Inflation with Massive Spin-2 Ghosts

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We consider a generic model of quadratic gravity coupled to a single scalar and investigate the effects of gravitational degrees of freedom on inflationary parameters. We find that quantum corrections arising from the massive spin-2 ghost generate significant contributions to the effective inflationary potential and allow for a realization of the spontaneous breakdown of global scale invariance without the need for additional scalar fields. We compute inflationary parameters, compare the resulting predictions to well-known inflationary models, and find that they fit well within the Planck collaboration's constraints on inflation.

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I. INTRODUCTION

Cosmic inflation is the most promising solution to many puzzles surrounding the big bang and offers a mechanism to generate cosmological perturbations from primordial quantum fluctuations. The approximate scale invariance of the corresponding inflationary power spectrum reported by the Planck collaboration [1] may also be hinting towards an underlying theory that is scale invariant before dynamical symmetry breaking; a possibility that we will embrace as others have done in the past [2–18].

Many successful models of inflation are constructed around f(R) gravitational sectors which may generically contain more than one power of the Ricci scalar e.g. the Starobinsky model [19] where the additional degree of freedom (DOF) due to the R^2 term plays the role of the inflaton. More general higher order terms built from the other independent contractions of the Riemann tensor are rarely included in the action, however, these terms are necessarily generated by quantum effects even if they are not included at the classical level from the start [20]. While it is usually accepted that such terms contribute to inflation only negligibly at the classical level [21, 22], it is not necessarily true that quantum corrections arising from the higher order contractions of the Riemann tensor are also negligible. Indeed, we find that the massive spin-2 ghost that originates from the Weyl tensor squared term (C^2) is particularly important as it can generate an inflationary potential that dynamically induces the Planck scale via radiative corrections \dot{a} la Coleman-Weinberg¹ [24]. It should be noted that the scalaron degree of freedom originating from the R^2 term alone is not sufficient for triggering radiative symmetry breaking in the Jordan frame. In this respect, our considerations will allow for the construction of the most minimal scale invariant model that yields a dynamical generation of the Planck scale and inflationary potential, as no additional bosonic degrees of freedom besides the inflaton scalar and the metric degrees of freedom are necessary.

It is well-known that the massive ghost DOFs that appear when one considers the C^2 term in the action threaten the unitarity of the resulting quantum theory. This quantum version of the Ostrogradsky instability, usually referred to as the "ghost problem", is a subtle and complicated topic that we will not address in this work. Rather, we refer the curious reader to a few of the most interesting attempts to solve this problem, namely, the work of Donoghue and Menezes that centers around the decay of the massive ghost [25, 26], as

¹ This fact also implies that modifications to the inflaton potential may be recognized at the level of the beta functions of the quartic couplings (see e.g. the beta functions in [23]).

well as the interesting possibility of \mathcal{PT} quantization championed by Bender and Mannheim [27, 28]. Though the details of these works are beyond the scope of the current paper, one important detail they share is that the massive spin-2 ghost is considered a genuine physical particle and is not merely some calculational relic as one might consider Faddeev-Popov ghosts, for example. As such, if one includes the C^2 term in the action, the physical effects of the massive ghost on inflationary predictions should not be neglected as they traditionally are.

We begin our investigations in the next section by establishing the full non-linear action, extracting the gravitational degrees of freedom, and deriving the propagators for said DOFs. We then calculate the Coleman-Weinberg one-loop effective potential, including contributions from the scalars and spin-2 ghost, which allows us to identify the dynamically generated Planck scale and establish the inflationary potential after transforming to the Einstein frame. Finally, we perform a numerical analysis of the predicted inflationary parameters and end with a discussion of the results.

II. THE MODEL

We consider the following general action describing globally scale invariant quadratic gravity non-minimally coupled to a single additional matter scalar S(x),

$$S_{\rm T} = S_{\rm QG} + S_{\rm S} \,, \tag{1}$$

$$S_{\rm QG} = \int d^4x \sqrt{-g} \Big(\gamma R^2 - \kappa C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \Big) \,, \tag{2}$$

$$S_{\rm S} = \int \mathrm{d}^4 x \sqrt{-g} \left(\frac{1}{2} \nabla_\mu S \nabla^\mu S - \frac{\beta}{2} S^2 R - \frac{\lambda}{4} S^4 \right),\tag{3}$$

where γ , κ , β , and λ are arbitrary dimensionless constants. As is standard practice in studies of quadratic gravity, the gravitational part of this action is parameterized in terms of the sum of squares of the Ricci scalar and Weyl tensor, which is equivalent to a general combination of the three independent contractions of the Riemann tensor after neglecting total derivatives [29]. The complete action (1) is invariant under infinitesimal local diffeomorphisms as well as the global scale transformations

$$g_{\mu\nu} \rightarrow \omega^2 g_{\mu\nu}, \qquad \qquad S \rightarrow \omega^{-1} S, \qquad (4)$$

where ω is a constant. The presence of this global symmetry is of particular interest because, as laid out in [5], the scalar S may form a condensate $\langle S \rangle = v_S$ that leads to the spontaneous breakdown of scale invariance and the subsequent generation of an Einstein-Hilbert term and identification of the Planck mass $M_{\rm Pl} \propto v_S$.

Since we are interested in the effects of gravitational DOFs on inflation, we separate out the dynamical part of the metric by linearizing the action around flat space with

$$g_{\mu\nu} \rightarrow \eta_{\mu\nu} + h_{\mu\nu},$$
 (5)

where $\eta_{\mu\nu}$ is the Minkowski metric and $h_{\mu\nu}(x)$ is a small perturbation. After performing this linearization up to second order in the graviton $h_{\mu\nu}$, integrating by parts, and dropping interaction terms, we arrive at the total action

$$S_{\rm T}^{\rm (lin)} = \int d^4x \left[\frac{\beta}{8} S_{\rm cl}^2 \left(h^{\mu\nu} \Box h_{\mu\nu} + 2h^{\mu\nu} \partial_\nu \partial^\rho h_{\mu\rho} - h_\mu{}^\mu \Box h_\nu{}^\nu - 2h_\mu{}^\mu \partial_\nu \partial_\rho h^{\nu\rho} \right) - \frac{\lambda}{4} S_{\rm cl}^4 \right. \\ \left. + \gamma \left(h^{\mu\nu} \partial_\mu \partial_\nu \partial_\rho \partial_\sigma h^{\rho\sigma} + h_\mu{}^\mu \Box^2 h_\nu{}^\nu + 2h_\mu{}^\mu \Box \partial_\nu \partial_\rho h^{\nu\rho} \right) \right. \\ \left. + \frac{\kappa}{6} \left(- 3h^{\mu\nu} \Box^2 h_{\mu\nu} - 6h^{\mu\nu} \Box \partial_\nu \partial^\rho h_{\mu\rho} - 2h^{\mu\nu} \partial_\mu \partial_\nu \partial_\rho \partial_\sigma h^{\rho\sigma} \right. \\ \left. + h_\mu{}^\mu \Box^2 h_\nu{}^\nu + 2h_\mu{}^\mu \Box \partial_\nu \partial_\rho h^{\nu\rho} \right) \right],$$

$$(6)$$

where $\Box = -\partial_{\mu}\partial^{\mu}$. Here we have also set *S* to its classical (approximately constant) background value S_{cl} since quantum fluctuations, in the standard Coleman-Weinberg sense, around S_{cl} make only negligible contributions to the inflationary potential at one-loop order [5].

We may further separate the gravitational DOFs according to their spin by performing a York decomposition in terms of transverse-traceless tensor modes $\tilde{h}_{\mu\nu}(x)$, transverse vector modes V(x), the scalar trace $h_{\mu}{}^{\mu}(x)$, and an additional scalar mode a(x) as

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} + \partial_{\mu}V_{\nu} + \partial_{\nu}V_{\mu} + \left(\partial_{\mu}\partial_{\nu} - \frac{1}{4}\eta_{\mu\nu}\Box\right)a + \frac{1}{4}\eta_{\mu\nu}h_{\rho}{}^{\rho}, \qquad (7)$$

where $\partial^{\mu}\tilde{h}_{\mu\nu} = \tilde{h}_{\mu}{}^{\mu} = 0$ and $\partial_{\mu}V^{\mu} = 0$ [30]. It is also instructive to redefine the graviton trace in terms of the gauge-invariant scalar quantity $\phi(x)$,

$$\phi = h_{\mu}{}^{\mu} - \Box a \,, \tag{8}$$

which may be identified as the well-known "scalaron" degree of freedom [31]. After applying these definitions, all of the quadratic terms containing V_{μ} and a cancel out leaving us with the simple action below.

$$S_{\rm Y} = \int \mathrm{d}^4 x \left[\phi \left(\frac{9\gamma}{16} \Box^2 - \frac{3\beta}{64} S_{\rm cl}^2 \Box \right) \phi - \tilde{h}_{\mu\nu} \left(\frac{\kappa}{2} \Box^2 - \frac{\beta}{8} S_{\rm cl}^2 \Box \right) \tilde{h}^{\mu\nu} - \frac{\lambda}{4} S_{\rm cl}^4 \right] \tag{9}$$

In this York-decomposed form it is straightforward to calculate the propagators and mass terms for each of the gravitational degrees of freedom. To do so, we perform a Fourier transform to identify the inverse propagators as the Hessians of (9) with respect to each field, which may then be inverted to yield the propagators

$$i \langle 0 | \mathcal{T}\phi\phi | 0 \rangle = \frac{32}{3\beta S_{\rm cl}^2} \left(-\frac{1}{p^2} + \frac{1}{p^2 - m_\phi^2} \right),$$
 (10)

$$i \langle 0 | \mathcal{T} \tilde{h}_{\mu\nu} \tilde{h}_{\rho\sigma} | 0 \rangle = \frac{4}{\beta S_{\rm cl}^2} \left(\frac{1}{p^2} - \frac{1}{p^2 - m_{\rm gh}^2} \right) \delta_{\mu\nu\rho\sigma} , \qquad (11)$$

where $p^2 = p_{\mu}p^{\mu}$, $\delta_{\mu\nu\rho\sigma} = \frac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho})$, and the masses are given by

$$m_{\phi}^2 = \frac{\beta}{12\gamma} S_{\rm cl}^2, \qquad \qquad m_{\rm gh}^2 = \frac{\beta}{4\kappa} S_{\rm cl}^2. \qquad (12)$$

III. INFLATION

A. The effective potential

After neglecting classical background contributions from the Weyl tensor squared term, the effective action for inflation may be written as

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left(\frac{1}{2} S \Box S - \frac{\beta}{2} S^2 R + \gamma R^2 - U_{\text{eff}}(S) \right), \tag{13}$$

where U_{eff} is the quantum effective one-loop potential. This term receives contributions from the massive spin-0 and spin-2 sectors, each of which may be calculated using standard Coleman-Weinberg (CW) methods [24].

The ϕ contribution to the CW potential is calculated by expanding the action (9) around the field's classical background as $\phi = \phi_{cl} + \delta \phi$ and integrating out the fluctuations $\delta \phi$. The part of the functional integral that is quadratic in $\delta \phi$ is Gaussian, leading to an effective potential that is proportional to

$$\ln\left[\det\left(\frac{\partial^2 S_{\rm Y}}{\partial\delta\phi\partial\delta\phi}\right)\right] = \operatorname{Tr}\left[\ln\left(\Box - m_{\phi}^2\right)\right] + \cdots, \qquad (14)$$

where the "…" stand for irrelevant constant terms that are independent of S. This trace may be written as a sum of the momentum space eigenvalues of the operator $\ln(\Box - m_{\phi}^2)$ and evaluated using dimensional regularization to give the scalaron's one-loop contribution to the the effective potential.

$$U_{\phi}(S) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \ln\left(\frac{p^2 - m_{\phi}^2}{p^2}\right) = \frac{1}{64\pi^2} m_{\phi}^4 \left[\ln\left(\frac{m_{\phi}^2}{\mu^2}\right) - \frac{3}{2}\right]$$
(15)

Here, we have employed MS, introducing the renormalization scale μ in the process, and we have absorbed the divergent terms into the renormalized constant λ . Performing the same calculation for the *S* contributions, which has already been performed in [5], yields the analogous result

$$U_S(S) = \frac{\lambda}{4}S^4 + \frac{1}{64\pi^2}m_S^4 \left[\ln\left(\frac{m_S^2}{\mu^2}\right) - \frac{3}{2}\right], \qquad m_S^2 = 3\lambda S^2, \qquad (16)$$

where the tree-level contributions have also been included.

Calculation of the spin-2 part follows in much the same way as the spin-0, with the nonzero contributions coming from the term $\tilde{h}^{\mu\nu}\delta_{\mu\nu\rho\sigma}(\Box - m_{\rm gh}^2)\tilde{h}^{\rho\sigma}$, i.e. only from the massive part of the inverse propagator. However, when going to momentum space, we must take advantage of the transverse-traceless nature of $\tilde{h}_{\mu\nu}$ to write

$$\tilde{h}^{\mu\nu}\delta_{\mu\nu\rho\sigma}\tilde{h}^{\rho\sigma} = \tilde{h}^{\mu\nu}P^{(2)}_{\mu\nu\rho\sigma}\tilde{h}^{\rho\sigma}, \qquad (17)$$

where

$$P_{\mu\nu\rho\sigma}^{(2)} = \frac{1}{2} \left(\theta_{\mu\rho} \theta_{\nu\sigma} + \theta_{\mu\sigma} \theta_{\nu\rho} \right) - \frac{1}{d-1} \theta_{\mu\nu} \theta_{\rho\sigma} \quad \text{with} \quad \theta_{\mu\nu} = \eta_{\mu\nu} - \frac{p_{\mu} p_{\nu}}{p^2} \,, \tag{18}$$

is a spin-2 projection operator [32]. Making this replacement ensures that we count the correct number of degrees of freedom, which is five for a massive spin-2 field in four dimensions, after noting that

$$\operatorname{Tr}(P^{(2)}_{\mu\nu\rho\sigma}) = \delta^{\mu\nu\rho\sigma} P^{(2)}_{\mu\nu\rho\sigma} = \frac{1}{2}(d+1)(d-2).$$
(19)

With these considerations, we find that the massive spin-2 contributes

$$U_{h}(S) = \lim_{d \to 4} \left[\mu^{4-d} \int \frac{\mathrm{d}^{d}p}{(2\pi)^{d}} \frac{1}{2} (d+1)(d-2) \ln\left(\frac{p^{2}-m_{\mathrm{gh}}^{2}}{p^{2}}\right) \right]$$
$$= \frac{5}{64\pi^{2}} m_{\mathrm{gh}}^{4} \left[\ln\left(\frac{m_{\mathrm{gh}}^{2}}{\mu^{2}}\right) - \frac{1}{10} \right]$$
(20)

to the effective potential, where we have subtracted the divergent part according to the MS scheme.

Finally, the entire effective potential is then given by

$$U_{\rm eff}(S) = U_{\phi}(S) + U_S(S) + U_h(S) + U_0, \qquad (21)$$

where U_0 is an arbitrary constant background that may be tuned in order to ensure that the classical zero-point energy vanishes, provided that scale invariance is broken spontaneously, which, as we will see in the next section, is indeed the case here.

B. The inflationary action

To calculate predictions for inflationary parameters, we need the proper inflationary potential in the Einstein frame. We must therefore calculate the symmetry breaking behavior of the Jordan frame potential given in (13) and (21), ensuring a vanishing zero-point energy both in Jordan and Einstein frame in the process. The effective one-loop potential may be

written as

$$U_{\rm eff}(S,0) = U_0 + \left[C_1 + C_2 \ln\left(\frac{S^2}{\mu^2}\right)\right] S^4, \qquad (22)$$

where

$$C_{1} = \frac{\lambda}{4} + \frac{9\lambda^{2}}{128\pi^{2}} \left(2\ln\left(3\lambda\right) - 3\right) - \frac{\beta^{2}}{2048\pi^{2}} \left[\frac{1}{9\gamma^{2}} \left(2\ln\left(\frac{12\gamma}{\beta}\right) + 3\right) + \frac{1}{\kappa^{2}} \left(10\ln\left(\frac{4\kappa}{\beta}\right) + 1\right) \right], \quad (23)$$

$$C_2 = \frac{9\lambda^2}{64\pi^2} + \frac{\beta^2}{1024\pi^2} \left(\frac{1}{9\gamma^2} + \frac{5}{\kappa^2}\right),$$
(24)

are dimensionless constants that depend only on the coupling constants.

We may now solve for the vacuum expectation value (VEV) of S, v_S , which is defined as the minimum of this potential

$$\frac{\partial U_{\text{eff}}(S)}{\partial S}\Big|_{S=v_S} = 0, \qquad v_S = \mu \exp\left(-\frac{1}{4} - \frac{C_1}{2C_2}\right). \tag{25}$$

The non-zero value of this minimum indicates a spontaneous breakdown of global scale symmetry, as advertised. We may also easily calculate the explicit value of U_0 by requiring that the effective potential vanishes in the broken phase which yields

$$U_{\text{eff}}(v_S) = 0,$$
 $U_0 = \frac{\mu^4}{2} C_2 \exp\left(-1 - \frac{2C_1}{C_2}\right).$ (26)

Finally, we obtain the explicit value of the Planck mass that is generated by the breaking of scale invariance by identifying the canonical Einstein term in (13) as

$$-\frac{1}{2}\beta S^2 R \bigg|_{S=v_S} = -\frac{1}{2}M_{\rm Pl}^2 R, \qquad \qquad M_{\rm Pl}^2 = \beta v_S^2.$$
(27)

In analogy to [5], this relates $M_{\rm Pl}$ to the renormalization scale μ via (25). In contrast to the aforementioned work ([5]), we do not need two external scalars to achieve successful Coleman-Weinberg symmetry breaking of scale invariance in the Jordan frame. This is due to the additional contributions from the scalar and tensor degrees of freedom to the effective scalar potential which is a novel consideration. To calculate predictions of the inflationary parameters that result from spontaneous symmetry breaking, we follow the procedure outlined in [5]. Since we have shown that scale invariance is spontaneously broken, we may introduce an auxiliary field to remove the R^2 term, then transform to the Einstein frame with a Weyl rescaling. There we find two dynamical scalar fields, S and the scalaron. The corresponding potential exhibits a valley structure [5], a flat direction with steep perpendicular potential lines, and the fields will thus always fall into a trajectory along that flat direction. After solving the minimum equations for the scalaron², the final inflationary potential can be rewritten to depend only on the original external scalar S and the coupling constants λ , β , γ , and κ . This effective inflationary action in the Einstein frame has the form

$$S_{\rm inf}^{\rm E} = \int d^4x \sqrt{-g} \left(-\frac{1}{2} M_{\rm Pl}^2 R + \frac{1}{2} F(S)^2 S \Box S - U_{\rm inf}(S) \right), \tag{28}$$

where F(S) denotes the modification to the kinetic term for S and is given by

$$F(S) = \frac{1}{(1+4A)B} \left[(1+4A)B + \frac{3}{2}M_{\rm Pl}^2 ((1+4A)B' + 4A'B)^2 \right]^{1/2},$$
(29)

where A and B are functions of the scalar field S given by

$$A(S) = \frac{4\gamma U_{\rm inf}(S)}{B(S)^2 M_{\rm Pl}^2}, \qquad B(S) = \frac{\beta S^2}{M_{\rm Pl}^2}, \qquad (30)$$

and primes denote derivatives with respect to S. With these definitions, the full inflationary potential $U_{inf}(S)$ is thus determined to be

$$U_{\rm inf}(S) = \frac{U_{\rm eff}(S)}{B(S)^2 + 16\gamma U_{\rm eff}(S)/M_{\rm Pl}^4} \,. \tag{31}$$

One may also obtain the canonically normalized field \hat{S} via a simple integration.

$$\hat{S}(S) = \int_{v_S}^{S} \mathrm{d}x \, F(x) \tag{32}$$

² Depending on the parameter configuration, solving for S rather than the scalaron may result in a better description of the flat direction. However, in our case the choice of contour has only a minor influence on the inflationary parameter predictions as both contours are valid for all calculated points. For an extended discussion we refer the reader to section 4.2 and Appendix A of [5].

C. Numerical analysis of slow-roll inflation

Inflationary CMB observables, namely, the scalar spectral index n_s and the tensor-toscalar ratio r, may be expressed in terms of the slow-roll parameters ε and η as

$$n_S = 1 + 2\eta_* - 6\varepsilon_*, \qquad r = 16\varepsilon_*, \qquad (33)$$

where the asterisks indicate quantities evaluated at $S = S_*$, the value of S at time of photon decoupling (CMB horizon exit). Since our inflationary potential depends only on the scalar field S, we can apply the well known formulas for ϵ , η , and N_e of one-field slow-roll inflation, modified to depend on the non-normalized field S using the relation (32).

$$\varepsilon(S) = \frac{M_{\rm Pl}^2}{2F^2(S)} \left(\frac{U_{\rm inf}'(S)}{U_{\rm inf}(S)}\right)^2 \tag{34}$$

$$\eta(S) = \frac{M_{\rm Pl}^2}{F^2(S)} \left(\frac{U_{\rm inf}'(S)}{U_{\rm inf}(S)} - \frac{F'(S)}{F(S)} \frac{U_{\rm inf}'(S)}{U_{\rm inf}(S)} \right)$$
(35)

$$N_e = \int_{S_*}^{S_{\text{end}}} \frac{F^2(S)}{M_{\text{Pl}}^2} \frac{U_{\text{inf}}(S)}{U'_{\text{inf}}(S)}$$
(36)

Here, S_{end} denotes the value of S at the end of inflation which is defined by max { $\epsilon(S = S_{\text{end}})$, $|\eta(S = S_{\text{end}})|$ } = 1. With this we may calculate expressions of n_S and r that depend only on the dimensionless couplings λ , β , γ and κ , as μ is fixed after demanding the correct value for M_{Pl} as in (27). To constrain this model we use the latest data from the Planck satellite mission [1] and assume an inflation duration of $N_e \approx 50 - 60$ e-folds. To ensure our predictions are consistent with the Planck data, we constrain the parameter space of the dimensionless couplings so that it ultimately fulfills the scalar power spectrum amplitude A_s constraints below.

$$\ln(10^{10}A_s) = 3.044 \pm 0.014 \qquad A_s = \frac{U_{\inf *}}{24\pi^2 \epsilon_* M_{\rm Pl}} \tag{37}$$

Predictions corresponding to the resulting coupling values below are displayed in (Fig. 1).

$$\lambda = 0.005 \qquad \beta \in [10^3, 10^4] \qquad \gamma \in [10^7, 10^9] \qquad \kappa \in [10^2, 10^{3.25}]$$
(38)



FIG. 1: Predictions for the scalar spectral index n_s and the tensor-to-scalar ratio r with varying numbers of e-folds N_e are displayed. For the points shown, λ is fixed, while β, γ , and κ are taken randomly from (38) while satisfying (37). We include the Planck TT,TE,EE+lowE+lensing+BK15+BAO 68% and 95% CL regions from [1], as well as predictions of the Starobinsky model (green) and linear inflation (red).

The ranges for the dimensionless couplings in (38) result from incorporating the Planck constraint on A_s (37). More parameter space was explored but did not yield promising predictions while being compatible with this constraint. We see that for the full range of possible e-folds, there are points that are compatible with even the tightest Planck constraints. We also see that the upper limit of our predictions for r approaches the upper limits of linear inflation $(m\phi^3)$, while the lower limits match those of Starobinsky inflation. The circles for linear and Starobinsky inflation in (Fig. 1) represent the predictions for $N_e = 50$ (left) and $N_e = 60$ (right) e-folds respectively. The point labelled "B1" in (Fig. 1) corresponds to the following benchmark values.

B1:
$$\lambda = 0.005$$
 $\beta = 5.62 \times 10^2$ $\gamma = 1.22 \times 10^8$ $\kappa = 837$ (39)

In order to get an order of magnitude estimate, we calculate the field masses m_{ϕ} , $m_{\rm gh}$ via the relation (12) evaluated at the non-zero VEV of S,

$$m_{\phi}^{\text{B1}}(S=v_S) \simeq 6.35 \times 10^{13} \,\text{GeV}\,, \qquad m_{\text{gh}}^{\text{B1}}(S=v_S) \simeq 4.21 \times 10^{16} \,\text{GeV}\,.$$
(40)

These masses are representative for the most points, while the field masses of all points displayed in (1) are roughly contained in the ranges $m_{\phi} \in [10^{13} \text{ GeV}, 10^{16} \text{ GeV}]$ and $m_{\text{gh}} \in [10^{16} \text{ GeV}, 10^{17} \text{ GeV}]$. Here, high m_{ϕ} goes hand in hand with small γ and therefore relatively large tensor-to-scalar ratios (see Fig. 1). Additionally, we take into account classical corrections to the inflation parameters due to the presence of the C^2 -term. Two different contributions are introduced in [21] (2.24) and [22] (7.4) respectively. To calculate the latter correction we need to use the slow-roll approximation during inflation³, $H^2 \approx V/(3M_{\text{Pl}}^2)$. Here we find that the correction is largest for large κ with a maximum of $\approx 22\%$, increasing the predicted tensor-to-scalar ratio. For smaller κ and large γ , we get a correction towards smaller r with a maximum of $\approx 11\%$. This leaves us with even the corrected predictions being fully compatible with the currently strongest cosmological constraints from Planck18.

³ To ensure a field value that is representative for inflation we choose $S = S_*$ to calculate V(S) and $m_{\rm gh}(S)$.

IV. CONCLUSION

We have investigated a classical scale invariant framework that dynamically generates the Planck mass via spontaneous symmetry breaking in the Jordan frame in the most minimal way i.e. with only one external scalar in addition to the quantum contributions of the graviton degrees of freedom. Given that higher powers of curvature tensors are necessarily generated via quantum corrections even if these terms are not considered at tree-level, we include the Weyl tensor squared term from the start and find that the resulting quantum contributions allow for spontaneous symmetry breaking via the Coleman Weinberg mechanism in the Jordan frame with only the one external scalar. Specifically, it is the massive spin-2 ghost DOF originating from the Weyl squared term that allows for spontaneous symmetry breaking, a role that is usually filled by additional external scalars in other scale invariant models.

Starting with a scale invariant Lagrangian, we are able to explicitly cancel the cosmological constant in both frames if scale invariance is spontaneously broken in the Jordan frame which is the one of the foremost advantages of our model when compared to other one-scalar models with symmetry breaking in the Einstein frame [4, 33]. Furthermore, the potential resulting from symmetry breaking in our framework leads to an inflationary potential that is in perfect agreement with the current strongest constraints from the Planck collaboration, as seen in (Fig. 1).

We also include the classical corrections to the predicted inflationary parameters due to the presence of the C^2 -term, which are calculated in [21, 22]. These come out to be of order 10% - 20% and reduce the available parameter space of predictions in agreement with the current limits, though at the same time, they improve the predictions of the parameters close to the limit of Starobinsky inflation. Therefore, though the impact of the C^2 -term's classical corrections is minor, its quantum contributions turn out to be important due to the fact that they lead to a one-loop scalar potential that enables symmetry breaking in the Jordan frame with only one external scalar.

To conclude, we note that though the primordial non-Gaussianities in cosmological fluctuations are suppressed in single-field systems of inflation [34], they can be generated in a multi-field system and appear in the CMB anisotropy as well as in measurements of the large scale structure of the Universe (see for instance [35] and [36]). Future experimental projects such as LiteBird [37], Euclid [38], LSST [39], etc. will be able to measure the magnitude of these non-Gassianities and constrain their existence. Though our model contains only one scalar field at the beginning, the scalaron which originates from the R^2 term in the action (2) makes the system behave as an effectively two-field system. In [40] it is outlined how one may compute the non-Gaussianities in such models. Furthermore, as we have mentioned with reference to [22] in section III.C, the massive spin-2 mode can contribute to the metric perturbations during inflation, thus altering the inflationary parameters. This correction has turned out to be very small in our model, but the size of its contribution to the non-Gaussianities is not known as of yet. We thus plan to put the focus of our future investigations on the primordial non-Gaussianities in inflationary models based on scale invariance.

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