{k}$ and $z = \frac{1}{T}$ are independent and read

$$(\partial_5 \phi)(\delta\phi)\big|_{1/k} = 0, \qquad (\partial_5 \phi)(\delta\phi)\big|_{1/T} = 0\,. \tag{5.5}$$

We focus on two possible choices of BCs [79], which both lead to vanishing boundary terms,

$$(+) \quad (\partial_5\phi)\big|_{z=1/k,1/T} = 0, \qquad \text{or} \qquad (-) \quad \phi\big|_{z=1/k,1/T} = 0\,. \tag{5.6}$$

The BC $(+)$ is called a Neumann BC and the BC $(-)$ is called a Dirichlet BC. To properly define a QFT we need to choose the BC on each brane. This means there are four different types of scalar field we can have. Labeling them by their BCs via $(s_{\mathrm{UV}}, s_{\mathrm{IR}}), s_i \in \{\pm 1\}$ they are

$$\phi(+,+), \quad \phi(+,-), \quad \phi(-,+), \quad \phi(-,-)\,. \tag{5.7}$$

After choosing one of the BCs we can in the future always integrate by parts and the boundary terms vanish. A similar situation will arise in the case for fermions and gauge bosons. Note that for the most general action there can also be operators localized on the branes at the endpoints of the interval. These can then modify the above BCs.

5.2 5D Fermions

The QFT for fermions can be done in a similar way as in the scalar case above, but there are important differences due to the non-trivial Lorentz structure. Other then in 4D the smallest irreducible representation for fermions in 5D is not a two-component Weyl-fermion but a four-component Dirac fermion. This means every bulk field contains LH as well as RH components and is therefore non-chiral. It is useful to define a generalization Γ^M of the gamma matrices which connects the fermion spinor indices with the external space-time indices. In general warped space they are related to the ordinary flat space ones γ^a by the so called vielbein e_a^M, where a denotes a flat 5d space index. It is defined by [96, 79]

$$e_a^M \eta^{ab} e_b^N = G^{MN}\,, \tag{5.8}$$

and explicitly given for the RS metric by $e_M^a = \frac{1}{kz}\delta_M^a$. The warped space gamma matrices are then calculated by

$$\Gamma^M = e_a^M \gamma^a\,, \tag{5.9}$$

and the flat space gamma matrices are determined similar to the ones in 4D by the Clifford Algebra (see Appendix A)

$$\{\gamma^a, \gamma^a\} = 2\eta^{ab}\,. \tag{5.10}$$

The solution to this are the familiar Dirac matrices γ^μ together with the familiar matrix γ^5 (multiplied by a factor i). The necessary inclusion of γ^5 reflects the fact, that both chiralities need to be included. We will use the following representation of them if needed

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} i\mathbf{1} & 0 \\ 0 & -i\mathbf{1} \end{pmatrix}, \tag{5.11}$$

where $\sigma^0 = \bar\sigma^0 = -\mathbf{1}$ and $\sigma^i = -\bar\sigma^i$ are the usual Pauli spin matrices. Furthermore in warped space the covariant derivative include also a connection, the spin-connection, which in AdS_5 acting on a fermion Ψ reads [79]

$$D_\mu \Psi = \left(\partial_\mu + \frac{1}{4z}\gamma_\mu\gamma_5\right)\Psi\,, \tag{5.12}$$

$$D_5\Psi = \partial_5\Psi\,. \tag{5.13}$$

The spin connection for scalars has been ignored in the last section, since scalars are in the trivial representation of the space-time group and therefore their corresponding connection is zero. In general warped spaces the bulk action for a fermion Ψ including only quadratic operators reads [79]

$$S = \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z\sqrt{G}\left(\frac{i}{2}\bar\Psi\Gamma^M D_M\Psi - \frac{i}{2}D_M\bar\Psi\Gamma^M\Psi - m\bar\Psi\Psi\right). \tag{5.14}$$

For AdS_5 this simplifies to[1] [79]

$$S = \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 \frac{1}{2}\left[\bar\Psi\left(i\gamma^\mu\partial_\mu + i\gamma^5\partial_5 - i\frac{2}{z}\gamma^5 - \frac{c}{z}\right)\Psi + \text{h.c.}\right], \tag{5.15}$$

where we introduced the dimensionless variable $c = \frac{m}{k}$. The bulk equation and boundary terms are best written down in terms of the LH and RH components $\Psi = \begin{pmatrix}\Psi_L & \Psi_R\end{pmatrix}^T$

$$\begin{aligned} S = \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 \frac{1}{2}\Big[&- i\bar\Psi_L\bar\sigma^\mu\partial_\mu\Psi_L - i\bar\Psi_R\sigma^\mu\partial_\mu\Psi_R \\ &+ \bar\Psi_R\left(\partial_5 - \frac{2}{z}\right)\Psi_L - \bar\Psi_L\left(\partial_5 - \frac{2}{z}\right)\Psi_R + \frac{c}{z}\left(\bar\Psi_R\Psi_L + \bar\Psi_L\Psi_R\right) + \text{h.c.}\Big]. \end{aligned} \tag{5.16}$$

[1]In usual 4D space the hermitian conjugate (h.c.) can be integrated by parts, thereby giving the same as the first term, and thus the familiar Dirac action is recovered. As we have seen for scalars integration by parts on a finite interval leads to important boundary terms which we will discuss below.

Varying the action and integrating by parts we find

$$\begin{aligned}\delta S = \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 \frac{1}{2}\Big[&\delta\bar{\Psi}_R \left(-i\sigma^\mu\partial_\mu\Psi_R + \partial_5\Psi_L - \frac{2}{z}\Psi_L + \frac{c}{z}\Psi_L\right) \\ &+ \delta\bar{\Psi}_L \left(-i\bar{\sigma}^\mu\partial_\mu\Psi_L - \partial_5\Psi_R + \frac{2}{z}\Psi_R + \frac{c}{z}\Psi_R\right) + \text{h.c.}\Big] \\ &+ \int \mathrm{d}^4x \left(\frac{1}{kz}\right)^4 \frac{1}{2}\left(\delta\bar{\Psi}_R\Psi_L - \delta\bar{\Psi}_L\Psi_R + \text{h.c.}\right)\Big|_{1/k}^{1/T} .\end{aligned} \tag{5.17}$$

The variation of $\delta\bar{\Psi}_R$ and $\delta\bar{\Psi}_L$ are independent so we get two coupled bulk EoM [79]

$$-i\sigma^\mu\partial_\mu\Psi_R + \left(\partial_5 - \frac{2}{z} + \frac{c}{z}\right)\Psi_L = 0\,, \tag{5.18}$$

$$-i\bar{\sigma}^\mu\partial_\mu\Psi_L + \left(-\partial_5 + \frac{2}{z} + \frac{c}{z}\right)\Psi_R = 0\,. \tag{5.19}$$

Looking at the boundary condition we need to require

$$\left(\delta\bar{\Psi}_R\Psi_L - \delta\bar{\Psi}_L\Psi_R + \text{h.c.}\right)\Big|_{1/k}^{1/T} = 0\,. \tag{5.20}$$

Considering the case for the UV boundary we notice that all factors vanish by choosing for the LH component $\Psi_R\big|_{1/k} = 0$. But through the EoM this fixes the BC for the RH component to $\left(\partial_5 - \frac{2}{z} + \frac{c}{z}\right)\Psi_L\big|_{1/k} = 0$. In fact these two conditions are equivalent. Similarly the conditions $\Psi_L\big|_{1/k} = 0 \iff \left(-\partial_5 + \frac{2}{z} + \frac{c}{z}\right)\Psi_R\big|_{1/k} = 0$ also let the boundary term vanish. Summarizing [79]

$$(+) : \begin{cases} (+)_L & \left(\partial_5 - \frac{2}{z} - \frac{c}{z}\right)\Psi_L\big|_{1/k} = 0 \\ (-)_R & \Psi_R\big|_{1/k} = 0 \end{cases}\,, \tag{5.21}$$

$$(-) : \begin{cases} (-)_L & \Psi_L\big|_{1/k} = 0 \\ (+)_R & \left(-\partial_5 + \frac{2}{z} - \frac{c}{z}\right)\Psi_R\big|_{1/k} = 0 \end{cases}\,. \tag{5.22}$$

Again we refer to Neumann (+) or Dirichlet (−) BC whether the BC contains a derivative or not. Thus the BCs of LH and RH fermions are opposite to each other. If we specify a BC of a general Dirac fermion we always give the LH BC, with an opposite BC for the RH component implied. If we need to make the BC explicit for the components we use the indices L/R on the BC. The same applies to the IR BC. Although we were forced to include a Dirac fermion we can still distinguish LH and RH components by their BCs. As for the scalar there are four different kinds of fermions

$$\Psi(+,+) = \begin{cases} \Psi_L(+,+)_L \\ \Psi_R(-,-)_R \end{cases}\,, \quad \Psi(+,-) = \begin{cases} \Psi_L(+,-)_L \\ \Psi_R(-,+)_R \end{cases}\,, \tag{5.23}$$

$$\Psi(-,+) = \begin{cases} \Psi_L(-,+)_L \\ \Psi_R(+,-)_R \end{cases}\,, \quad \Psi(-,-) = \begin{cases} \Psi_L(-,-)_L \\ \Psi_R(+,+)_R \end{cases}\,, \tag{5.24}$$

and boundary operators can again change the form of the BCs.

5.3 5D Gauge Bosons

In this section we discuss the QFT for vector fields A_M in RS space. Different from the two sections above we consider an internal symmetry group $SU(N)$ for which A_M will be a gauge field with 5D coupling constant g_5. The case of a $U(1)$ gauge field follows from this trivially. The field A_M is a matrix in the Lie algebra of $SU(N)$ and can therefore be expanded in the generators $A_M = A_M^A T^A$. We will use upper case letters for the gauge group index for reasons which will become clear later. The gauge fields transform under an infinitesimal 5D gauge transformations as

$$A_M^A \to A_M^A + \frac{1}{g_5} D_M^{AC} \alpha^C \,, \tag{5.25}$$

with the covariant derivative in the adjoint representation

$$D_M^{AC} = \delta^{AC} \partial_M + g_5 f^{ABC} A_M^B \,, \tag{5.26}$$

as opposed to the covariant derivative in the fundamental representation $D_M = \partial_M - i g_5 T^A A_M^A$. Here g_5 is the 5D gauge coupling, how it is related to the 4D gauge couplings of the SM is presented in Chapter 6. As the gauge transformation is similar to the 4D case we can define a 5D field strength tensor by

$$F_{MN} = \frac{i}{g_5} [D_M, D_N] \,. \tag{5.27}$$

Note that any warped space connection vanishes in the commutator. Expanding the field strength in terms of generators $F_{MN} = F_{MN}^A T^A$ we find

$$F_{MN}^A = \partial_M A_N^A - \partial_N A_M^A + g_5 f^{ABC} A_M^B A_N^C \,. \tag{5.28}$$

From this we can construct a bulk action by [84]

$$\begin{aligned} S &= \int \mathrm{d}^4 x \int_{1/k}^{1/T} \mathrm{d}z \sqrt{G} G^{MN} G^{PQ} \left(-\frac{1}{4} F_{MP}^A F_{NQ}^A \right) \\ &= \int \mathrm{d}^4 x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz} \right) \left(-\frac{1}{4} \eta^{\mu\nu} \eta^{\rho\lambda} F_{\mu\rho}^A F_{\nu\lambda}^A + \frac{1}{2} \eta^{\mu\nu} F_{\mu 5}^A F_{\nu 5}^A \right) \,. \end{aligned} \tag{5.29}$$

This action is invariant under the above gauge transformations. To properly define a QFT one can perform the Faddeev-Popov procedure, similar to the 4D case, by adding a gauge fixing action S_{gf} and the corresponding ghost action S_{gh}. For this we look at the quadratic parts, after integrating by parts one finds

$$
\begin{aligned}
S = \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right) \Bigg(&-\frac{1}{2}\eta^{\mu\nu}\eta^{\rho\lambda} \left(\partial_\mu A^A_\rho \partial_\nu A^A_\lambda - \partial_\nu A^A_\mu \partial_\rho A^A_\lambda\right) \\
&+ \frac{1}{2}\eta^{\mu\nu} \left(\partial_\mu A^A_5 \partial_\nu A^A_5 - 2kz\partial_5 \frac{1}{kz} A^A_5 \partial_\mu A^A_\nu + \partial_5 A^A_\mu \partial_5 A^A_\nu\right)\Bigg) \\
&+ \int \mathrm{d}^4x \left(\frac{1}{kz}\right) \left(\eta^{\mu\nu}\left(A^A_5 \partial_\mu A^A_\nu\right)\right)\Bigg|_{1/k}^{1/T} .
\end{aligned}
\tag{5.30}
$$

It is common to distinguish between the field A_μ and A_5. A_μ behaves like a 4D Lorentz vector and A_5 as 4D Lorentz scalar under 4D Lorentz transformations. Gauge fixing should also get rid of the mixing term between A_μ and A_5 in the bulk. This can be achieved by the following gauge fixing action, inspired by the R_ξ gauges [89]

$$
S_{\mathrm{gf}} = \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(-\frac{1}{2\xi kz}\left[\eta^{\mu\nu}\partial_\mu A^A_\nu - \xi kz \partial_5 \left(\frac{1}{kz}A^A_5\right)\right]^2\right) . \tag{5.31}
$$

Adding this to the action gives

$$
\begin{aligned}
S = \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right) \Bigg(&-\frac{1}{2}\eta^{\mu\nu}\eta^{\rho\lambda} \left(\partial_\mu A^A_\rho \partial_\nu A^A_\lambda - \left(1-\frac{1}{\xi}\right)\partial_\nu A^A_\mu \partial_\rho A^A_\lambda\right) \\
&+ \frac{1}{2}\eta^{\mu\nu} \left(\partial_\mu A^A_5 \partial_\nu A^A_5 + \partial_5 A^A_\mu \partial_5 A^A_\nu - \xi kz\partial_5 \left(\frac{1}{kz} A^A_5\right) kz\partial_5 \left(\frac{1}{kz}A^A_5\right)\right)\Bigg) \\
&+ \int \mathrm{d}^4x \left(\frac{1}{kz}\right) \left(\eta^{\mu\nu}\left(A^A_5 \partial_\mu A^A_\nu\right)\right)\Bigg|_{1/k}^{1/T} .
\end{aligned}
\tag{5.32}
$$

Now we are in a position to vary the action with respect to the fields A_μ and A_5

$$
\begin{aligned}
\delta S = \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)& \\
\Bigg(\Bigg(&\eta^{\rho\lambda}\partial^2 A^A_\lambda - \left(1-\frac{1}{\xi}\right)\eta^{\rho\nu}\eta^{\mu\lambda}\partial_\nu\partial_\mu A^A_\lambda - \eta^{\rho\lambda} kz\partial_5 \frac{1}{kz}\partial_5 A^A_\lambda\Bigg)\delta A^A_\rho \\
&+ \left(-\partial^2 A^A_5 + \xi\partial_5 z\partial_5\left(\frac{1}{z}A^A_5\right)\right)\delta A^A_5\Bigg) \\
&+ \int \mathrm{d}^4x \left(\frac{1}{kz}\right)\left(\left(\eta^{\rho\lambda}\partial_5 A^A_\lambda - \eta^{\mu\rho}\partial_\mu A^A_5\right)\delta A^A_\rho\right)\Bigg|_{1/k}^{1/T} \\
&+ \int \mathrm{d}^4x \left(\frac{1}{kz}\right)\left(\left(\eta^{\mu\lambda}\partial_\mu A^A_\lambda - \xi z\partial_5\left(\frac{1}{z}A^A_5\right)\right)\delta A^A_5\right)\Bigg|_{1/k}^{1/T} .
\end{aligned}
\tag{5.33}
$$

The variations with respect to A_μ and A_5 are independent so we get the two bulk EoM (neglecting interaction terms) [84]

$$\left(\eta^{\mu\nu}\partial^2 - \left(1-\frac{1}{\xi}\right)\eta^{\mu\lambda}\eta^{\rho\nu}\partial_\lambda\partial_\rho - \eta^{\mu\nu}kz\partial_5\frac{1}{kz}\partial_5\right)A^A_\nu = 0\,, \tag{5.34}$$

$$\left(-\partial^2 + \xi\partial_5 z\partial_5\frac{1}{z}\right)A^A_5 = 0\,. \tag{5.35}$$

There are also two independent boundary terms which need to vanish

$$\begin{aligned}
&\left(\left(\eta^{\mu\nu}\partial_5 A^A_\nu - \eta^{\nu\mu}\partial_\nu A^A_5\right)\delta A^A_\mu\right)\Big|_{1/k}^{1/T} = 0,\\
&\left(\left(\eta^{\nu\mu}\partial_\nu A^A_\mu - \xi z\partial_5\left(\frac{1}{z}A^A_5\right)\right)\delta A^A_5\right)\Big|_{1/k}^{1/T} = 0\,.
\end{aligned} \tag{5.36}$$

Considering first the UV BC we can make the first term in the first equation vanish by demanding $\partial_5 A^A_\mu\big|_{1/k} = 0$. The first equation then implies $A^A_5\big|_{1/k} = 0$ and from these it follows that the second equation is also zero. Alternatively, the first term in the second equation vanishes for $A^A_\mu\big|_{1/k} = 0$ which on the one hand implies that the first equation vanishes and on the other hand implies $\partial_5\left(\frac{1}{z}A^A_5\right)\big|_{1/k} = 0$. Labeling these two by [84]

$$(+):\begin{cases}(+) & \partial_5 A^A_\mu\big|_{1/k} = 0\\ (-)_5 & A^A_5\big|_{1/k} = 0\end{cases}\quad,\qquad (-):\begin{cases}(-) & A^A_\mu\big|_{1/k} = 0\\ (+)_5 & \partial_5\left(\frac{1}{z}A^A_5\right)\big|_{1/k} = 0\end{cases}\quad. \tag{5.37}$$

Again we refer to Neumann $(+)$ or Dirichlet $(-)$ BC, respectively, when the BC contains a derivative or not. Thus the BCs of A^A_μ and A^A_5 are opposite to each other, like in the fermion case. If we specify a BC of a general A^A_M we always give the A^A_μ BC, with an opposite BC for the A^A_5 component implied. If we need to make the BC explicit for the components we use the index 5 for A^A_5. Doing the same for the IR BC we have again four possibilities $A_M(+,+), A_M(+,-), A_M(-,+), A_M(-,-)$. Like for scalars and fermions boundary operators can change the form of the BCs.

Since we also included an internal symmetry there is an interesting feature: We can assign different BCs for fields with different gauge group indices. This splits the gauge fields and their corresponding generators into four different sets. The generators of fields that have $(+)$ BC on the UV brane form a subgroup H_0 of $SU(N)$ and the the generators of fields that have $(+)$ BC on the IR brane form a subgroup H_1 of $SU(N)$. The four groups can then be organized by [78]

$$(+,+)\quad A^a_M \qquad T^a \in \mathrm{Alg}\{H = H_0 \cap H_1\}\,, \tag{5.38}$$

$$(+,-)\quad A^{\bar a}_M \qquad T^{\bar a} \in \mathrm{Alg}\{H_0/H\}\,, \tag{5.39}$$

$$(-,+)\quad A^{\dot a}_M \qquad T^{\dot a} \in \mathrm{Alg}\{H_1/H\}\,, \tag{5.40}$$

$$(-,-)\quad A^{\hat a}_M \qquad T^{\hat a} \in \mathrm{Alg}\{SU(N)/H_0\} \cap \mathrm{Alg}\{SU(N)/H_1\}\,. \tag{5.41}$$

Thus on the UV brane the symmetry is reduced to H_0 and on the IR brane to H_1. Similarly, if we also charge scalar and fermion fields under $SU(N)$ we can assign different BCs to different components[2].

To complete the Faddeev-Popov procedure we derive the ghost action S_{gh}. This can be done in a similar way as in the 4D case for the R_σ gauges [6]. The gauge fixing condition from the above gauge fixing action (5.31) is [89]

$$G^A(A) = \partial^\mu A^A_\mu - \xi k z \partial_5 \left(\frac{1}{kz} A^A_5 \right) . \tag{5.42}$$

Note that $\partial_5 \left(\frac{1}{kz} A^A_5 \right)$ has always the same BCs as A^A_μ. Thus also the ghost fields will have the same BCs as A^A_μ. Varying (5.42) with respect to gauge transformations (5.25) gives

$$\frac{\delta G^A}{\delta \alpha^C} = \frac{1}{g_5} \left(\partial^\mu D^{AC}_\mu - \xi k z \partial_5 \left(\frac{1}{kz} D^{AC}_5 \right) \right) , \tag{5.43}$$

and results in the ghost action[3]

$$S_{\text{gh}} = \int \mathrm{d}^4 x \int_{1/k}^{1/T} \mathrm{d}z \left[\frac{1}{kz} \bar{c}^A \left(-\partial^\mu D^{AC}_\mu + \xi k z \partial_5 \left(\frac{1}{kz} D^{AC}_5 \right) \right) c^C \right] , \tag{5.44}$$

where $c^A, \bar{c}^A$ are Lorentz scalar, anticommuting, 5D fields with the same BC as A^A_μ.

6 Kaluza-Klein-Decomposition of 5D Fields

This and the next chapter are focused on connecting the 5D theories from the last chapter with the usual 4D fields of the SM. Here we integrate out the extra dimension in the action. To do this it is useful to expand the dependence of the 5D fields on the extra dimension in in terms of a set of eigenfunctions, known as a Kaluza-Klein (KK) decomposition. This is possible since the extra dimension is compact. Consider first the case of a real scalar field ϕ, which can be expanded into [79]

$$\phi(x, z) = \sum_{n=0}^{\infty} \phi^{(n)}(x) f^{(n)}_\phi(z) . \tag{6.1}$$

[2]The scalar/fermion BCs must be consistent with the assignments of the gauge fields to ensure gauge invariance on the branes

[3]To perform the manipulations in the Faddeev–Popov procedure it is best to do a KK expansion (see next chapter) of the gauge fields.

The fields $\phi^{(n)}$ are called KK modes and the functions $f_\phi^{(n)}(z)$ are called KK wavefunctions. Looking at the scalar action and bulk equation of motion we want these eigenfunctions to satisfy the differential equation [79]

$$
\begin{aligned}
&\left(-z^3\partial_5\frac{1}{z^3}\partial_5+\frac{1}{k^2z^2}m^2\right)f_\phi^{(n)}(z)=(m_n^\phi)^2f_\phi^{(n)}(z)\\
\implies &\left(-\partial_5^2+\frac{3}{z}\partial_5+\frac{1}{k^2z^2}m^2\right)f_\phi^{(n)}(z)=(m_n^\phi)^2f_\phi^{(n)}(z)\,,
\end{aligned}
\tag{6.2}
$$

as well as the orthonormality condition

$$
\int_{1/k}^{1/T}\mathrm{d}z\,\left(\frac{1}{kz}\right)^3 f_\phi^{(n)}(z)f_\phi^{(m)}(z)=\delta_{nm}\,. \tag{6.3}
$$

Note that we can interpret the above integral as defining a scalar product on this space. With this identification one can show that the differential operator above is a hermitian operator. This guarantees that the eigenfunctions $f_\phi^{(n)}$ exist, form a complete (orthogonal) set and the corresponding eigenvalues $(m_n^\phi)^2$ are positive. The BCs of ϕ translate now to conditions on the $f_\phi^{(n)}$s, which means also the eigenvalues $(m_n^\phi)^2$ depend on the BCs. If needed, we will add a subscript $f_{\phi,(s,s')}^{(n)}$, $s,s'\in\{+,-\}$ to indicate the different choices.

Plugging this expansion in the action we get

$$
S_\phi=\int\mathrm{d}^4x\sum_{n=0}^{\infty}\frac{1}{2}\left[\phi^{(n)}(-\partial^2-(m_n^\phi)^2)\phi^{(n)}\right]\,. \tag{6.4}
$$

One can see that we now have an infinite spectrum of 4D scalar fields each with a different mass. Since we can order the eigenvalues $(m_n^\phi)^2$, such that they increase with n this is also called a tower of KK-modes. Explicit expressions for the KK wavefunctions and KK masses are given in Appendix B. We reserve the index $n=0$ for modes with $(m_{n=0}^\phi)^2=0$ and call this mode a zero mode. It turns out, without boundary operators, that for the BCs in the last chapter only in the case for a bulk mass $m=0$ and $(+,+)$ BC such a zero mode exists [80]. The KK wavefunction in this case is constant along the fifth dimension and reads

$$
f_{\phi,(+,+)}^{(0)}(z)=\sqrt{\frac{2k^3}{k^2-T^2}}=\sqrt{k}\sqrt{\frac{2}{1-\left(\frac{T}{k}\right)^2}}\,. \tag{6.5}
$$

That only $(+,+)$ fields have a zero mode will also be true for fermions and gauge bosons. The first KK mode will in general have a mass around $(m_{n=1}^\phi)^2\sim T^2=\mathcal{O}(\mathrm{TeV}^2)$. This means at low energies, like the ones probed at past colliders, only the zero mode plays a phenomenological role. If one tries to model the SM with these 5D fields, the SM fields should then be identified with the zero modes of the 5D fields. Of course most of these fields should then acquire a small mass (small compared to

$\mathcal{O}$(TeV)) through some mechanism. This will be discussed in a later chapter. Note also that the KK decomposition is defined by the free action, interactions, like ϕ^4 have to be calculated perturbatively and the interaction strengths are determined by overlap integrals over the respective KK wavefunctions.

We can do a similar KK decomposition for a fermion $\Psi = \begin{pmatrix}\Psi_L & \Psi_R\end{pmatrix}^T$ [79]

$$\Psi_L(x,z) = \sum_{n=0}^{\infty} \Psi_L^{(n)}(x) f^{(n)}_{\Psi L,(s,s')}(z)\,, \tag{6.6}$$

$$\Psi_R(x,z) = \sum_{n=0}^{\infty} \Psi_R^{(n)}(x) f^{(n)}_{\Psi R,(s,s')}(z)\,. \tag{6.7}$$

Looking at the fermion action and bulk equation of motions we want these eigenfunctions to satisfy the following differential equations [79]

$$\left(\partial_5 - \frac{2}{z} - \frac{c}{z}\right) f^{(n)}_{\Psi R,(s,s')}(z) = -m_n f^{(n)}_{\Psi L,(s,s')}(z)\,, \tag{6.8}$$

$$\left(-\partial_5 + \frac{2}{z} - \frac{c}{z}\right) f^{(n)}_{\Psi L,(s,s')}(z) = -m_n f^{(n)}_{\Psi R,(s,s')}(z)\,, \tag{6.9}$$

as well as the orthonormality conditions

$$\int_{1/k}^{1/T} \mathrm{d}z\, \left(\frac{1}{kz}\right)^4 f^{(n)}_{\Psi L,(s,s')}(z) f^{(m)}_{\Psi L,(s,s')}(z) = \delta_{mn}\,, \tag{6.10}$$

$$\int_{1/k}^{1/T} \mathrm{d}z\, \left(\frac{1}{kz}\right)^4 f^{(n)}_{\Psi R,(s,s')}(z) f^{(m)}_{\Psi R,(s,s')}(z) = \delta_{mn}\,. \tag{6.11}$$

Again we can interpret the the above integral as defining a scalar product on this space. With this identification one can show that the differential operator above is a hermitian operator. This guarantees that the eigenfunctions $f^{(m)}_{\Psi L/R,(s,s')}$ exist, form a complete (orthogonal) set and the corresponding eigenvalues $(m_n^{\Psi})^2$ are positive. The BCs of Ψ_L, Ψ_R now translate to conditions on the $f^{(m)}_{\Psi L/R,(s,s')}$s. We will add a subscript $f^{(m)}_{\Psi L/R,(s,s')}$, $s, s' \in \{+,-\}$ to indicate the different choices. Remember that Ψ_L and Ψ_R have opposite BCs, but we will specify by (s,s') the BCs for Ψ_L. This means $f^{(m)}_{\Psi R,(-,-)}$ refers to the KK wavefunction of a RH fermion with $(-,-) = (+,+)_R$ BCs. Explicit expressions for the KK wavefunctions and KK masses are given in Appendix B. Note that if we have a solution for $f^{(m)}_{\Psi L,(s,s')}$ the above equation completely determine $f^{(m)}_{\Psi R,(s,s')}$. In fact from the symmetry of the equations we can get the solution for $f^{(m)}_{\Psi R,(s,s')}$ from $f^{(m)}_{\Psi L,(s,s')}$ by replacing $(s,s') \to (-s,-s')$ and $c \to -c$.

Plugging this expansion in the action we get

$$S_\Psi = \int \mathrm{d}^4x \sum_{n=0}^{\infty} \Big(- \overline{\Psi_L^{(n)}} i\bar{\sigma}^\mu \partial_\mu \Psi_L^{(n)} - \overline{\Psi_R^{(n)}} i\sigma^\mu \partial_\mu \Psi_R^{(n)} \\ - m_n^\Psi \overline{\Psi_R^{(n)}} \Psi_L^{(n)} - m_n^\Psi \overline{\Psi_L^{(n)}} \Psi_R^{(n)} \Big) . \tag{6.12}$$

One can see that we now have an infinite spectrum of 4D Dirac fermion fields each with a different mass. Looking first at the LH modes it turns out that a zero mode solution only exists for $(+,+)$ BCs, but unlike the scalar case for every choice of the bulk mass parameter c. Because the RH components have the opposite BCs, there is no corresponding RH zero mode. The action for the zero mode is therefore an action for a LH massless Weyl fermion. Similarly for $(-,-)$ BCs there is only a RH zero mode. Even though we started with a non-chiral theory, using the BCs we can get a chiral theory in the low energy limit. For every SM fermion we thus need two Dirac fermions in the 5D theory, one for the LH and one for the RH fermions. Looking at the KK wavefunctions for the zero mode one finds [79]

$$f^{(0)}_{\Psi L,(+,+)}(z) = \sqrt{T}(kz)^2 (Tz)^{-c} f(+c) , \tag{6.13}$$

$$f^{(0)}_{\Psi R,(-,-)}(z) = \sqrt{T}(kz)^2 (Tz)^{+c} f(-c) , \tag{6.14}$$

with the flavor function

$$f(c) = \sqrt{\frac{1-2c}{1-\left(\frac{T}{k}\right)^{1-2c}}} . \tag{6.15}$$

The dependence of the zero mode wavefunctions $f^{(0)}_{\Psi L/R}$ on the c–parameter is very important for the running of gauge couplings (see Part IV) and also for flavor phenomenology (see Part V) thus it is instructive to take a closer look at this. By varying the c–parameter one changes the localization along the extra dimension. Since the notion of localization is tied to that of distance special care has to be given to the warped metric. To talk about localization one has to study the wavefunctions compared to a flat space metric and this is done more easily by working with the non–factorizable form of the metric (see Chapter 4) and define explicitly the fifth component $y \equiv r|\phi|$ [85]. The rescaled wavefunctions are plotted in Figure 6.1 for different values of c. One can see that the bulk mass parameter has an influence on the localization of the zero mode profiles. The LH profiles are localized towards the UV brane for $c > \frac{1}{2}$ and towards the IR brane for $c < \frac{1}{2}$. As the RH solution can be constructed by flipping $c \to -c$, the zero mode profile is UV localized for $c < -\frac{1}{2}$ and IR localized for $c > -\frac{1}{2}$.

Moving on to gauge bosons A_M. Suppressing the gauge index the KK decompo-

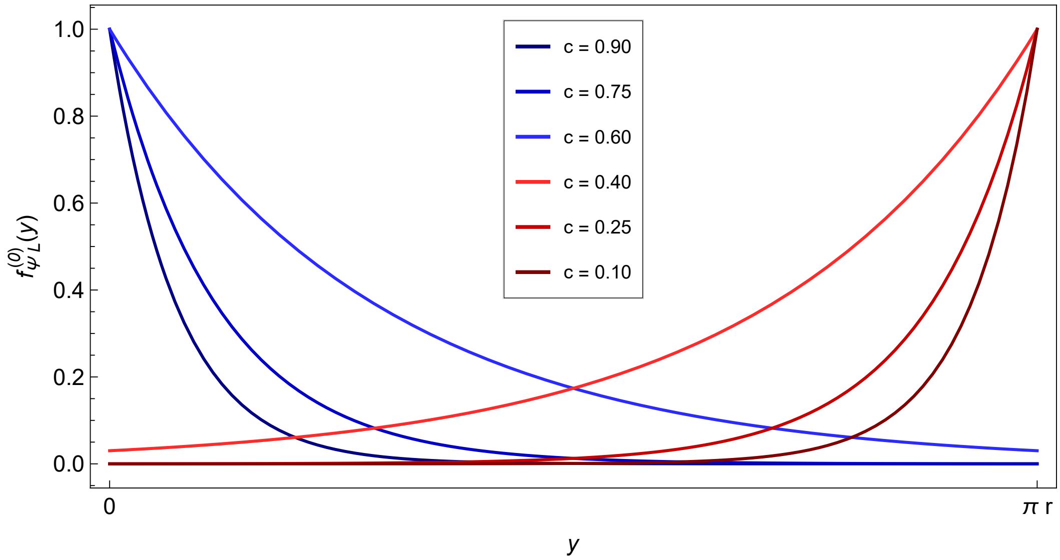


Figure 6.1: Depiction of the localization of fermion zero mode wavefunctions for the case of a LH zero mode for different values of the c–parameter. For values $c > 1/2$ the fermion wavefunction is localized towards the UV and for values $c < 1/2$ the fermion wavefunction is localized towards the IR. For $c = 1/2$ the fermion wavefunction would be flat.

sition for the components A_μ and A_5 read [79]

$$A_\mu(x,z) = \sum_{n=0}^{\infty} A_\mu^{(n)}(x) f^{(n)}_{A,(s,s')}(z)\,, \tag{6.16}$$

$$A_5(x,z) = \sum_{n=0}^{\infty} A_5^{(n)}(x) f^{(n)}_{A5,(s,s')}(z)\,. \tag{6.17}$$

Looking at the gauge boson action and bulk equation of motion we want these eigenfunctions to satisfy the following differential equations [84]

$$z\partial_5 \left(\frac{1}{z} \partial_5 f^{(n)}_{A,(s,s')}(z) \right) = -m_n^2 f^{(n)}_{A,(s,s')}(z)\,, \tag{6.18}$$

$$\partial_5 \left(z\partial_5 \left(\frac{1}{z} f^{(n)}_{A5,(s,s')}(z) \right) \right) = -m_{n,5}^2 f^{(n)}_{A5,(s,s')}(z)\,, \tag{6.19}$$

as well as the orthonormality conditions

$$\int_{1/k}^{1/T} \mathrm{d}z\, \frac{1}{kz} f^{(n)}_{A,(s,s')}(z) f^{(m)}_{A,(s,s')}(z) = \delta_{nm}\,, \tag{6.20}$$

$$\int_{1/k}^{1/T} \mathrm{d}z\, \frac{1}{kz} f^{(n)}_{A5,(s,s')}(z) f^{(m)}_{A5,(s,s')}(z) = \delta_{nm}\,. \tag{6.21}$$

Again we can interpret the the above integral as defining a scalar product on this space. With this identification one can show that the differential operators above are hermitian operators. This guarantees that the eigenfunctions $f^{(n)}_{A(5),(s,s')}$ exist, form a complete (orthogonal) set and the corresponding eigenvalues $(m^A_{n,(5)})^2$ are positive. The BCs of A_μ, A_5 now translate to conditions on the $f^{(n)}_{A(5),(s,s')}$s. We will add a subscript $f^{(n)}_{A(5),(s,s')}$, $s, s' \in \{+,-\}$ to indicate the different choices. Remember that $f^{(n)}_{A,(s,s')}$ and $f^{(n)}_{A5,(s,s')}$ have opposite BCs, but we will specify by (s,s') the BCs for A_μ. This means $f^{(n)}_{A5,(-,-)}$ refers to the KK wavefunction of A_5 with $(-,-) = (+,+)_5$ BCs. Explicit expressions for the KK wavefunctions and KK masses are given in the Appendix B. Note that if we have a solution for $f^{(n)}_{A,(s,s')}$ we can for $m_n \neq 0$ construct a solution for $f^{(n)}_{A5,(s,s')}$ by $f^{(n)}_{A5,(s,s')}(z) = \frac{1}{m_n}\partial_5 f^{(n)}_{A,(s,s')}(z)$ and set generally $m_{n,5} = m_n$.

Plugging this expansion in the action we get

$$
\begin{aligned}
S = \int \mathrm{d}^4x \sum_{n=0}^{\infty} \Bigg(& \frac{1}{2} A_{n,\mu} \left(\eta^{\mu\nu}\partial^2 - \left(1 - \frac{1}{\xi}\right)\eta^{\mu\lambda}\eta^{\rho\nu}\partial_\lambda\partial_\rho + \eta^{\mu\nu} m_n^2 \right) A_{n,\nu} \\
& + \frac{1}{2} A_{n,5}\left(-\partial^2 - \xi m_n^2\right) A_{n,5} \Bigg) .
\end{aligned} \tag{6.22}
$$

One can see that we now have an infinite spectrum of 4D massive gauge boson fields and an infinite spectrum of 4D massive scalar fields each with a different mass. Comparing this to 4D theories (like the Higgs mechanism) we see that the KK modes of A_5 provide the longitudinal polarizations of the massive gauge bosons A_μ. In unitary gauge $\xi \to \infty$ these scalars are removed from the theory. This of course is not true for a possible zero mode with $m_n = 0$. Again it turns out that a zero mode for A_μ only exists in the case of $(+,+)$ BCs and because A_5 has opposite BCs there is no corresponding scalar. Thus the zero mode consists only of a single massless gauge boson. Alternatively, for $(-,-) = (+,+)_5$ BCs there is no zero mode of A_μ but one of A_5. The low energy theory thus contains only a massless scalar A_5. Looking at the KK wavefunction for the zero fields we find

$$
f^{(0)}_{A,(+,+)}(z) = \sqrt{\frac{k}{\log\left(\frac{k}{T}\right)}} , \tag{6.23}
$$

$$
f^{(0)}_{A5,(-,-)}(z) = Tz\sqrt{\frac{2k^3}{k^2 - T^2}} . \tag{6.24}
$$

Equipped with this knowledge, one way to model the SM is to choose a bulk group $SU(3)_c \times SU(2)_L \times U(1)_Y$ and assign each field $(+,+)$ BCs such that only the A_μ's have zero modes, which can be identified with the SM gauge fields. Then the A_5's have $(+,+) = (-,-)_5$ BCs and thus there are no additional scalars in the theory.

But this is not the only way to generate the SM fields. As we have seen in Chapter 5, we can assign different BCs to different components of the gauge field in gauge group space. By giving $(+,+)$ BCs to only some components of a general bulk gauge group G, there will not be a A_μ zero mode for every component of G, but only of a subgroup H. This has (at least) two interesting applications. First, if it is possible to realize $H = G_{\text{SM}}$, but still start with a larger group G in the bulk, one also reproduced the SM gauge field at low energies, allowing for the realization of a GUT in this context [97], with the GUT group G and an alternative breaking mechanism instead of the usual Higgs mechanism. This will be further explored in Chapter 10. Second, by starting with a bulk gauge group G one can deliberately assign $(-,-)$ BCs to some components such that there are some scalar zero modes from A_5. If one can choose the BCs such that the quantum numbers of these scalars are the same as the quantum numbers of the Higgs doublet, these A_5's can play the role of the SM Higgs. This possibility will be further explored in Chapter 9.

Going back to general considerations of RS gauge fields, we can also determine the relation between the 5D gauge coupling g_5 and the 4D gauge couplings of SM fields. For this one has to look at the covariant derivative acting on fermion or scalar fields, perform a KK decomposition of all fields involved, integrate out the extra dimension and look at the zero mode fields. Exemplary, we do this here for an abelian gauge field acting on fermions with a LH zero mode. Starting point is the following part of the action

$$\begin{aligned} S &\supseteq \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 \bar{\Psi}\left(i\gamma^\mu D_\mu\right)\Psi \\ &\supseteq g_5 \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 \bar{\Psi}\gamma^\mu\Psi A_\mu\,, \end{aligned} \tag{6.25}$$

which results from (5.15) with the covariant derivative. Doing a KK expansion one finds the following terms for some of the lowest modes

$$\begin{aligned} S \supseteq \int \mathrm{d}^4x \Bigg(& g_5 \left[\int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 \left[f_{\psi_L}^{(0)}(z)\right]^2 f_A^{(0)}(z)\right] \bar{\Psi}_L^{(0)}\gamma^\mu\Psi_L^{(0)}A_\mu^{(0)} \\ & + g_5 \left[\int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 \left[f_{\psi_L}^{(0)}(z)\right]^2 f_A^{(1)}(z)\right] \bar{\Psi}_L^{(0)}\gamma^\mu\Psi_L^{(0)}A_\mu^{(1)} + \ldots \Bigg)\,. \end{aligned} \tag{6.26}$$

Although each gauge boson KK mode will have the same 5D gauge coupling g_5 due to gauge invariance, the resulting observed 4D gauge couplings depend on the overlap integrals between the KK profiles and thus on their localization. The zero mode 4D gauge coupling is given by [89]

$$\begin{aligned} g \equiv g_5 &\left[\int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 \left[f_{\psi_L}^{(0)}(z)\right]^2 f_A^{(0)}(z)\right] \\ &= g_5\sqrt{\frac{k}{\log\left(\frac{k}{T}\right)}} \left[\int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 \left[f_{\psi_L}^{(0)}(z)\right]^2\right] \\ &= g_5\sqrt{\frac{k}{\log\left(\frac{k}{T}\right)}} \,. \end{aligned} \tag{6.27}$$

Since the gauge boson zero mode wavefunction is flat the integral over the extra dimension reduces to the normalization of the fermion profiles. This will not be true for the first and higher KK modes of the gauge boson, leading to couplings which depend on the localization of the fermions along the extra dimension. They will be discussed further in Part V, where we also use the relation above to convert from 5D to 4D couplings when needed.

7 AdS/CFT correspondence

Apart from the KK decomposition there exist another method to relate the 5D RS theory to a 4D one: holography [78, 85]. This is directly motivated by the AdS/CFT correspondence [98], which states that a 5D gravitational theory in anti de-Sitter space (AdS) is dual to a strongly coupled 4D conformal field theory (CFT). But one can also be see this more directly in the following way: We adopt the point of view of a 4D observer located on the Planck brane. In terms of a functional integral we need to integrate out the bulk and IR brane degrees of freedom while keeping the values of the fields at the UV brane. In this way one finds a weakly interacting theory with local gauge invariance H_0, which is the symmetry on the UV brane. This sector is then weakly coupled to a 4D strongly interacting sector (CFT) with global symmetry G, the bulk gauge symmetry. Through the presence of the IR brane the CFT confines at the TeV scale. Furthermore, if the IR brane symmetry H_1 is a true subgroup of G the global symmetry is spontaneously broken down to H_1. In this way we can construct a "holographic dictionary" between quantities in the 5D and quantities in the 4D theory, for which we list a few in Table 7.1 [82]. The relation between this interpretation and the KK picture is the following. The KK states, which are of order TeV, are the mass states resulting from the admixture of the massive resonances of the strong sector with the fields of the elementary sector.

Additionally, the holographic interpretation allows one also to relate RS theories to Composite Higgs scenarios (see Chapter 2). In general, the Higgs will be localized towards the IR brane implying that it is a composite particle of the 4D CFT. This

5D theory	4D theory
5D bulk gauge symmetry G	4D global symmetry G of strong sector
UV symmetry H_0	weakly coupled 4D gauge symmetry
IR symmetry H_1	breaking of G to H_1 in composite sector
fields localized towards the UV brane	mostly elementary fields
fields localized towards the IR brane	mostly composite fields
motion along z	rescaling 4D coordinates
...	...

Table 7.1: Overview of the AdS/CFT dictionary

picture becomes even more illuminating, if one considers GHU, see Chapter 9. In this way the Higgs can be identified as the pNGB associated with the coset G/H_1. We can also comment on the compositness of other particles. In general, as we will see in Chapter 14, the higher the mass of a fermion the more it will localized towards the IR brane. Since the top quark is the heaviest of all the fermions, it is localized the most towards the IR brane, thus being the most composite fermion. For more information see e.g. [78, 85].

8 5D Propagators and Vertices

A key ingredient of any QFTs are the Feynman rules. In this chapter we derive the propagators and vertices for scalars, fermions and gauge bosons for every possible BC. In general, there are two approaches to this. First, one can also do KK decomposition for the propagators with the same KK wavefunctions as for the fields itself. In this way one gets am infinite series over 4D propagators, with their masses given by the respective KK masses. Here we opt for the second approach and work in the full 5D space. This has the advantage to result in a closed form for the propagators, which simplifies some of the renormalization techniques in Part IV considerably.

8.1 5D Propagators

In this section we give the propagators for scalar, fermion and gauge fields extending the works of [99, 100, 89]. We start with a scalar field ϕ, with general BCs, described by the action (5.2). As we noted in Chapter 5, having chosen any BCs we can freely integrate by parts, resulting in[1]

$$S = \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \frac{1}{2k^3z^3} \left[\phi \left(-\partial^2 + k^3z^3\partial_z \left(\frac{1}{k^3z^3}\partial_z \right) - \frac{1}{k^2z^2}m^2 \right) \phi \right] . \quad (8.1)$$

[1] We will also use $\partial_z = \partial_5$ to explicitly show on which coordinate the derivative is acting.

Denoting the propagator by[2] $\langle\phi(x,z)\phi(x',z')\rangle = \Delta_\phi(x,z;x',z') = iG_\phi(x,z;x',z')$ the following differential equation follows

$$\frac{1}{k^3z^3}\left(-\partial^2 + k^3z^3\partial_z\left(\frac{1}{k^3z^3}\partial_z\right) - \frac{1}{k^2z^2}m^2\right)G_\phi(x,z;x',z') = \delta^{(4)}(x-x')\delta(z-z')\,. \tag{8.2}$$

The BCs on the field ϕ now translate to analogous BCs. For the 4D coordinates x, x' we can do a Fourier transform like usual, but because of the metric dependence on the fifth coordinate z this will not be useful for z, z'. We use in this thesis the following 4D Fourier transformation

$$G_\phi(x,x';z,z') = \int\frac{\mathrm{d}^4p}{(2\pi)^4}e^{-ip(x-x')}G_{\phi,p}(z,z'),\quad \delta^{(4)}(x-x') = \int\frac{\mathrm{d}^4p}{(2\pi)^4}e^{-ip(x-x')}\,. \tag{8.3}$$

Thus we work with the so called *position/momentum space* propagators. Doing this transformation, the differential equation becomes

$$\frac{1}{k^3z^3}\left(p^2 + k^3z^3\partial_z\left(\frac{1}{k^3z^3}\partial_z\right) - \frac{1}{k^2z^2}m^2\right)G_{\phi,p}(z,z') = \delta(z-z')\,. \tag{8.4}$$

Notice that the momentum independent part of this differential operator is the same as for the KK decomposition in (6.2), thus we get directly the following solution

$$G_{\phi,p}(z,z') = \sum_{n=0}^{\infty} f_\phi^{(n)}(z)\frac{1}{p^2-(m_n^\phi)^2}f_\phi^{(n)}(z')\,. \tag{8.5}$$

It is useful to have an explicit expression for the propagator, which can be obtained by solving (8.4) directly. The solution is also given in terms of Bessel functions, explicitly we find (see Appendix C)

$$G_{\phi,p}(u,u') = \frac{\pi(ku)^{d/2}(ku')^{d/2}}{2k(AD-BC)}\left(A\,\mathrm{J}_\alpha(pu)+B\,\mathrm{Y}_\alpha(pu)\right)\left(C\,\mathrm{J}_\alpha(pu')+D\,\mathrm{Y}_\alpha(pu')\right). \tag{8.6}$$

Here $u = \min(z,z'), u' = \max(z,z')$, $\alpha = \sqrt{4+\frac{m^2}{k^2}}$ and the coefficients A, B, C, D are given in Appendix C for every possible BC.

We can do a similar analysis for a fermion ψ. Denoting the propagator by $\left\langle\psi_\xi(x,z)\bar{\psi}_{\bar{\xi}}(x',z')\right\rangle = \Delta_{\psi,\xi\bar{\xi}}(x,z;x',z') = iG_{\psi,\xi\bar{\xi}}(x,z;x',z')$ we get the following differential equation from (5.15)

$$\left(\frac{1}{kz}\right)^4\left[\not{p} + i\gamma^5\partial_z - i\frac{2}{z}\gamma^5 - \frac{c}{z}\right]G_{\psi,p}(z,z') = \delta(z-z')\,, \tag{8.7}$$

[2]Throughout this thesis all expectation values are time–ordered.

where we already went to Fourier space. Using the projectors $P_\pm = \frac{1}{2}(1 \mp i\gamma^5)$, the Dirac structure can be simplified by the following ansatz

$$\begin{aligned} G_{\psi,p}(z,z') &= P_+ \left[S_p^+(z,z') + \not{p}V_p^+(z,z')\right] \\ &\quad + P_- \left[S_p^-(z,z') + \not{p}V_p^-(z,z')\right] \\ &= \begin{pmatrix} S_p^+ & \sigma^\mu p_\mu V_p^+ \\ \bar{\sigma}^\mu p_\mu V_p^- & S_p^- \end{pmatrix} . \end{aligned} \tag{8.8}$$

Explicit expressions for the closed forms of these functions are given in Appendix C for all possible BC.

Now onto the gauge bosons. Denoting the propagator by $\langle A_\mu(x,z)A_\nu(x',z')\rangle = \Delta_{A,\mu\nu}(x,z;x',z') = -iG_{A,\mu\nu}(x,z;x',z')$ and $\langle A_5(x,z)A_5(x',z')\rangle = \Delta_5(x,z;x',z') = iG_5(x,z;x',z')$ we get the following differential equations from (5.32)

$$\begin{aligned} \left(\frac{1}{kz}\right) \Bigg[\left(kz\partial_z \frac{1}{kz}\partial_z + p^2\right) \left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) \\ + \left(kz\partial_z \frac{1}{kz}\partial_z + \frac{p^2}{\xi}\right) \left(\frac{p^\mu p^\nu}{p^2}\right) \Bigg] G_{A,p,\nu\lambda}(z,z') = \delta^\mu_\lambda \delta(z-z')\,, \end{aligned} \tag{8.9}$$

$$\left(\frac{1}{kz}\right) \left[p^2 + \xi \partial_z kz \partial_z \frac{1}{kz}\right] G_{5,p}(z,z') = \delta(z-z')\,, \tag{8.10}$$

where we again did a Fourier transformation and split the differential operator into its different tensor structures. The ansatz

$$G_{A,p,\nu\lambda} = G^0_{A,p}(z,z') \left(\eta_{\nu\lambda} - \frac{p_\nu p_\lambda}{p^2}\right) + G^0_{A,\frac{p}{\sqrt{\xi}}}(z,z') \left(\frac{p_\nu p_\lambda}{p^2}\right)\,, \tag{8.11}$$

$$G_{5,p} = \frac{1}{\xi} G^i_{A,\frac{p}{\sqrt{\xi}}}(z,z')\,, \tag{8.12}$$

simplifies the tensor structure of (8.9) and explicit expressions of $G^{0,i}_{A,p}(z,z')$ can be found in Appendix C.

Using (5.44), we can also derive the ghost propagator. Denoting the propagator by $\langle c(x,z)\bar{c}(x',z')\rangle = \Delta_c(x,z;x',z') = iG_c(x,z;x',z')$ the differential equation reads

$$\frac{1}{kz}\left(p^2 + \xi kz\partial_z \left(\frac{1}{kz}\partial_z\right)\right) G_{c,p}(z,z') = \delta(z-z')\,, \tag{8.13}$$

which is solved by

$$G_{c,p}(z,z') = \frac{1}{\xi} G^0_{A,\frac{p}{\sqrt{\xi}}}(z,z')\,. \tag{8.14}$$

8.2 5D Interactions

In this section we give look at some of the possible interactions in RS models. We focus on the interactions used in Part IV. As a starting point we will consider the

RS version of scalar Quantum electrodynamics (QED), for which the action reads

$$S = \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^3 \left[\eta^{\mu\nu}(D_\mu\phi)^*(D_\nu\phi) - (D_z\phi)^*(D_z\phi)\right] . \tag{8.15}$$

From this the coupling to the vector part A_μ of the gauge boson follows as

$$S \supseteq \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^3 \eta^{\mu\nu} \left[igA_\mu \left(\phi^* \left(\partial_\nu\phi\right) - \left(\partial_\nu\phi^*\right)\phi\right) + g^2 A_\mu A_\nu \phi^*\phi\right] . \tag{8.16}$$

Note that this has the same form as the 4D scalar QED except for the extra z integral.

The main theory we consider here is that of a RS non-abelian gauge theory coupled to a RS fermion. The fermion action reads

$$S = \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 \bar{\Psi} \left(i\gamma^\mu D_\mu + i\gamma^5 D_z - i\frac{2}{z}\gamma^5 - \frac{c}{z}\right) \Psi . \tag{8.17}$$

From which the interaction term coupling to the vector part A_μ follows as

$$S \supseteq \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 \left[gA_\mu^a \bar{\Psi}\left(\gamma^\mu T^a\right)\Psi\right] . \tag{8.18}$$

Again this is the same as in 4D, except the extra integration over the extra dimension.

Lastly, we look at the self interactions between the gauge bosons. Their action reads

$$S = \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right) \left[-\frac{1}{4}\eta^{\mu\lambda}\eta^{\nu\sigma}F^a_{\mu\nu}F^a_{\lambda\sigma} + \frac{1}{2}\eta^{\mu\lambda}F^a_{\mu 5}F^a_{\lambda 5}\right] + S_{\mathrm{gf}} + S_{\mathrm{gh}} , \tag{8.19}$$

for which the cubic and quartic terms simplify to

$$\begin{aligned} S \supseteq \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right) \Bigg[&- \eta^{\mu\lambda}\eta^{\nu\sigma}\Big(gf^{abc}(\partial_\lambda A^a_\sigma)A^b_\mu A^c_\nu \\ &+ \frac{1}{4}g^2 f^{abc}f^{ade}A^b_\mu A^c_\nu A^d_\lambda A^e_\sigma\Big) + \eta^{\mu\lambda}\Big(gf^{abc}(\partial_\lambda A^a_5)A^b_\mu A^c_5 \\ &- gf^{abc}(\partial_z A^a_\lambda)A^b_\mu A^c_5 + \frac{1}{2}g^2 f^{abc}f^{ade}A^b_\mu A^c_5 A^d_\lambda A^e_5\Big)\Bigg] . \end{aligned} \tag{8.20}$$

Whereas the first bracket has the same form as for a non-abelian gauge theory in 4D, there are additional terms involving the A_5.

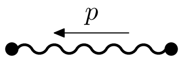
The last interaction we need is the ghost vertex, which stems from the following action

$$S \supseteq \int_x \int_z \left(\frac{1}{kz}\right) \bar{c}^a \left(-\partial^\mu D^{ac}_\mu + \xi kz\partial_z\left(\frac{1}{kz}D^{ac}_z\right)\right) c^c . \tag{8.21}$$

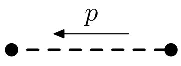
Again we see the similarities with its 4D equivalent for the interactions with the A_μ

8.3 Feynman rules

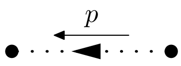
The Feynman rules following from the propagators and interactions given above can be derived in a similar manner as in 4D. Additionally, to integrating over internal momenta one should also integrate over internal z-positions. Using the methods of [6] we find the following rules. First we list the propagators

(z_1, μ_1, a_1) 

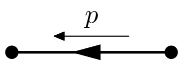
$(z_2, \mu_2, a_2) = \Delta^{a_1 a_2}_{A,p,\mu_1\mu_2}(z_1, z_2)$ (8.22)

(z_1, a_1) 

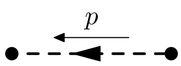
$(z_2, a_2) = \Delta^{a_1 a_2}_{5,p}(z_1, z_2)$ (8.23)

(z_1, a_1) 

$(z_2, a_2) = \Delta^{a_1 a_2}_{c,p}(z_1, z_2)$ (8.24)

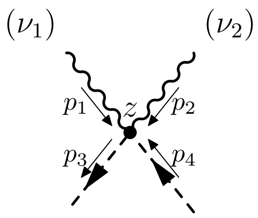
(z_1, α_1, a_1) 

$(z_2, \alpha_2, a_2) = \Delta^{a_1 a_2}_{\psi,p,\alpha_1\alpha_2}(z_1, z_2)$ (8.25)

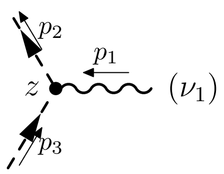
(z_1) 

$(z_2) = \Delta_{\phi,p}(z_1, z_2)$ (8.26)

Next we give the vertices for RS scalar QED

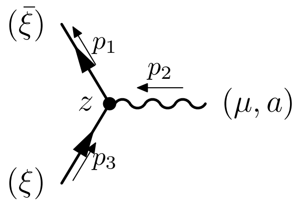


$$= 2ig^2 \eta^{\nu_1\nu_2} \left(\frac{1}{kz}\right)^3 \tag{8.27}$$



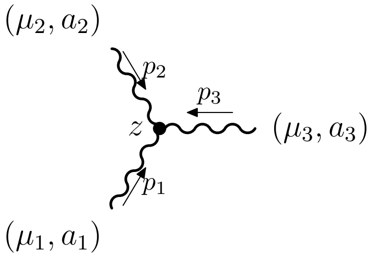
$$= ig(p_3 + p_2)^{\nu_1} \left(\frac{1}{kz}\right)^3 \tag{8.28}$$

We move now on to interactions with fermions. The fermion-A_μ vertex for a non-abelian gauge theory is given by

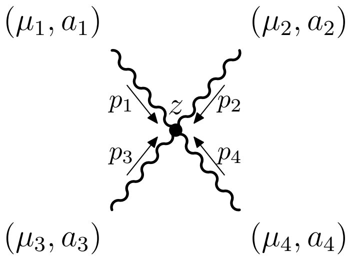


$$= ig\gamma^{\mu}_{\bar{\xi}\xi} t^a \left(\frac{1}{kz}\right)^4 \tag{8.29}$$

Now we can focus on the non-abelian structure itself. We can split the vertices of the RS Yang-Mills part into three groups. First there are two vertices involving only A_μ

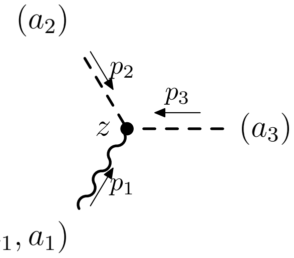


$$
\begin{aligned}
&= g f^{a_1 a_2 a_3} \left(\frac{1}{kz}\right) \Big[\eta_{\mu_1\mu_2}(p_1 - p_2)_{\mu_3} + \eta_{\mu_2\mu_3}(p_2 - p_3)_{\mu_1} \\
&\qquad + \eta_{\mu_3\mu_1}(p_3 - p_1)_{\mu_2} \Big] \\
&\equiv P^{a_1 a_2 a_3}_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3)
\end{aligned}
\tag{8.30}
$$

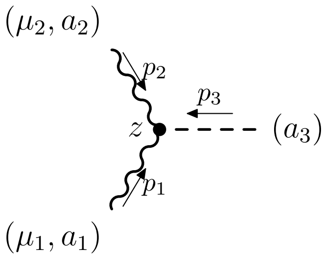


$$
\begin{aligned}
&= -ig^2 \left(\frac{1}{kz}\right) \Big[f^{a_1 a_2 a_5} f^{a_3 a_4 a_5} (\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4} - \eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}) \\
&\qquad + f^{a_1 a_3 a_5} f^{a_2 a_4 a_5} (\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4} - \eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}) \\
&\qquad + f^{a_1 a_4 a_5} f^{a_2 a_3 a_3} (\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4} - \eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}) \Big] \\
&\equiv N^{a_1 a_2 a_3 a_4}_{\mu_1\mu_2\mu_3\mu_4}
\end{aligned}
\tag{8.31}
$$

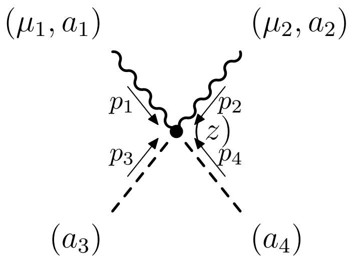
Second, there are in total three vertices involving A_5



$$
\begin{aligned}
&= g f^{a_1 a_2 a_3} \left(\frac{1}{kz}\right) (p_2 - p_3)_{\mu_1} \\
&\equiv Q^{a_1 a_2 a_3}_{\mu_1}(p_2, p_3)
\end{aligned}
\tag{8.32}
$$

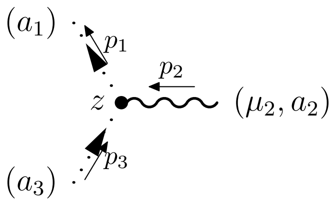


$$
= -ig\eta^{\mu_1\mu_2} f^{a_1 a_2 a_3} \left(\frac{1}{kz}\right) \left(\partial_z^{(a_1)} - \partial_z^{(a_2)}\right)
\tag{8.33}
$$



$$
= ig^2 \eta^{\mu_1\mu_2} \left(\frac{1}{kz}\right) \left[f^{a_1 a_3 a_5} f^{a_2 a_4 a_5} + f^{a_1 a_4 a_5} f^{a_2 a_3 a_5} \right]
\tag{8.34}
$$

Here $\partial_z^{(a_1)}$ is a z-derivative acting on the propagator attached to the leg with the gauge index a_1. Lastly, there is one vertex involving the ghosts



$$= -g f^{a_1 a_2 a_3} \left(\frac{1}{kz} \right) p_1^{\mu_2} \tag{8.35}$$

Note there are also vertices between A_5 and the fermions/scalars which are not listed here, as they will not play a role in the renormalization of the gauge couplings.

9 Gauge-Higgs Unification

We noted in Chapter 6 that the zero mode of the fifth component of a 5D gauge field $A_5^{(0)}$ is a massless 4D scalar. To identify this component with the SM Higgs is known as Gauge-Higgs Unification (GHU) [101, 102, 103]. But we know that the Higgs is not massless, or more explicitly that it has a potential, which allows for a non-trivial minimum. The fact that $A_5^{(0)}$ has no potential and is massless is a consequence of locality and 5D gauge invariance, but this is only valid at tree level. At the one-loop level a potential for $A_5^{(0)}$ can arise from non-local operators. This is because of the finite interval for the extra dimension, similar to the Casimir effect [102]. This potential will be finite, since it arises from non-local operators for which no local counterterms could cancel a possible divergence. Thus $A_5^{(0)}$ will be massless at tree-level and acquires a finite mass radiatively. It has been shown that this also works in warped extra dimensions [64, 65, 104]. In this context the AdS/CFT correspondence allows us to identify the $A_5^{(0)}$ with a pNGB in the 4D dual. Here the bulk gauge group corresponds to global symmetry of the CFT, which is spontaneously broken at the TeV scale by the IR brane. The associated Nambu-Goldstone boson (NGB) is the $A_5^{(0)}$. If the gauge symmetry of the elementary sector (the UV brane symmetry) only gauges a real subgroup of the global symmetry, the global symmetry is explicitly broken and the $A_5^{(0)}$ acquires a finite mass. Similar to the QCD pion the $A_5^{(0)}$ can be viewed as a composite state of the strong dynamics and thus does not receive corrections above the TeV scale. Moreover, in the 5D theory this potential is in fact calculable, by decomposing all fields in their KK modes and resumming the series of one-loop diagrams induced by the virtual exchange of them [79, 84]. This will then give a VEV to the zero mode $A_5^{(0)}$. But this in turn changes the KK wavefunctions of the fermion and gauge fields. The KK decomposition can be written as

$$\Phi(x,z) = \sum_{n=0}^{\infty} \Phi^{(n)}(x) f^{(n)}(z,h) \tag{9.1}$$

$$A_5^{\hat{a}}(x,z) = A_5^{(0),\hat{a}}(x) f_h^{(0)}(z) + \sum_{n=0}^{\infty} A_5^{(n),\hat{a}}(x) f_h^{(n)}(z,h)\,, \tag{9.2}$$

where Φ is a generic fermion or gauge boson field and $A_5^{(0),\hat{a}}$ are the zero modes of the components in the coset which get a VEV $h = \left\langle (A_5^{(0),\hat{a}} A_5^{(0),\hat{a}})^{1/2} \right\rangle$. The VEV dependent KK wavefunctions $f^{(n)}(z,h)$ satisfy the $(\pm)$ BCs specified in Chapter 5. To calculate the VEV one needs to solve the coupled equations of the wavefunctions and generated potential, which is very involved. Fortunately, there exists a gauge transformation which removes the VEV from the bulk [105]

$$A_M^A T^A \to \Omega A_M^A T^A \Omega^\dagger - \frac{i}{g_5} \partial_M \Omega \Omega^\dagger\,, \tag{9.3}$$

$$\Psi \to \Omega \Psi\,, \tag{9.4}$$

with the Wilson line $\Omega(z)$ defined as

$$\Omega(z) = \exp\left(-i g_5 \left\langle A_5^{(0),\hat{a}} \right\rangle T^{\hat{a}} \int_{1/k}^{z} dz' f_h^{(0)}(z') \right) . \tag{9.5}$$

This removes the VEV from the differential equations of the KK profiles such that we can use the wavefunctions derived previously. What changes are the BCs. Note that $\Omega\left(z = \frac{1}{k}\right) = 1$, so the UV brane transforms trivially, but on the IR the transformation is given by

$$\Omega\left(z = \frac{1}{T}\right) = \exp\left(-i \frac{\left\langle A_5^{(0),\hat{a}} \right\rangle}{f} \sqrt{2} T^{\hat{a}} \right), \qquad f = \frac{2T}{g_5 \sqrt{k}}\,, \tag{9.6}$$

where we defined the Higgs decay constant f. Thus the KK masses m_n, which follow from the IR BC will change too.

Using these modified masses it is now considerably easier to solve for the generated Coleman-Weinberg potential

$$V = \frac{N_r}{2} \sum_n \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \log\left(p^2 + m_n^2(h)\right) . \tag{9.7}$$

N_r is the number of degrees of freedom for each level of the KK tower (with $N_r = 3$ for gauge bosons, $N_r = -4$ for fermions) and $m_n(h)$ are the VEV dependent KK masses. It is useful to regulate the integral and, since the result will be finite, its values does not depend on the regularization scheme. To calculate the infinite sum one can use the spectral function ρ which zeros are given by $\rho(m_n^2) = 0$. The integrand will then have poles at the position of the masses and by going to the complex plane, deforming the contour and using the residue theorem the sum can be converted into an integral, resulting in [105]

$$V = \frac{N_r}{(4\pi)^{d/2}\Gamma(d/2)} \int \mathrm{d}q\ q^{d-1} \log\big(\rho_r(-q^2)\big)\,. \tag{9.8}$$

One can now show that the spectral function takes the form [105]

$$\rho_r(-q^2) = 1 + F_r(-q^2)\sin^2\left(\frac{\lambda_r h}{f}\right)\,, \tag{9.9}$$

where λ_r is numerical factor depending on the representation of the fields. In general the potential will then take the form

$$V_{\text{eff}} = V_{\text{gauge}} + V_{\text{fermion}}\,, \tag{9.10}$$

with the gauge and fermion potential taking e.g. for $SO(5)$ the form [79]

$$V_{\text{gauge}} = \alpha \sin^2\left(\frac{h}{f}\right)\,, \tag{9.11}$$

$$V_{\text{fermion}} = \beta_1 \sin^2\left(\frac{h}{f}\right) + \beta_2 \sin^4\left(\frac{h}{f}\right)\,. \tag{9.12}$$

For suitable coefficients α, β_i this can then give a non-trivial minimum h, which in turn then leads to EWSB.

Part III

Gauge-Higgs Grand Unification

10 Grand Unified Theories in Randall-Sundrum Set-Ups

In Chapter 6 we saw an interesting aspect of gauge fields in RS set-ups: Assigning different BCs to different components of 5D bulk gauge fields based on a bulk gauge symmetry G leads to a reduced gauge symmetry H at low energies [97]. To show that this allows for a GUT model, let us choose $G = SU(5)$ [106, 107, 108] and compare this with the Georgi–Glashow $SU(5)$ model (see Chapter 3) as an example of 4D GUTs. One way to obtain the SM symmetry at low energies is to choose $H_0 = SU(5)$ as the UV brane symmetry and $H_1 = G_{\mathrm{SM}}$ as the IR brane symmetry such that $H = SU(5) \cap G_{\mathrm{SM}} = G_{\mathrm{SM}}$. In terms of the BCs of the vector part of the 5D gauge field this can be written as

$$A_\mu = \left(\begin{array}{cc|ccc} (++) & (++) & (+-) & (+-) & (+-) \\ (++) & (++) & (+-) & (+-) & (+-) \\ \hline (+-) & (+-) & (++) & (++) & (++) \\ (+-) & (+-) & (++) & (++) & (++) \\ (+-) & (+-) & (++) & (++) & (++) \end{array}\right) \quad . \tag{10.1}$$

The diagonal blocks and the diagonal generator T^{24} have the same commutation relations as in the Georgi–Glashow model and, since they have $(+,+)$ BCs, their zero modes can be identified with the SM gauge fields. There are also off diagonal 5D fields with $(+,-)$ BCs. They have the same quantum numbers as the X/Y gauge bosons. Since their BCs are $(+,-) = (-,+)_5$ they neither have a vector field zero mode nor a scalar zero mode. Note that by doing this we also have a different breaking mechanism compared to the Higgs mechanism, which is usually used in 4D GUTs. But this set-up also implies that the masses of the first KK modes of the $(+,-)$ fields are of the order $\mathcal{O}(\mathrm{TeV})$ and not around the GUT scale as in the Georgi–Glashow model. This would then lead to excessive proton decays in sharp contrast with observation. One way to avoid this is to impose an additional baryon number symmetry, but we will see in the next chapter, that such symmetries can naturally arise in certain models.

To include fermions in such a model one can use the same $SU(5)$ representations as the Georgi–Glashow model in Chapter 3 and choose the BCs such that the zero modes are the same LH and RH fields as in the SM. The theory then also contains KK states of these fermions.

The last field which needs to be added is the Higgs doublet. One simple way to do this, is to add a 4D localized scalar field on the IR brane, instead of a 5D bulk gauge field. Since the symmetry on the IR brane is $H_1 = G_{\mathrm{SM}}$ and not $G = SU(5)$ one is not forced to include a $SU(5)$ multiplet as in the Georgi–Glashow model. One can simply take a doublet with the same quantum numbers as the SM Higgs. Thus this naturally solves the doublet–triplet splitting problem: there is not triplet to begin with. Another way to incorporate the Higgs doublet is to use the concepts of GHU which will be explored in the next chapter.

Since the group structure is the same as in the Georgi–Glashow model one also finds the same tree–level relations (3.18) between the three gauge couplings. This relation holds for the 5D gauge couplings as well as for the zero mode gauge couplings defined in (6.27), as they are rescaled by a universal factor independent of the gauge group structure. Like in 4D renormalization effects have to be considered to accurately predict how the measured low energy gauge couplings are related to the common high scale value. As RS models have, additionally to the zero modes, also KK excitations of them, the RGE might differ considerably. Moreover, the effective cut-off of RS models is at the TeV scale and it is not clear if the usual 4D renormalization techniques can be use to go above this scale. This thesis studies in detail the renormalization of gauge couplings in RS scenarios in Part IV.

Of course there are many more aspects one can analysis in this model, and the same mechanisms can also be applied to other possible GUT groups like $SO(10)$ or E_6, but here we will not pursue this further and instead extend the unification to also include the Higgs via the methods of GHU.

11 $SU(6)$ Gauge-Higgs Grand Unification

During the last chapters we saw that two different types of unification are possible in RS scenarios: The unification of the Higgs and 4D gauge fields into a 5D gauge field (GHU) and the unification of the SM interactions in a single gauge group (GUT). Moreover, these types of unification are not mutually exclusive allowing to to both in one single model: This is known as a Gauge-Higgs Grand Unified Theory (GHGUT). Like in GHU, this is possible by minimally extending the GUT group to incorporate the Higgs doublet. For example, one can extend the GUT group $SU(5)$ to a gauge group $SU(6)$, or $SO(10)$ to $SO(11)$. GHGUT have been studied based on various groups in warped [109, 110, 111] as well as flat [112, 113, 114, 115, 116] extra dimensions, but to illustrate the important points we focus here on the minimal $SU(6)$, put forward in [1]. There the following breaking pattern, extending the breaking pattern of the last chapter, was achieved: The UV brane symmetry is $H_0 = SU(5)$ and the IR brane symmetry $H_1 = G_{\mathrm{SM}}$. Again this implies that the low energy gauge symmetry is $H = SU(5) \cap G_{\mathrm{SM}} = G_{\mathrm{SM}}$. In terms of the BCs of the vector part of the 5D gauge field this can be written as

$$A_\mu = \left(\begin{array}{cc|ccc|c} (++) & (++) & (+-) & (+-) & (+-) & (--) \\ (++) & (++) & (+-) & (+-) & (+-) & (--) \\ \hline (+-) & (+-) & (++) & (++) & (++) & (--) \\ (+-) & (+-) & (++) & (++) & (++) & (--) \\ (+-) & (+-) & (++) & (++) & (++) & (--) \\ \hline (--) & (--) & (--) & (--) & (--) & (--) \end{array}\right)_{IR-model} . \tag{11.1}$$

Note that the upper left block can be identified with the $SU(5)$ group of (10.1), but this structure features additional fields with $(-,-)$ BCs in the last row/column. Of these new gauge fields only the lower right one will play a role in this thesis: It is a single gauge field Z'_μ and corresponds to a gauge group $U(1)_X$ and we will study its mixing with the Z in Chapter 16. Moreover, since $(-,-) = (+,+)_5$, these additional fields correspond to scalar zero modes in the A_5 components. Looking at the transformation properties of these fields under the SM gauge group one finds that the top right and bottom left components transform as $(\mathbf{1},\mathbf{2})_{1/2}$ such that it is possible to identify this component with the Higgs doublet. Moreover, this model predicts two additional scalar fields: one scalar leptoquark transforming as $(\mathbf{3},\mathbf{1})_{-1/3}$ and a scalar singlet transforming as $(\mathbf{1},\mathbf{1})_0$. These can have important phenomenological consequences, for example in [117] it was shown that many of the flavor anomalies and the anomalies in the measurement of the muon $(g-2)_\mu$ can be explained by a leptoquark with exactly these quantum numbers. Additionally, it is well known that new scalar singlets are useful contributors to electroweak baryogenesis (see e.g. [118]).

In contrast to the model of the last chapter, we do not need to add the Higgs on the IR brane, as it is already included in the 5D gauge fields. But this also means that we are not forced to choose $G_{\rm SM}$ as the IR symmetry, allowing for an alternative breaking pattern. Choosing $H_0 = G_{SM}$ and $H_1 = SU(5)$ also leads to $H = G_{\rm SM} \cap SU(5) = G_{\rm SM}$. In terms of the BCs of the vector part of the 5D bulk gauge field this can be written as

$$A_\mu = \left(\begin{array}{cc|ccc|c} (++) & (++) & (-+) & (-+) & (-+) & (--) \\ (++) & (++) & (-+) & (-+) & (-+) & (--) \\ \hline (-+) & (-+) & (++) & (++) & (++) & (--) \\ (-+) & (-+) & (++) & (++) & (++) & (--) \\ (-+) & (-+) & (++) & (++) & (++) & (--) \\ \hline (--) & (--) & (--) & (--) & (--) & (--) \end{array}\right)_{UV-model} . \tag{11.2}$$

The field content will be the same as before, with the only difference that off diagonal fields, corresponding to the X/Y bosons, have $(-,+)$ instead of $(+,-)$ BCs, in both cases not giving a zero mode. But as we will see in Chapter 13 these models have very different RGEs. Labeling these according to the brane which is broken to G_{SM}, they are

$$\begin{aligned} &IR\text{–}model: \quad H_0 = SU(5), \quad H_1 = G_{\text{SM}} \quad \Longrightarrow\; H = G_{\text{SM}}\,, \\ &UV\text{–}model: \quad H_0 = G_{\text{SM}}, \quad\;\; H_1 = SU(5) \;\Longrightarrow\; H = G_{\text{SM}}\,. \end{aligned}$$

In Chapter 13 it is shown that the *IR–model* requires unification of the gauge couplings at the IR scale (TeV), whereas the unification scale for the *UV–model* can be much higher.

In [1] it has also been shown that one can reproduce the spectrum of SM fermions by a minimal set of 5D fermions consisting of the lowest representation of $SU(6)$. By choosing the BCs accordingly, they decompose under $SU(5)$ and G_{SM} in the following form

$$\begin{aligned} \mathbf{20} \to \mathbf{10} &= q'_L(\mathbf{3},\mathbf{2})_{1/6} \oplus (\mathbf{3}^*,\mathbf{1})_{-2/3} \oplus e'^c_L(\mathbf{1},\mathbf{1})_1 \\ \mathbf{10}^* &= (\mathbf{3}^*,\mathbf{2})_{-1/6} \oplus u_R(\mathbf{3},\mathbf{1})_{1/3} \oplus (\mathbf{1},\mathbf{1})_{-1}, \\ \mathbf{15} \to \mathbf{10} &= q_L(\mathbf{3},\mathbf{2})_{1/6} \oplus (\mathbf{3}^*,\mathbf{1})_{-2/3} \oplus e^c_L(\mathbf{1},\mathbf{1})_1 \\ \mathbf{5} &= d'_R(\mathbf{3},\mathbf{1})_{-1/3} \oplus l'^c_R(\mathbf{1},\mathbf{2})_{1/2}, \\ \mathbf{6} \to \mathbf{5} &= d_R(\mathbf{3},\mathbf{1})_{-1/3} \oplus l^c_R(\mathbf{1},\mathbf{2})_{1/2} \\ \mathbf{1} &= \nu^c_L(\mathbf{1},\mathbf{1})_0, \\ \mathbf{1} \to \mathbf{1} &= \nu'^c_L(\mathbf{1},\mathbf{1})_0\,. \end{aligned} \tag{11.3}$$

The fields which have a LH or RH zero mode correspond to SM fields and are labeled with their usual SM symbols in (11.3). Note that there are additional fields, which do not have a LH or RH zero modes as the correspond to $(+,-)$ or $(-,+)$ BCs. Some of them are also labeled with the SM symbols with an additional prime. They have a special role, which we will now explain. Since fermion zero modes are massless, the SM fields have to acquire their mass by some mechanism. As in the SM this will be provided by the Higgs doublet, which acquires a VEV (see Chapter 9). But, as the Higgs doublet is part of the 5D gauge field, its interactions are given by the covariant derivative acting on the fermion fields (see Chapter 14). This implies that only fields in the same $SU(6)$ multiplet can be connected via the Higgs. As an example let us see how the mass of the up–quark is generated. The A_5 Higgs will lead to an interaction term between the u_R and the q'_L. Ideally we would like to identify the q'_L with the SM quark doublet q_L, but the field q'_L has no zero mode[1]. The solution is now to connect the q'_L with q_L in the **15**, which does contain a zero mode. This can be done for example by a brane mass on the UV or IR brane, which implies that the KK modes will be linear combinations of both fields [96]. In that way the resulting zero mode can be coupled to u_R via the Higgs doublet. Once the Higgs doublet acquires its VEV this gives a mass to the u_R like in the SM. This mechanism will be further explored in Chapter 14.

Thus this model contains all the necessary SM fields and in [1] it has been shown that one can get the correct Higgs VEV and the correct masses for all SM particles

[1]Note that this is due to the GHU aspect of GHGUT and not due to the GUT part, see e.g. the fermion incarnation of [65]

including neutrino masses. This thesis, together with upcoming papers, e.g. [119], investigate this model further. In Part IV the groundwork for the renormalization and thus for the evolution of the gauge couplings is laid out. In Part V and the upcoming paper [119] flavor phenomenology is studied in more detail and the model is confronted with EWPT.

Let us focus again on the GUT aspect of this $SU(6)$ GHGUT model. Like in Chapter 10 the first KK modes of the X/Y bosons in both the *IR–* and *UV–model* have masses of the order $\mathcal{O}(\mathrm{TeV})$. This is not nearly heavy enough to suppress dangerous proton decay. As has been demonstrated in [1], this model features a hidden baryon number symmetry, rendering the proton stable. Thus there is no issue with having these KK states at such low energies. Furthermore, the doublet–triplet splitting problem is not a problem in this model, although one can identify the scalar leptoquark with the corresponding $SU(5)$ triplet of the Higgs doublet. Again, since baryon number is conserved, also the leptoquark does not mediate any baryon number violating decays, allowing it to be light. For more details on this see [1]. If one takes $SU(5)$ symmetric boundary masses the masses of the down–quark and electron would be degenerate as was the case in 4D GUT (see (3.14)). But if one opts for brane masses on the brane which only has G_{SM} as it its symmetry, the brane masses can be chosen independently giving different masses. Alternatively, one can add gauge kinetic terms to split them (see [119] for details on both).

In total, we see that this minimal $SU(6)$ GHGUT solves many of the GUT problems presented in Chapter 3. What remains to be studied are the predictions for the measured low energy gauge couplings. As in the last chapter, the relations (3.18) hold at tree–level at the unification scale, but how the couplings run in this $SU(6)$ GHGUT model will be studied in the next part.

Part IV

Renormalization of Randall-Sundrum Models and Unification in $SU(6)$ Gauge-Higgs Grand Unification

This part discusses the one-loop renormalization of gauge couplings in RS models. After an introduction, which highlights the key difficulties in trying to properly account for loop diagrams in the determination of a renormalized gauge coupling, different methods proposed in the literature are examined. Their validity domain as well as advantages and disadvantages are discussed and the methods are compared to each other. At the end the most promising method is investigated throughly and applied to study the unification of gauge couplings in GHGUTs.

12 Evolution of gauge couplings in Randall-Sundrum models

12.1 Challenges with Renormalization

This chapter introduces the key features of renormalization in (warped) 5D theories and what difficulties can arise in the process of renormalization itself. First, it has to be clarified precisely what the object of interest is. What has been studied at colliders is the coupling of the SM fields, which is not the same as the coupling of the full 5D field in extra dimensional theories. One can identify the SM fields with the zero modes of the KK decomposition of a 5D field with and thus the coupling of this zero mode as defined in (6.27) corresponds to the measured coupling. Of course gauge invariance implies that the 5D gauge coupling of each KK mode is equal at tree level, but this might no longer be true if one considers loop effects. This has already been seen in the case of the Gauge-Higgs mass. Although the Gauge-Higgs mass is zero at tree-level due to gauge invariance, at loop-level a finite potential is generated. The first methods discussed in this part, will therefore explicitly single out the zero mode and calculate loops to its propagator.

An alternative way to relate the 4D and 5D pictures is to use the holographic dictionary. The SM fields are here identified with the 5D field value at the Planck brane. Calculating loops to this Planck brane field will be discussed in the method in Section 12.5.

We will focus here on methods which directly work with the 5D RS set-up, but there are also approaches using deconstruction [120, 121]. In [121] it is argued, that this method is equivalent to the method in Section 12.3.

The start of all methods is the 5D action

$$S \supseteq \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \sqrt{G} G^{MN} G^{PQ} \left(-\frac{1}{4g_5^2} F_{MP} F_{NQ} \right) , \tag{12.1}$$

where we normalized the 5D gauge field such that the 5D gauge coupling g_5 is included in the kinetic terms of the gauge fields. It turns out that the process of renormalization can also induce brane localized terms of the form

$$
\begin{aligned}
S &\supseteq \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \sqrt{G} G^{MN} G^{PQ} \left(\frac{\lambda_k}{4} \delta \left(z - \frac{1}{k} \right) + \frac{\lambda_T}{4} \delta \left(z - \frac{1}{T} \right) \right) F_{MP} F_{NQ} \\
&\supseteq \int \mathrm{d}^4x \left[\frac{\lambda_k}{4} F_{\mu\nu} \left(x, \frac{1}{k} \right) F^{\mu\nu} \left(x, \frac{1}{k} \right) + \left(\frac{T}{k} \right) \frac{\lambda_T}{4} F_{\mu\nu} \left(x, \frac{1}{T} \right) F^{\mu\nu} \left(x, \frac{1}{T} \right) \right] .
\end{aligned}
\tag{12.2}
$$

Note that the full 5D action is not renormalizable and the naive cut-off is of the order $T \sim \mathcal{O}(\mathrm{TeV})$. In fact, also the KK picture is only a good description for low energies and this means the zero mode gauge coupling g can only be properly defined for low energies. Note that the coupling has a mass dimension $\left[g_5^{-2} \right] = 1$ and thus diverges linearly with the cut-off Λ. In flat extra dimensions this is the well-known *power-law* divergence [122, 123]. At low energies the coupling evolves with the SM RGE evolution and thus changes logarithmically, but at the TeV scale this power-law divergence kicks in and spoils predictivity. Surprisingly, this will not be true for all observables in RS models, because of the warping along the extra dimension. There will then be an effective cut-off depending on the position along the extra dimension: On the UV brane the cut-off is of the order $k \sim \mathcal{O}(M_{Pl})$ and on the IR plane $T \sim \mathcal{O}(\mathrm{TeV})$. Thus it is in principle possible to still talk about a high scale unification as was first noted in [124] using Pauli-Villars (PV) fields as a regulator.

This problem can also be seen in a different way. Since renormalization considers UV effects of arbitrary high energies in principle all KK modes in a KK decomposition have to be included in the calculation. Each mode is a 4D field and thus contributes logarithmically to the running of the gauge coupling. Summing then over all modes gives again power-law corrections. Naively, this is also true for RS models, but here we can also look at the 4D theory from a holographic perspective [86]. Since the theory is dual to a CFT weakly coupled to an elementary sector, the couplings are expected to still run logarithmically. As an analogy one can take the running of the electric charge in the SM: Although the (electrically charged) quarks confine due to QCD, the running of the electric charge is still logarithmically. The resolution of the discrepancy between these two interpretations is presented in the next chapter using PV fields and it is shown that the running is indeed logarithmically.

Furthermore, we will find that the logarithmic divergencies in RS can be absorbed by renormalizing the boundary terms $\lambda_{k/T}$, with the mass dimensions $\left[\lambda_{k/T} \right] = 0$. By naive dimensional analysis their values are given by $\lambda_{k/T} \sim \frac{1}{16\pi^2}$ at their respective energy scales. Since they are suppressed by a loop factor compared to coupling g_5 we only need to include the boundary terms at tree level in our calculation.

Before looking at loop effects, let us take a closer look at the tree–level value for the gauge coupling. The tree–level gauge coupling is given by (6.27)

$$
g^2 = \frac{g_5^2 k}{\log\left(\frac{k}{T}\right)} . \tag{12.3}
$$

Since the gauge coupling g depends logarithmically on the TeV scale T this might be interpreted as a coupling run down to the scale T from a Planckian value. That this interpretation has some merits is confirmed by looking at the CFT dual of the theory [125]. Here (12.3) describes the corrections to a 4D gauge boson propagtor from the conserved CFT currents coupled to this gauge boson [126]. In the RS picture this can be computed at tree–level and in this context this "running" is referred to as *tree–level running*.

In the following chapters we look at loop contributions to this coupling in RS set–ups and how one can regularize and renormalize the resulting expressions.

12.2 Renormalization using Pauli-Villars Fields

We consider first scalar QED with a 5D massless scalar field ϕ in flat extra dimension [86]. One can do a KK decompostition similar to the one described in Chapter 6, but the eigenfunctions in flat space will be much simpler sine and cosine functions. All KK modes contribute in loops to the zero mode gauge boson propagator and thus to the photon self energy $\Pi_{\mu\nu}(p^2) = \left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right)\Pi(p^2)$. These corrections are diverging and can be regulated by introducing a 5D PV field Φ with bulk-mass Λ. Doing a KK decomposition of these fields results also in a tower of KK modes with masses shifted by an amount $\sim \Lambda$. Λ then regulates two types of divergencies in the self-energy

$$\Pi(0) \propto \sum_n \int \mathrm{d}^4p \left[\frac{1}{(p^2-(m_n^\phi)^2)^2} - \frac{1}{(p^2-(m_n^\Phi)^2)^2}\right]. \tag{12.4}$$

The first divergence is the diverging momentum integration, as one encounters also in 4D, and the second one is the infinite sum over all KK modes. Above $\sim \Lambda$ we can pair up modes of each field cancelling both divergences. The remaining KK modes lead to a power-law divergence at the TeV scale [86]

$$\Pi(0) \simeq \frac{b_\phi}{8\pi^2}\Lambda R. \tag{12.5}$$

Here $b_\phi = \frac{1}{3}$ is the β-function coefficient for a 4D scalar and R is the size of the extra dimension.

This can be contrasted with the result one obtains in RS scenarios. The KK modes of the PV are not all shifted by an amount $\sim \Lambda$. This will only be true for the zero mode, all higher KK modes stay approximately equal. Thus the self energy is dominated by the contribution of the zero mode and only for large values $\Lambda \simeq k$ are corrections from the KK modes relevant. Since for high energies the warping becomes irrelevant these corrections will again follow a power-law behavior [124]

$$\Pi(0) \simeq \frac{b_\phi}{8\pi^2}\log\left(\frac{\mu}{\Lambda}\right) - \frac{b_\phi}{64\pi^2}\frac{\Lambda^2}{k^2}(\pi k R). \tag{12.6}$$

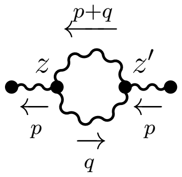
Since the photon is massless we introduced an infrared cut-off μ. The zero mode contribution is thus the same contribution as one expects from a massless 4D scalar field. For the SM gauge fields embedded in a RS space this means that the coupling evolution is logarithmically and approximately the same as in the SM. This allows one to study high scale unification, similar to 4D GUTs also in this theory [124, 127, 128]. But the extension of this PV regularization to non-abelian gauge groups and fermion fields is not straight forward. As in 4D the gauge invariant application of PV to non-abelian gauge bosons is more involved and beyond the scope of this thesis, but for fermions a more fundamental issue arises. We have seen in Chapter 6 that a bulk mass for fermions does not change the mass of a zero mode, but only the localization of the zero mode profiles. This implies that the PV field does not decouple in the limit $\Lambda \to \infty$. Thus we will not pursue this method further and investigate another scheme in the next section.

12.3 Renormalization using a 5D-Position-Dependent Cut-Off

Another regularization method which also works for non-abelian gauge fields has been put forward in [89] by using a cut-off procedure. As explained above the warping changes energy scales along the extra dimension. This motivates a *position-dependent* cut-off of the momentum by $\Lambda/(kz)$ at the position z in the bulk. On the UV brane ($z = 1/k$) the cut-off is $\Lambda \sim M_{Pl}$. This then gets gradually reduced to a value of $\Lambda T/k \sim \mathcal{O}(\mathrm{TeV})$ at the IR brane ($z = 1/T$). Equivalently, one can cut-off the z integration at a value $\Lambda/(kq)$ for a given internal momentum q. This can be seen as an *effective IR brane* which moves closer to the UV brane for increasing momentum. But it turns out this also captures less and less contributions from the propagators in the loop. The correct way to proceed is to renormalize the propagators to this effective brane, which is done best by resolving the differential equation with $\Lambda/(kq)$ as the IR brane position [89]. We follow the approach of [89] using the background field method [6] for a non-abelian gauge field. Splitting the 5D gauge field in a background value A_M and quantum fluctuations $\mathcal{A}_M$ the quadratic part of the action can be calculated by similar manipulations (gauge fixing, integration by parts, etc.) as in the last part. One finds [89]

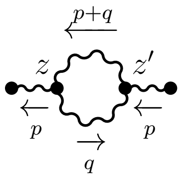
$$
\begin{aligned}
S = \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \frac{1}{kz}\Bigg[&- \frac{1}{2g^2}\Bigg(\mathcal{A}_\mu^A \left[-(D^2)^{AB}\eta^{\mu\nu} + \left(1 - \frac{1}{\xi}\right)(D^\mu D^\nu)^{AB}\right] \mathcal{A}_\nu^B \\
&+ \eta^{\mu\nu} z \mathcal{A}_\mu^A D_z^{AC}\left(\frac{1}{z} D_z^{CE}\mathcal{A}_\nu^E\right) + 2F^{A,\mu\nu} f^{ABC}\mathcal{A}_\mu^B \mathcal{A}_\nu^C \Bigg) \\
&+ \frac{1}{2g^2}\left[\mathcal{A}_5^A \left(-(D^2)^{AB}\right)\mathcal{A}_5^B + \xi \mathcal{A}_5^A D_z^{AC}\left(z D_z^{CE}\left(\frac{1}{z}\mathcal{A}_5^E\right)\right)\right] \\
&+ \frac{1}{g^2}\bar{c}^A\left(-(D^2)^{AE} + \xi z D_z^{AC}\left(\frac{1}{z}D_z^{CE}\right)\right) c^E \Bigg] .
\end{aligned} \tag{12.7}
$$

Here D_M is the covariant derivative and $F^{A,\mu\nu}$ the field strength tensor both using the constant background field A_M. From this one can derive the one-loop effective action $\Gamma^{(1)}$ by integrating out the fluctuating quantum fields. As in 4D the resulting functional determinants can be expanded in the background field and the quadratic term can be given in terms of three diagrams [6]. For example the diagram involving $F^{A,\mu\nu}$ reads



$$= -\frac{1}{2}(4C_r C(j)) \int \frac{\mathrm{d}^4 p}{(2\pi)^4} A^A_\mu(-p)(p^2\eta^{\mu\nu} - p^\mu p^\nu) A^A_\nu(p)$$
$$\times \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \int_{1/k}^{\Lambda/(kq)} \frac{\mathrm{d}z}{kz} \int_{1/k}^{\Lambda/(kq)} \frac{\mathrm{d}z'}{kz'} G^0_{A,q}(z,z') G^0_{A,q+p}(z',z)\,, \tag{12.8}$$

where $C(j) = 2$ for vectors (A_μ) and zero for scalars (A_5) and C_r is the Dynkin index for the representation of the field in the loop. This is similar to the result in 4D with the propagators $\frac{1}{q^2}$ replaced by the 5D propagators $G_q(z,z')$ and an integration over the extra dimensional position of the internal vertices. In fact one can split the integral in two, by inserting the corresponding 4D propagators



$$= -\frac{1}{2}(4C_r C(j)) \int \frac{\mathrm{d}^4 p}{(2\pi)^4} A^A_\mu(-p)(p^2\eta^{\mu\nu} - p^\mu p^\nu) A^A_\nu(p)$$
$$\times \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{1}{q^2(p+q)^2}$$
$$\times\, q^2(p+q)^2 \int_{1/k}^{\Lambda/(kq)} \frac{\mathrm{d}z}{kz} \int_{1/k}^{\Lambda/(kq)} \frac{\mathrm{d}z'}{kz'} G^0_{A,q}(z,z') G^0_{A,q+p}(z',z)\,. \tag{12.9}$$

The first lines are exactly the same integral as in 4D and could be calculated with standard 4D regularization techniques if the last line would not depend on the integration variable q. The last line is a dimensionless quantity and captures the 5D aspect. As we need to take $p \to 0$ for the vacuum expectation value, we can already set $p = 0$ in this integral and we will denote its value by $I(\Lambda, q)$. It turns out that the dependence on q is very weak, in fact $I_0(\Lambda) \equiv I(\Lambda, q=0)$ is a good approximation for $q \ll k$. In this approximation one can pull $I_0(\Lambda)$ out of the q integral and what remains gives the same contribution to the β-function as in 4D. Including also the other diagrams for the fields A_μ, A_5 and the ghosts c gives the following 1-loop β-function for the 4D gauge coupling g

$$\beta(g) = -\frac{g^3}{4\pi^2} C_2(G) \left(\frac{11}{3} I_0^{(A_\mu)}(\Lambda) - \frac{1}{6} I_0^{(A_5)}(\Lambda) \right). \tag{12.10}$$

Here, $C_2(G)$ is the quadratic Casimir operator, $I_0^{(A_\mu)}$ is the contribution from A_μ and the ghosts c, which have the same propagators, and $I_0^{(A_5)}$ using the A_5 propagators. For $\Lambda \lesssim k$ we have $I_0^{(A_\mu)}(\Lambda) \approx 1$ and $I_0^{(A_5)}(\Lambda) \approx 0$, thus the evolution of the β-function is similar to the 4D case.

There are several issues with this regularization scheme. First, the parameter Λ, introduced to regulate the divergence, is still present in the final equation and it is unclear how to interpret this result. Second, there are problems, when one applies this analysis to the fermion case. Extending the results from [89] to fermions we get the following diverging parts of the effective action

$$\begin{aligned} i\Gamma^{(1)}[A] \supseteq\, & i\frac{1}{2g^2}\int \frac{\mathrm{d}^4p}{(2\pi)^4} A^A_\mu(-p)A^A_\nu(p)\left(\eta^{\mu\nu}p^2 - p^\mu p^\nu\right) \\ & + iI_0^{(\Psi,V)}(\Lambda)\left(\frac{8}{3}\right)\frac{1}{(4\pi)^2}\frac{1}{\epsilon}\frac{1}{2}\int \frac{\mathrm{d}^4p}{(2\pi)^4} A^A_\mu(-p)A^A_\nu(p)\left(\eta^{\mu\nu}p^2 - p^\mu p^\nu\right) \\ & + iI_0^{(\Psi,S)}(\Lambda)\left(-\frac{1}{4}\right)\frac{1}{(4\pi)^2}\frac{1}{\epsilon}\frac{1}{2}\int \frac{\mathrm{d}^4p}{(2\pi)^4} A^A_\mu(-p)A^A_\nu(p)\left(\eta^{\mu\nu}p^2 + 0\right) . \end{aligned} \tag{12.11}$$

Here, the first line is the tree level contribution and the next two are the divergent contributions from the one-loop diagrams. The second line is what one finds also in 4D multiplied by a factor

$$\begin{aligned} I_0^{(\Psi,V)}(\Lambda) = q^2(p+q)^2 & \int_{1/k}^{\Lambda/(kq)} \frac{\mathrm{d}z}{(kz)^4}\int_{1/k}^{\Lambda/(kq)} \frac{\mathrm{d}z'}{(kz')^4} \\ & \frac{1}{2}\left[V^+_q(z,z')V^+_{q+p}(z',z) + V^-_q(z,z')V^-_{q+p}(z',z)\right] . \end{aligned} \tag{12.12}$$

Note that this term comes from the off-diagonal elements in (8.8), but there is also a term coming from the diagonal ones, which results in the last line of (12.11) with the factor

$$\begin{aligned} I_0^{(\Psi,S)}(\Lambda) = |q||p+q| & \int_{1/k}^{\Lambda/(kq)} \frac{\mathrm{d}z}{(kz)^4}\int_{1/k}^{\Lambda/(kq)} \frac{\mathrm{d}z'}{(kz')^4} \\ & \frac{1}{2}\left[S^+_q(z,z')S^-_{q+p}(z',z) + S^-_q(z,z')S^+_{q+p}(z',z)\right] . \end{aligned} \tag{12.13}$$

Note that we find that in general $I_0^{(\Psi,S)}(\Lambda) \neq 0$ such that the last line is (12.11) is in fact a real contribution. This term is not present in 4D, where gauge invariance implies that these terms cancel. Moreover, the last line in (12.11) gives a contribution, which is not gauge invariant. In fact, to cancel its divergence one has to add non gauge invariant counterterms to the action. In [89] only gauge bosons have been considered and it was concluded that this position-dependent cut-off does not break gauge invariance. In contrast we find here that when considering fermions one indeed breaks gauge invariance with this regularization procedure. As in 4D one should look for a regulator which does not break gauge invariance, like dimensional regularization. If this can also be applied to the 5D case will be explored in the next chapter.

12.4 Renormalization using Dimensional Regularization

In this chapter we investigate how one can use dimensional regularization to regulate the infinities arising in 5D loops. This was done in the KK picture by [126, 129, 130], which we will review here. Again we look at scalar QED with a 5D scalar field ϕ. Doing a KK decomposition one can use dimensional regularization to get the one-loop scalar correction

$$\Pi(p^2,\mu) = -\mu^{d-4}\sum_n \int_0^1 \mathrm{d}x(2x-1)^2 \int \frac{\mathrm{d}^d q}{(2\pi)^d}\frac{1}{(q^2+(m_n^\phi)^2-x(1-x)p^2)^2}\,. \quad (12.14)$$

$d=4-2\epsilon$ is the regularization parameter of dimensional regularization and μ the corresponding regularization scale. Though the 4D integral is easy to perform, the problem here is the infinite sum over KK modes. The trick is to change the sum into a complex integration using the residue theorem [126, 129, 130]. For example

$$\sum_n \frac{1}{p^2+m_n^2} = \int_C \frac{\mathrm{d}z}{2\pi i}\frac{1}{p^2+z^2}P(z)\,. \quad (12.15)$$

Here C is a closed contour that encloses all KK masses and $P(z)$ can be given by $P(z)=\frac{N'(z)}{N(z)}$, where $N(z)$ is a function which has zeros at $z=m_n$. Deforming the contour allows one to perform this integral and expanding around $\epsilon=0$ gives [126]

$$\begin{aligned}\Pi(p^2,\mu) = \frac{b_\phi}{16\pi^2}\Bigg[&-\frac{1}{\epsilon}+\log\left(\frac{\sqrt{-p^2}}{\sqrt{kT}}\right)+\log\left(\frac{\sqrt{-p^2}}{\mu}\right)\\ &+3\int_0^1 \mathrm{d}y y\sqrt{1-y^2}\log\left(N\left(\frac{iy\sqrt{-p^2}}{2}\right)\right)\\ &+\frac{\gamma_E}{2}+\log(4\pi)-\frac{8}{3}\Bigg]\,,\end{aligned} \quad (12.16)$$

with $b_\phi=\frac{1}{3}$. Note that the exact form changes slightly with the chosen BC. In [130] this result was extended to also include fermions and gauge bosons.

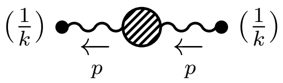
As has been pointed out in [126], (12.16) is only valid for momenta $p\lesssim\mathcal{O}(\mathrm{TeV})$, since the zero-mode becomes strongly coupled above the TeV scale. Expanding (12.16) for low momenta gives

$$\Pi(p^2,\mu) \simeq \frac{b_\phi}{8\pi^2}\left[\log\left(\frac{k}{T}\right)+\log\left(\frac{\sqrt{-p^2}}{k}\right)-\frac{1}{4}\log\left(\frac{\mu}{k}\right)-\frac{1}{4}\log\left(\frac{\mu}{T}\right)\right]. \quad (12.17)$$

With this object the evolution of the coupling can only be studied up to the TeV. To go to higher energies one needs to study the gauge coupling using a different observable. This is the approach of the method in the next section. Additionally, the interpretation of the logarithms in (12.17) is not entirely clear. In principle they can be viewed as running effects starting on the Planck brane [126], but again strictly speaking the formula is only applicable for lower energies. Alternatively, the origin of these logarithms is naturally explained by the method in the next section.

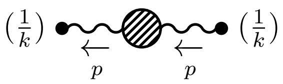
12.5 Renormalization using Planck-Brane Correlators

In [131, 132] it was realized that there are other useful observables besides the ones derived from the zero mode, namely observables defined via the so called Planck brane correlator



$$= \int \mathrm{d}^4 x e^{ip\cdot x} \left\langle A_\mu(x, 1/k) A_\nu(0, 1/k) \right\rangle . \tag{12.18}$$

This essetntially is the full 5D propagator for the gauge field with start- and end-points on the Planck brane. It is more directly inspired by the AdS/CFT correspondence (see Chapter 7) compared to the KK picture. Fields located on the UV brane are elementary and the bulk and IR brane correspond to composite states. Furthermore, one can define a gauge coupling $g(p^2)$ using this correlator via

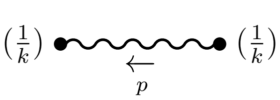


$$\equiv i \frac{g^2(p^2)}{p^2} \eta_{\mu\nu} + \dots , \tag{12.19}$$

where we suppressed gauge dependent parts. At low energies the 5D propagator is given predominately by the propagator of the zero mode, so the gauge coupling defined in (12.19) is equal to the zero mode gauge coupling at low energies. But because the start and end points are on the Planck brane this definition is even valid for energies $p \gg \mathcal{O}(\text{TeV})$. In this section we follow the discussions in [131, 132] and extend on it.

First let us look what the tree level contribution to this gauge coupling is. Throughout this section we are interested in the regime $T \ll p \ll k$ and we will Wick rotate all results, as opposed to directly work in euclidean signature as was done in [131, 132]. For simplicity we also employ to work in Feynman gauge ($\xi = 1$).

Using the euclidean propagator from (8.11) one finds [132]



$$= -ig_5^2 G^0_{A,p}\left(\frac{1}{k}, \frac{1}{k}\right)\eta_{\mu\nu}\,. \tag{12.20}$$

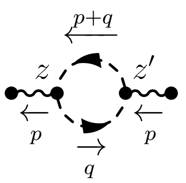
Comparing this with the definition (12.19) we see that the gauge coupling is given by

$$\frac{g^2(p^2)}{p^2} = -g_5^2 G^0_{A,p}(1/k, 1/k) \approx \frac{g_5^2}{p}\frac{\mathrm{K}_1\left(\frac{p}{k}\right)}{\mathrm{K}_0\left(\frac{p}{k}\right)} \simeq \frac{g_5^2 k}{\log\left(\frac{2k}{p}\right)}\frac{1}{p^2}\,. \tag{12.21}$$

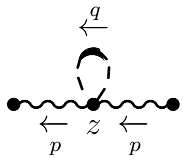
Note that for energies $p \gg T$ the value of the gauge coupling is independent from the behavior near TeV brane. This will also be true at the loop level. The result is the tree level running seen from the point of view of a Planck brane observer [125]. Up to small corrections in the matching scale, this agrees with the findings of Section 12.1 [131, 132]. The Planck brane correlator gives an explanation for the large logarithms encountered also in the KK picture. At low energies the coupling is given by the Planck brane coupling "run down" to an energy $\sim$ TeV. It is also worth noting that this term is uncalculable, but since its contribution is universal to all gauge couplings it will cancel in differences and not influence unification.

Next we look at Planck brane correlator at one-loop for different fields. Starting with contributions from massive scalar with mass $m < k$, there are two diagrams to consider

$L^{(1)}_{\mu\nu} =$



, $L^{(2)}_{\mu\nu} =$



. (12.22)

To regulate these expressions we work in $d = 4-2\epsilon$ dimensions. Using the euclidean version of the Feynman rules from Section 8.3 and dropping the purely longitudinal components we get

$$\begin{aligned} L^{(1)}_{\mu\nu} = -ig_5^4 \int \frac{\mathrm{d}^d q}{(2\pi)^d} \int_{1/k}^{1/T} \frac{\mathrm{d}z}{(kz)^{d-1}} \int_{1/k}^{1/T} \frac{\mathrm{d}z'}{(kz')^{d-1}} G^0_{A,p}(1/k, z)(2q+p)_\mu \\ G_{\phi,q}(z,z')G_{\phi,p+q}(z',z)(2q+p)_\nu G^0_{A,p}(z', 1/k), \end{aligned} \tag{12.23}$$

$$L^{(2)}_{\mu\nu} = 2ig_5^4\eta_{\mu\nu} \int \frac{\mathrm{d}^d q}{(2\pi)^d} \int_{1/k}^{1/T} \frac{\mathrm{d}z}{(kz)^{d-1}} G^0_{A,p}(1/k, z) G_{\phi,q}(z,z) G^0_{A,p}(z, 1/k)\,. \tag{12.24}$$

To calculate these integrals we focus first on the external propagators. They have one endpoint on the Planck brane and for $T \ll p \ll k$ they are given by

$$G^0_{A,p}(1/k, z) \approx -\frac{(kz)^{d/2-1}}{p}\frac{\mathrm{K}_{d/2-1}(pz)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\,. \tag{12.25}$$

Since $\mathrm{K}_\nu(y) \sim \sqrt{\frac{\pi}{2y}}\exp(-y)$, (12.25) is suppressed for values $pz \gg 1$. Thus the support for the z-integrals is mainly given by the region $\frac{1}{k} < z < \frac{1}{p}$. In fact one finds that the main contribution to these integrals comes from the region where $pz \ll 1$ is the smallest and thus for a fixed p, from z values near the Planck brane.

Next we look at the internal loop propagators. As in 4D, the main contribution to the running from the momentum integrals comes from the regions around the external momentum p. Since we take $p \ll k$ the dominant contributions should come from loop momenta l with $l \ll k$. Loop momenta greater than the Planck scale give only rise to analytic structures, i.e. terms analytic in the momentum p, which we will drop here. Note that this is an implicit choice of scheme, but, if used consistently throughout, cannot effect the result for the low energy gauge coupling. Since we concluded the main contribution for the integrals comes for values z, z' near the Plannck brane this implies $lz, lz' \ll 1$ for the loop momenta. Expanding the propagators in this limit one finds

$$G_{\phi,q}(z,z') \simeq -(kz)^{d/2-\alpha}(kz')^{d/2-\alpha}\frac{2k(\alpha-1)}{p^2+\left(1-\frac{2}{d}\right)m^2}\,. \tag{12.26}$$

Here $\alpha = \sqrt{(d/2)^2 + m^2/k^2}$ and m is the mass of the scalar field. Again we see that the exact dynamics on the IR brane are irrelevant. Moreover one can argue that the propagator in this limit is dominated by a single pole at a mass $m_d^2 \equiv \left(1-\frac{2}{d}\right)m^2$. This allows for a connection to the KK picture. For $m = 0$ the contribution is essentially given by the zero mode. Although we consider energies $p \gg T$, only this modes has a significant overlap with the Planck brane. Due to the AdS_5 curvature, the higher KK modes are localized towards the IR brane and are suppressed at the Planck brane. For $m \neq 0$ the situation is similar. We can identify one KK mode with mass m_d, which behaves like a zero mode, i.e. it is the only one with a non negligible overlap with the UV brane. In both cases there is only one single mode which has a significant overlap with the UV brane, and thus the running will turn out to be logarithmic rather than power law like.

Explicitly the above integrals simplify to

$$\begin{aligned} L^{(1)}_{\mu\nu} &= -ig_5^4\int\frac{\mathrm{d}^d q}{(2\pi)^d}\frac{(2q+p)_\mu(2q+p)_\nu}{(q^2+m_d^2)((p+q)^2+m_d^2)} \\ &\quad \left[\frac{2k(\alpha-1)}{p}\int_{1/k}^{\infty}\mathrm{d}z(kz)^{d/2-2\alpha}\frac{\mathrm{K}_{d/2-1}(pz)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\right]^2, \end{aligned} \tag{12.27}$$

$$\begin{aligned} L^{(2)}_{\mu\nu} &= -2ig_5^4\eta_{\mu\nu}\int\frac{\mathrm{d}^d q}{(2\pi)^d}\frac{1}{q^2+m_d^2} \\ &\quad \left[\frac{2k(\alpha-1)}{p}\int_{1/k}^{\infty}\mathrm{d}z(kz)^{d-1-2\alpha}\left(\frac{\mathrm{K}_{d/2-1}(pz)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\right)^2\right]. \end{aligned} \tag{12.28}$$

Although the full propagators mix the dependence on momentum and the extra dimension non-trivially, by the virtue of the expansion above we can separate the two.

Moreover, the momentum integrals have reduced to simple 4D one–loop integrals for a 4D scalar with mass m_d. Expanding the z–integrals in $pz, p/k \ll 1$, one finds for the leading terms

$$L^{(1)}_{\mu\nu}\big|_{z-\mathrm{int}} = \left[\frac{2k(\alpha-1)}{p}\int_{1/k}^{\infty}\mathrm{d}z(kz)^{d/2-2\alpha}\frac{\mathrm{K}_{d/2-1}(pz)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\right]^2 \simeq \left(\frac{1}{p}\frac{\mathrm{K}_{d/2-1}\left(\frac{p}{k}\right)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\right)^2, \tag{12.29}$$

$$L^{(2)}_{\mu\nu}\big|_{z-\mathrm{int}} = \left[\frac{2k(\alpha-1)}{p^2}\int_{1/k}^{\infty}\mathrm{d}z(kz)^{d-1-2\alpha}\left(\frac{\mathrm{K}_{d/2-1}(pz)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\right)^2\right] \simeq \left(\frac{1}{p}\frac{\mathrm{K}_{d/2-1}\left(\frac{p}{k}\right)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\right)^2. \tag{12.30}$$

Adding the two diagrams to the tree level contribution results in the one loop gauge coupling

$$\frac{g^2(p^2)}{p^2} = \frac{g_5^2}{p}\frac{\mathrm{K}_{d/2-1}\left(\frac{p}{k}\right)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\left[1 - g_5^2\frac{p\,\mathrm{K}_{d/2-1}\left(\frac{p}{k}\right)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\Pi(p^2)\right], \tag{12.31}$$

where the vacuum polarization $\Pi(p^2)$ is the result from the 4D momentum integrals as in the 4D case

$$\Pi(p^2) = \frac{\Gamma(2-d/2)}{(4\pi)^{d/2}}\int_0^1 \mathrm{d}x(2x-1)^2\left[x(1-x)p^2 + m_d^2\right]^{d/2-2}. \tag{12.32}$$

Like in 4D the result is diverging in the limit $d \to 4$ and needs to be renormalized. The functional dependence on $\frac{p}{k}$ of the Bessel functions suggests that this corresponds to a brane localized term on the UV brane. In fact the boundary term of (12.2) for the UV brane gives a contribution [131, 132]

$$\frac{g^2(p^2)}{p^2} \supseteq -g_5^4\lambda_k p^2\left(\frac{1}{p}\frac{\mathrm{K}_{d/2-1}\left(\frac{p}{k}\right)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\right)^2, \tag{12.33}$$

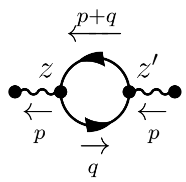
and thus has the right form to cancel the divergencies of (12.32). This means one only needs to renormalize λ_k to get a finite answer. Doing this the gauge coupling, after resumming the result, is given by [131, 132]

$$\frac{1}{g^2(p^2)} = \frac{\log\left(\frac{k}{p}\right)}{g_5^2 k} + \lambda_k(\mu) - \frac{1}{48\pi^2}\log\left(\frac{p^2}{\mu^2}\right), \tag{12.34}$$

where we took $m = 0$ for simplicity. The interpretation of the logarithmic term is again simple in this method: it corresponds to running of the brane coupling $\lambda_k(\mu)$.

Next we investigate how this result changes if one chooses different BC for the scalar field. From our analysis above we have an intuitive view what effects the running. Only fields with a non–negligible overlap with the UV brane contribute to the running and the dynamics on the IR brane is irrelevant. Thus a $(+,-)$ mode should give the same result as the above $(+,+)$ mode. Since the $(-,+)$ and $(-,-)$ modes are per definition zero at the UV brane they will give no contribution to the running. An explicit calculation confirms that this is indeed the case.

This analysis can be extended to fields with other spins. First, we focus on fermion fields and start with a field with $(+,+)$ BC, which has a LH zero mode. We know that the localisation of the zero mode and thus the overlap with the Planck brane depends on the bulk mass parameter c and later we will split our analysis into different cases depending on the value of c. There is only one diagram contributing at one-loop

$L_{\mu\nu} =$ 

. (12.35)

Using the Feynman rules from Section 8.3 this evaluates to

$$L_{\mu\nu} = -ig_5^4 \int \frac{\mathrm{d}^d q}{(2\pi)^d} \int_{1/k}^{1/T} \frac{\mathrm{d}z}{(kz)^d} \int_{1/k}^{1/T} \frac{\mathrm{d}z'}{(kz')^d} G^0_{A,p}(1/k,z)$$
$$\mathrm{Tr}[\gamma_\mu G_{\psi,p+q}(z,z')\gamma_\nu G_{\psi,q}(z',z)] G^0_{A,p}(z',1/k)\,. \tag{12.36}$$

Using the Dirac structure of the fermion propagator (8.8) the trace can be simplified to traces over gamma matrices

$$L_{\mu\nu} = -ig_5^4 \int \frac{\mathrm{d}^d q}{(2\pi)^d} \int_{1/k}^{1/T} \frac{\mathrm{d}z}{(kz)^d} \int_{1/k}^{1/T} \frac{\mathrm{d}z'}{(kz')^d} G^0_{A,p}(1/k,z)$$
$$\left[\mathrm{Tr}(\gamma^\mu\gamma^\nu)\tilde{S}(z,z') + p_\lambda(p+q)_\rho \,\mathrm{Tr}\big(\gamma^\lambda\gamma^\mu\gamma^\rho\gamma^\nu\big)\tilde{V}(z,z')\right] G^0_{A,p}(z',1/k)\,. \tag{12.37}$$

Here we already dropped a term which is anti-symmetric in the indices μ,ν as this will vanish when contracted with the resulting momentum structure $p^2\eta^{\mu\nu} - p^\mu p^\nu$. The functions $\tilde{S}$ and $\tilde{V}$ are given by

$$\tilde{V}(z,z') = \frac{V^+_{p+q}(z,z')V^+_q(z',z) + V^-_{p+q}(z,z')V^-_q(z',z)}{2}\,, \tag{12.38}$$
$$\tilde{S}(z,z') = \frac{S^+_{p+q}(z,z')S^-_q(z',z) + S^-_{p+q}(z,z')S^+_q(z',z)}{2}\,. \tag{12.39}$$

Now we can use the the same arguments as in the scalar case for the integral (12.37) and we expand the fermion propagators in $lz, lz' \ll 1$ for a generic loop momenta l. One finds that V^- and $S^\pm$ scale like $l^\alpha, \alpha \geq 0$ and thus give only rise to analytic structures which we will drop in this scheme. The remaining V^+ has to be approximated differently for different values of c

$$V_p^+(z,z') \simeq \begin{cases} -\frac{2k\left(c-\frac{1}{2}\right)}{p^2}(kz)^{\frac{d}{2}-c}(kz')^{\frac{d}{2}-c} & \text{for } c > 1/2 \\ -\frac{(2k)^{2c}}{(p^2)^{c+1/2}}\frac{\Gamma(1/2+c)}{\Gamma(1/2-c)}(kz)^{\frac{d}{2}-c}(kz')^{\frac{d}{2}-c} & \text{for } -1/2 < c < 1/2 \end{cases}. \tag{12.40}$$

For $c < -\frac{1}{2}$ the function V^+ scales like $l^\alpha, \alpha \geq 0$ and thus gives only rise to analytic structures. Like in the scalar case, the expansion allows us to disentangle the position and momentum integrals. Starting with the easier case $c > \frac{1}{2}$ the z integration result is

$$L_{\mu\nu}\Big|_{z-\text{int}}^{c>\frac{1}{2}} = \left[\frac{2k(c-\frac{1}{2})}{p}\int_{1/k}^{\infty} \mathrm{d}z (kz)^{d/2-1-2c}\frac{\mathrm{K}_{d/2-1}(pz)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\right]^2 \simeq \left(\frac{1}{p}\frac{\mathrm{K}_{d/2-1}\left(\frac{p}{k}\right)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\right)^2 . \tag{12.41}$$

Note that this is the same structure as in the scalar case and thus can be renormalized by the localized boundary term. The momentum integral simplifies to

$$L_{\mu\nu}\Big|_{q-\text{int}}^{c>\frac{1}{2}} = g_5^4 \,\mathrm{Tr}\big(\gamma^\lambda\gamma^\mu\gamma^\rho\gamma^\nu\big)\frac{1}{2}\int \frac{\mathrm{d}^d q}{(2\pi)^d}\frac{p_\lambda (p+q)_\rho}{q^2(p+q)^2} . \tag{12.42}$$

Again this is the same integral as one encounters is 4D, with the exception of an extra factor of $\frac{1}{2}$, which can be explained by the fact that the zero mode corresponds to a Weyl fermion. Now onto the case of $c < \frac{1}{2}$, the z integration gives

$$\begin{aligned} L_{\mu\nu}\Big|_{z-\text{int}}^{c<\frac{1}{2}} &= \left[\frac{(2k)^{2c}\Gamma(1/2+c)}{p\Gamma(1/2-c)}\int_{1/k}^{1/p} \mathrm{d}z (kz)^{d/2-1-2c}\frac{\mathrm{K}_{d/2-1}(pz)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\right]^2 \\ &\simeq \left(\frac{2^{2c-1}p^{2c-1}\Gamma(\frac{1}{2}+c)}{\Gamma(\frac{3}{2}-c)}\right)^2\left(\frac{1}{p}\frac{\mathrm{K}_{d/2-1}\left(\frac{p}{k}\right)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\right)^2 , \end{aligned} \tag{12.43}$$

and the corresponding momentum integral reads

$$L_{\mu\nu}\Big|_{q-\text{int}}^{c<\frac{1}{2}} = g_5^4 \,\mathrm{Tr}\big(\gamma^\lambda\gamma^\mu\gamma^\rho\gamma^\nu\big)\frac{1}{2}\int \frac{\mathrm{d}^d q}{(2\pi)^d}\frac{p_\lambda (p+q)_\rho}{[q^2(p+q)^2]^{c+\frac{1}{2}}} . \tag{12.44}$$

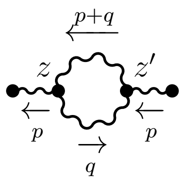
We see that the z–integral has a different structure compared to the scalar and the other fermion case above. Since it has an additional fractional power of the momentum, this is not renormalizable with the simple UV boundary term from (12.2). Luckily, the momentum integral only gives structures which are analytic in the momentum and thus no renormalization is required.

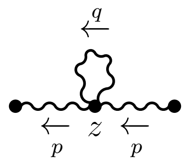
Finally, the case for $c = 1/2$ can be constructed from the two cases above by noting that the propagators are continuous in c. After the z–integration, but before the momentum integral, we can safely take the limit $c \to 1/2$ as the integrand is still continuous. Doing this we find that the running for $c = 1/2$ is the same as for $c > 1/2$.

To summarize the running induced by a 5D fermion is like that of a 4D (LH) Weyl fermion if it is an UV localized fermion ($c \geq \frac{1}{2}$) and it does not contribute if it is an IR localized fermion ($c < \frac{1}{2}$).

Now we can extend these results to other BCs. Starting with the case $(-,-) = (+,+)_R$, which corresponds to a RH zero mode. Instead of V^+ only V^- contributes to the running, for which $c < -\frac{1}{2}$ gives a UV localized zero mode and $c > -\frac{1}{2}$ an IR localized one. Correspondingly, we find that the $(-,-)$ fermion contributes like a 4D (RH) Weyl fermion for $c \leq -\frac{1}{2}$ and there is no contribution for $c > -\frac{1}{2}$. Furthermore, as for the scalar case the BC on the IR brane is irrelevant and the results for $(+.-)$ BCs are the same as for $(+,+)$ and the results for $(-.+)$ are the same as for $(-.-)$.

Finally, the analysis can also be applied to gauge fields in the loop. Looking at a gauge boson with $(+,+)$ BC we first focus on the contribution coming from the vector part A_μ. Again we work in the Feynman gauge ($\xi = 1$) also for the internal propagators. There are two diagrams contributing

$L^{(1)}_{\mu\nu} =$ 

, $L^{(2)}_{\mu\nu} =$ 

, (12.45)

which are explicitly given by

$$
\begin{aligned}
L^{(1),ab}_{\mu\nu} = ig_5^4 \int \frac{\mathrm{d}^d q}{(2\pi)^d} \int_{1/k}^{1/T} \frac{\mathrm{d}z}{(kz)^{d-3}} \int_{1/k}^{1/T} \frac{\mathrm{d}z'}{(kz')^{d-3}} \\
G^0_{A,p}(1/k, z) P^{a_1 a_2 a}_{\lambda_1\lambda_2\mu}(p+q, -q, -p) G^0_{A,p+q}(z, z') \eta^{\lambda_1\rho_1} \\
P^{a_1 a_2 b}_{\rho_1\rho_2\nu}(-p-q, q, p) G^0_{A,q}(z', z) \eta^{\lambda_2\rho_2} G^0_{A,p}(z', 1/k)\,, \qquad (12.46)\\
L^{(2),ab}_{\mu\nu} = ig_5^4 \int \frac{\mathrm{d}^d q}{(2\pi)^d} \int_{1/k}^{1/T} \frac{\mathrm{d}z}{(kz)^{d-3}} G^0_{A,p}(1/k, z) N^{accb}_{\mu\lambda_1\lambda_2\nu} \\
G^0_{A,q}(z, z) \eta^{\lambda_1\lambda_2} G^0_{A,p}(z', 1/k)\,. \qquad (12.47)
\end{aligned}
$$

Again we expand the propagators for loop momenta l in the regime $lz, lz' \ll 1$ finding

$$
G^0_{A,q}(z, z') \simeq -\frac{2k(\frac{d}{2} - 2)}{q^2}\,. \qquad (12.48)
$$

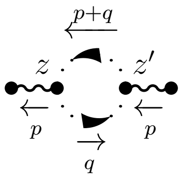
This allows us to perform the momentum and z integration separately, with the z

integration given by

$$L^{(1)}_{\mu\nu}\big|_{z-\mathrm{int}} = \left[\frac{2k(\frac{d}{2}-2)}{p}\int_{1/k}^{\infty}\mathrm{d}z(kz)^{-d/2+2}\frac{\mathrm{K}_{d/2-1}(pz)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\right]^2 \simeq \left(\frac{1}{p}\frac{\mathrm{K}_{d/2-1}\left(\frac{p}{k}\right)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\right)^2 \tag{12.49}$$

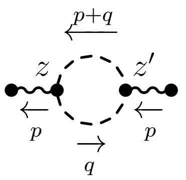
$$L^{(2)}_{\mu\nu}\big|_{z-\mathrm{int}} = \left[\frac{2k(\frac{d}{2}-2)}{p^2}\int_{1/k}^{\infty}\mathrm{d}z(kz)^{1}\left(\frac{\mathrm{K}_{d/2-1}(pz)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\right)^2\right] \simeq \left(\frac{1}{p}\frac{\mathrm{K}_{d/2-1}\left(\frac{p}{k}\right)}{\mathrm{K}_{d/2-2}\left(\frac{p}{k}\right)}\right)^2 . \tag{12.50}$$

Again these integrals allow for renormalization via the boundary action and one finds that the momentum integrals are also the same as in 4D. Since in Feynman gauge ($\xi = 1$) the ghost propagator is the same as for A_μ the same analysis also holds for the ghost diagram

$L^{(3)}_{\mu\nu} =$  . (12.51)

In total this means that A_μ and the ghosts c contribute as in a 4D non-abelian gauge theory. Before we look at the scalar part A_5, let us first discuss what happens to the contribution of A_μ and c for different BCs. As for the other fields the IR brane is irrelevant and thus a field with $(+,-)$ BCs gives the same contribution as one with $(+,+)$ BC. Like in the cases before $(-,-)$ and $(-,+)$ fields are zero at the Planck brane and will not contribute the the running.

Now looking at scalar parts involving loops of A_5 we start with the BCs $(-,-) = (+,+)_5$. For example there is the diagram

$L^{(4)}_{\mu\nu} =$  , (12.52)

which gives

$$\begin{aligned} L^{(4)}_{\mu\nu} = i g_5^4 \int \frac{\mathrm{d}^d q}{(2\pi)^d}\int_{1/k}^{1/T}\frac{\mathrm{d}z}{(kz)^{d-3}}\int_{1/k}^{1/T}\frac{\mathrm{d}z'}{(kz')^{d-3}} G^0_{A,p}(1/k,z) Q^{aa_1a_2}_{\mu}(p+q,-q) \\ G^i_{A,p+q}(z,z') Q^{ba_1a_2}_{\nu}(-p-q,q) G^i_{A,q}(z',z) G^0_{A,p}(z',1/k)\,. \end{aligned} \tag{12.53}$$

As before we expand the propagators for loop momenta l in the regime $lz, lz' \ll 1$

$$G^i_{A,q}(z,z') \simeq \frac{2(kz)(kz')(1-(kz^{(\prime)})^{d-4})}{k(d-4)}\,. \tag{12.54}$$

Different than the other cases these propagators have no momentum poles and thus will not give any contribution to the running. Similarly, fields with $(-,+)$, $(+,-)$ and $(+,+)$ will not contribute. Thus, at one loop, the A_5 can be completely dismissed in the evolution of the gauge coupling.

13 Unification in $SU(6)$ Gauge-Higgs Grand Unified Theories

Having discussed different renormalization techniques in the last chapter, we can apply them to the study of unification in GHGUT models. For this we choose the method of the Planck brane correlator, since it allows one to study the running of the couplings up to the Planck scale. We apply this analysis to the minimal $SU(6)$ model from Chapter 11, but it can be easily adopted for other models. The $SU(6)$ model comes in two incarnations: One *UV-model* and one *IR-model*, which differ in their breaking to the SM group (see Chapter 11). Depending on the incarnation the evolution of the coupling constants will be very different and this will give us useful insights on the conditions of any GHGUT.

Before presenting the results let us take a moment to reflect on our expectations from the breaking structure and from holographic considerations. In the *IR-model* $SU(6)$ is broken to G_{SM} on the IR brane and to $SU(5)$ on the UV brane. Holographically this corresponds to a strong sector, which is invariant under a global $SU(6)$ symmetry, with a gauged $SU(5)$ subgroup. At the scale $T \sim$ TeV this group is broken further down to G_{SM}. Turning this around above the TeV scale the full $SU(5)$ gauge group is recovered. Thus above the TeV scale there will be no differential running and consequently the couplings have to unify before that. But, below the TeV scale the running is SM–like, which is not enough to unify at the TeV scale (see Figure 3.1). We conclude that this model is not consistent with the observed low energy values for the three gauge couplings. Turning now to the *UV-model*, $SU(6)$ is broken to G_{SM} on the UV brane and to $SU(5)$ on the IR brane. In the holographic picture this corresponds to gauging the G_{SM} subgroup of $SU(6)$, and this gauge group is not broken down further at the TeV scale. In contrast to the case above, G_{SM} will thus also be the gauge group above the TeV scale and there is a non trivial differential running, allowing the couplings to unify at one point. In the following we will calculate if this actually happens. In the following we focus on the running above the TeV scale and neglect the running between m_Z and the TeV scale, since this has only a relatively small effect on the numerical values of the gauge couplings.

First we look at the calculation for the *IR-model*. Beginning with the contribution from the vector part of the 5D gauge field, we know from our results in Section 12.5 that all fields with $(+,+)$ and $(+,-)$ BCs contribute. From (11.1) we see that these are the components of the gauge field, which contain the SM gauge boson, $((+,+)$ BCs), and the components of the gauge field, which contain the X/Y gauge bosons $((+,-)$ BCs). As seen in Section 12.5 they both contribute like corresponding 4D fields even though the X/Y have no zero mode. Note also that the SM gauge bosons together with the X/Y bosons form a complete $SU(5)$ multiplet, which means that there will be contribution to the differential running from these gauge bosons. The effect of the scalar part of the 5D gauge fields is very simple. In Section 12.5 we concluded that no matter how the BCs are, they do not contribute. Thus the fields containing the SM Higgs as well as the scalar leptoquark and singlet can be ignored. Since the model does not contain any scalar fields, the last thing to look at are the fermion fields. Here we encounter a problem since all fermion fields in the $SU(6)$ model from Chapter 11 are connected via brane masses. We leave the extension of the methods of Section 12.5 to fermions connected via brane masses to a future work, but we believe that they will have a small effect and not change the result of this analysis considerably. This is based on the fact that the fermions of the same BC form again complete representations of $SU(5)$ implying that they will not contribute to the differential running. Here we take the fields containing the SM fermions as they have in general the largest overlap with the UV brane. In Section 12.5 we have seen that only fields which are UV localized contribute to the running. To reproduce the fermion masses (see Chapter 14) we need to take the $SU(5)$ multiplet of $t_R, Q_L^3 = (t_L, b_L)^T$ and τ_R to be IR localized, whereas the rest are UV localized. Thus the contribution will be the same as in the SM without the $t_R, Q_L^3 = (t_L, b_L)^T$ and τ_R fermions. In total we find the following running couplings

$$\alpha_3^{-1}(\mu) = \alpha_3^{-1}(m_Z) - \frac{b_3}{2\pi}\log\left(\frac{\mu}{m_Z}\right), \qquad b_3 = -\frac{46}{3}\,, \tag{13.1}$$

$$\alpha_2^{-1}(\mu) = \alpha_2^{-1}(m_Z) - \frac{b_2}{2\pi}\log\left(\frac{\mu}{m_Z}\right), \qquad b_2 = -\frac{46}{3}\,, \tag{13.2}$$

$$\alpha_1^{-1}(\mu) = \alpha_1^{-1}(m_Z) - \frac{b_1}{2\pi}\log\left(\frac{\mu}{m_Z}\right), \qquad b_1 = -\frac{46}{3}\,. \tag{13.3}$$

Note that all three coefficients have the same value, which is the result of all contributing fields forming $SU(5)$ multiplets. Thus there is no differential running and the unification has to be at the TeV scale. This matches our expectation from above, rendering this model inconsistent with the observed low energy values of the gauge couplings. We can illustrate this inconsistency by using the SM values and the above runnings to extrapolate the couplings to higher energies. This is depicted in Figure 13.1. The fact that these lines are not on top of each other, reflects the fact that unification is not possible in this scenario.

Next, we investigate how this changes in the *UV-model*. The fields with the SM gauge bosons still have $(+,+)$ BCs, but the BCs of the fields containing the X/Y bosons are now $(-,+)$. This means they have zero overlap with the Planck brane and according to Section 12.5 do not contribute to the running. Like in the case for the *IR-model* the scalar components of the 5D gauge fields do not contribute in this

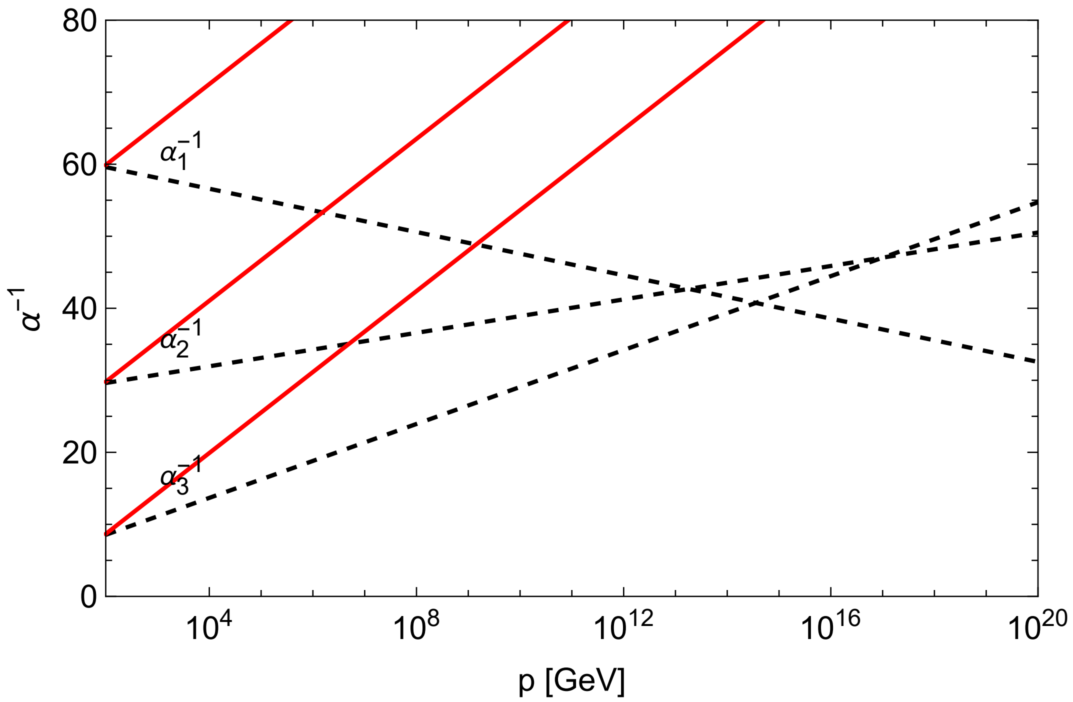


Figure 13.1: Running of the three SM couplings $\alpha_3, \alpha_2, \alpha_1$ at one-loop according to the *IR-model* (13.1)–(13.3) (red) and SM (3.8)–(3.10) (black, dashed). The initial values are given by (1.10). Note the inconsistency of the IR model: Above the TeV scale there should be only one unified gauge coupling, the three curves should be on top of each other, which does not happen due to the large difference in the measured low energy gauge couplings. See text for details.

model either. This means, concerning the gauge sector, the evolution of the gauge couplings is like in the SM. There is also no difference in the fermion sector between the models. Thus we take the running of the first few SM fermions and exclude $t_R, Q_L^3 = (t_L, b_L)^T$ and τ_R. In total this results in the following running couplings

$$\alpha_3^{-1}(\mu) = \alpha_3^{-1}(m_Z) - \frac{b_3}{2\pi}\log\left(\frac{\mu}{m_Z}\right), \qquad b_3 = -8\,, \tag{13.4}$$

$$\alpha_2^{-1}(\mu) = \alpha_2^{-1}(m_Z) - \frac{b_2}{2\pi}\log\left(\frac{\mu}{m_Z}\right), \qquad b_2 = -\frac{13}{3}\,, \tag{13.5}$$

$$\alpha_1^{-1}(\mu) = \alpha_1^{-1}(m_Z) - \frac{b_1}{2\pi}\log\left(\frac{\mu}{m_Z}\right), \qquad b_1 = 3\,. \tag{13.6}$$

Since all three coefficients are different, two couplings each are equal at some energy. If this happens in one single point, can be determined by running the measured low energy couplings with the above equations to higher energies. One finds the behavior depicted in Figure 13.2. As seen in this figure the three couplings do not

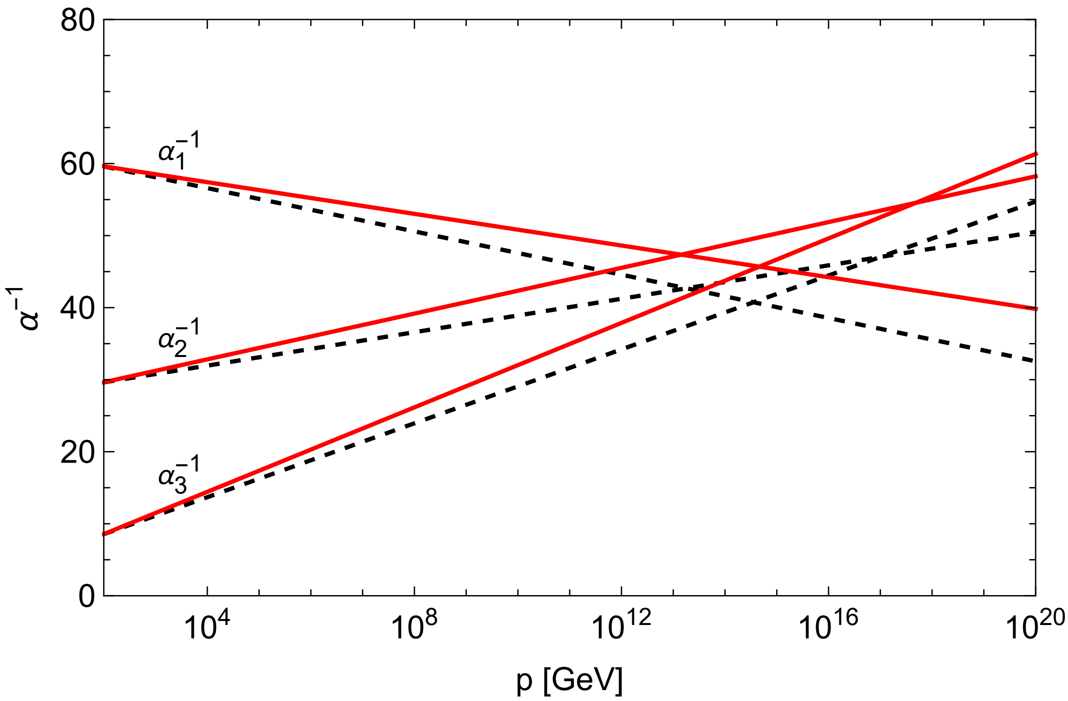


Figure 13.2: Running of the three SM couplings $\alpha_3, \alpha_2, \alpha_1$ at one-loop according to *UV-model* (13.4)–(13.6) (red) and SM (3.8)–(3.10) (black, dashed). The initial values are given by (1.10). See text for details.

meet exactly in one point. In fact the degree of mismatch is comparable to the one in the SM. This is not surprising since the differential running is (almost) the same in the *UV-model* and in the SM. In both cases the SM gauge fields are the main contribution to the running and in both cases the fermions form complete $SU(5)$ multiplets dropping out of the differential running. The only difference is the Higgs, which contributes only in the SM, but its contribution is small. As described in Chapter 3 the meeting points in the SM case are too far apart to accurately account for the observed low energy gauge couplings. The same is true for the *UV-model* presented here. We can quantify this as in Chapter 3 by postdicting $\sin^2(\theta_W)(m_Z)$, using (13.4)–(13.6) we find $\sin^2(\theta_W)(m_Z) = 0.203$. Comparing this to the value $\sin^2(\theta_W)(m_Z) = 0.23120$ from measurement and the original Georgi–Glashow $SU(5)$ postdiction $\sin^2(\theta_W)(m_Z) = 0.207$ [7], our value is off by an amount of 10% compared to the measurement and on a similar level as the Georgi–Glashow $SU(5)$ value.

One way to improve the unification is to add brane localized gauge kinetic terms for each SM field on the UV brane. Since the UV brane symmetry is G_{SM} the coefficients of each of these terms can be chosen independently such that the unification can be achieved exactly and consistent with the observed low energy gauge couplings, but a deeper understanding of their origins would be preferable. We leave the investigation of this possibility for a future work.

Part V

Flavor Phenomenology and Precision Tests in Gauge-Higgs Grand Unified Theories

Having discussed the unification of gauge couplings in the last part we focus in this part on selected phenomenological topics. In particular we study some consequences of EWSB in GHGUT. Starting with fermions it is shown how the hierarchies of the SM fermion masses can naturally arise in the 5D set-up. Since this mechanism is different compared to the SM one, this might change some of the FCNCss significantly, as they are naturally suppressed in the SM by the GIM mechanism. In Chapter 15 it is shown that there exists an analogous mechanism in RS set–ups and the important contributions in GHGUT are presented. In the last chapter, the effect of EWSB on gauge fields is discussed. Again, since the nature of EWSB in GHGUT and SM are different, this might change some relations involving the W and Z-bosons. Here we will calculate the effect of GHGUT on the oblique parameters S, T, U and compare it with experimental results. In this part we will assume that the low energy gauge couplings are given by their experimental values if needed. The work presented here is part of a larger collaboration, whose result will appear in a future work [119]. On the one hand this means that there will be some overlap between the results presented here and in [119] and on the other hand we will refer for further details to [119].

14 Flavor Hierarchies

In this chapter the generation of fermion masses and the hierarchies in the CKM matrix in GHGUT models are discussed. Note that this mechanism is similar as for a bulk Higgs field [133, 134] just with a different nature of the Higgs. As an example we work again with the $SU(6)$ model of Chapter 11.

Although the masses can be calculated exactly by the procedure outlined in Chapter 9, we will use here a more intuitive perturbative approach. We will not make the gauge transformation to move the Higgs VEV to the IR brane, but instead work with the regular KK expansion and investigate the effect of EWSB on the full KK tower. We start by looking at the interaction of the Gauge-Higgs, which is part of the covariant derivative, with fermions. The relevant part of the action is

$$\begin{aligned} S &\supseteq \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 \bar{\Psi}\left(i\gamma^5 D_5\right)\Psi \\ &\supseteq g_5 \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 \left[i\bar{\Psi}_R A_5^{\hat{a}} T^{\hat{a}} \Psi_L + \text{h.c.}\right] . \end{aligned} \tag{14.1}$$

From this we see that only fermion fields which are in the same multiplet can be connected via the Higgs doublet and get a mass via the Higgs VEV. Let us take as an example the mass of the up quark. In the $SU(6)$ model of Chapter 11 the right handed up quark is contained in the the **20**. After taking into account the group structure and doing a KK expansion (see Chapter 6) one finds for the lowest states involving u_R

$$S \supseteq \int \mathrm{d}^4x \left(\frac{g_5}{\sqrt{2}}\bar{u}_R H^c q'_L\right)\left[\int_{1/k}^{1/T}\mathrm{d}z\left(\frac{1}{kz}\right)^4 f_h^{(0)}(z) f_{\Psi_{20}R}^{(0)}(z) f_{\Psi_{20}L}^{(1)}(z)\right] + \text{h.c.}\,, \quad (14.2)$$

where we collected the components of the real $A_5^{\hat{a}}$ into the complex Higgs doublet H (see Appendix A). Note that the 4D operator in (14.2) is similar to usual 4D Yukawa interaction term, except it involves q'_L instead of q_L. Because also the fermions are unified in $SU(6)$ multiplets in GHGUT, q'_L has the same quantum numbers as q_L, but does not have a zero mode. The solution is to embed the q_L in a different multiplet, in this case the **15**, and connect the two multiplets via a brane mass M_u. The resulting (would be) zero mode can then be viewed as a linear combination of q'_L and q_L and thus allows on the one hand to be connected to u_R via the Higgs and on the other hand to lie below the TeV scale (see Chapter 11). To illustrate the most important concepts of GHU let us for the moment assume that q'_L does have a zero mode with zero mode wavefunction $f_{\Psi_{20}L}^{(0)}(z)$ and discuss the effect of brane masses afterwards. After the Higgs acquires its VEV one would get for the mass of the up quark

$$m_u = \frac{g_5 v}{2}\left[\int_{1/k}^{1/T}\mathrm{d}z\left(\frac{1}{kz}\right)^4 f_h^{(0)}(z) f_{\Psi_{20}L}^{(0)}(z) f_{\Psi_{20}R}^{(0)}(z)\right] \approx \frac{g_* v}{2\sqrt{2}} f(c_{20}) f(-c_{20})\,, \quad (14.3)$$

where $g_* = g_5\sqrt{k}$. Note that also the KK states get a mass contribution from the Higgs VEV, but since they also have KK masses of the order TeV this effect is negligible for them. Furthermore, the expansion leads to mixing between the zero mode and all higher KK states and to get the true mass of the zero mode one has to diagonalize this infinite mass matrix. Taking then only the zero mode, as we do here, gives the up quark mass up to an order $\mathcal{O}((v/T)^2)$. An exact result can be achieved as outlined in Chapter 9.

What appears as Yukawa coupling in 4D is in GHU given by the gauge coupling g_5 and the overlap between fermion wavefunctions and the Higgs wavefunction. The Higgs wavefunction is localized towards the IR brane, but the Fermion localization changes with the c-parameter c_{20} as explained in Chapter 6. The more the fermion wavefunctions are localized towards the IR the greater their mass. In fact the mass is proportional to the flavor functions $f(\pm c_{20})$. For $c_{20} > \frac{1}{2}$ or $c_{20} < -\frac{1}{2}$ which corresponds to the LH and RH components to be UV localized respectively, the flavor functions $f(\pm c_{20})$ are suppressed resulting in an up quark mass which is much smaller than the EW scale. Alternatively, for $-\frac{1}{2} < c_{20} < \frac{1}{2}$ the flavor functions $f(\pm c_{20})$ are of the order $\mathcal{O}(1)$ and can give rise to large fermion masses. In this way a large hierarchy of fermion masses can be generated by small changes in the value of the c–parameter, thereby offering a solution to the flavor puzzle. In general, we can reproduce the SM by localizing fermions with small masses more towards to the UV brane and fermions with large masses more towards the IR brane [95, 94].

This result can be extended to the actual case where q'_L and q_L are connected via a brane mass M_u. For simplicity we take an IR brane mass here, for UV masses see [119]. The up quark mass in this case can be given by

$$m_u = \frac{g_* v}{2\sqrt{2}} f(c_{15}) M_u^* f(-c_{20})\,. \tag{14.4}$$

Note that there will then also be mixing between the kinetc terms of q_L and q'_L, known as kinetic mixing. Canonically normalizing these terms changes (14.4) by a factor K_q. The same arguments about the localization as given above are true, but now there are two c–parameters c_{20} and c_{15} to adjust giving more flexibility. This flexibility can then be used to choose the parameters c_{20}, c_{15}, c_6, c_1 such that correct fermion masses for u, d, e, ν_e can be achieved.

Having discussed how one can successfully generate the masses of one generation we can now extend this analysis to three generations to explain the the flavor structure. As the Higgs itself can only couple fields in the same multiplet, the terms which actually mixes different generations are the brane masses. Again we work with the example of the up type quarks. In general the brane mass M_u will then be a 3×3 matrix in generation space and we will take its entries to be anarchic to the flavor structure. Equation (14.4) will then turn in a matrix equation

$$m_u = \frac{g_* v}{2\sqrt{2}} f_{c_{15}} M_u^* f_{-c_{20}}\,, \tag{14.5}$$

where $f_{c_{15}} = \mathrm{diag}(f(c_{15}^1), f(c_{15}^2),\ f(c_{15}^3)), f_{-c_{20}} = \mathrm{diag}(f(-c_{20}^1), f(-c_{20}^2), f(-c_{20}^3))$ are diagonal matrices. The $c^i_{15/20}$ are the c–parameters of the different generations. Hierarchies in this matrix can be achieved by hierarchies in the flavor functions $f(c)$, which in turn can be achieved by $\mathcal{O}(1)$ values for c. Note again that one has to include the effect of kinetic mixing, but the 3×3 matrix K_q is close to the identity and thus its effect is generally small [119].

The mass matrix of (14.5) can then be diagonalized as in the SM by the unitary matrices $U_{u,L}, U_{u,R}$ and the hierarchies of m_u translate to the hierarchies of the rotation matrices [134]

$$U_{u,L} \sim \begin{pmatrix} 1 & \frac{f^1_{c_{15}}}{f^2_{c_{15}}} & \frac{f^1_{c_{15}}}{f^3_{c_{15}}} \\ \frac{f^1_{c_{15}}}{f^2_{c_{15}}} & 1 & \frac{f^2_{c_{15}}}{f^3_{c_{15}}} \\ \frac{f^1_{c_{15}}}{f^3_{c_{15}}} & \frac{f^2_{c_{15}}}{f^3_{c_{15}}} & 1 \end{pmatrix}, \quad U_{u,R} \sim \begin{pmatrix} 1 & \frac{f^1_{-c_{20}}}{f^2_{-c_{20}}} & \frac{f^1_{-c_{20}}}{f^3_{-c_{20}}} \\ \frac{f^1_{-c_{20}}}{f^2_{-c_{20}}} & 1 & \frac{f^2_{-c_{20}}}{f^3_{-c_{20}}} \\ \frac{f^1_{-c_{20}}}{f^3_{-c_{20}}} & \frac{f^2_{-c_{20}}}{f^3_{-c_{20}}} & 1 \end{pmatrix}, \tag{14.6}$$

where $f^i_{\pm c_{15/20}}$ are the components of the flavor function matrices. Similarly, one finds for the rotation matrices of the down quark

$$U_{d,L} \sim \begin{pmatrix} 1 & \frac{f^1_{c_{15}}}{f^2_{c_{15}}} & \frac{f^1_{c_{15}}}{f^3_{c_{15}}} \\ \frac{f^1_{c_{15}}}{f^2_{c_{15}}} & 1 & \frac{f^2_{c_{15}}}{f^3_{c_{15}}} \\ \frac{f^1_{c_{15}}}{f^3_{c_{15}}} & \frac{f^2_{c_{15}}}{f^3_{c_{15}}} & 1 \end{pmatrix}, \tag{14.7}$$

from which follows, to first order, the following CKM matrix

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & \frac{f^1_{c_{15}}}{f^2_{c_{15}}} & \frac{f^1_{c_{15}}}{f^3_{c_{15}}} \\ \frac{f^1_{c_{15}}}{f^2_{c_{15}}} & 1 & \frac{f^2_{c_{15}}}{f^3_{c_{15}}} \\ \frac{f^1_{c_{15}}}{f^3_{c_{15}}} & \frac{f^2_{c_{15}}}{f^3_{c_{15}}} & 1 \end{pmatrix} . \tag{14.8}$$

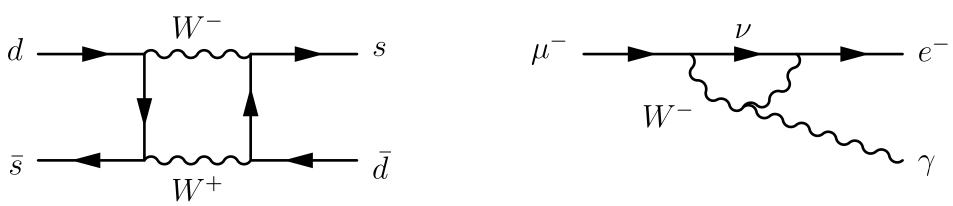
Comparing this with the measured hierarchies of the SM in equation (1.37) one can see that the by choosing

$$\frac{f^1_{c_{15}}}{f^2_{c_{15}}} \sim \lambda, \qquad \frac{f^2_{c_{15}}}{f^3_{c_{15}}} \sim \lambda^2, \qquad \frac{f^1_{c_{15}}}{f^3_{c_{15}}} \sim \lambda^3\,, \tag{14.9}$$

the hierarchies in the CKM matrix can be reproduced. By choosing the other c–parameters appropriately one can also get the hierarchies in the quark masses. For further details and a discussion of the other fermion masses as well as of the PMNS matrix we refer to [119]

15 Flavor Changing Neutral Currents in Randall-Sundrum Models

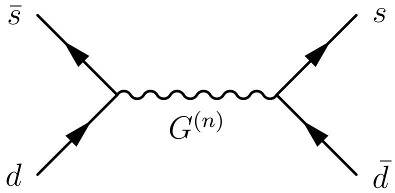
As the nature of the fermion masses and of the CKM matrix is considerably different compared to the SM, this could allow for large FCNCs. In the SM, there are no FCNCs at tree–level as the only field, which allows for a flavor change, is the W-boson. At loop level the W-boson can mediate FCNCs, for example in the following diagrams



(15.1)

Let us take a closer look the first diagram, the box diagram, which leads to K–$\bar{K}$ mixing, and see how this process changes in RS set–ups. In the SM all three up–type quarks are allowed to propagate in between, the vertex for each of these possibilities is proportional to the corresponding entry in the CKM matrix. Expanding in the fermion masses, one finds that the first order term, which is independent of the masses, vanishes when one sums over all three up–type fermions, as a consequence of the unitarity of the CKM matrix. This is the well–known SM GIM mechanism which suppresses these kinds of diagrams. The next term depends then on the difference of the quark masses divided by the W mass, such that it would vanish for equal masses. Thus the most important contribution to these processes stems from the top quark as it is considerably heavier than the other two.

The situation changes in RS models. Although the CKM matrix is still unitary, one has to take the into account the overlap integrals between the fermion and gauge boson wavefunctions. This can then lead to tree-level diagrams for these processes. For example for K–$\bar{K}$ mixing we can consider the following diagram



(15.2)

The strength of the interaction between the j-th and k-th generation is determined by

$$g^{jk} \sim g_5 \sum_i (U^\dagger)^{ji} \left[\int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 f_G^{(n)}(z) f_{\Psi_i}^{(0)}(z) f_{\Psi_i}^{(0)}(z) \right] U^{ik} \,, \tag{15.3}$$

where U is the matrix for the rotation to the mass basis. Let us first consider the zero mode which corresponds to the SM gluon. Since the wavefunction $f_G^{(0)}(z)$ is constant the overlap integral reduces to the normalization of the fermion wavefunction (6.10)

$$g^{jk} \sim g_5 \sum_i (U^\dagger)^{ji} \left[\int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 f_{\Psi_i}^{(0)}(z) f_{\Psi_i}^{(0)}(z) \right] U^{ik} = g_5 \sum_i (U^\dagger)^{ji} U^{ik} \,. \tag{15.4}$$

The unitarity of the rotation matrix U implies that off-diagonal entries vanish for this diagram and consequently, the gluon itself leads to no FCNCs at tree level. In contrast, the wavefunction of the first KK mode of the gluon $G^{(1)}$ does depend on the position along the extra dimension and thus the off-diagonal elements do not vanish. Weather these lead to dangerous FCNCs depends on the value of the overlap integral (15.3). Now, the nature of RS space implies that the wavefunctions of KK modes are IR localized and, since the light zero modes are UV localized, the overlap integral will be very suppressed. This will be true for any FCNCs and is, in analogy with the SM GIM mechanism, called RS–GIM mechanism [135, 136]. We also see that, like in the SM, this mechanism is broken by the top as its large mass implies a larger IR localization (see Chapter 14). Higher KK modes will be even more IR localized and have additionally higher masses. Thus in tree-level diagrams the dominant contribution comes from the first KK modes. On top of the contribution of the gluon, also the KK modes of Z-boson and the photon have to included. One finds that the largest contribution comes from the KK gluon as the strong coupling is the largest of the three coupling constants.

Additionally, one has to consider higher dimensional operators. The EFT scale with which they are suppressed depends on the position along the extra dimension.

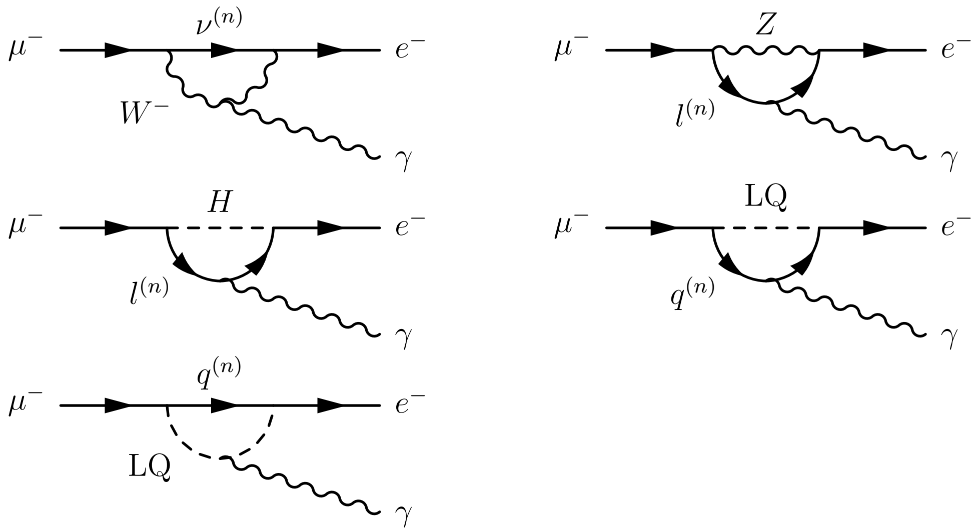
For the IR brane it would correspond to a TeV scale and thus the suppression might be too small. Luckily, the RS–GIM mechanism also ensures that the effect of these operators on FCNCs is small. The EFT cut-off depends on the localization of the fermions and again since light fermions are UV localized the cut-off will be much higher.

It will turn out that constraints coming from K–$\bar{K}$ mixing will be much smaller compared to the process $\mu \to e\gamma$, which we will discuss below. Thus we will not go into more details of the calculation here, but a detailed analysis will be found in [119], together with a discussion on other meson mixings.

We move on to discuss the second type of process, $\mu \to e\gamma$, which will appear at loop level in RS. In the SM it is induced through penguin diagrams, like the second diagram in (15.1). Like before it is suppressed by the SM GIM mechanism this time following from the the unitary PMNS matrix. The first order term is proportional to the mass differences of the neutrinos compared to the W mass. As neutrinos are very light and thus their mass differences very small, this process is almost absent in the SM ($\mathrm{BR}(\mu \to e\gamma) \approx 10^{-40}$), which implies that any significant observation would be a clear signal of BSM physics.

Now, in RS scenarios one can also have KK neutrino states and KK W-bosons propagating in the loop. As the suppression grows with the mass of the KK W-boson we can focus on the (would be) zero mode, which corresponds to the SM W-boson. Note that we can not ignore the KK states of the neurtinos as their masses are of the order TeV and the diagram is proportional to their mass difference.

Note that in general the interaction of zero mode fermions with their KK states is not diagonal in flavor space such that also the Higgs and Z-boson can contribute to this process. Additionally, in $SU(6)$ GHGUT also the scalar leptoquark contributes. In summary, the following diagrams have to be considered



(15.5)

Like for K–$\bar{K}$ mixing the suppression of these diagrams comes from the RS-GIM mechanism. The interaction strength is given by the overlap between a fermion zero mode and of one fermion KK mode together with the bosonic particle in the loop. As the zero mode fermions are UV localized and the KK states IR localized the corresponding integral will be small. Again this is in general broken by the top quark which is IR localized. In the following we investigate if the suppression is large enough to satisfy the current experimental constraints.

Using the Ward identities the general structure of these diagrams is given by [137]

$$M^\mu = \bar{u}_{p'} i \frac{\sigma^{\mu\nu} q_\nu}{m_\mu} (C_L P_L + C_R P_R) u_p \,, \tag{15.6}$$

where p, p', q are the momenta of the muon, electron and photon respectively. From this the decay width for the flavour violating process can be given by

$$\Gamma(\mu \to e\gamma) = \frac{(m_\mu^2 - m_e^2)^3 (|C_L|^2 + |C_R|^2)}{16\pi m_\mu^5} \,. \tag{15.7}$$

This decay width has to be compared to the dominating $\mu \to e\nu\bar{\nu}$ decay with the decay width $\Gamma(\mu \to e\nu\bar{\nu}) = m_\mu^5 G_F^2 / 192\pi^3$. The branching ratio for the flavor violating $\mu \to e\gamma$ is thus

$$\mathrm{Br}(\mu \to e\gamma) = \frac{12\pi^2 (|C_L|^2 + |C_R|^2)}{(G_F m_\mu^2)^2} \,. \tag{15.8}$$

The most stringent constraint on this branching ratio comes from the MEG experiment [138]

$$\mathrm{Br}(\mu \to e\gamma) = 4.2 \times 10^{-13} \,, \tag{15.9}$$

at the 90% confidence level. In the upcoming years it will receive an update from MEG II [139] with a projected sensitivity of

$$\mathrm{Br}(\mu \to e\gamma) = 6 \times 10^{-14} \,. \tag{15.10}$$

The full calculation of this process has been done in collaboration with the authors of [119] and the results are plotted in Figure 15.1

As one can see in Figure 15.1, $\mu \to e\gamma$ gives an excellent observable to constrain the parameter space. With the current measurement, this puts the IR scale to $T = 1/R' \gtrsim 10\,\mathrm{TeV}$.

16 Electroweak Precision Parameters in Randall-Sundrum Models

In this chapter we study in more detail the effect of EWSB in GHGUT on the gauge boson states. We will pay special attention on the implications on electroweak precision parameters, namely the oblique parameters S, T, U. These oblique corrections

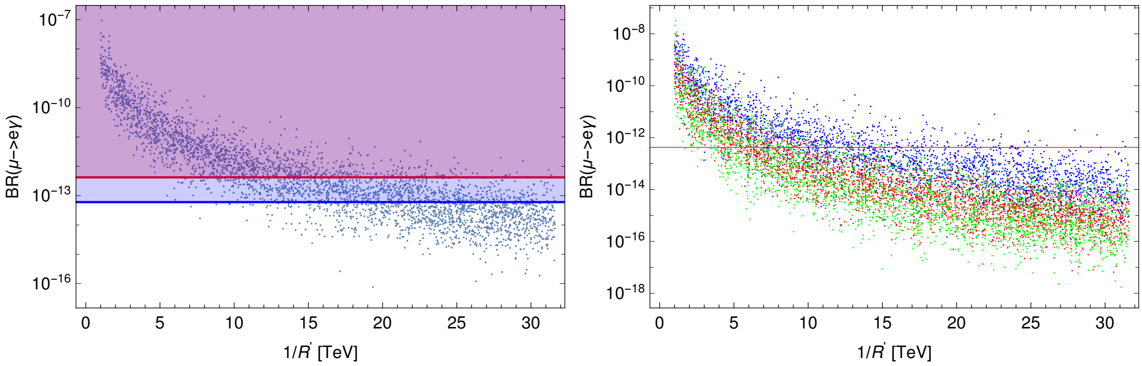


Figure 15.1: Lepton flavor violation constraints: (left) current and future constraints on $\mu \to e\gamma$ decay (right) relative size of the leptoquark (blue), Higgs (red) and Z-boson (green) loop contributions. The W-boson contribution is negligible and not shown. We used the definition $1/R' \equiv T$ to better compare with [119] in the future.

have already been studied in several models of warped extra dimensions, for example in the context of brane fermions [140], IR brane localized Higgs scenarios with and without custodial symmetry [141, 142, 90], bulk Higgs scenarios [143], and also GHU models with custodial symmetry [65], but none of these apply directly to our model. In the following, we show an explicit calculation for the oblique parameters in our model, but the obtained results apply also to other GHU scenarios without custodial symmetry.

16.1 Gauge Boson Masses

Like for fermions the exact mass states can be calculated via the method in Chapter 9. For this one would rotate the Higgs VEV on the brane and solve again for the KK wavefunctions now with modified BCs coming from the Higgs. For illustrative purposes, we choose a perturbative approach here. To start, the mass of the gauge bosons comes from the following term in the action (5.29)

$$
\begin{aligned}
S &\supset \frac{1}{2}\int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right) \eta^{\mu\nu} F_{\mu 5} F_{\nu 5} \\
&\supset \frac{1}{2}\int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right) D_\mu A_5 D^\mu A_5 \,. \qquad (16.1)
\end{aligned}
$$

Once the Higgs acquires its VEV this results in masses/mixings between the different gauge bosons. For easier comparison with the SM masses we can already rotate the fields exactly like in the SM to get the W-, Z-bosons and the photon field, which we also denote by A. Note that this is possible since all these fields have $(+,+)$

BCs and as a consequence the KK wavefunctions of these fields will also be given by the expressions given in Chapter 6. For now we only do a KK decomposition of the Higgs as described in (9.1) and leave the fields W_μ, Z_μ, Z'_μ as 5D fields. Inserting the Higgs VEV one finds

$$\begin{aligned}S \supset \frac{1}{k}\log\left(\frac{k}{T}\right)\int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)\Bigg[\frac{1}{2}\left(\frac{g_Z^2 v^2}{4}\right) Z_\mu Z^\mu + \left(\frac{g_W^2 v^2}{4}\right) W_\mu W^\mu \\ + \frac{1}{2}\left(\frac{g_X^2 v^2}{36}\right) Z'_\mu Z'^{,\mu} + \frac{1}{2}\left(-\frac{g_Z g_X v^2}{6}\right) Z_\mu Z'^{,\mu}\Bigg]\left[f_h^{(0)}(z)\right]^2 .\end{aligned} \tag{16.2}$$

Here $g_i, i \in \{Z, W, X\}$ are the 4D gauge couplings, as defined in (6.27), which explains the factor $\frac{1}{k}\log\left(\frac{k}{T}\right)$. The additional numerical factors in the brackets come from the group generators. For g_Z, g_W we use their measured values at low energies, thus they are related by $g_Z = g_W/\cos(\theta_W)$, with the weak mixing angle θ_W. Z'_μ corresponds to the gauge boson of $U(1)_X$ (see Chapter 11) with coupling strength g_X. As the value of this coupling has to be determined by the RGE evolution we leave an exact account of this for a future work. Conservatively, we use here $g_X = g_Z$ for calculations, as g_Z is the largest of the electroweak couplings.

Apart from the mass terms of the 5D W and Z fields, which will result in mass terms for the zero modes, there is also a mass term for the Z'_μ and a mixing term between Z and Z'_μ.[1] Consequently, the Z field used here is not the physical field. Since in our perturbative approach there will be mixing between the zero modes and their KK states, this is also true for the W-boson. Thus, to get the physical fields one has to do an additional field rotation to the mass basis. This will change the relations between parameters of the W- and Z-bosons compared to the SM, which can be summarized by the oblique parameters S, T, U. In the next sections we calculate the corrections to these parameters, deriving the gauge boson mass eigenstates along the way.

16.2 Deriving the effective Lagrangian

To connect to the electroweak parameters we need to match our model onto an 4D effective theory. The most general 4D Lagrangian for the electroweak gauge bosons is given by [140]

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}\left(1 - \Pi'_{\gamma\gamma}(0)\right) F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\left(1 - \Pi'_{WW}(0)\right) W_{\mu\nu}W^{\mu\nu} \\ &- \frac{1}{4}\left(1 - \Pi'_{ZZ}(0)\right) Z_{\mu\nu}Z^{\mu\nu} + \frac{s_W c_W}{2}\Pi'_{\gamma Z}(0) F_{\mu\nu}Z^{\mu\nu} \\ &+ \left(m_W^2 + \Pi_{WW}(0)\right) W_\mu^+ W^{-,\mu} + \frac{1}{2}\left(m_Z^2 + \Pi_{ZZ}(0)\right) Z_\mu Z^\mu .\end{aligned} \tag{16.3}$$

[1]The mass terms only play a role if there is a massless zero modes. For higher KK states this mass contribution gets overshadowed by the much greater KK mass. Note that Z'_μ has no zero mode, but still the mixing of it with Z_μ has to be taken into account.

Here $m_{W,Z}$ are the SM masses at tree level and $s_W = \sin(\theta_W)$ and $c_W = \cos(\theta_W)$ are the sine and cosine of the weak mixing angle θ_W, respectively. The vacuum polarization amplitudes $\Pi(0)$ and their derivatives $\Pi'(0) = \frac{\partial}{\partial q^2}\Pi|_{q^2=0}$ incorporate the effects of BSM physics, with the fermion vertices normalized to their (tree–level) SM values. We consider the case of oblique corrections in which all vertex corrections are equal and can therefore be absorbed in the common gauge boson polarizations Π. This is in general not the case in RS scenarios, but the RS–GIM mechanism ensures that the differences are small [90]. An exception to this could be IR localized fermions, and their effects, for example of the $Zb\bar{b}$ coupling, have to be considered independently. In fact, as will be shown in [119], we find that the coupling of IR localized fermions are in line with the current constraints. Thus we focus only on the common oblique part and neglect the differences between fermions.

In RS models the coefficients Π can be computed to first order in a tree–level calculation by integrating over the extra dimension. All effects result from the fact that EWSB in GHU mixes the gauge bosons with their KK states and, in the case of the Z-boson, also mixes the Z-boson with the Z' associated with $U(1)_X$ (see above). In this section we will explicitly derive the corrections for EWPT from the Z-boson, with the calculation for the W-boson proceeding similarly. Then in the next section we compare the corrections coming from the Z- and W-bosons with the current bounds on the oblique parameters S, T and U.

Because the wavefunction of the photon zero mode is flat, it gives no contribution, i.e. $\Pi'_{\gamma\gamma}(0) = \Pi_{\gamma\gamma}(0) = 0$, and in this model there is no kinetic mixing between the Z-boson and the photon at tree level, $\Pi'_{\gamma Z}(0)$=0.

Doing a KK decomposition of the 5D fields Z and Z' as given by (6.16), including the terms from (16.2) we find

$$\begin{aligned} S \supset \int \mathrm{d}^4x \Bigg(\sum_n \Bigg[&-\frac{1}{4} Z^{(n)}_{\mu\nu} Z^{(n),\mu\nu} - \frac{1}{4} Z'^{,(n)}_{\mu\nu} Z'^{,(n),\mu\nu} + \frac{1}{2} m^2_{n,(+,+)} Z^{(n)}_\mu Z^{(n),\mu} \\ &+ \frac{1}{2} m^2_{n,(-,-)} Z'^{,(n)}_\mu Z'^{,(n),\mu} \Bigg] + \sum_{n,m} \Bigg[\frac{1}{2} f_{nm} \left(\frac{g_Z^2 v^2}{4} \right) Z^{(n)}_\mu Z^{(m),\mu} \\ &+ \frac{1}{2} f^{XX}_{nm} \left(\frac{g_X^2 v^2}{36} \right) Z'^{,(n)}_\mu Z'^{,(m),\mu} + \frac{1}{2} f^X_{nm} \left(-\frac{g_Z g_X v^2}{6} \right) Z^{(n)}_\mu Z'^{,(m),\mu} \Bigg] \Bigg) . \end{aligned} \tag{16.4}$$

Here we defined f_{nm} as the overlap between the n–th and m–th wavefunctions of the Z field with the Higgs wavefunction

$$f_{mn} = \frac{1}{k} \log\left(\frac{k}{T}\right) \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right) f_Z^{(n)}(z) f_Z^{(m)}(z) \left[f_h^{(0)}(z)\right]^2 , \tag{16.5}$$

and f^X_{nm}, f^{XX}_{nm} are similar overlaps with one and two Z wavefunctions replaced with Z' wavefunctions respectively. The $m_{n,(+,+)}$ are the KK masses of the Z field, which has $(+,+)$ BCs and $m_{n,(-,-)}$ are the KK masses of the Z' field, which has $(-,-)$ BCs.

We will calculate the oblique parameters to first order in $(v/T)^2$ and to this order the most relevant terms come from the zero and first modes of each field, as the higher overlap integrals quickly diminish. Note that only Z has a zero mode, which implies $m_{0,(+,+)} = 0$ and thus the term quadratic in $Z^{(0)}$ is given solely by $f_{00}(g_Z^2 v^2/4)$. Since $f_{00} = 1$, we can identify this expression with the SM relation $m_Z^2 \equiv g_Z^2 v^2/4$. Furthermore, the first KK masses are given by $m_{(+,+)} \equiv m_{1,(+,+)} \approx 2.45\ T$ and $m_{(-,-)} \equiv m_{1,(-,-)} \approx 3.83\ T$ (see Appendix B). Thus to first order in our calculation we can neglect the diagonal terms from EWSB on the higher KK mode masses, as they are of order v^2. With this the above expression simplifies to

$$\begin{aligned} S \supset \int \mathrm{d}^4x \Bigg[&-\frac{1}{4} Z^{(0)}_{\mu\nu} Z^{(0),\mu\nu} - \frac{1}{4} Z^{(1)}_{\mu\nu} Z^{(1),\mu\nu} - \frac{1}{4} X^{(1)}_{\mu\nu} X^{(1),\mu\nu} \\ &+ \frac{1}{2} \begin{pmatrix} Z^{(0)}_\mu & Z^{(1)}_\mu & Z'^{,(1)}_\mu \end{pmatrix} \begin{pmatrix} m_Z^2 & f_{01} m_Z^2 & -f^X_{01} \frac{g_Z g_X v^2}{12} \\ f_{01} m_Z^2 & m^2_{(+,+)} & -f^X_{11} \frac{g_Z g_X v^2}{12} \\ -f^X_{01} \frac{g_Z g_X v^2}{12} & -f^X_{11} \frac{g_Z g_X v^2}{12} & m^2_{(-,-)} \end{pmatrix} \begin{pmatrix} Z^{(0),\mu} \\ Z^{(1),\mu} \\ Z'^{,(1),\mu} \end{pmatrix} \Bigg]. \end{aligned} \tag{16.6}$$

From this one can determine the mass basis of these fields to order $(vR')^2$. Denoting the mass eigenstate of the physical Z-boson by Z_0 one finds the following relevant part of the action:

$$S \supset \int \mathrm{d}^4x \Bigg[-\frac{1}{4} Z_{0,\mu\nu} Z_0^{\mu\nu} + \frac{1}{2} m_Z^2 \left(1 - f_{01}^2 \frac{m_Z^2}{m^2_{(+,+)}} - \left(f^X_{01}\right)^2 \frac{g_X^2 v^2}{36 m^2_{(-,-)}} \right) Z_{0,\mu} Z_0^\mu \Bigg]. \tag{16.7}$$

Additionally, one has to correctly normalize the fermion interaction terms to their SM values. As an example, let us take the interaction with a LH electron. In the gauge basis this interaction reads[2]

$$\begin{aligned} S &\supset \sqrt{\frac{1}{k} \log\left(\frac{k}{T}\right)} g_Z c_L \int \mathrm{d}^4x \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 \bar{e}_L \gamma^\mu e_L Z_\mu \\ &\supset g_Z c_L \int \mathrm{d}^4x \left(\bar{e}^{(0)}_L \gamma^\mu e^{(0)}_L Z^{(0)}_\mu \lambda_0 + \bar{e}^{(0)}_L \gamma^\mu e^{(0)}_L Z^{(1)}_\mu \lambda_1 + \ldots \right), \end{aligned} \tag{16.8}$$

where $c_L = T^3 - Q s_W^2$ is the same linear combination of the third component of weak isospin and electric charge as in the SM and we defined the overlap functions

$$\lambda_n = \sqrt{\frac{1}{k} \log\left(\frac{k}{T}\right)} \int_{1/k}^{1/T} \mathrm{d}z \left(\frac{1}{kz}\right)^4 \left[f^{(0)}_{e_L}(z)\right]^2 f^{(n)}_Z(z)\,. \tag{16.9}$$

[2]We neglect here the rotation of the electron to the mass basis, which is a small effect at this order in (vR')

Note that $\lambda_0 = 1$ and we find with some numerical approximations

$$\lambda_1 \approx \frac{N_1 m_{(+,+)}}{T^2}\left(\frac{0.05}{0.48-0.35c}f^2(c) - \frac{1}{\left(\frac{m_{(+,+)}}{T}\right)^2 \log\left(\frac{2k}{m_{(+,+)}}\right)}\right), \tag{16.10}$$

where $N_1 \propto m_{(+,+)}$ is the normalization of the first KK wavefunction as given in Appendix B. In general λ_1 depends on the c–parameter of the fermion. For the UV localized fermions we consider here, the c dependent term will be negligible and thus λ_1 independent of c. This justifies the assumptions of obliqueness we did in the beginning.

There is a corresponding overlap λ_n^X for the Z'_μ, but because $(-,-)$ fields are even more IR localized compared to $(+,+)$ fields the overlap will be very suppressed and thus negligible for our analysis here.

Rotating to the mass basis gives for the interaction with the physical Z-boson

$$S = g_Z c_L \int \mathrm{d}^4x \left[\left(1 - \lambda_1 f_{01}\frac{m_Z^2}{m_{(+,+)}^2}\right)\bar{e}_L^{(0)}\gamma^\mu e_L^{(0)} Z_{0,\mu} + ...\right]. \tag{16.11}$$

Thus to normalize the interaction to the SM value we can rescale the field as

$$Z_{0,\mu} \to \left(1 - \lambda_1 f_{01}\frac{m_Z^2}{m_{(+,+)}^2}\right)^{-1} Z_{0,\mu}\,. \tag{16.12}$$

In total this gives then the action

$$\begin{aligned} S \supset \int \mathrm{d}^4x \Bigg[&-\frac{1}{4}\left(1 - \left(-2\lambda_1 f_{01}\frac{m_Z^2}{m_{(+,+)}^2}\right)\right) Z_{0,\mu\nu}Z_0^{\mu\nu} \\ &+ \frac{1}{2}m_Z^2\left(1 + \left(-f_{01}^2 + 2\lambda_1 f_{01}\right)\frac{m_Z^2}{m_{(+,+)}^2} - \left(f_{01}^X\right)^2\frac{g_X^2 v^2}{36 m_{(-,-)}^2}\right) Z_{0,\mu}Z_0^\mu\Bigg]. \end{aligned} \tag{16.13}$$

Comparing this with (16.3), we can read off

$$\Pi'_{ZZ}(0) = -2\lambda_1 f_{01}\frac{m_Z^2}{m_{(+,+)}^2}\,, \tag{16.14}$$

$$\Pi_{ZZ}(0) = m_Z^2\left[\left(-f_{01}^2 + 2\lambda_1 f_{01}\right)\frac{m_Z^2}{m_{(+,+)}^2} - \left(f_{01}^X\right)^2\frac{g_X^2 v^2}{36 m_{(-,-)}^2}\right]. \tag{16.15}$$

Doing a similar calculation for the W-boson results in

$$\Pi'_{WW}(0) = -2\lambda_1 f_{01}\frac{m_W^2}{m_{(+,+)}^2}\,, \tag{16.16}$$

$$\Pi_{WW}(0) = m_W^2\left(-f_{01}^2 + 2\lambda_1 f_{01}\right)\frac{m_W^2}{m_{(+,+)}^2}\,. \tag{16.17}$$

16.3 S, T, U parameters

Rescaling the Π's by $\Pi_{WW} = g^2\Pi_{11}, \Pi_{ZZ} = g_Z^2\Pi_{33}$, etc., we can use the standard definitions of the S, T, U parameters [55]

$$S = 16\pi(\Pi'_{33}(0) - \Pi'_{3Q}(0))\,, \tag{16.18}$$

$$T = \frac{4\pi}{s_W^2 c_W^2 m_Z^2}(\Pi_{11}(0) - \Pi_{33}(0))\,, \tag{16.19}$$

$$U = 16\pi(\Pi'_{11}(0) - \Pi'_{33}(0))\,. \tag{16.20}$$

To avoid confusion between the oblique parameter T and the RS IR scale parameter T we rename the latter to $R' \equiv 1/T$ in this section. Thus to leading order in $(vR')^2$ we obtain

$$S = \frac{4\pi v^2}{m^2_{(+,+)}}\left(-2\lambda_1 f_{01}\right)\,, \tag{16.21}$$

$$T = \frac{4\pi v^2}{4c_W^2 m^2_{(+,+)}}\left(f_{01}^2 - 2\lambda_1 f_{01}\right) + \frac{4\pi v^2}{36 s_W^2 c_W^2 m^2_{(-,-)}}\left(f_{01}^X\right)^2 \frac{g_X^2}{g_Z^2}\,, \tag{16.22}$$

$$U = 0\,. \tag{16.23}$$

Note that S, T are both positive since $\lambda_1 f_{01} < 0$, which is the case for all other RS scenarios (except when the fermions are on the IR brane [140]). The result depends on the coupling g_X, which has to be calculated by running down the unified coupling from the unifying scale. To compare with the experimental constraint we use here again conservatively $g_X = g_Z$ and leave the exact analysis of the running for a future work. Note that slight variations of this value will not change the result drastically since the contribution of the $Z'^{,(1)}$ is only about 10% that of $Z^{(1)}$.

From (16.10) it follows to leading order, for the UV localized fermions we consider here, $\lambda_1 \sim -\frac{1}{\sqrt{L}}$, with $L = \log(R'k)$ the logarithm of the warp factor. In fact it is convenient to use the same formula to estimate the scaling of f_{01}, by realizing that the integral can be recovered by replacing the Higgs wavefunction in (16.5) with a LH zero mode fermion wavefunction with $c = -1/2$. This leads to $f_{01} \sim \sqrt{L}$ and one can show that similarly $f_{01}^X \sim \sqrt{L}$, but with a smaller numerical coefficient. Together, these imply $S \sim L^0$ and $T \sim L^1$ to leading order, as in the brane Higgs scenarios without custodial symmetry [90]. In fact one could use the above formula for a brane localized Higgs by replacing in (16.5) the Higgs wavefunction by a delta function on the IR brane. However, in brane Higgs scenarios one can no longer neglect the contribution of higher KK modes, as their overlap with the Higgs is not decreasing [144]. Taking this into account, we find that the constraints in GHU are significantly weaker. More quantitatively, in our scenario, the T (S) parameter gets reduced by a factor approximately 0.4 (0.8) with respect to non–custodial brane Higgs scenarios.

The experimental bounds on the S and T parameters and their correlation matrix are given by [145]

$$S = 0.04 \pm 0.08, \qquad \rho = \begin{pmatrix} 1.00 & 0.92 \\ 0.92 & 1.00 \end{pmatrix}, \qquad (16.24)$$
$$T = 0.08 \pm 0.07,$$

where in the global fit the parameter U is set to zero. Using these bounds, we give in Figure 16.1 the regions allowed at 68%, 95%, and 99% confidence level (CL) in the $S-T$ plane. On the left we plotted the corrections to the S, T parameters from GHU as given by (16.21) and (16.22) for different values of $1/R'$. On the right the corrections from a standard Brane–Higgs scenario according to the formulas in [90] are plotted for comparison. As one can see from the position and values of the blue dots, the constraint from GHU on the scale $1/R'$ is in general less than from the Brane–Higgs case. In GHU the RS contributions from (16.21) and (16.22) pass the

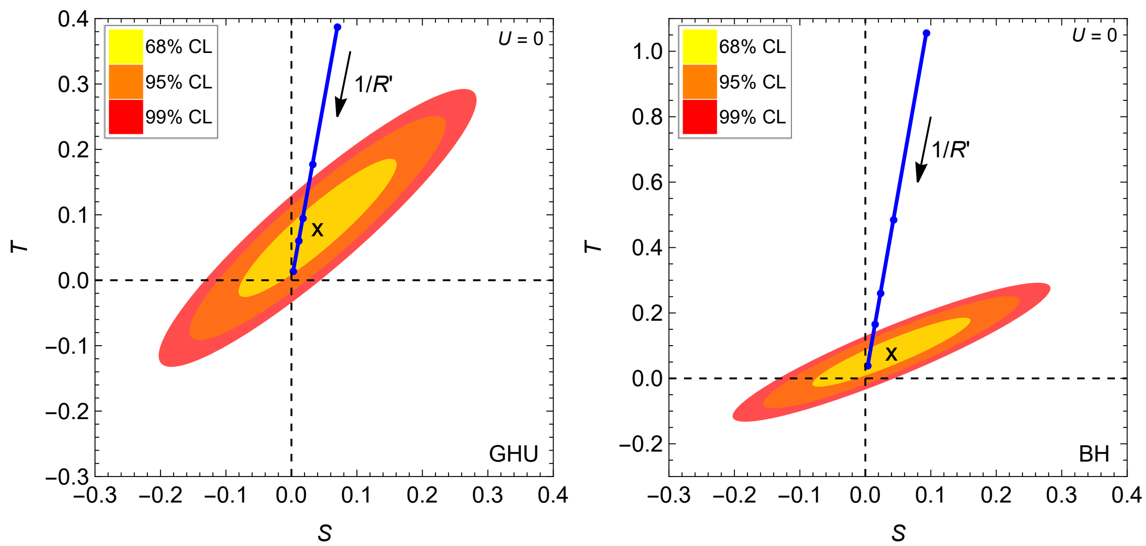


Figure 16.1: Regions allowed at 68%, 95%, and 99% confidence level (CL) in the $S-T$ plane with $U = 0$ ($m_{t,\mathrm{ref.}} = 172.5\,\mathrm{GeV}$, $m_{H,\mathrm{ref.}} = 125\,\mathrm{GeV}$) [145]. The blue line represents the contributions in our GHU (left) and in a Brane–Higgs (BH) scenario [90] (right) for $1/R' \in [2, 10]\,\mathrm{TeV}$ and $k = 10^{18}\,\mathrm{GeV}$. Note that both models are without custodial protection. $1/R'$ increases in the direction of the arrow and the blue dots represent the values $1/R' = 2, 3, 4, 5, 10\,\mathrm{TeV}$. See text for details.

CL thresholds at

$$\frac{1}{R'} > 3.2\,\mathrm{TeV} \quad (99\%\,\mathrm{CL})\,, \qquad (16.25)$$
$$\frac{1}{R'} > 3.4\,\mathrm{TeV} \quad (95\%\,\mathrm{CL})\,, \qquad (16.26)$$
$$\frac{1}{R'} > 3.9\,\mathrm{TeV} \quad (68\%\,\mathrm{CL})\,. \qquad (16.27)$$

Comparing these with the results from Chapter 15, we see that the constraints coming from flavor observables are much more stringent. We can also note that for moderately large values of $1/R' \sim 4 - 5\,\text{TeV}$ the fit of the oblique parameters S, T is improved compared to the SM.

Part VI

Conclusions and Outlook

RS models are one of the most exciting directions for BSM physics. In this thesis, we have seen that they offer solutions to many open questions of the SM, which include the HP and the flavor puzzle. Moreover, these extra dimensional models allow for a new approach to the unification of the interactions and an interesting account of the nature of the Higgs boson in the context of GHU. In particular, we studied a fascinating new $SU(6)$ GHGUT model [1], which combines GHU aspects with a GUT.

After giving an overview over the SM and GUTs in Part I, we reviewed in Part II the theory of general RS models. Here we laid the groundwork for our analysis, and, extending the literature, derived the propagators for 5D scalar, fermion, gauge and ghost fields for arbitrary BCs in generalized AdS_{d+1}. This part closed with a brief introducing to the concepts of GHU.

In Part III an account of GUT theories in RS models in was given, such that we could review afterwards, how GHU and GUT are successfully combined to a GHGUT in warped extra dimensions. We gave a short overview over the recently developed $SU(6)$ GHGUT [1] as a benchmark for the analysis in the following parts.

Equipped with this knowledge, a detailed study of renormalization techniques in RS scenarios has been performed in Part IV. We carved out the advantages and disadvantages of several methods and settled on one approach, which allows for a consistent study of unification above the TeV scale. The contributions of scalar, fermion, gauge fields as well as ghost fields have been worked out, going beyond what was done the literature. In Chapter 13 these results were applied for the first time to two incarnations of $SU(6)$ GHGUTs, where the importance of the breaking patterns on the boundaries was seen. It has been shown that one of these scenarios offers a unification of gauge couplings with a similar postdiction of the weak mixing angle as the original Georgi–Glashow $SU(5)$ model.

Complementary to this, in Part V a study of selected phenomenological aspects of $SU(6)$ GHGUT has been presented. It was reviewed that RS models offer a natural way to explain the hierarchies in the fermion masses as well as the hierarchies in the CKM matrix. Moreover, we saw that RS models have an analog of the SM GIM mechanism, the RS–GIM mechanism, which suppresses FCNCs. We looked at diagrams for these processes coming from KK particles and studied the flavor violating process $\mu \to e\gamma$ in detail. Here the results of simple RS theories have been extended by the new contribution of a scalar leptoquark to be applicable to $SU(6)$ GHGUTs, therby allowing to determine constraints on the IR scale. Lastly, we studied in detail how the KK states in GHGUT models influence the EWPT observables. It has been shown that GHU models can improve on the standard RS scenarios, with the Higgs on the IR brane. We also found that, with moderately large values of the IR scale, the fit compared to the SM can be improved.

There are several interesting directions for which further research would be desirable. First the $SU(6)$ GHGUT model of [1] predicts additionally to the SM particles also a scalar leptoquark and a scalar singlet. A next goal could be to study if they explain the present flavor anomalies, the muon $(g-2)_\mu$ anomalies and EW baryogenesis.

Furthermore, the renormalization of RS models should be explored further. The method of the Planck brane correlator needs to be extended to also include mixing between fermions via brane masses to be applicable to realistic GHGUTs. Additionally, as these theories are dual to four dimensional broken CFTs, one could study renormalization in a concrete UV model of Composite Higgs theories as a complementary project and compare the results with the ones obtained here.

The topics of Part V are currently further investigated and extended, and will appear in an upcoming paper [119]. There, further constraints coming from FCNCs are presented and discussed. One could also perform the analysis of Chapter 16 to all orders of (v/T) by rotating the Higgs VEV on the IR brane and calculating exact expressions for the KK wavefunctions with this modified boundary condition. This could then be compared with the expressions derived in this thesis.

Acknowledgements

I want to thank Dr. Florian Goertz for supervising this master thesis and his marvelous support on all topics, also well beyond the actual research. I am happy to have joined his research group and to be part of the lively group culture he established there. Special thanks to my colleagues Dr. Andrei Angelescu and Andreas Bally for the many fruitful discussions we had. This thesis would have not been possible without the constant exchange with them. Furthermore, I want to thank Prof. Dr. Joerg Jaeckel for agreeing to be the second examiner of this master thesis.

Additionally, I want to thank the whole Max Planck Institute for Nuclear Physics (MPIK) for allowing to do my research there. Especially, many thanks to Prof. Dr. Dr. h.c. Manfred Lindner and his whole division for the many seminars they organized and the many discussions in which they led. In general, thanks to all members of the MPIK, also for ensuring an excellent work environment even in the pandemic. For the few times we could also meet in person, I want to thank my officemates Andreas Bally, Sophie Klett, Sven Fabian and María Dias Astros for the lively atmosphere they provided.

I am also very grateful for the useful insights of Prof. Dr. Kaustubh Sadanand Agashe, Prof. Dr. Emilian Dudas and Dr. Javier Fuentes-Martin they exchanged with us.

Lastly, I want to thank my family for their love and support, not only during this master thesis, but during my entire life.

Part VII

Appendix

A Notation and Conventions

In this thesis the following convenient representation for the gamma matrices [96, 79]

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} , \qquad \gamma^5 = \begin{pmatrix} i\mathbf{1} & 0 \\ 0 & -i\mathbf{1} \end{pmatrix} , \tag{A.1}$$

with the usual Pauli matrices $\sigma^i = -\bar{\sigma}^i$ (see below) and $\sigma^0 = \bar{\sigma}^0 = -1$, such that $\left\{\gamma^a, \gamma^b\right\} = 2\eta^{ab}\mathbf{1}$, where $(\eta^{ab}) = \mathrm{diag}(+1, -1, -1, -1, -1)$ are used. We can also define projection operators as $P_\pm = \frac{1}{2}\left(1 \mp i\gamma^5\right)$.

The generators of $SU(2)_L$ and $SU(3)_c$ are given by $T^i = \sigma^i/2$ and $t^a = \lambda^a/2$, with the Pauli matrices σ^i and Gell–Mann matrices λ^a defined as

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \tag{A.2}$$

$$\begin{aligned}
\lambda^1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \\
\lambda^4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} , \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} , \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} , \\
\lambda^7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} , \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} .
\end{aligned} \tag{A.3}$$

For $SU(6)$ we use the following generators $T^A, A = 1, ..., 35$, which we list in blocks with the dimensions $2, 3$ and 1 for the rows and columns

$$\begin{aligned}
T^{1-3} &= \frac{1}{2} \left(\begin{array}{c|c|c} \sigma^{1-3} & & \\ \hline & 0 & \\ \hline & & 0 \end{array} \right) , \quad T^{4-11} = \frac{1}{2} \left(\begin{array}{c|c|c} 0 & & \\ \hline & \lambda^{1-8} & \\ \hline & & 0 \end{array} \right) , \\
T^{24} &= \frac{1}{2} \left(\begin{array}{c|c|c} \sqrt{3/5} & & \\ \hline & -\sqrt{4/15} & \\ \hline & & 0 \end{array} \right) , \\
T^{35} &= \frac{1}{2} \left(\begin{array}{c|c|c} \sqrt{1/15} & & \\ \hline & \sqrt{1/15} & \\ \hline & & -\sqrt{5/3} \end{array} \right) .
\end{aligned} \tag{A.4}$$

The remaining generators T^{12-23} and T^{25-34} have exactly two non-zero entries for every entry in every off–diagonal block in the same pattern as can be seen by λ^{4-7}. Important for this thesis is the combination

$$\sum_{\hat{a}=25}^{28} A_5^{\hat{a}} T^{\hat{a}} = \frac{1}{2} \left(\begin{array}{cc|c|c} \multicolumn{2}{c|}{0} & & \begin{matrix} A_5^{25} + iA_5^{26} \\ A_5^{27} + iA_5^{28} \end{matrix} \\ \hline & & 0 & \\ \hline A_5^{25} - iA_5^{26} & A_5^{27} - iA_5^{28} & & 0 \end{array} \right), \tag{A.5}$$

which can be identified with the complex Higgs doublet from (1.22).

B KK Wavefunctions and Masses

In this chapter we list the KK wavefunctions for scalars, fermions and gauge fields for general BCs.

B.1 Scalar

The scalar KK wavefunctions satisfy the differential equation (6.2) and the appropriate BCs from (5.6). Without boundary operators, a zero mode only exists for a bulk mass $m = 0$ and $(+,+)$ BCs and is given

$$f^{(0)}_{\phi,(+,+)}(z) = \sqrt{\frac{2k^3}{k^2 - T^2}}\,. \tag{B.1}$$

The higher KK modes ($n \geq 1$) can be described by

$$f^{(n)}_{\phi,(s,s')}(z) = N_n^{(s,s')} z^2 \left(\mathrm{J}_\alpha \left(m^\phi_{n,(s,s')} z \right) - \frac{A_n^{(s)}}{C_n^{(s)}} \mathrm{Y}_\alpha \left(m^\phi_{n,(s,s')} z \right) \right), \tag{B.2}$$

where (s, s') are the UV and IR BCs respectively, and the normalization constant $N_n^{(s,s')}$ can be determined from (6.3). Furthermore, the KK masses can be determined from the equation

$$A_n^{(s)} D_n^{(s')} - B_n^{(s')} C_n^{(s)} = 0\,. \tag{B.3}$$

Here, we defined the BC dependent coefficients

$$\begin{aligned}
A_n^{(+)} &= \mathrm{J}_{\alpha-1}\left(\frac{m_n^\phi}{k}\right) - (\alpha-2)\frac{k}{m_n^\phi}\,\mathrm{J}_\alpha\left(\frac{m_n^\phi}{k}\right), &\qquad A_n^{(-)} &= \mathrm{J}_\alpha\left(\frac{m_n^\phi}{k}\right)\,, \\
C_n^{(+)} &= \mathrm{Y}_{\alpha-1}\left(\frac{m_n^\phi}{k}\right) - (\alpha-2)\frac{k}{m_n^\phi}\,\mathrm{Y}_\alpha\left(\frac{m_n^\phi}{k}\right), &\qquad C_n^{(-)} &= \mathrm{Y}_\alpha\left(\frac{m_n^\phi}{k}\right)\,, \\
B_n^{(+)} &= \mathrm{J}_{\alpha-1}\left(\frac{m_n^\phi}{T}\right) - (\alpha-2)\frac{T}{m_n^\phi}\,\mathrm{J}_\alpha\left(\frac{m_n^\phi}{T}\right), &\qquad B_n^{(-)} &= \mathrm{J}_\alpha\left(\frac{m_n^\phi}{T}\right)\,, \\
D_n^{(+)} &= \mathrm{Y}_{\alpha-1}\left(\frac{m_n^\phi}{T}\right) - (\alpha-2)\frac{T}{m_n^\phi}\,\mathrm{Y}_\alpha\left(\frac{m_n^\phi}{T}\right), &\qquad D_n^{(-)} &= \mathrm{Y}_\alpha\left(\frac{m_n^\phi}{T}\right)\,,
\end{aligned} \tag{B.4}$$

where $\alpha = \sqrt{4 + \left(\frac{m}{k}\right)^2}$. For $m = 0$ the masses of the first KK modes are approximately given by

$$\begin{aligned}
m^\phi_{1,(+,+)} &= 3.83\,T, \quad m^\phi_{1,(+,-)} = 5.14\,T, \\
m^\phi_{1,(-,+)} &= 3.83\,T, \quad m^\phi_{1,(-,-)} = 5.14\,T\,.
\end{aligned} \tag{B.5}$$

B.2 Fermion

The fermionic KK wavefunctions satisfy the differential equation (6.8) and the appropriate BCs from (5.21). The LH component has a zero mode for $(+,+)$ BCs and the RH component for $(-,-) = (+,+)_R$ BC, these are given by

$$f^{(0)}_{\Psi L,(+,+)}(z) = \sqrt{T} f(+c)(kz)^2 (Tz)^{-c}\,, \tag{B.6}$$

$$f^{(0)}_{\Psi R,(-,-)}(z) = \sqrt{T} f(-c)(kz)^2 (Tz)^{+c}\,, \tag{B.7}$$

with the flavor function

$$f(c) = \sqrt{\frac{1-2c}{1-\left(\frac{T}{k}\right)^{1-2c}}}\,. \tag{B.8}$$

The higher KK modes ($n \geq 1$) are given by

$$f^{(n)}_{\Psi L,(s,s')}(z) = N_n^{(s,s')} z^{\frac{5}{2}} \left(\mathrm{J}_{c+1/2}\left(m^\Psi_{n,(s,s')} z\right) - \frac{A_n^{(s)}}{C_n^{(s)}}\,\mathrm{Y}_{c+1/2}\left(m^\Psi_{n,(s,s')} z\right)\right)\,, \tag{B.9}$$

and $f^{(n)}_{\Psi R,(s,s')}(z)$ can be constructed from $f^{(n)}_{\Psi L,(s,s')}(z)$ by switching the BCs and also flipping $c \to -c$. The normalization constant is given by (6.10). Furthermore, the KK masses can be determined from the equation

$$A_n^{(s)} D_n^{(s')} - B_n^{(s')} C_n^{(s)} = 0\,. \tag{B.10}$$

Here, we defined the BC dependent coefficients as

$$
\begin{aligned}
A_n^{(+)} &= \mathrm{J}_{c-1/2}\left(\frac{m_n^\Psi}{k}\right), \qquad & A_n^{(-)} &= \mathrm{J}_{c+1/2}\left(\frac{m_n^\Psi}{k}\right), \\
C_n^{(+)} &= \mathrm{Y}_{c-1/2}\left(\frac{m_n^\Psi}{k}\right), \qquad & C_n^{(-)} &= \mathrm{Y}_{c+1/2}\left(\frac{m_n^\Psi}{k}\right), \\
B_n^{(+)} &= \mathrm{J}_{c-1/2}\left(\frac{m_n^\Psi}{T}\right), \qquad & B_n^{(-)} &= \mathrm{J}_{c+1/2}\left(\frac{m_n^\Psi}{T}\right), \\
D_n^{(+)} &= \mathrm{Y}_{c-1/2}\left(\frac{m_n^\Psi}{T}\right), \qquad & D_n^{(-)} &= \mathrm{Y}_{c+1/2}\left(\frac{m_n^\Psi}{T}\right).
\end{aligned}
\tag{B.11}
$$

In general the KK masses depend on the c–parameter but they are always of the order $\mathcal{O}(T)$, for $c = 0$ the masses of the first KK modes are approximately given by

$$
\begin{aligned}
m^\Psi_{1,(+,+)} &= 3.14\,T, \quad m^\Psi_{1,(+,-)} = 1.57\,T, \\
m^\Psi_{1,(-,+)} &= 1.57\,T, \quad m^\Psi_{1,(-,-)} = 3.14\,T.
\end{aligned}
\tag{B.12}
$$

B.3 Gauge Boson

The gauge boson KK wavefunctions satisfy the differential equation (6.18) and the appropriate BCs from (5.37). The vector part A_μ has a zero mode for $(+,+)$ BCs and the scalar part A_5 for $(-,-) = (+,+)_5$ BCs, which are given by

$$
f^{(0)}_{A,(+,+)}(z) = \sqrt{\frac{k}{\log\left(\frac{k}{T}\right)}}\,, \tag{B.13}
$$

$$
f^{(0)}_{A5,(-,-)}(z) = Tz\sqrt{\frac{2k^3}{k^2 - T^2}}\,. \tag{B.14}
$$

The higher KK modes ($n \geq 1$) have the KK wavefunctions

$$
f^{(n)}_{A,(s,s')}(z) = N_n^{(s,s')} z\left(J_1(m^A_{n,(s,s')}z) - \frac{A_n^{(s)}}{C_n^{(s)}} Y_1(m^A_{n,(s,s')}z)\right), \tag{B.15}
$$

$$
f^{(n)}_{A5,(s,s')}(z) = N_n^{(s,s')} z\left(J_0(m^A_{n,(s,s')}z) - \frac{A_n^{(s)}}{C_n^{(s)}} Y_0(m^A_{n,(s,s')}z)\right), \tag{B.16}
$$

where the normalization constant $N_n^{(s,s')}$ is given by (6.20). Furthermore, the KK masses can be determined from the equation

$$
A_n^{(s)} D_n^{(s')} - B_n^{(s')} C_n^{(s)} = 0\,. \tag{B.17}
$$

Here we defined the BC dependent coefficients as

$$\begin{aligned}
A_n^{(+)} &= \mathrm{J}_0\left(\frac{m_n^A}{k}\right), \quad A_n^{(-)} = \mathrm{J}_1\left(\frac{m_n^A}{k}\right), \\
C_n^{(+)} &= \mathrm{Y}_0\left(\frac{m_n^A}{k}\right), \quad C_n^{(-)} = \mathrm{Y}_1\left(\frac{m_n^A}{k}\right), \\
B_n^{(+)} &= \mathrm{J}_0\left(\frac{m_n^A}{T}\right), \quad B_n^{(-)} = \mathrm{J}_1\left(\frac{m_n^A}{T}\right), \\
D_n^{(+)} &= \mathrm{Y}_0\left(\frac{m_n^A}{T}\right), \quad D_n^{(-)} = \mathrm{Y}_1\left(\frac{m_n^A}{T}\right).
\end{aligned} \tag{B.18}$$

The KK masses of the first KK modes are approximately given by

$$\begin{aligned}
m^A_{1,(+,+)} &= 2.45\,T, \quad m^A_{1,(+,-)} = 0.25\,T, \\
m^A_{1,(-,+)} &= 2.40\,T, \quad m^A_{1,(-,-)} = 3.83\,T.
\end{aligned} \tag{B.19}$$

We also give explicitly the normalization of the first KK wavefunction, as we need it in Chapter 16

$$N_1 = -\frac{\pi m_{1,(+,+)}\,\mathrm{Y}_0\left(\frac{m_{1,(+,+)}}{k}\right)\sqrt{2\log\frac{k}{T}}}{2\sqrt{\frac{\mathrm{Y}_0\left(\frac{m_{1,(+,+)}}{k}\right)^2}{\mathrm{Y}_0\left(\frac{m_{1,(+,+)}}{T}\right)^2}-1}}. \tag{B.20}$$

C 5D Propagators in d-dimensions

In this chapter we give the solutions for the differential equations for the propagators of scalars, fermions and gauge bosons for all possible BCs. As we use these propagators in Section 12.5, we need to determine them for AdS_{d+1}, where $d = 4 - 2\epsilon$ is an arbitrary dimension. Note that we already factored out a factor $\pm i$ according to the definitions in Chapter 8.

C.1 Scalar

In d dimensions the differential equation (8.4) for the propagator of a scalar with (s, s') BCs becomes

$$\left(\frac{1}{kz}\right)^{d-1}\left(p^2 + z^{d-1}\partial_z\left(\frac{1}{z^{d-1}}\partial_z\right) - \frac{m^2}{k^2z^2}\right)G_p(z,z') = \delta(z-z'). \tag{C.1}$$

To solve this equation it is useful to split the propagator into two parts

$$G_p(z,z') = \begin{cases} G_p^<(z,z') & \text{for} \quad z < z' \\ G_p^>(z,z') & \text{for} \quad z > z' \end{cases} . \tag{C.2}$$

Solving this explicitly we find

$$G_p^<(z,z') = \frac{\pi (kz)^{d/2}(kz')^{d/2}}{2k(AD-BC)} \left(A\,\mathrm{J}_\alpha(pz) + B\,\mathrm{Y}_\alpha(pz)\right)\left(C\,\mathrm{J}_\alpha(pz') + D\,\mathrm{Y}_\alpha(pz')\right) , \tag{C.3}$$

$$G_p^>(z,z') = \frac{\pi (kz)^{d/2}(kz')^{d/2}}{2k(AD-BC)} \left(C\,\mathrm{J}_\alpha(pz) + D\,\mathrm{Y}_\alpha(pz)\right)\left(A\,\mathrm{J}_\alpha(pz') + B\,\mathrm{Y}_\alpha(pz')\right) , \tag{C.4}$$

where $\alpha = \sqrt{\left(\frac{d}{2}\right)^2 + \left(\frac{m}{k}\right)^2}$ and the BC dependent coefficients $A^{(s)}, B^{(s)}, C^{(s')}, D^{(s')}$ are given by

$$\begin{aligned}
A^{(+)} &= -\,\mathrm{Y}_{\alpha-1}\Big(\frac{p}{k}\Big) - \frac{k}{p}\left(\frac{d}{2}-\alpha\right)\mathrm{Y}_\alpha\Big(\frac{p}{k}\Big), & A^{(-)} &= -\,\mathrm{Y}_\alpha\Big(\frac{p}{k}\Big) , \\
B^{(+)} &= \mathrm{J}_{\alpha-1}\Big(\frac{p}{k}\Big) + \frac{k}{p}\left(\frac{d}{2}-\alpha\right)\mathrm{J}_\alpha\Big(\frac{p}{k}\Big) & B^{(-)} &= \mathrm{J}_\alpha\Big(\frac{p}{k}\Big) , \\
C^{(+)} &= -\,\mathrm{Y}_{\alpha-1}\Big(\frac{p}{T}\Big) - \frac{T}{p}\left(\frac{d}{2}-\alpha\right)\mathrm{Y}_\alpha\Big(\frac{p}{T}\Big), & C^{(-)} &= -\,\mathrm{Y}_\alpha\Big(\frac{p}{T}\Big) , \\
D^{(+)} &= \mathrm{J}_{\alpha-1}\Big(\frac{p}{T}\Big) + \frac{T}{p}\left(\frac{d}{2}-\alpha\right)\mathrm{J}_\alpha\Big(\frac{p}{T}\Big), & D^{(-)} &= \mathrm{J}_\alpha\Big(\frac{p}{T}\Big) .
\end{aligned} \tag{C.5}$$

We also need the Wick rotated versions of the propagators, which are easily obtained by resolving the differential equation with the replacement $p^2 \to -q^2$

$$\left(\frac{1}{kz}\right)^{d-1}\left(-q^2 + z^{d-1}\partial_z\left(\frac{1}{z^{d-1}}\partial_z\right) - \frac{c}{z^2}\right) G_q(z,z') = \delta(z-z') . \tag{C.6}$$

This equation is solved by modified Bessel functions and the general solution is given by

$$G_q^<(z,z') = -\frac{(kz)^{d/2}(kz')^{d/2}}{k(AD-BC)} \left(A_n\,\mathrm{I}_\alpha(qz) + B\,\mathrm{K}_\alpha(qz)\right)\left(C\,\mathrm{I}_\alpha(qz') + D\,\mathrm{K}_\alpha(qz')\right) , \tag{C.7}$$

$$G_q^>(z,z') = -\frac{(kz)^{d/2}(kz')^{d/2}}{k(AD-BC)} \left(C\,\mathrm{I}_\alpha(qz) + D\,\mathrm{K}_\alpha(qz)\right)\left(A\,\mathrm{I}_\alpha(qz') + B\,\mathrm{K}_\alpha(qz')\right) , \tag{C.8}$$

where the BC dependent coefficients $A^{(s)}, B^{(s)}, C^{(s')}, D^{(s')}$ are given by

$$
\begin{aligned}
A^{(+)} &= \mathrm{K}_{\alpha-1}\Big(\frac{q}{k}\Big) - \frac{k}{q}\left(\frac{d}{2}-\alpha\right)\mathrm{I}_\alpha\Big(\frac{q}{k}\Big)\,, & A^{(-)} &= -\,\mathrm{K}_\alpha\Big(\frac{q}{k}\Big)\,,\\
B^{(+)} &= \mathrm{I}_{\alpha-1}\Big(\frac{q}{k}\Big) + \frac{k}{q}\left(\frac{d}{2}-\alpha\right)\mathrm{I}_\alpha\Big(\frac{q}{k}\Big)\,, & B^{(-)} &= \mathrm{I}_\alpha\Big(\frac{q}{k}\Big)\,,\\
C^{(+)} &= \mathrm{K}_{\alpha-1}\Big(\frac{q}{T}\Big) - \frac{T}{q}\left(\frac{d}{2}-\alpha\right)\mathrm{K}_\alpha\Big(\frac{q}{T}\Big)\,, & C^{(-)} &= -\,\mathrm{K}_\alpha\Big(\frac{q}{T}\Big)\,,\\
D^{(+)} &= \mathrm{I}_{\alpha-1}\Big(\frac{q}{T}\Big) + \frac{T}{q}\left(\frac{d}{2}-\alpha\right)\mathrm{I}_\alpha\Big(\frac{q}{T}\Big)\,, & D^{(-)} &= \mathrm{I}_\alpha\Big(\frac{q}{T}\Big)\,.
\end{aligned}
\tag{C.9}
$$

C.2 Fermion

In d dimensions the differential equation (8.7) for the propagator of a fermion with $(s, s') = (-s, -s')_R$ BCs becomes

$$
\left(\frac{1}{kz}\right)^d \left(\not{p} + i\gamma^5\left(\partial_z - \frac{d}{2z}\right) - \frac{c}{z}\right) G_p(z, z') = \delta(z - z')\,. \tag{C.10}
$$

Using the ansatz (8.8) the coupled differential equations for the components are solved by

$$
S^+(z, z') = -\left(\partial_z - \frac{d}{2z} - \frac{c}{z}\right) V^-(z, z')\,, \tag{C.11}
$$

$$
S^-(z, z') = \left(\partial_z - \frac{d}{2z} + \frac{c}{z}\right) V^+(z, z')\,, \tag{C.12}
$$

if $V^\pm$ satisfy the differential equations

$$
\left[\partial_z^2 - \frac{d}{z}\partial_z + \frac{-c^2 + c + \frac{d(d+2)}{4}}{z^2} + p^2\right] V^-(z, z') = (kz)^d \delta(z - z')\,, \tag{C.13}
$$

$$
\left[\partial_z^2 - \frac{d}{z}\partial_z + \frac{-c^2 - c + \frac{d(d+2)}{4}}{z^2} + p^2\right] V^+(z, z') = (kz)^d \delta(z - z')\,. \tag{C.14}
$$

Note that V^- can be constructed out of a solution for V^+ by switching the BCs and using the replacement $c \to -c$. Thus we focus on the solution for V^+ for which we split again

$$
V_p^+(z, z') = \begin{cases} V_p^{+,<}(z, z') & \text{for} \quad z < z' \\ V_p^{+,>}(z, z') & \text{for} \quad z > z' \end{cases}. \tag{C.15}
$$

We find the solutions

$$V_p^{+,<}(z,z') = \frac{\pi (kz)^{\frac{d+1}{2}} (kz')^{\frac{d+1}{2}}}{2k(AD-BC)} \Big(A\,\mathrm{J}_{\frac{1}{2}+c}(pz) + B\,\mathrm{Y}_{\frac{1}{2}+c}(pz)\Big) \Big(C\,\mathrm{J}_{\frac{1}{2}+c}(pz') + D\,\mathrm{Y}_{\frac{1}{2}+c}(pz')\Big)\,, \tag{C.16}$$

$$V_p^{+,>}(z,z') = \frac{\pi (kz)^{\frac{d+1}{2}} (kz')^{\frac{d+1}{2}}}{2k(AD-BC)} \Big(C\,\mathrm{J}_{\frac{1}{2}+c}(pz) + D\,\mathrm{Y}_{\frac{1}{2}+c}(pz)\Big) \Big(A\,\mathrm{J}_{\frac{1}{2}+c}(pz') + B\,\mathrm{Y}_{\frac{1}{2}+c}(pz')\Big)\,, \tag{C.17}$$

where the BC dependent coefficients $A^{(s)}, B^{(s)}, C^{(s')}, D^{(s')}$ are given by

$$\begin{aligned}
A^{(+)} &= -\,\mathrm{Y}_{-\frac{1}{2}+c}\Big(\frac{p}{k}\Big)\,, & A^{(-)} &= -\,\mathrm{Y}_{\frac{1}{2}+c}\Big(\frac{p}{k}\Big)\,,\\
B^{(+)} &= \mathrm{J}_{-\frac{1}{2}+c}\Big(\frac{p}{k}\Big)\,, & B^{(-)} &= \mathrm{J}_{\frac{1}{2}+c}\Big(\frac{p}{k}\Big)\,,\\
C^{(+)} &= -\,\mathrm{Y}_{-\frac{1}{2}+c}\Big(\frac{p}{T}\Big)\,, & C^{(-)} &= -\,\mathrm{Y}_{\frac{1}{2}+c}\Big(\frac{p}{T}\Big)\,,\\
D^{(+)} &= \mathrm{J}_{-\frac{1}{2}+c}\Big(\frac{p}{T}\Big)\,, & D^{(-)} &= \mathrm{J}_{\frac{1}{2}+c}\Big(\frac{p}{T}\Big)\,.
\end{aligned} \tag{C.18}$$

We also need the Wick rotated versions of the propagators, which are easily obtained by resolving the differential equation with the replacement $p^2 \to -q^2$

$$\left(\frac{1}{kz}\right)^{d-1} \left(-q^2 + \partial_z^2 - \frac{d}{z}\partial_z + \frac{-c^2 - c + \frac{d(d+2)}{4}}{z^2}\right) G_q(z,z') = \delta(z-z')\,. \tag{C.19}$$

Using the same methods as before one finds

$$V_q^{+,<}(z,z') = -\frac{(kz)^{\frac{d+1}{2}} (kz')^{\frac{d+1}{2}}}{k(AD-BC)} \Big(A\,\mathrm{I}_{\frac{1}{2}+c}(qz) + B\,\mathrm{K}_{\frac{1}{2}+c}(qz)\Big) \Big(C\,\mathrm{I}_{\frac{1}{2}+c}(qz') + D\,\mathrm{K}_{\frac{1}{2}+c}(qz')\Big)\,, \tag{C.20}$$

$$V_q^{+,>}(z,z') = -\frac{(kz)^{\frac{d+1}{2}} (kz')^{\frac{d+1}{2}}}{k(AD-BC)} \Big(C\,\mathrm{I}_{\frac{1}{2}+c}(qz) + D\,\mathrm{K}_{\frac{1}{2}+c}(qz)\Big) \Big(A\,\mathrm{I}_{\frac{1}{2}+c}(qz') + B\,\mathrm{K}_{\frac{1}{2}+c}(qz')\Big)\,, \tag{C.21}$$

where the BC dependent coefficients $A^{(s)}, B^{(s)}, C^{(s')}, D^{(s')}$ are given by

$$\begin{aligned}
A^{(+)} &= \mathrm{K}_{-\frac{1}{2}+c}\Big(\frac{q}{k}\Big)\,, & A^{(-)} &= -\,\mathrm{K}_{\frac{1}{2}+c}\Big(\frac{q}{k}\Big)\,,\\
B^{(+)} &= \mathrm{I}_{-\frac{1}{2}+c}\Big(\frac{q}{k}\Big)\,, & B^{(-)} &= \mathrm{I}_{\frac{1}{2}+c}\Big(\frac{q}{k}\Big)\,,\\
C^{(+)} &= \mathrm{K}_{-\frac{1}{2}+c}\Big(\frac{q}{T}\Big)\,, & C^{(-)} &= -\,\mathrm{K}_{\frac{1}{2}+c}\Big(\frac{q}{T}\Big)\,,\\
D^{(+)} &= \mathrm{I}_{-\frac{1}{2}+c}\Big(\frac{q}{T}\Big)\,, & D^{(-)} &= \mathrm{I}_{\frac{1}{2}+c}\Big(\frac{q}{T}\Big)\,.
\end{aligned} \tag{C.22}$$

C.3 Gauge Boson

In d dimensions the differential equation (8.9) for the vector and scalar part of the gauge boson propagator as well as the differential equation (8.13) for the ghost propagator, all with $(s, s') = (-s, -s')_5$ BCs, become

$$\left(\frac{1}{kz}\right)^{d-3}\left(\eta^{\mu\nu}p^2-\left(1-\frac{1}{\xi}\right)p^\mu p^\nu+\eta^{\mu\nu}\left(kz\right)^{d-3}\partial_z\left(\frac{1}{kz}\right)^{d-3}\partial_z\right)G_{p,\nu\lambda}(z,z')=\delta^\mu_\lambda\delta(z-z')\,, \tag{C.23}$$

$$\left(\frac{1}{kz}\right)^{d-3}\left(p^2+\xi\partial_z\left(kz\right)^{d-3}\partial_z\left(\frac{1}{kz}\right)^{d-3}\right)G_p(z,z')=\delta(z-z')\,, \tag{C.24}$$

$$\left(\frac{1}{kz}\right)^{d-3}\left(p^2+\xi\left(kz\right)^{d-3}\partial_z\left(\left(\frac{1}{kz}\right)^{d-3}\partial_z\right)\right)G^{(c)}_p(z,z')=\delta(z-z')\,, \tag{C.25}$$

where we suppressed gauge group indices. Using the ansatz (8.11) for the gauge bosons and (8.14) for the ghosts the differential equations are solved if the function $G^m_p(z,z')$ satisfies the differential equation

$$\left[\partial_z^2-\frac{d-3}{z}\partial_z+p^2-\frac{(d-3)m^2}{z^2}\right]G^m_p(z,z')=(kz)^{d-3}\delta(z-z')\,. \tag{C.26}$$

Again it is useful to split the propagators into

$$G^m_p(z,z')=\begin{cases}G^{m,<}_p(z,z') & \text{for}\quad z<z'\\ G^{m,>}_p(z,z') & \text{for}\quad z>z'\end{cases}. \tag{C.27}$$

For $m=0,i$ the solutions are particularly simple and given by

$$G^{0,<}_p(z,z')=\frac{\pi(kz)^{\frac{d}{2}-1}(kz')^{\frac{d}{2}-1}}{2k(AD-BC)}\Big(A\,\mathrm{J}_{\frac{d}{2}-1}(pz)+B\,\mathrm{Y}_{\frac{d}{2}-1}(pz)\Big)\Big(C\,\mathrm{J}_{\frac{d}{2}-1}(pz')+D\,\mathrm{Y}_{\frac{d}{2}-1}(pz')\Big)\,, \tag{C.28}$$

$$G^{0,>}_p(z,z')=\frac{\pi(kz)^{\frac{d}{2}-1}(kz')^{\frac{d}{2}-1}}{2k(AD-BC)}\Big(C\,\mathrm{J}_{\frac{d}{2}-1}(pz)+D\,\mathrm{Y}_{\frac{d}{2}-1}(pz)\Big)\Big(A\,\mathrm{J}_{\frac{d}{2}-1}(pz')+B\,\mathrm{Y}_{\frac{d}{2}-1}(pz')\Big)\,, \tag{C.29}$$

$$G^{i,<}_p(z,z')=\frac{\pi(kz)^{\frac{d}{2}-2}(kz')^{\frac{d}{2}-2}}{2k(AD-BC)}\Big(A\,\mathrm{J}_{\frac{d}{2}-2}(pz)+B\,\mathrm{Y}_{\frac{d}{2}-2}(pz)\Big)\Big(C\,\mathrm{J}_{\frac{d}{2}-2}(pz')+D\,\mathrm{Y}_{\frac{d}{2}-2}(pz')\Big)\,, \tag{C.30}$$

$$G^{i,>}_p(z,z')=\frac{\pi(kz)^{\frac{d}{2}-2}(kz')^{\frac{d}{2}-2}}{2k(AD-BC)}\Big(C\,\mathrm{J}_{\frac{d}{2}-2}(pz)+D\,\mathrm{Y}_{\frac{d}{2}-2}(pz)\Big)\Big(A\,\mathrm{J}_{\frac{d}{2}-2}(pz')+B\,\mathrm{Y}_{\frac{d}{2}-2}(pz')\Big)\,, \tag{C.31}$$

where the BC dependent coefficients $A^{(s)}, B^{(s)}, C^{(s')}, D^{(s')}$ are given by

$$
\begin{aligned}
A^{(+)} &= -\,\mathrm{Y}_{\frac{d}{2}-2}\Big(\frac{p}{k}\Big)\,, & A^{(-)} &= -\,\mathrm{Y}_{\frac{d}{2}-1}\Big(\frac{p}{k}\Big)\;,\\
B^{(+)} &= \mathrm{J}_{\frac{d}{2}-2}\Big(\frac{p}{k}\Big)\,, & B^{(-)} &= \mathrm{J}_{\frac{d}{2}-1}\Big(\frac{p}{k}\Big)\;,\\
C^{(+)} &= -\,\mathrm{Y}_{\frac{d}{2}-2}\Big(\frac{p}{T}\Big)\,, & C^{(-)} &= -\,\mathrm{Y}_{\frac{d}{2}-1}\Big(\frac{p}{T}\Big)\;,\\
D^{(+)} &= \mathrm{J}_{\frac{d}{2}-2}\Big(\frac{p}{T}\Big)\,, & D^{(-)} &= \mathrm{J}_{\frac{d}{2}-1}\Big(\frac{p}{T}\Big)\;.
\end{aligned}
\tag{C.32}
$$

We also need the Wick rotated versions of the propagators, which are easily obtained by resolving the differential equation with the replacement $p^2 \to -q^2$

$$
\left(\frac{1}{kz}\right)^{d-3}\left(-q^2+\partial_z^2-\frac{d}{z}\partial_z-\frac{(d-3)m^2}{z^2}\right)G^m_q(z,z')=\delta(z-z')\,. \tag{C.33}
$$

This is solved by modified Bessel functions and the general solution is given by

$$
G^{0,<}_q(z,z') = -\frac{(kz)^{\frac{d}{2}-1}(kz')^{\frac{d}{2}-1}}{k(AD-BC)}\Big(A\,\mathrm{I}_{\frac{d}{2}-1}(qz)+B\,\mathrm{K}_{\frac{d}{2}-1}(qz)\Big)\Big(C\,\mathrm{I}_{\frac{d}{2}-1}(qz')+D\,\mathrm{K}_{\frac{d}{2}-1}(qz')\Big)\;, \tag{C.34}
$$

$$
G^{0,>}_q(z,z') = -\frac{(kz)^{\frac{d}{2}-1}(kz')^{\frac{d}{2}-1}}{k(AD-BC)}\Big(C\,\mathrm{I}_{\frac{d}{2}-1}(qz)+D\,\mathrm{K}_{\frac{d}{2}-1}(qz)\Big)\Big(A\,\mathrm{I}_{\frac{d}{2}-1}(qz')+B\,\mathrm{K}_{\frac{d}{2}-1}(qz')\Big)\;, \tag{C.35}
$$

$$
G^{i,<}_q(z,z') = \frac{(kz)^{\frac{d}{2}-2}(kz')^{\frac{d}{2}-2}}{k(AD-BC)}\Big(A\,\mathrm{I}_{\frac{d}{2}-2}(qz)-B\,\mathrm{K}_{\frac{d}{2}-2}(qz)\Big)\Big(C\,\mathrm{I}_{\frac{d}{2}-2}(qz')-D\,\mathrm{K}_{\frac{d}{2}-2}(qz')\Big)\;, \tag{C.36}
$$

$$
G^{i,>}_q(z,z') = \frac{(kz)^{\frac{d}{2}-2}(kz')^{\frac{d}{2}-2}}{k(AD-BC)}\Big(C\,\mathrm{I}_{\frac{d}{2}-2}(qz)-D\,\mathrm{K}_{\frac{d}{2}-2}(qz)\Big)\Big(A\,\mathrm{I}_{\frac{d}{2}-2}(qz')-B\,\mathrm{K}_{\frac{d}{2}-2}(qz')\Big)\;, \tag{C.37}
$$

where the BC dependent coefficients $A^{(s)}, B^{(s)}, C^{(s')}, D^{(s')}$ are given by

$$
\begin{aligned}
A^{(+)} &= \mathrm{K}_{\frac{d}{2}-2}\Big(\frac{q}{k}\Big)\,, & A^{(-)} &= -\,\mathrm{K}_{\frac{d}{2}-1}\Big(\frac{q}{k}\Big)\;,\\
B^{(+)} &= \mathrm{I}_{\frac{d}{2}-2}\Big(\frac{q}{k}\Big)\,, & B^{(-)} &= \mathrm{I}_{\frac{d}{2}-1}\Big(\frac{q}{k}\Big)\;,\\
C^{(+)} &= \mathrm{K}_{\frac{d}{2}-2}\Big(\frac{q}{T}\Big)\,, & C^{(-)} &= -\,\mathrm{K}_{\frac{d}{2}-1}\Big(\frac{q}{T}\Big)\;,\\
D^{(+)} &= \mathrm{I}_{\frac{d}{2}-2}\Big(\frac{q}{T}\Big)\,, & D^{(-)} &= \mathrm{I}_{\frac{d}{2}-1}\Big(\frac{q}{T}\Big)\;.
\end{aligned}
\tag{C.38}
$$

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Erklärung:

Ich versichere, dass ich diese Arbeit selbstständig verfasst habe und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Heidelberg, den 12.05.2022 S. Weber