Absolute neutrino mass scale and dark matter stability from flavour symmetry

Salvador Centelles Chuliá,[1,*] Ricardo Cepedello,[2,†] and Omar Medina[3,‡]

[1] *Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany*

[2] *Institut für Theoretische Physik und Astrophysik, University of Würzburg, Campus Hubland Nord, D-97074 Würzburg, Germany*

[3] *AHEP Group, Institut de Física Corpuscular – C.S.I.C./Universitat de València, Parc Científic de Paterna. C/ Catedrático José Beltrán, 2 E-46980 Paterna (Valencia), Spain*

We explore a simple but extremely predictive extension of the scotogenic model. We promote the *scotogenic symmetry* $\mathbb{Z}_2$ to the flavour non-Abelian symmetry $\Sigma(81)$, which can also automatically protect dark matter stability. In addition, $\Sigma(81)$ leads to striking predictions in the lepton sector: only Inverted Ordering is realised, the absolute neutrino mass scale is predicted to be $m_{\text{lightest}} \approx 7.5 \times 10^{-4}$ eV and the Majorana phases are correlated in such a way that $|m_{ee}| \approx 0.018$ eV. The model also leads to a strong correlation between the solar mixing angle θ_{12} and δ_{CP}, which may be falsified by the next generation of neutrino oscillation experiments. The setup is minimal in the sense that no additional symmetries or flavons are required.

I. INTRODUCTION

Motivated by two fundamental problems of particle and astroparticle physics, namely the origin of neutrino masses [1–9] and the nature of dark matter [10], there has been a great effort to relate them within a single, predictive framework. Unarguably, they both point towards the presence of physics beyond the Standard Model (SM), presumably with the addition of new particles and symmetries that account for a mass mechanism for neutrinos, a viable dark matter candidate and its stability.

An *economical* approach to combine all these appealing properties is to consider radiative neutrino mass models [11–18] (for a review see [19]). In this kind of models, fields running in a loop generate neutrino masses, giving rise to two clearly distinguishable particle sectors, one of which can be associated to a dark sector by means of a symmetry. The stability of the dark matter candidate, i.e. the lightest of the particles belonging to the dark sector, is determined by the transformation properties of the SM fields and the dark sector under symmetries [20–22]. In the most simple scenarios, the SM fields transform only under an invariant subgroup of the symmetry, while any particle beyond those of the SM not belonging to this subgroup will not be able to decay solely to the SM, i.e. it will be part of the dark sector. A popular implementation of this principle is the scotogenic model [23] and its many variants (see for instance [24–33]).

While a large number of models built following the described approach are consistent with experimental data from neutrino oscillations and bounds from dark matter searches, there are further unknowns about fundamental particles that are also important to address. The SM lacks a suitable theoretical explanation for the masses and the mixing pattern of fermions. Furthermore, the majority of input parameters of the SM are

* chulia@mpi-hd.mpg.de
† ricardo.cepedello@physik.uni-wuerzburg.de
‡ omar.medina@ific.uv.es

directly related to this *flavour puzzle*. The lepton mixing angles, being large and with a completely different structure in comparison to their analogues in the quark sector, manifest the lack of a first principle explanation of the flavour phenomenology [34–36]. Here is where *flavour symmetries* can play a major role in explaining such mixing patterns and mass hierarchies.[1] By means of imposing a flavour symmetry between the three generations it is possible to predict strong correlations between different observables. This is essential for a flavour symmetry model to be verifiable.

In this paper we build a model for radiative neutrino masses with a flavour symmetry $\Sigma(81)$. We focus on such a discrete group due to an interesting feature: $\Sigma(81)$ contains a non-trivial subgroup formed by the singlets and one of the triplet representations. This ensures, as we will show in section IV, that for a reasonable choice of the transformation properties of the field content under the flavour symmetry, one can straightforwardly obtain a stable dark matter candidate. Thus, providing a *natural* framework to account for dark matter stability along with light radiative Majorana neutrino masses through a scotogenic-like mechanism. Other works with *flavoured stability* are, for example, [39–42].

A more conventional role played by $\Sigma(81)$ symmetry is to strongly constrain the structure of the mass matrices of fermions, leading to strong predictions that can be tested in the following years by the next generation of neutrino oscillation [43–47], and neutrinoless double beta decay experiments [48–53]. While the idea of imposing a flavour symmetry is certainly not new, we will show that our setup has a series of attractive and unique features, namely explaining the lepton mixing pattern, as well as predicting the absolute mass scale of neutrinos, their ordering and the Majorana phases, and therefore leading to a definite prediction for neutrinoless double beta decay ($0\nu ee$). Moreover, this is obtained without the need of extra *flavons*, i.e. extra scalars that further break the flavour symmetry. In our setup the breaking of the flavour symmetry is done by extending the number of Higgs doublets, as a variant of a 3HDM [54–60] and giving them non-trivial charges under $\Sigma(81)$.

The paper is structured as follows: in section II we present the model setup, i.e. the field content, the charges under the SM gauge group and flavour symmetry and discuss some of its most important attributes. In section III we delve into its most important phenomenological predictions: absolute neutrino mass scale and ordering, strong correlations between oscillation observables and the implications for neutrinoless double beta decay. In section IV we explicitly flesh out the non-Abelian stability mechanism provided by $\Sigma(81)$. The paper then closes with a short summary and conclusions. Details about the symmetry group $\Sigma(81)$ are relegated to Appendix A.

II. THE MODEL SETUP

We extend the Standard Model gauge symmetry $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ by a global, discrete flavour group $\Sigma(81)$. This group is of the type $\Sigma(3N^3)$ and contains 9 singlets and 4 complex triplets, denoted as $\mathbf{1}_{(i,j)}$ with $i, j = 0, 1, 2$ and $\mathbf{3}_X(\bar{\mathbf{3}}_X)$ with $X = A, B, C, D$ (see Appendix A for details). The irreducible representations $\mathbf{3}_D$ ($\bar{\mathbf{3}}_D$), together with the singlets, form a closed set under tensor products, implying that if every Standard Model field transforms as $\mathbf{3}_D$, $\bar{\mathbf{3}}_D$ or as one of the singlets, then any field transforming as $\mathbf{3}_{A,B,C}$ and their conjugates, will belong to the *dark sector*. The lightest among them will then be a dark matter candidate. This relation between $\Sigma(81)$ and dark matter will be further discussed in section IV.

[1] While there is a vast bibliography on this topic, we direct the interested readers to the reviews [37, 38].

	Fields	$\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$	$\Sigma(81)$
Visible	L	$(\mathbf{1}, \mathbf{2}, -1/2)$	$\mathbf{3}_D$
	ℓ_R	$(\mathbf{1}, \mathbf{1}, -1)$	$\bar{\mathbf{3}}_D$
	H	$(\mathbf{1}, \mathbf{2}, 1/2)$	$\bar{\mathbf{3}}_D$
Dark	$N_{L,R}$	$(\mathbf{1}, \mathbf{1}, 0)$	$\mathbf{3}_A$
	η	$(\mathbf{1}, \mathbf{2}, 1/2)$	$\mathbf{3}_A$
	ϕ	$(\mathbf{1}, \mathbf{2}, 1/2)$	$\bar{\mathbf{3}}_A$

TABLE I. Particle content and symmetry transformation properties under the SM gauge group and the flavour symmetry $\Sigma(81)$. Note that the fields of the visible sector transform as $\mathbf{3}_D$, $\bar{\mathbf{3}}_D$ or $\mathbf{1}_{(i,j)}$, while the dark sector transforms as $\mathbf{3}_A$ or $\bar{\mathbf{3}}_A$. The lightest particle of the dark sector will be automatically stable. See text for details.

The field content of the SM is extended by adding a vector-like singlet N and two Higgs-like scalars, transforming non-trivially under $\Sigma(81)$. All the fields and charges are given in table I. Comparing to the original scotogenic model [23], new fields were also required to generate neutrino masses at one-loop with $\Sigma(81)$. While for the simple $\mathbb{Z}_2$ symmetry of the scotogenic model, any product of an odd field under $\mathbb{Z}_2$ times itself transforms as a singlet under $\mathbb{Z}_2$, this is not the case for any of the triplet representations of $\Sigma(81)$. For this reason, one needs to promote the right-handed neutrino to a vector-like fermion and, on a similar footing, two copies of the inert doublet Higgs are required, η and ϕ.

For simplicity, we split the most relevant parts of the Lagrangian as,

$$\mathcal{L} \supset \mathcal{L}_{\mathrm{Y}}^{V} + \mathcal{L}_{\mathrm{Y}}^{D} - \mathcal{V}_{\mathrm{s}}\,, \tag{1}$$

where the scalar potential is further divided into parts, for convenience, as $\mathcal{V}_s = \mathcal{V}_\nu + \mathcal{V}_{\mathrm{soft}} + \ldots$. The first part of the potential contains the scalar interactions that enter in the neutrino mass, the second of soft breaking terms of mass dimension 2 and "..." denotes the rest of the usual four-scalar interactions, that are not interesting for the purpose of our discussion.

The soft breaking terms in $\mathcal{V}_{\mathrm{soft}}$ are of the form,

$$\mathcal{V}_{\mathrm{soft}} = \mu_{ij}^2\, H_i^\dagger H_j\,, \tag{2}$$

and break the flavour symmetry $\Sigma(81)$ at the dimension two level. These terms are necessary in order to satisfy phenomenological bounds. On the one hand, the addition of these terms allow a deviation from symmetrical vacuum expectation value (VEV) alignments. On the other hand, they are needed in order to decouple the heavier physical Higgs scalars and suppress FCNC [61].

A. Charged lepton masses

In this section, we derive the mass matrix for the charged leptons given the particle content of table I. The relevant piece of the Lagrangian in equation (1) is the first term, which contains the Yukawa interaction terms among fields of the visible sector. To make the derivation clearer, all terms have been expanded in $\Sigma(81)$ components, for example, $L = (L_1, L_2, L_3)^{\mathrm{T}}$, and similarly for the other triplets, following the tensor products given in the second edition of the book "An Introduction to Non-Abelian Discrete Symmetries for Particle Physicists" [62] (see also appendix A for more details). In this way, it is made explicit in the Lagrangian itself

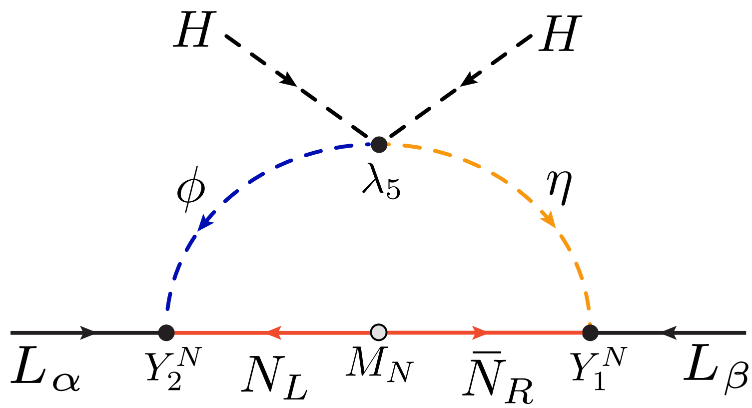

FIG. 1. Diagram generating neutrino masses at one-loop in our model, analogous to the original scotogenic model. Fields running in the loop are charged as $\mathbf{3}_A$ or $\bar{\mathbf{3}}_A$ under $\Sigma(81)$, while fields in the external legs (SM) transform as $\mathbf{3}_D$ or $\bar{\mathbf{3}}_D$.

that several contractions may lead to a singlet under $\Sigma(81)$. For instance, three triplets $\mathbf{3}_D$ have three different contractions to an invariant singlet $\mathbf{1}_{(0,0)}$.

The Yukawa interactions among the visible sector are given by,

$$\begin{aligned} \mathcal{L}_{\rm Y}^V \; = \; & Y_1^e \sum_{i=1}^{3} \bar{L}_i \ell_{R_i} H_i \\ & + Y_2^e \left(\bar{L}_1 \ell_{R_3} H_2 + \bar{L}_2 \ell_{R_1} H_3 + \bar{L}_3 \ell_{R_2} H_1 \right) \\ & + Y_3^e \left(\bar{L}_1 \ell_{R_2} H_3 + \bar{L}_2 \ell_{R_3} H_1 + \bar{L}_3 \ell_{R_1} H_2 \right) \\ & + \text{h.c.}\,, \end{aligned} \tag{3}$$

where $SU(2)_L$ indices and contractions have been omitted for simplicity.

After electroweak symmetry breaking (EWSB) the Lagrangian $\mathcal{L}_{\rm Y}^V$ in last equation gives rise to the mass matrix for charged leptons,

$$M_e = \frac{1}{\sqrt{2}} \begin{pmatrix} Y_1^e\, v1 & Y_3^e\, v3 & Y_2^e\, v2 \\ Y_2^e\, v3 & Y_1^e\, v2 & Y_3^e\, v1 \\ Y_3^e\, v2 & Y_2^e\, v1 & Y_1^e\, v3 \end{pmatrix}, \tag{4}$$

in the basis $\{(L_1, L_2, L_3), (\ell_{R_1}, \ell_{R_2}, \ell_{R_3})\}$, with the vacuum expectation values of the Higgs defined as, $\langle H_i \rangle = v_i/\sqrt{2}$ and $\sum_i v_i^2 = v_{\rm SM}^2$ the Standard Model VEV. The mass matrix M_e is then diagonalised by the unitary rotations U_ℓ and V_ℓ as,

$$\hat{M}_e = U_\ell^\dagger M_e V_\ell, \quad \text{with} \quad L \to U_\ell\, L, \qquad \ell_R \to V_\ell\, \ell_R\,, \quad \hat{M}_e = \text{diag}(m_e, m_\mu, m_\tau)\,. \tag{5}$$

B. Neutrino masses

The term $\mathcal{L}_{\rm Y}^D$ in the RHS of equation (1), describes the interactions with the fields of the dark sector. This piece, together with the scalar potential, will give rise to the one-loop neutrino mass diagram depicted in figure 1 and its corresponding mass matrix.

This term in the Lagrangian is given by,

$$\begin{aligned}\mathcal{L}_{\rm Y}^{D} \;=\;& M_N \left(\bar{N}_{L_1} N_{R_1} + \bar{N}_{L_2} N_{R_2} + \bar{N}_{L_3} N_{R_3}\right) \\ &+ Y_1^N \left(L_1 \bar{N}_{R_2} \eta_1 + L_2 \bar{N}_{R_3} \eta_2 + L_3 \bar{N}_{R_1} \eta_3\right) \\ &+ Y_2^N \left(L_1 N_{L_1} \phi_2 + L_2 N_{L_2} \phi_3 + L_3 N_{L_3} \phi_1\right) \\ &+ \text{h.c.}\,. \end{aligned} \tag{6}$$

While the relevant scalar couplings, analogous to the λ_5 interaction from the original scotogenic model [23], are

$$\begin{aligned}\mathcal{V}_\nu \;=&\; \lambda_5^{(1)} \left[(H_1 \eta_2^\dagger)(H_1 \phi_1^\dagger) + (H_2 \eta_3^\dagger)(H_2 \phi_2^\dagger) + (H_3 \eta_1^\dagger)(H_3 \phi_3^\dagger)\right] \\ &+ \lambda_5^{(2)} \left[(H_1 \eta_1^\dagger)(H_2 \phi_3^\dagger) + (H_1 \eta_3^\dagger)(H_3 \phi_2^\dagger) + (H_2 \eta_2^\dagger)(H_3 \phi_1^\dagger)\right] \\ &+ \text{h.c.}\,. \end{aligned} \tag{7}$$

The expansion in components of $\Sigma(81)$ makes explicit that not every entry of the neutrino mass matrix will be generated. In fact, there are only six possible diagrams with different $\Sigma(81)$ components outside the loop and running in it. After EWSB, the resultant neutrino mass matrix is of the form,

$$M_\nu \sim \frac{1}{2} \begin{pmatrix} 0 & C_1 v_3^2 + C_2 v_1 v_2 & C_1 v_2^2 + C_2 v_1 v_3 \\ C_1 v_3^2 + C_2 v_1 v_2 & 0 & C_1 v_1^2 + C_2 v_2 v_3 \\ C_1 v_2^2 + C_2 v_1 v_3 & C_1 v_1^2 + C_2 v_2 v_3 & 0 \end{pmatrix}. \tag{8}$$

For the sake of clarity, we have assigned colours to each entry of the matrix and to its corresponding terms in the Lagrangian $\mathcal{L}_{\rm Y}^{D}$ and the scalar potential $\mathcal{V}_\nu$ in equations (6) and (7) respectively. The coefficients C_a are obtained by computing the different diagrams of the type of figure 1 that contribute,

$$C_a \sim \frac{1}{16\pi^2} \, \frac{\lambda_5^{(a)} \, (Y_1^N)\, (Y_2^N)}{M_N}\,. \tag{9}$$

A very remarkable feature of the UV-realisation with $\Sigma(81)$ that we present here, is the fact that the neutrino matrix is exactly traceless with vanishing diagonal entries. This feature is protected by the symmetry and yields several strong predictions in the neutrino sector, as we will discuss in the next section.

The matrix in equation (8) coefficients C_a correspond to the dominant contribution. The neutrino mass matrix is, in general, given by,

$$(M_\nu)_{\alpha\beta} = \frac{1}{16\pi^2} \, (Y_1^N)_{\beta ij} \, (Y_2^N)_{\alpha ij} \, M_N \sum_{X=R,I} \sigma_X \, (U_X^\alpha)_{1i} \, (U_X^\alpha)_{i2} \, B_0(0, M_N, m_{X_i}^2)\,, \tag{10}$$

where $\sigma_{R,I} = \pm 1$. The expression for the neutrino mass matrix (10) is very similar to that of the original scotogenic model, where after electroweak symmetry breaking the neutral part of the scalar doublet in the loop splits into its CP-even and CP-odd components (denoted as R and I, respectively) due to the quartic coupling λ_5. The result is the sum of two B_0 Passarino-Veltman loop functions [63] with a relative minus sign. Also, similar to the generalised scotogenic models with several scalars [64], the mixing among the different scalar doublets in the loop need to be considered. The main subtlety is that, given the flavour symmetry $\Sigma(81)$, not every coupling is allowed. The only non-zero Yukawa couplings are $(Y_1^N)_{112} = (Y_1^N)_{223} = (Y_1^N)_{331} = Y_1^N$ and $(Y_2^N)_{121} = (Y_2^N)_{232} = (Y_2^N)_{313} = Y_2^N$. While the mass matrices mixing the neutral components of the scalars can be trivially obtained from (7), with diagonalising matrices U_R^α and U_I^α, for the CP-even and odd components respectively, in the basis (η_α, ϕ_k). Note that again $\Sigma(81)$ only allows the mixing among specific pairs of η and ϕ (see the scalar potential (7)).

III. PREDICTIONS

The neutrino mass matrix in equation (8) is diagonalised as,

$$U_\nu^T\, M_\nu\, U_\nu \;=\; \mathrm{diag}(m_1, m_2, m_3)\,, \tag{11}$$

where U_ν is the neutrino unitary mixing matrix and m_i are the neutrino masses. In the Normal Ordering case $m_1 > m_2 > m_3$, while in the Inverted Ordering case $m_2 > m_1 > m_3$.

Considering both equations (5) and (11) we obtain the lepton mixing matrix,

$$U_{\rm lep} = U_\ell^\dagger\, U_\nu\,. \tag{12}$$

$U_{\rm lep}$ is constrained by neutrino oscillation experiments. We choose the so-called symmetric parametrisation of a general unitary matrix [5, 65],

$$U_{\rm lep} = P(\delta_1,\delta_2,\delta_3)\, U_{23}(\theta_{23},\phi_{23})\, U_{13}(\theta_{13},\phi_{13})\, U_{12}(\theta_{12},\phi_{12})\,, \tag{13}$$

where $P(\delta_1,\delta_2,\delta_3)$ is a diagonal matrix of unphysical phases and the U_{ij} are complex rotations in the ij plane, as for example,

$$U_{23}(\theta_{23},\phi_{23}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23}\, e^{-i\phi_{23}} \\ 0 & -\sin\theta_{23}\, e^{i\phi_{23}} & \cos\theta_{23} \end{pmatrix}\,. \tag{14}$$

The phases ϕ_{12} and ϕ_{13} are relevant for neutrinoless double beta decay while the combination $\delta_{CP} = \phi_{13} - \phi_{12} - \phi_{23}$ is the usual Dirac CP phase measured in neutrino oscillations.

Before going into the numerical results, let us note an interesting analytical property of the matrix (8). The shape of this mass matrix, due to the $\Sigma(81)$ flavour symmetry, implies that the neutrino masses satisfy the relation,

$$\frac{1}{2}\sum m_i \;=\; m_{\rm heaviest}\,, \tag{15}$$

where $m_{\rm heaviest}$ is the heaviest neutrino mass. Equation (15) is actually a general prediction for a complex, symmetric, diagonal-less neutrino mass matrix. If we call such a mass matrix A and define it in general as,

$$A = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}\,, \qquad \text{with } a,\, b,\, c \in \mathbb{C}\,, \tag{16}$$

diagonalised as usual by

$$U^T A\, U = m_d = \mathrm{diagonal}(m_1, m_2, m_3)\,, \tag{17}$$

$$U^\dagger A^\dagger A\, U = m_d^2\,, \tag{18}$$

where m_d is real, diagonal and positive. With this definition the traces of the matrices $A^\dagger A$ and $(A^\dagger A)^2$ can be computed straightforwardly,

$$Tr(A^\dagger A) = 2(|a|^2 + |b|^2 + |c|^2) = m_1^2 + m_2^2 + m_3^2\,, \tag{19}$$

$$Tr\left[(A^\dagger A)^2\right] = 2(|a|^2 + |b|^2 + |c|^2)^2 = m_1^4 + m_2^4 + m_3^4\,. \tag{20}$$

These traces fulfill the general relation,

$$\frac{1}{2}\left[Tr(A^\dagger A)\right]^2 = Tr\left[(A^\dagger A)^2\right] , \tag{21}$$

which translated to the mass eigenvalues reads,

$$m_3^2 = (m_1 \pm m_2)^2 . \tag{22}$$

Since m_i are real and positive, only one solution survives after specifying the ordering. In particular,

$$m_3^{\rm NO} = m_1^{\rm NO} + m_2^{\rm NO} , \tag{23}$$

$$m_2^{\rm IO} = m_1^{\rm IO} + m_3^{\rm IO} , \tag{24}$$

or in general, irrespective of the ordering, the sum rule (15).

Neutrino oscillations measure the mass squared differences of neutrino masses [66–71], which in combination with the mass sum rules (23) and (24), lead to the prediction of the absolute scale of the neutrino masses:

$$m_{\rm lightest}^{\rm NO} \approx 2.8 \times 10^{-2}\,{\rm eV} , \tag{25}$$

$$m_{\rm lightest}^{\rm IO} \approx 7.5 \times 10^{-4}\,{\rm eV} . \tag{26}$$

Both values are well below cosmological bounds [72] and direct measurements of neutrino mass [73, 74].

Note, however, that the neutrino mass matrix in equation (8) is more restricted than the matrix in equation (16). In particular, the strong hierarchy in the masses of the charged leptons implies a strong hierarchy between the VEVs of the Higgs doublets, further restricting the neutrino mass matrix. We have performed a numerical scan and found the following results and predictions for both orderings.

A. Inverted ordering

In the Inverted Ordering case, a strong correlation appears between θ_{12} and δ_{CP} when the charged lepton and neutrino masses, as well as the angles θ_{13} and θ_{23}, are fitted to the experimental values. As can be seen in figure 2, the model can accommodate all oscillation observables inside their 3σ ranges, with a slight tension in the θ_{12} vs δ_{CP} plane. However, it is worth noting that the best fit point of δ_{CP} in the global fit of is very sensible to new data sets and new data from the Nova collaboration [69] may change the picture in 2022. Moreover, we are using the global fit [75] to produce the plots, although the other two global fits [76, 77] yield slightly lower values for δ_{CP}, thus reducing the tension of the model. Taking θ_{12} alone, we can see that the model can accommodate $\theta_{12} \approx \theta_{12}^{\rm best\ fit}$ if $\delta_{CP} \approx \pi$. In other words, this model prediction may be tested in the following data releases of neutrino oscillation experiments. Furthermore, the Majorana phases ϕ_{12} and ϕ_{13}, relevant for neutrinoless beta decay experiments, also obtain a strong correlation, as seen in figure 3.

The striking similarities between these correlations and the ones in [78, 79] may indicate that our setup leads to the partial conservation of some of the TBM symmetries of the neutrino mass matrix.

For neutrinoless double beta decay, if the Majorana neutrino mass mechanism is the dominant contribution to $0\nu\beta\beta$, its rate will be proportional to the quantity $|m_{ee}|$, given by,

$$|m_{ee}| = \left|\sum_i U_{ei}^2 m_i\right| = |c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{2i\phi_{12}} m_2 + s_{13}^2 e^{2i\phi_{13}} m_3| . \tag{27}$$

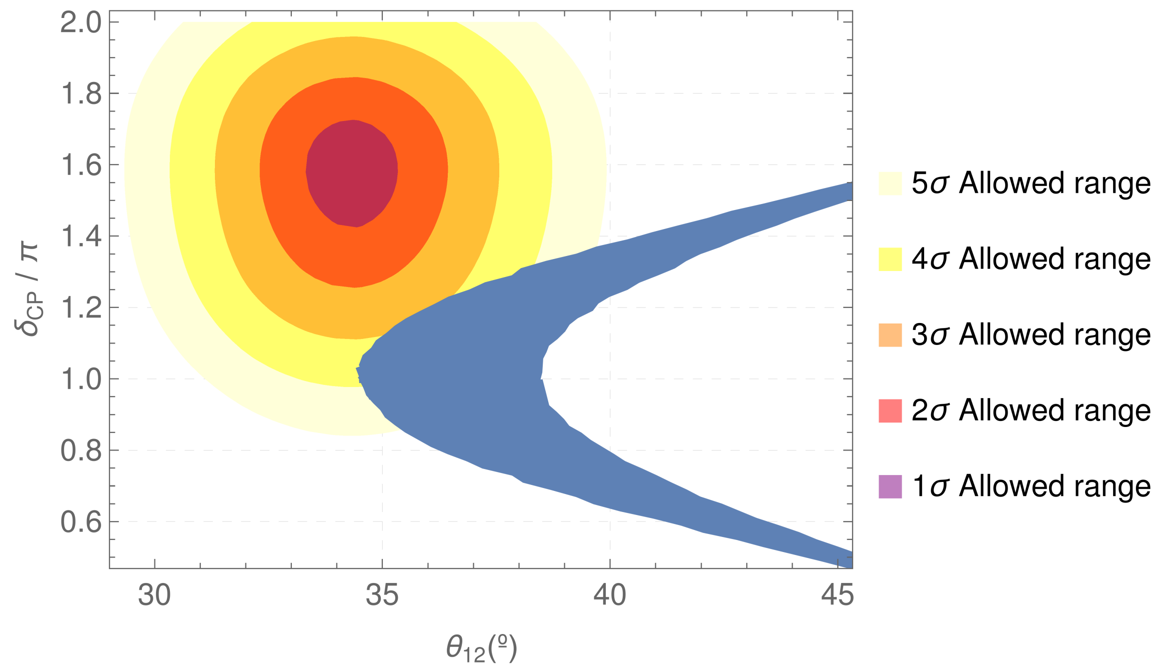

FIG. 2. Correlation between θ_{12} and δ_{CP} in the IO case when the other observables are fitted inside their 3σ experimental ranges: charged lepton masses, neutrino squared mass differences, θ_{13} and θ_{23}. The 3σ tension in the combined $\theta_{12}-\delta_{CP}$ plane may be relieved if δ_{CP} is measured to be around CP conserving values. In that case, θ_{12} would lie in its 1σ experimental region. χ^2 profiles extracted from the global fit [75].

In our model the Majorana phases are approximately fixed as $\phi_{12} \approx 0.45\pi$ and $\phi_{13} \approx 0.12\pi$, while the neutrino masses are also predicted to be around $m_3 \approx 7.51 \times 10^{-4}$ eV, $m_1 \approx 4.95 \times 10^{-2}$ eV, $m_2 \approx 5.02 \times 10^{-2}$ eV. Small deviations from these values are possible due to the experimental uncertainty on Δm^2_{ij} and the variance in ϕ_{ij}. This automatically leads to a definite prediction of $|m_{ee}|$ in our model:

$$|m_{ee}^{\text{model}}| \approx 0.018 \text{ eV}\,, \tag{28}$$

Note that the term with ϕ_{13} in (27) interferes constructively to $|m_{ee}|$ but is strongly suppressed by $s^2_{13}m_3$, while the term with ϕ_{12} interferes destructively. This is why the allowed points in the model are in the lower region of $|m_{ee}|$ as seen in figure 4. The nEXO experiment is expected to test this model prediction in the future [80, 81].

B. Normal ordering

In the Normal Ordering case, after imposing the correct charged lepton and neutrino masses at 3σ, a strong correlation appears between the mixing angles θ_{23} and θ_{13} in the neutrino sector. This correlation is not compatible with experimental constraints by more than 7σ, as can be seen in figure 5. Therefore, **Normal Ordering of neutrino masses cannot be realised in this model.**

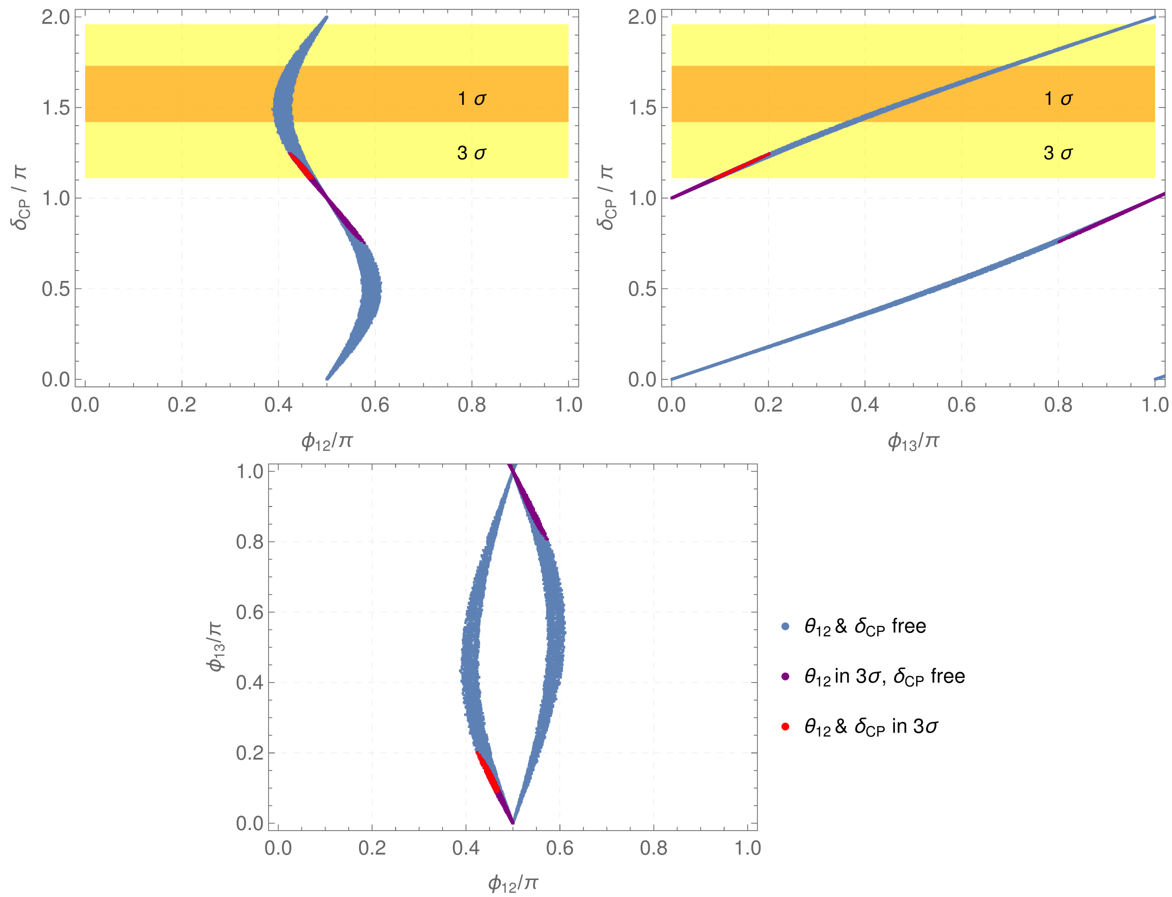

FIG. 3. Correlations between physical phases in the IO case. Left up: Correlation between ϕ_{12} and δ_{CP}. Right up: Correlation between ϕ_{12} and δ_{CP}. Down: Correlation between the Majorana phases ϕ_{13} and ϕ_{13}. In all the plots blue dots arise when the other observables are fitted inside their 3σ experimental ranges except for θ_{12} and δ_{CP}, which are free. In addition, purple dots fit θ_{12} and red dots also fit δ_{CP} at the 3σ level. By imposing all the experimental constraints, the model predicts $\delta_{CP} \approx 1.2\pi$, $\phi_{12} \approx 0.45\pi$ and $\phi_{13} \approx 0.12\pi$ plus a small variance.

IV. DARK MATTER STABILITY

The $\Sigma(81)$ flavour symmetry has the additional property of stabilizing the lightest of the dark sector fields. In order to see how this mechanism works, we must first note that the singlets $\mathbf{1}_{(i,j)}$ and the $\mathbf{3}_D$, $\bar{\mathbf{3}}_D$ triplets form a closed subset under the tensor products, i.e.

$$\mathbf{1}_{(i,j)} \times \mathbf{1}_{(k,l)} = \mathbf{1}_{(i+k,\, j+l)}, \qquad \mathbf{1}_{(i,j)} \times \mathbf{3}(\bar{\mathbf{3}})_D = \mathbf{3}(\bar{\mathbf{3}})_D, \tag{29}$$

$$\mathbf{3}(\bar{\mathbf{3}})_D \times \mathbf{3}(\bar{\mathbf{3}})_D = \bar{\mathbf{3}}(\mathbf{3})_D, \qquad \mathbf{3}_D \times \bar{\mathbf{3}}_D = \mathbf{1}_{(i,j)}\,. \tag{30}$$

We start by imposing the condition that all of the *visible sector* fields, i.e. the Standard Model fermions and Higgs, transform as either $\mathbf{1}_{(i,j)}$, $\mathbf{3}_D$ or $\bar{\mathbf{3}}_D$. This automatically implies that any effective operator formed by any arbitrary combination of SM particles, $\mathcal{O}_{\text{visible}}$, will still transform under the same subgroup, i.e. as $\mathbf{1}_{(i,j)}$, $\mathbf{3}_D$ or $\bar{\mathbf{3}}_D$.

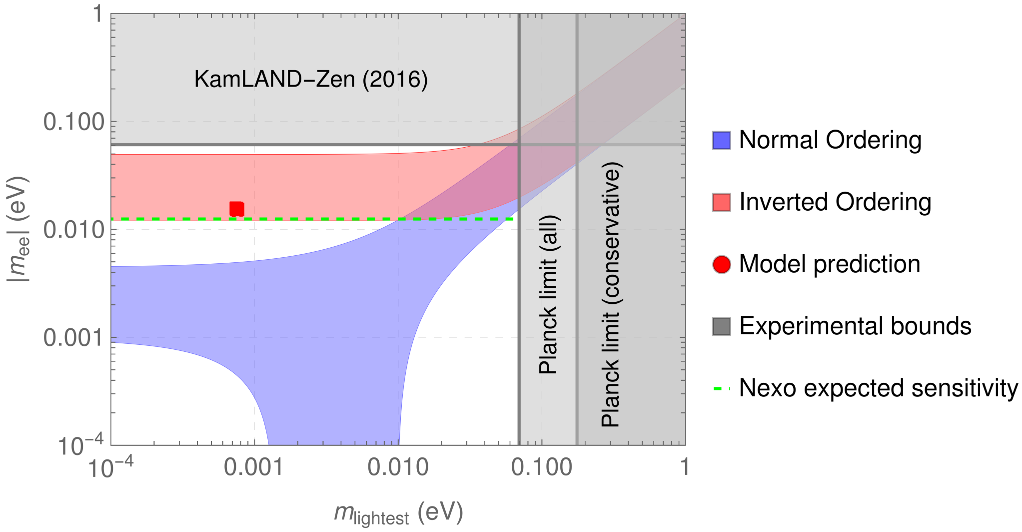

FIG. 4. $|m_{ee}|$ is restricted to a small region in this model. The reason is that m_{light} and the Majorana phases are predicted, as well as the ordering. The deviation from a single point comes from the experimental uncertainties in Δm_{ij}^2 and a small variance in ϕ_{ij}. The current experimental constraints are given by KamLAND [48] and Planck [72]. In the future, nEXO is expected to have enough sensitivity to completely rule out the inverted ordering region [81].

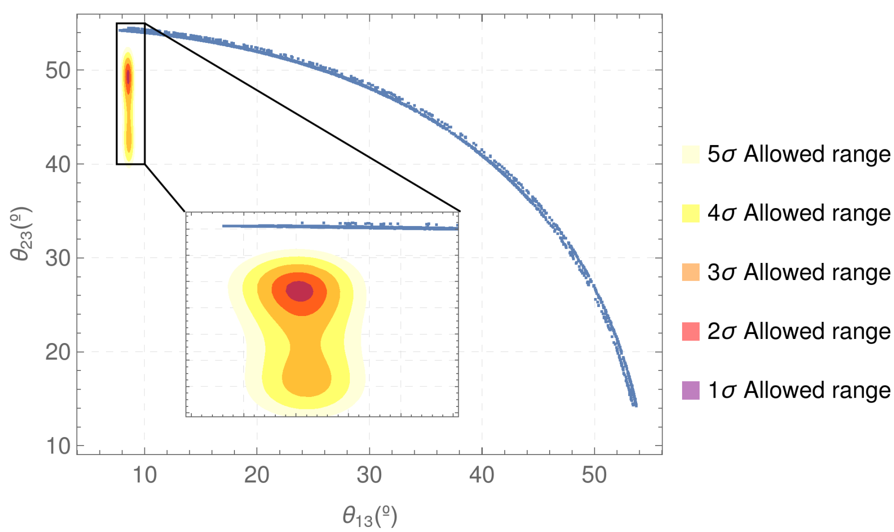

FIG. 5. Model prediction in the NO case. The correlation between θ_{13} and θ_{23} is incompatible with current experimental constraints at more than 7σ. χ^2 profiles extracted from the global fit [75].

Consider now a field η belonging to the *dark sector* and transforming as, for example, $\mathbf{3}_A$. It is clear that the effective operator $\eta \,.\, \mathcal{O}_{\text{visible}}$ cannot be invariant under $\Sigma(81)$, because no operator of the type $\mathcal{O}_{\text{visible}}$ transforms as $\bar{\mathbf{3}}_A$.

In conclusion, any symmetry invariant decay operator of a particle belonging to the dark sector must involve, at least, one dark sector particle in the final state and, thus, the lightest of them will necessarily be stable (see

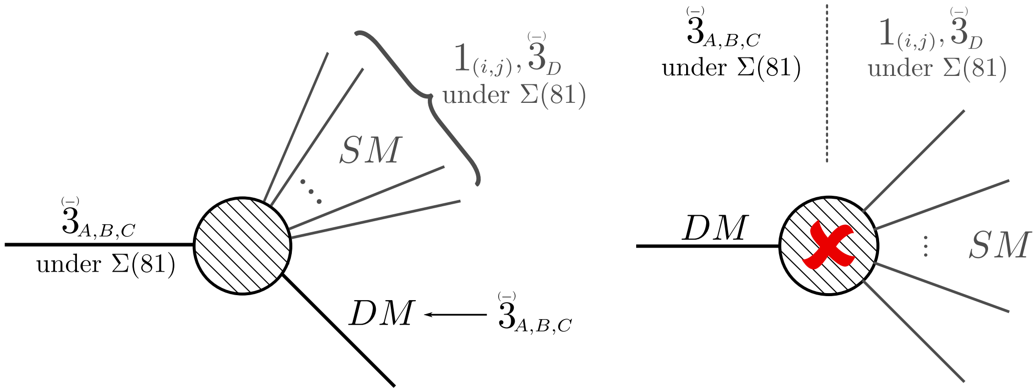

FIG. 6. Stability diagrams of the dark matter sector in the model. The lightest particle charged as $3(\bar{3})_{A,B,C}$ cannot decay into only Standard Model fields. Left: allowed decay channels for a dark sector particle will necessarily include at least one dark sector particle in the final state (if kinematically allowed). Right: the lightest dark sector particle cannot decay into Standard Model particles due to the flavour symmetry, thus ensuring its stability.

figure 6). In our model the dark matter could be either the lightest neutral mass eigenstate of the scalars η, ϕ or the vector-like fermion N, if lighter than the scalars.

Note that this is a generalised, non-Abelian version of the original scotogenic mechanism of [23], where the stability of the dark matter candidate is enforced by a $\mathbb{Z}_2$ symmetry. This mechanism was extended to Abelian symmetries in [20, 21].

V. CONCLUSIONS

We have presented a simple but extremely predictive variant of the scotogenic model. We promoted the scotogenic $\mathbb{Z}_2$ symmetry of the original work to a non-Abelian $\Sigma(81)$ symmetry, which will satisfy the same role of stabilizing the dark matter candidates running in the neutrino mass loop. We considered that leptons, as well as the Higgs doublet H, transform as triplets under the flavour symmetry, thus, resembling a 3HDM. These three scalar doublets are responsible for all the spontaneous symmetry breaking, which implies that the model does not need extra flavons in order to fit the experimental data. We found that such a model can, not only satisfy the current experimental constraints, but also lead to very strong and testable predictions in the close future. Fitting the charged lepton masses, θ_{13} and θ_{23} inside their 3σ allowed ranges, we automatically obtained the following predictions:

- Neutrino mass sum rule: $\frac{1}{2}\sum m_i = m_{\text{heaviest}}$.
- Only Inverted Ordering is realised.
- These two conditions together lead to $m_{\text{lightest}} \approx 7.5 \times 10^{-4}$ eV, with some small deviations due to experimental uncertainty in Δm^2_{ij}.
- Strong correlation between θ_{12} and δ_{CP} as shown in figure 2, testable in the near future [82, 83].
- Majorana phases predicted to be around $\phi_{12} \approx 0.45\pi$ and $\phi_{13} \approx 0.12\pi$, when all the other observables are in their experimental allowed ranges (see figure 3).

- The prediction of the Majorana phases and $m_{\rm lightest}$ lead to $|m_{ee}| \approx 0.018$ eV, testable in future neutrinoless double beta decay experiments [81] (see figure 4).
- The flavour symmetry $\Sigma(81)$ ensures the stability of the dark matter candidate, which could be either fermionic or scalar. No other symmetries are required apart from the Standard Model gauge symmetries and the spontaneous symmetry breaking comes solely from the three Higgs gauge doublets arranged into a flavour triplet.

Appendix A: $\Sigma(81)$ Group

The group $\Sigma(81)$ is a discrete, non-Abelian subgroup of $SU(3)$ and belongs to the family of groups $\Sigma(3N^3)$. It has four generators denoted by a, a', a'', and b, which fulfill the relations,

$$a^3 = a'^3 = a''^3 = 1, \qquad aa' = a'a, \qquad aa'' = a''a, \qquad a'a'' = a''a', \tag{A1}$$

$$b^3 = 1, \qquad b^2ab = a'', \qquad b^2a''b = a', \qquad b^2a'b = a. \tag{A2}$$

All the elements of $\Sigma(81)$ can be written in terms of the four generators as,

$$\forall g \in \Sigma(81), \qquad g = b^k a^n a'^n a''^l, \qquad \text{with} \quad k,n,n,l = 0,1,2. \tag{A3}$$

The representations of $\Sigma(81)$ used for the fields multiplets in this model are $\mathbf{3}_A$, $\bar{\mathbf{3}}_A$, $\mathbf{3}_D$, and $\bar{\mathbf{3}}_D$. We choose the following basis for these representations: in the $\bar{\mathbf{3}}_A$,

$$b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad a = \begin{pmatrix} \omega & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad a' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad a'' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{A4}$$

where $\omega = e^{i2\pi/3}$. In the $\bar{\mathbf{3}}_D$ representation,

$$b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad a = \begin{pmatrix} \omega^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad a' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad a'' = \begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{A5}$$

Notice that the generators in the $\mathbf{3}_A$ representation are the complex conjugate of the generators in the $\bar{\mathbf{3}}_A$, and similarly between the $\mathbf{3}_D$, and $\bar{\mathbf{3}}_D$.

It is worth showing explicitly one of the key properties of $\Sigma(81)$ that give rise to dark matter stability in the model presented, i.e. the singlet irreps together with $\mathbf{3}_D$ and $\bar{\mathbf{3}}_D$ form a close subgroup. This can be seen by looking at the products (A7)-(A11).

$$\mathbf{1}_{(k,l)} \times \mathbf{3}_D(\bar{\mathbf{3}}_D) = \mathbf{3}_D(\bar{\mathbf{3}}_D)\,, \quad \mathbf{3}_D \times \mathbf{3}_D = \bar{\mathbf{3}}_D + \bar{\mathbf{3}}_D + \bar{\mathbf{3}}_D\,, \quad \mathbf{3}_D \times \bar{\mathbf{3}}_D = \mathbf{1}_{(k,l)}\,, \tag{A6}$$

with $k, l = 0, 1, 2$.

Expanding in components in the basis defined by (A4) and (A5), we have the tensor products,

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{\mathbf{3}_D} \otimes \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{\bar{\mathbf{3}}_D} = \sum_{k=0,1,2} [(x_1y_1 + \omega^{2k}x_2y_2 + \omega^k x_3y_3)_{\mathbf{1}_{(k,0)}} \oplus (x_2y_3 + \omega^{2k}x_3y_1 + \omega^k x_1y_2)_{\mathbf{1}_{(k,2)}} \oplus (x_3y_2 + \omega^{2k}x_1y_3 + \omega^k x_2y_1)_{\mathbf{1}_{(k,1)}}]. \tag{A7}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{\mathbf{3}_D} \otimes \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{\mathbf{3}_D} = \begin{pmatrix} x_1 y_1 \\ x_2 y_2 \\ x_3 y_3 \end{pmatrix}_{\bar{\mathbf{3}}_D} \oplus \begin{pmatrix} x_2 y_3 \\ x_3 y_1 \\ x_1 y_2 \end{pmatrix}_{\bar{\mathbf{3}}_D} \oplus \begin{pmatrix} x_3 y_2 \\ x_1 y_3 \\ x_2 y_1 \end{pmatrix}_{\bar{\mathbf{3}}_D}, \tag{A8}$$

$$(x)_{\mathbf{1}_{(k,0)}} \otimes \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{\mathbf{3}(\bar{\mathbf{3}})_D} = \begin{pmatrix} x y_1 \\ \omega^k x y_2 \\ \omega^{2k} x y_3 \end{pmatrix}_{\mathbf{3}(\bar{\mathbf{3}})_D} \tag{A9}$$

$$(x)_{\mathbf{1}_{(k,1)}} \otimes \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{\mathbf{3}(\bar{\mathbf{3}})_D} = \begin{pmatrix} x y_3 \\ \omega^k x y_1 \\ \omega^{2k} x y_2 \end{pmatrix}_{\mathbf{3}(\bar{\mathbf{3}})_D} \tag{A10}$$

$$(x)_{\mathbf{1}_{(k,2)}} \otimes \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{\mathbf{3}(\bar{\mathbf{3}})_D} = \begin{pmatrix} x y_2 \\ \omega^k x y_3 \\ \omega^{2k} x y_1 \end{pmatrix}_{\mathbf{3}(\bar{\mathbf{3}})_D} \tag{A11}$$

The label $\mathbf{1}_{(k,l)}$, with $k, l = 0, 1, 2$, represent the nine different one dimensional irreps of $\Sigma(81)$, being $\mathbf{1}_{(0,0)}$ the invariant singlet.

For further details of the properties of the $\Sigma(81)$ group and the explicit expressions of the tensor products of dark sector fields we refer the reader to the **second** edition of the book "An Introduction to Non-Abelian Discrete Symmetries for Particle Physicists" [62], since the first edition had inconsistencies in the representations used in the tensor products.

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