

## Strong circular dichroism for the HE<sub>11</sub> mode in twisted single-ring hollow-core photonic crystal fiber: erratum

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Recent work has revealed that the dispersion relation, given in Optica 5, 1315 (2018), for helical Bloch modes in a ring of capillaries surrounding a central hollow core, is incorrect. Here we correct this error and provide a revised version of Fig. 2. The overall conclusions of the original paper are unaffected. © 2022 Optica Publishing Group under the terms of the Optica Open Access Publishing Agreement

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The dispersion relation in Eq. (6) of the publication [1] does not take proper account of the transformation from the helicoidal to the cartesian frame. The correct dispersion relation, as derived in detail in a recent paper [2], takes the form:

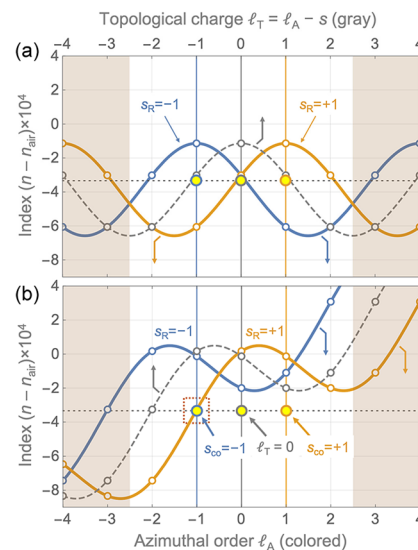
$$n(\ell_A) = n_0 \sqrt{1 + \alpha^2 \rho^2} - \frac{\alpha \lambda (\ell_A - s)}{2\pi} - \frac{s \alpha^3 \rho^2 \lambda}{4\pi} + 2n_\kappa \sqrt{1 + \alpha^2 \rho^2} \cos(2\pi(\ell_A - s - \beta_0 \alpha \rho^2)/N) \quad (1)$$

for perfectly circular-polarized helical Bloch modes (HBMs). Apart from the spin-dependent term (the third on the right-hand side), which causes a very small amount of circular birefringence that can usually be neglected, the propagation constant depends solely on the topological charge  $\ell_T = \ell_A - s$ , all other quantities being kept constant. Note that we have changed the sign from  $+s$  to  $-s$  (which depends on the sign convention for the twist rate  $\alpha$ ) so as to standardize the analysis with recent papers. The need to satisfy an azimuthal resonance condition restricts  $\ell_A$  to integer values, and each value of  $\ell_A = \ell_A^{(0)}$  within the first Brillouin zone  $-N/2 < \ell_A < N/2$  is accompanied by Bloch harmonics with azimuthal order  $\ell_A^{(m)} = \ell_A^{(0)} + mN$  where  $m$  is an integer.

The corrected version of Fig. 2 is shown in Fig. 1 of this Erratum. When plotted versus topological charge (dashed gray curve), the blue and orange curves collapse exactly on top of each other in the untwisted case [Fig. 1(a)]. The perfect symmetry in this case means that there is no possibility of circular dichroism. In the twisted case, apart from the very small circular birefringence, the curves also collapse on top of each other when plotted versus topological charge [Fig. 1(b)].

At a certain twist rate the  $s_{co} = -1, \ell_A = -1$  core mode phase-matches to the  $s_R = +1, \ell_A = -1$  HBM in the ring (the

point inside the red-dotted box), while the  $s_{co} = +1, \ell_A = +1$  core mode is not phase-matched to any ring mode. Although the



**Fig. 1.** Illustrative plots of modal refractive index versus azimuthal mode order within the first Brillouin zone  $-2.5 < \ell_A < +2.5$  for (a) an untwisted and (b) a twisted five-fold rotationally symmetric single-ring PCF. Subscript R = “ring” and co = “core”. The azimuthal mode orders allowed in the ring are marked with open circles, and the core modes by the yellow-filled circles (the horizontal dotted line marks the core modal index). In the twisted case the  $s_{co} = -1, \ell_A = -1$  core mode phase-matches to the  $s_R = +1, \ell_A = -1$  HBM in the ring (inside the red-dotted box), while the  $s_{co} = +1, \ell_A = +1$  core mode is not phase-matched to any ring mode. The dashed gray curves are plots against topological charge  $\ell_T = \ell_A - s_R$  for both polarization states; note that the circular birefringence of the core modes is very small:  $\sim 10^{-7}$ .

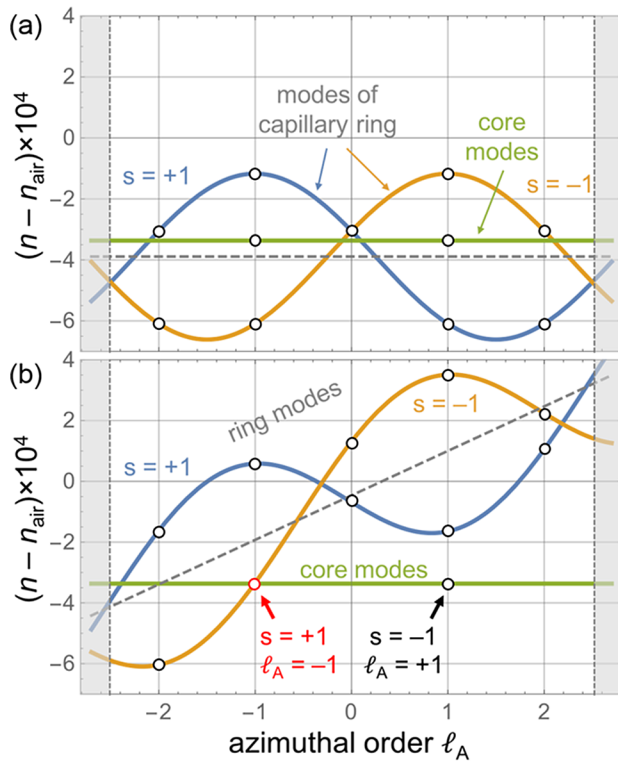


Fig. 2. Figure 2 in the original paper [1].

polarization states of these phase-matched modes are circular and orthogonal in this simple model, so would be unable to couple, the much more complicated structure in the SR-PCF means that

the actual polarization state is not perfectly circular, allowing some coupling to occur and giving rise to circular dichroism. On the diagram this means that a proportion of the “ring” light inside the red-dashed box ends up phase-matched to the yellow  $\ell_T = 0$  point.

Note that the overall conclusions of the paper are unaffected by these corrections.

### APPENDIX A

For reference, the incorrect equation (Eq. (6) in [1]) is

$$n(\ell_A^m) = n_0 \sqrt{1 + \rho^2 \alpha^2} - \frac{\alpha \ell_A^m \lambda}{2\pi} + 2n_c \cos\left(\frac{2\pi}{N}(\ell_A^m + s - \beta_0 \alpha \rho^2)\right), \quad (A1)$$

and the uncorrected figure is shown in Fig. 2 of this Erratum.

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### REFERENCES

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2. Y. Chen and P. St.J. Russell, “Frenet-Serret analysis of helical Bloch modes in N-fold rotationally symmetric rings of coupled spiralling optical waveguides,” *J. Opt. Soc. Am. B* **38**, 1173–1183 (2021).