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Breaking electric charge conservation with charged Higgs vacua

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Abstract

Breaking electric charge conservation with charged Higgs vacua

We introduce additional Higgs fields with charged vacuum expectation values to break the conservation of electric charge. However the phenomenology of such models is very strongly constrained and limits such theories to regions of the parameter space that are already well explored. The main obstacle turns out to be the emergence of a photon rest mass, further studies would need to find a way of dealing this with this to have a reasonable prospect of success.

Verletzung elektrischer Ladungserhaltung durch geladene Higgs-Vakua

Wir führen zusätzliche Higgs-Felder mit geladenen Vakuums-Erwartungswerten ein, um die Erhaltung elektrischer Ladung zu brechen. Die Phänomenologie solcher Modelle ist allerdings bereits sehr stark eingeschränkt und begrenzt solche Theorien auf Regionen des Parameterraums, die bereits sehr gut erkundet sind. Das Aufkommen einer Photonen-Ruhemasse stellt sich als das gröSSte Problem in dieser Hinsicht heraus, zukünftige Studien müssten vermutlich einen Weg finden mit diesen umzugehen um ernsthafte Erfolgsaussichten zu haben.

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Contents

Chapter 1

Introduction

Most methods in physics rely on the conservation of certain physical quantities, famously energy and momentum, mass in most low energy processes and, depending on the field of study, others. The conservation of these quantities is not self-evident and through Noether's theorem a connection between this quality and the mathematical notion of symmetry has been established, allowing a more formal study of these properties.

Particle physics is one of the fields that has profited greatly from this formalism. However since the discovery of spontaneous symmetry breaking and the Higgs mechanism there is also a way in which locally conserved currents do not lead to a globally conserved charge (often referred to as a breaking of the charge conservation). Examples for this would be the electroweak symmetry breaking after which hypercharge and isospin are not conserved quantities anymore, but only the electromagnetic (EM) charge.

A very naive question at this point would be: if some charge conservations are broken in the Standard Model of particle physics, why not all or at least some more of them? Utilising a similar Higgs mechanism as in the SM it is possible to introduce a charged Higgs vacuum that would also lead to the violation of EM charge conservation.

Conventionally, this is considered to be an unattractive prospect, since the experimental evidence for conservation of EM charge is very strong and since such a violation would invariably lead to a photon rest mass, upon which there are also extremely stringent limits.

Another obstacle is that up to now charge has only been observed as a quantised property in multiples of 1/3 of the elementary charge e. However, since EM charge is a sum of the, by definition, quantised isospin and hypercharge, upon which no such conceptual restriction can be placed, its quantization is not a theoretical necessity. Therefore, so-called milli-charged particles (MCPs) can be studied upon which there are less stringent limits than on processes violating charge conservation at $\sim 1e$.

In the following we consider a few possible motivations for the study of theories predicting such MCPs.

1.1 Cosmology

We consider whether a uniform distribution of MCPs in the universe could contribute to its expansion that in the ΛCDM model is attributed to the cosmological constant Σ or 'dark energy' (DE). Naively, this seems possible since Λ is understood to be a 'vacuum energy density' throughout the universe. A distribution of particles effectively decoupled from the SM possessing an electrostatic energy density could account for this.

However, one can find that the behaviour of such MCPs in an expanding universe is not compatible with what is required of DE. To see this we consider the acceleration Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3p). \tag{1.1}$$

To explain the observed accelerating expansion of the universe $\ddot{a} \geq 0$ one needs $\rho+3p \leq 0$ or expressed through the equation of state parameter $p = \omega\rho$ as $\omega \leq -1/3$. Without explicit calculation it is clear that for a distribution of slightly charged particles one would have $\omega > 0$; one simple way of thinking about this is that the energy density due to MCPs should decline with expansion a as a^{-3} due to the expansion of space and then some more due to the weakening of the electrostatic field and since $\rho \sim a^{-3(1+\omega)}$ we have $\omega > 0$. So while an intriguing idea, it is clear from the outset that MCPs alone cannot account for the expansion of the universe.

1.2 Particle physics

It has been noted that in experiments a millicharged neutrino would be very hard to distinguish from an electrically neutral neutrino with a magnetic moment [11]. Since the XENONnt experiment discovered an excess in electron recoil events that could be explained through a neutrino magnetic moment [2] the study of MCPs in this light might bring interesting insights. However, upon further data-taking and analysis carried out by the XENONnt collaboration during the time of writing of this thesis the excess disappeared [3], so this can also not serve as an immediate motivation.

Instead the aim of this thesis was to study additional Higgs fields with charged vacuum expectation values and to try to find an effective field theory (EFT) description for such theories of additional Higgs multiplets. In chapter 2 we go through some of the relevant theoretical background pertaining to symmetries, conserved quantities and spontaneous symmetry breaking. In chapter 3 we work out the theories for the singlet, doublet and triplet case and look at some of the phenomenological with regard to an EFT formulation and chapter 4 summarises these efforts.

Chapter 2

Theory background

Although we frequently use mathematical terminology in this chapter, it does not aspire to be a mathematically rigorous treatment of the subject nor is the level of rigour applied throughout it consistent. We rather aim for a mode of presentation that is useful to readers with a graduate-level background in theoretical particle physics and go into as much nuance as is deemed useful for later applications; accordingly, proofs for statements will be given where instructive and not otherwise.

The contents of this chapter can be found in many textbooks and for the most part will be attributed summarily at this point: Aside from the usual famous choices [12, 9], we want to especially highlight the clear and thorough treatment of symmetries and spontaneous symmetry breaking (SSB) in [6].

2.1 Noether's theorem

We start our description of physics with defining the action S

$$S = \int \mathrm{d}t\, L = \int \mathrm{d}^4 x\, \mathcal{L} \tag{2.1}$$

as the integral over time of the Lagrangian L or as space-time integral over the Lagrangian density $\mathcal{L}$. Assuming that $\mathcal{L}$ describes a field $\phi(x)$, the time derivative of this field will only appear in terms of $\partial_\mu \phi(x)$ and so $\mathcal{L}$ can be written as a function of $\mathcal{L}(\phi(x), \partial_\mu \phi(x))$. We derive the equations of motions for this field through the stationary-action principle

$$\begin{aligned} 0 &= \delta S \\ &= \int \mathrm{d}^4 x \left(\frac{\partial \mathcal{L}(\phi(x), \partial_\mu \phi(x))}{\partial \phi(x)} \delta\phi(x) + \frac{\partial \mathcal{L}(\phi(x), \partial_\mu \phi(x))}{\partial(\partial_\mu \phi(x))} \delta \partial_\mu \phi(x) \right) \\ &= \int \mathrm{d}^4 x \left(\frac{\partial \mathcal{L}(\phi(x), \partial_\mu \phi(x))}{\partial \phi(x)} - \partial_\mu \frac{\partial \mathcal{L}(\phi(x), \partial_\mu \phi(x))}{\partial(\partial_\mu \phi(x))} \right) \delta\phi(x) \end{aligned} \tag{2.2}$$

where we used $\delta \partial_\mu \phi(x) = \partial_\mu \delta \phi(x)$ and integrated by parts with vanishing boundary terms in the last step. Since the integrand has to be zero for all $\delta\phi(x)$ we find the Euler-Lagrange equations

$$\frac{\partial \mathcal{L}(\phi(x), \partial_\mu \phi(x))}{\partial \phi(x)} - \partial_\mu \frac{\partial \mathcal{L}(\phi(x), \partial_\mu \phi(x))}{\partial(\partial_\mu \phi(x))} = 0\,. \tag{2.3}$$

The equations of motions are the first physically measureable manifestations of our theory while S is undetermined up to a constant c and $\mathcal{L}$ is undetermined up to a total derivative $\partial_\mu F^\mu$ because

$$\delta\left(\int \mathrm{d}^4x\,(\mathcal{L}+\partial_\mu F^\mu)\right)=\delta(S+c)=\delta(S)\,. \tag{2.4}$$

We use this notion to define symmetries of the action S.

Definition 1 (Symmetry of the action) *A transformation Λ_α with a parameter $\alpha\in\mathbb{R}$ of a field $\phi(x)\to\Lambda_\alpha(\phi)=\phi'(x)$ is called a continuous symmetry transformation if it leaves the equations of motion of ϕ invariant or, equivalently, changes the Lagrangian $\mathcal{L}$ by at most a total derivative of some function F_μ s.t. $\mathcal{L}(\phi'(x),\partial_\mu\phi'(x))=\mathcal{L}(\phi(x),\partial_\mu\phi(x))+\partial_\mu F^\mu$ for all $\alpha\in\mathbb{R}$.*

If this is true for some smooth function $\alpha:\mathbb{R}^n\to\mathbb{R}$ it is called a local symmetry and if it is only true for some constant function α it is called a global symmetry.

Theories that posses such symmetries proved very fruitful to study, especially those that can be variied continuously (in contrast to discrete symmetries), because of the powerful consequences of Noether's theorem.

Theorem 1 (Noether's first theorem) *Every continuous global symmetry gives rise to a Noether current $j^\mu(x)$ such that $\partial_\mu j^\mu(x)=0$ on-shell.*

The proof for this theorem can be found in many textbooks and we will not present it here. Note however that we did not make any statement about conserved charges yet and that it is indeed impossible to do so without at least one more assumption. Many textbook derivations are imprecise on this point and imply that a conserved current in the above sense also produces a charge that is conserved in time. Since this aspect is relevant for our problem we will illuminate it in greater detail:

Theorem 2 (Conserved charges) *Every continuous global symmetry whose associated Noether current satisfies $j^i(t,x)\to 0$ sufficiently fast for $|x|\to\infty$ gives rise to a conserved charge*

$$Q=\int_{\mathbb{R}^3} d^3x j^0(t,x)$$

with

$$\frac{d}{dt}Q=0\,.$$

We outline the proof of this theorem to understand why the requirement of a sufficiently fast fall-off of j^i is necessary for charge conservation:

$$\begin{aligned}\frac{d}{dt}Q&=\int_{\mathbb{R}^3} d^3x\partial_0 j^0(t,x)\\&=\int_{\mathbb{R}^3} d^3x\partial_0 j^0(t,x)+\partial_i j^i(t,x)-\partial_i j^i(t,x)\\&=-\int_{\mathbb{R}^3} d^3x\partial_i j^i(t,x)\\&=-\oint_{S^2(|x|\to\infty)} dS\, j^i(t,x)=0\end{aligned} \tag{2.5}$$

Here we first used the conservation of the Noether current, then Gauß' theorem and finally the fall-off of j^i at $|x| \to \infty$. It should be noted that the fall-off at infinity is not necessary to get rid of some boundary terms, as it is most of the time, but that it directly concerns the expression in the last line. In many derivations of Noether's theorem the conservation of charge is treated as an implicit by-product of Noether's theorem, but here we see that it is possible to have a Lagrangian with conserved currents due to Noether's theorem while the associated charges are not conserved.

2.2 Gauge theories

Apart from such global symmetries, Lagrangians can also be symmetric under local transformations, which are more general because they can transform the Lagrangian differently at each space-time-point. A special case of local transformations are the so-called gauge transformations. [6] outlines a more nuanced differentiation between local and gauge symmetries, but while these considerations are informative for physical understanding they are not necessary for our considerations.

Such gauge symmetries are at the heart of modern quantum field theory (QFT) formulations of particle physics and questions about them frequently arise when dealing with the mass of particles. One common argument states that the Higgs mechanism is necessary for a gauge-invariant description of particle masses. While it is straightforward to see that this is true, it implicitly assumes that gauge invariance is indeed a necessary ingredient of physically useful QFTs; we will try to illustrate their role in a bit more detail here. We will follow a standard textbook approach in introducing gauge invariance ad-hoc and start with quantum electrodynamics (QED) as an example:

Constructing a theory describing the interaction of photons and Dirac fermions we start off with the Lagrangian of free particles (which we consider as given here)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi \tag{2.6}$$

with the field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ of the vector field A_μ and Dirac fermion field ψ. We note that this Lagrangian is invariant under a global $U(1)$ transformation of ψ (without explicit effort to make it so)

$$\psi(x) \to e^{-ie\alpha}\psi(x)\,, \quad \overline{\psi}(x) \to \overline{\psi}(x) e^{ie\alpha} \tag{2.7}$$

and the corresponding Noether current is $j^\mu = e\overline{\psi}\gamma^\mu\psi$. A physical way of thinking about this $U(1)$-symmetry is to interpret it as a global phase of a wavefunction which can be chosen arbitrarily since only differences in phase affect physically measurable quantities and the conserved current turns out to be the electric charge we would expect electrons to have[1]. We will now try to make Eq. 2.6 invariant under the transformation in 2.7 in local form, so with $\alpha(x)$ as a function of coordinates. We find that this is not the case, instead

$$\overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi \to \overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi + (\partial_\mu \alpha(x)) j^\mu\,. \tag{2.8}$$

Conveniently, one finds that the Lagrangian extended by an interaction term between A_μ and ψ through the covariant derivative $D_\mu = \partial_\mu + ieA_\mu$

[1] Note that to make the step from a conserved current to a charge we need $j^i(x) \to 0$ for $|x| \to \infty$ as noted previously. This is the case here since we assume ψ to be localized.

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\gamma^\mu\partial_\mu - m)\psi - A_\mu j^\mu \\
&= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\gamma^\mu D_\mu - m)\psi
\end{aligned}
\tag{2.9}
$$

is invariant under the combined gauge transformation

$$
\psi(x) \to e^{-ie\alpha(x)}\psi(x)\,, \quad A_\mu \to A_\mu + \partial_\mu\alpha(x)\,. \tag{2.10}
$$

Since the global $U(1)$-symmetry is a special case of this gauge symmetry for $\alpha(x) = const.$ all statements about conservation of electric charge are still true. We thus find that in our search for a gauge invariant formulation we have inadvertently also found an expression that contains everything we desire for our interacting theory, i.e. coupling of photons to a conserved electric charge and the gauge transformation of A_μ is consistent with the gauge freedom that also exists in classical electrodynamics.

For QED this is not as coincidental as has been suggested up to this point. As taken from chapter 8.1 of [12] the Lorentz transformation Λ of a general massless vector field a_μ is

$$
U(\Lambda)a_\mu(x)U^{-1}(\Lambda) = \Lambda_\mu^\nu a_\nu(\Lambda x) + \partial_\mu\alpha(x,\Lambda) \tag{2.11}
$$

where α is a linear combination of creation and annihilation operators of a_μ. By identifying the α in this expression with the gauge transformation we find that for all massless vector fields demanding Lorentz invariance and gauge invariance are equivalent. And because Lorentz invariance is a physically very well motivated requirement using gauge invariance as a building principle for QED turns out to be not just useful but also well justified. While we used this equivalence to argue that a Lorentz invariant theory of massless vector bosons has to be gauge invariant it can just as well be used to explain why Lorentz and gauge invariant vector bosons have to be massless (and this second line of reasoning is more common in the literature).

For massive vector fields (like the $W^\pm$- and Z-bosons of the SM) however, this argument does not hold true, there are Lorentz invariant theories which are not gauge invariant. Still it is commonly argued that it is not possible to naively introduce mass terms, like $-m_A^2 A_\mu A^\mu$, supposedly necessitating the Higgs mechanism as a gauge invariant way of introducing particle masses.

While gauge invariance may not be strictly necessary to build an interacting theory with massive vector fields, there are still arguments for why it is desirable to do so. The most important is surely that gauge invariant Yang-Mills theories have been shown to be renormalizable to all orders[1]; this has not been achieved for other theories.

2.3 Spontaneous symmetry breaking

Up to now we have only considered whether symmetry transformations leave the Lagrangian (or equivalently the action or the e.o.m.) invariant but not whether the same is true for the physical ground state. Theories where this mechanism of spontaneous symmetry breaking (SSB) occurs have first been described in condensed matter physics (e.g. Ginzburg-Landau theory) but it has subsequently found fruitful application in particle physics as well.

To gain some understanding we will study SSB at the example of the linear sigma model of N real scalar fields ϕ^i

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^i)^2 + \frac{1}{2}\mu^2(\phi^i)^2 - \frac{\lambda}{4}(\phi^i)^4 \tag{2.12}$$

where μ and λ are real, positive parameters. This Lagrangian is evidently invariant under global $O(N)$ rotations. However, the state which minimises the potential

$$V(\phi^i) = -\frac{1}{2}\mu^2(\phi^i)^2 + \frac{\lambda}{4}(\phi^i)^4 \tag{2.13}$$

is not determined uniquely, only its absolute value, the vacuum expectation value (VEV), is

$$(\phi_0^i)^2 = \frac{\mu^2}{\lambda} =: v^2 \tag{2.14}$$

and in contrast to most other potentials this minimum is not zero. It is now possible to find infinitely many different ground states simply by rotating any one state fulfilling Eq. 2.14 by an $O(N)$-transformation under which the theory is invariant. For further analysis we pick the state pointing in the Nth direction $\phi_0^i = (0, 0, \ldots, 0, v)$ and reparametrize the fields around this

$$\phi^i(x) = (\pi^k(x), v + \sigma(x)), \quad k = 1, \ldots, N-1\,. \tag{2.15}$$

Expressing the Lagrangian in Eq. 2.14 in terms of these new fields

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \pi^k)^2 + \frac{1}{2}(\partial_\mu \sigma^k)^2 - \frac{1}{2}(2\mu^2)\sigma^2 - \sqrt{\lambda}\mu\sigma^3 - \sqrt{\lambda}\mu(\pi^k)^2\sigma - \frac{\lambda}{4}\sigma^4 - \frac{\lambda}{2}(\pi^k)^2\sigma^2 - \frac{\lambda}{4}(\pi^k)^2 \tag{2.16}$$

we find a field σ with mass $m_\sigma^2 = 2\mu^2$ and $N-1$ massless π fields. This is not coincidental but a consequence of Goldstone's theorem.

Theorem 3 (Goldstone's theorem) *For each spontaneously broken continuous symmetry a theory will contain exactly one massless particle, a Goldstone boson.*

While SSB provides us as with a way to generate particle masses without introducing gauge invariance breaking explicit mass terms, it creates a new problem: if the masses of the $W^\pm$- and Z-bosons were generated through SSB, the SM would need to contain at least three massless Goldstone bosons. However, such particles have never been experimentally observed so SSB alone only replaces one problem, gauge invariance, with another, the missing Goldstone bosons.

2.4 The Higgs mechanism

The combination of a gauge invariant theory with SSB, known as the Higgs mechanism, turns out to solve all the previously stated problems concurrently. To avoid misunderstandings we start off with a short note about terminology:

It is frequently stated that the Higgs mechanism is about "spontaneous breaking of a gauge symmetry" but this phrasing is misleading. It is mathematically impossible to spontaneously break a gauge symmetry, a fact known as *Elitzur's theorem* [8]. Instead of trying to find a more suitable turn of phrase we will lay out how it should be understood:

Assume that we have a theory that is - by construction - gauge invariant under some group of transformations. As noted previously, this theory cannot have explicit mass terms for gauge bosons, but it is necessarily invariant under global transformations of this group (since a global transformation is just one particular case of a local transformation). While it is not possible to spontaneously break the gauge symmetry, this global symmetry can be broken and generate mass terms for previously massless particles. It might seem that the problem of the Goldstone bosons would persist, but the beauty of the Higgs mechanism is that it does not.

We demonstrate this by constructing a theory of a massive photon (this is an example, not a general proof but that would be significantly more involved). Consider the following Lagrangian of a complex scalar field ϕ coupled to a gauge field A_μ through a covariant derivative $D_\mu = \partial_\mu + ieA_\mu$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(\phi) \tag{2.17}$$

which has a $U(1)$ gauge invariance similar to that of QED in Eq. 2.7. If the potential has the form

$$V(\phi) = -\frac{1}{2}\mu^2(\phi^i)^2 + \frac{\lambda}{4}(\phi^i)^4 \tag{2.18}$$

the resulting global $U(1)$ symmetry is broken spontaneously; if we expand the Lagrangian around the VEV as in the previous section we find

$$\phi(x) = \phi_0 + \frac{1}{\sqrt{2}}(\phi_1(x) + i\phi_2(x))$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 + \sqrt{2}e\phi_0 A_\mu\partial^\mu\phi_2 + \frac{1}{2}2e^2\phi_0^2 A_\mu A^\mu + \frac{1}{2}2\mu^2\phi_1^2 + \dots$$

omitting terms either linear or higher than second order in the fields. We find a mass term for the photon with $m_A^2 = 2e^2\phi_0^2$, a scalar particle with mass $m_{\phi_1}^2 = 2\mu^2$ and a massless Goldstone boson ϕ_2. However, there is a term coupling the photon to the Goldstone boson which contributes to the vacuum polarization of the photon

$$im_A^2 g^{\mu\nu} + (m_A k^\mu)\frac{i}{k^2}(m_A k^\nu) = im_A^2(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2})\,. \tag{2.19}$$

This is exactly the form of a propagator of a massive gauge boson that obeys the ward identity and so we find that the Goldstone boson has become part of the newly massive photon, a fact commonly being referred to as the 'Goldstone boson having been eaten by the photon'. While this has only been shown exemplary here this turns out to always happen in gauge theories with SSB and explains why in such theories no massless Goldstone bosons can be detected as independent particles.

There is also another way of thinking about the role of the Goldstone boson: As is known from classical electrodynamics, a massless vector boson has two transverse degrees of freedom (dofs). It is possible to construct two further formal dofs, commonly referred to as longitudinal and time-like dofs, but these do not contribute to any physically observable processes. Massive particles however, posses three dofs, two transverse and one longitudinal; an extremely naive way of thinking about this would be that a longitudinal dof requires a variation in the velocity of the particle along its axis of propagation which is impossible for massless particles which, by definition, move with exactly the speed of light. This also makes it impossible to introduce mass terms for gauge bosons in a naive fashion: if a vector boson has two dofs before introducing an explicit mass term and three afterwards we would need to account for the origin of this new dof. Again, the Higgs mechanism provides such an explanation through the absorption of the Goldstone bosons, which have exactly one scalar dof each, into the massive gauge bosons.

Summarizing, we find that there is no single reason necessitating the Higgs mechanism in the outlined form, but that it achieves several a priori independent properties that are desirable for a consistent theory:

- Introducing gauge invariant mass terms and thereby ensuring renormalizability.
- Explaining the absence of massless Goldstone bosons expected after SSB.
- Accounting for additional longitudinal degrees of freedom of massive gauge bosons.

2.5 The Standard Model

In the SM these mechanisms are applied in a way known as the electroweak unification: The electroweak sector has a $SU(2)\times U(1)$ gauge symmetry with the covariant derivative

$$D_\mu = \partial_\mu - ig'\, B_\mu Y - ig\, T^i W^i_\mu \tag{2.20}$$

with hypercharge Y and weak isospin T and a Higgs field Φ

$$\mathcal{L}_s = (D_\mu \Phi)^\dagger \, (D^\mu \Phi) - V(\Phi)\,, \tag{2.21}$$

and $V(\Phi)$ is the scalar potential

$$V(\Phi, \Sigma) = -\mu^2 |\Phi|^2 + \frac{\lambda}{2}|\Phi|^4\,. \tag{2.22}$$

Due to this form of the potential for $\mu^2,\ \lambda > 0$ the Higgs field acquires a non-zero vacuum expectation value v. Since a similar computation is done in the following sections we limit ourselves to merely stating that this leads to a massless photon γ and three massive gauge bosons, the $W^\pm$- and Z-bosons,

$$\begin{aligned}
\gamma &= \sin\theta_W W^3 + \cos\theta_W B \\
Z &= \cos\theta_W W^3 - \sin\theta_W B \\
W^\pm &= \frac{1}{\sqrt{2}}\left(W^1 \mp i W^2\right) \\
m_Z^2 &= \frac{(g^2 + g'^2)v^2}{4} \\
m_W^2 &= \frac{g^2 v^2}{4}
\end{aligned} \tag{2.23}$$

where the mixing between the W^3- and B-bosons is given by the Weinberg mixing angle $\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$.

2.6 Example: Gauge invariance and the meaning of charge

As a demonstration, we will now examine how gauge invariance determines the physical meaning of theoretical quantities at the example of conserved EM charge Q in the SM. For this consider a SM VEV rotated by some angle α

$$\langle\Phi_r\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{v}{\sqrt{2}}\begin{pmatrix} \sin\alpha \\ \cos\alpha \end{pmatrix} \tag{2.24}$$

with hypercharge $Y(\Phi_r) = 1/2$. Due to the Gell-Mann-Nishijima formula $Q = T_3 + Y$, one might assume on the first glance that this VEV has both an EM-neutral and EM-charged component, resulting in a presumably changed phenomenology.

Let us state right away that this hunch is unequivocally incorrect and that there is a straightforward way to see this: The rotation matrix above is obviously an element of $SU(2)$ - it has a determinant $= 1$ and is orthogonal and therefore unitary - and the SM is defined to be gauge invariant under $SU(2)$-transformations. In principle, no further computations are needed to know that this supposedly modified VEV will result in the known SM phenomenology. We will proceed anyway in the spirit of a pedagogical demonstration that will demonstrate the benefit of using the conventional form of the VEV[2].

Using the covariant derivative in Eq. (2.20) we compute the masses of the gauge bosons[3]

$$\begin{aligned}\mathcal{L}_s \supset \frac{1}{4}\langle\Phi_r\rangle^T &\left[g'^2 B_\mu B^\mu + g^2\left(W^1_\mu W^{1,\mu}\right.\right.\\ &+ \left. W^2_\mu W^{2,\mu} + W^3_\mu W^{3,\mu}\right) - 2gg'\left(W^1_\mu B^\mu \sigma^1\right.\\ &+ \left.\left. W^2_\mu B^\mu\sigma^2 + W^3_\mu B^\mu \sigma^3\right)\right]\langle\Phi_r\rangle\end{aligned} \tag{2.25}$$

A difference to the SM occurs for the product of the first Pauli matrix σ^1 with the VEV

$$\begin{aligned}\langle\Phi_r\rangle^T\sigma^1\langle\Phi_r\rangle &= 2\sin\alpha\cos\alpha\\ \langle\Phi_r\rangle^T\sigma^2\langle\Phi_r\rangle &= 0\\ \langle\Phi_r\rangle^T\sigma^3\langle\Phi_r\rangle &= \sin^2\alpha - \cos^2\alpha\end{aligned} \tag{2.26}$$

which is non-zero in general but is zero in the SM ($\alpha = 0$). Using the geometric identities $2\sin\alpha\cos\alpha = \sin 2\alpha \equiv s_\alpha$ and $\sin^2\alpha - \cos^2\alpha = -\cos 2\alpha \equiv -c_\alpha$ we can write the mass matrix

$$\frac{v^2}{4}\begin{pmatrix} g^2 & 0 & 0 & gg'\sin 2\alpha \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg'\cos 2\alpha \\ gg'\sin 2\alpha & 0 & -gg'\cos 2\alpha & g'^2 \end{pmatrix}. \tag{2.27}$$

By taking out the block-diagonal part of W^2 we get

$$\frac{v^2}{4}\begin{pmatrix} g^2 & 0 & gg'\sin 2\alpha \\ 0 & g^2 & -gg'\cos 2\alpha \\ gg'\sin 2\alpha & -gg'\cos 2\alpha & g'^2 \end{pmatrix}. \tag{2.28}$$

[2] to which $\langle\Phi_r\rangle$ will reduce for $\alpha = 0$

[3] Terms of the kind $W^i_\mu W^{j,\mu}$ cancel because of the anti-symmetric tensor in $\sigma^a\sigma^b = 2i\epsilon^{abc}\sigma^c$.

The eigenvalues of this matrix are, unsurprisingly, independent of α and accordingly the same as in the SM

$$\begin{aligned} m_X^2 &= \frac{g^2 v^2}{4} \\ m_Z^2 &= \frac{\left(g^2 + g'^2\right) v^2}{4} \\ m_A^2 &= 0 \end{aligned} \tag{2.29}$$

but the mass eigenstates are not

$$\begin{aligned} X_\mu &= \cos 2\alpha \, W_\mu^1 + \sin 2\alpha \, W_\mu^3 \\ Z_\mu &= \frac{1}{\sqrt{g^2 + g'^2}} \left(g \left(-\sin 2\alpha \, W_\mu^1 + \cos 2\alpha \, W_\mu^3 \right) - g' B_\mu \right) \\ A_\mu &= \frac{1}{\sqrt{g^2 + g'^2}} \left(g' \left(-\sin 2\alpha \, W_\mu^1 + \cos 2\alpha \, W_\mu^3 \right) + g B_\mu \right) . \end{aligned} \tag{2.30}$$

We identified these expressions with the Z-boson and photon of the SM and notice that everything is consistent for $\alpha = 0$. However, the interpretation of the X-boson and a useful way of building new $W^{\pm}$-bosons from it seem less straightforward.

2.6.1 Gell-Mann-Nishijima formula

In the SM the Gell-Mann-Nishijima formula $Q = T^3 + Y$ relates electric charge Q, isospin T^3 and hypercharge Y. One textbook approach to deriving this relation is to consider the effect of the four generators of $SU(2)_L \times U(1)_Y$ on the Higgs VEV $\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$:

$$\begin{aligned} T^1 \langle \Phi \rangle &\neq 0 \\ T^2 \langle \Phi \rangle &\neq 0 \\ \left(T^3 - Y \right) \langle \Phi \rangle &\neq 0 \\ \left(T^3 + Y \right) \langle \Phi \rangle &= 0 \end{aligned} \tag{2.31}$$

This illustrates that the Higgs VEV breaks the gauge group $SU(2)_L \times U(1)_Y \to U(1)_Q$ with conserved charge $Q = T^3 + Y$.

Applying the same procedure to the rotated VEV $\langle \Phi_r \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} \sin\alpha \\ \cos\alpha \end{pmatrix}$ we find

$$\begin{aligned} T^1 \langle \Phi_r \rangle &\neq 0 \\ T^2 \langle \Phi_r \rangle &\neq 0 \\ \left(T^3 - Y \right) \langle \Phi_r \rangle &\neq 0 \\ \left(T^3 + Y \right) \langle \Phi_r \rangle &= \begin{pmatrix} \frac{1}{2} + \frac{1}{2} & 0 \\ 0 & \frac{1}{2} - \frac{1}{2} \end{pmatrix} \langle \Phi_r \rangle \\ &= \frac{v}{\sqrt{2}} \begin{pmatrix} \sin\alpha \\ 0 \end{pmatrix} \neq 0 \end{aligned} \tag{2.32}$$

and so we find that the charge Q_r has to have some different form to still be conserved.

We now work out an expression for the electric charge Q_r. First we express the fields[4] in Eq. (**??**) in terms of the mass eigenstates

$$\begin{aligned} W^1_\mu &= \frac{-s_\alpha}{\sqrt{g^2+g'^2}}\left(g'A_\mu + gZ_\mu\right) + c_\alpha X_\mu \\ W^3_\mu &= \frac{c_\alpha}{\sqrt{g^2+g'^2}}\left(g'A_\mu + gZ_\mu\right) + s_\alpha X_\mu \\ B_\mu &= \frac{1}{\sqrt{g^2+g'^2}}\left(gA_\mu - g'Z_\mu\right) . \end{aligned} \tag{2.33}$$

By inserting these back into the covariant derivative in Eq. (3.2) we get the following expression where we can identify Q_r as the charge associated with the photon A_μ

$$\begin{aligned} D_\mu = \partial_\mu &- iA_\mu \frac{gg'}{\sqrt{g^2+g'^2}}\left(c_\alpha T_3 + Y - s_\alpha T_1\right) \\ &- iZ_\mu \frac{1}{\sqrt{g^2+g'^2}}\left(c_\alpha g^2 T_3 - g'^2 Y - s_\alpha g^2 T_1\right) \\ &- iX_\mu g\left(c_\alpha T_1 - s_\alpha T_3\right) \\ Q_r = &\left(c_\alpha T_3 + Y - s_\alpha T_1\right) . \end{aligned} \tag{2.34}$$

As a consistency check we work out how this generator acts on the VEV

$$\begin{aligned} Q_r\langle\Phi_r\rangle &= \begin{pmatrix} \frac{1}{2}\left(1+\cos 2\alpha\right) & -\frac{1}{2}\sin 2\alpha \\ -\frac{1}{2}\sin 2\alpha & \frac{1}{2}\left(1-\cos 2\alpha\right) \end{pmatrix}\langle\Phi_r\rangle \\ &= \frac{v}{\sqrt{2}}\begin{pmatrix} \cos^2\alpha & -\cos\alpha\sin\alpha \\ -\cos\alpha\sin\alpha & \sin^2\alpha \end{pmatrix}\begin{pmatrix} \sin\alpha \\ \cos\alpha \end{pmatrix} \\ &= 0 \end{aligned} \tag{2.35}$$

and find that this works out as we want it to. At this point it has become obvious that the original misconception about the charge of the vacuum arose because the concrete manifestation of the Gell-Mann-Nishijma formula depends on the form of the vacuum and is different for our non-conventional choice of the vacuum. Rather it turns out that the VEV breaking the electroweak symmetry is always EM-neutral and that the definition of EM charge adjusts to make sure that it stays so.

2.6.2 Fermions

We now consider the implications of this model on the fermions; nothing new is expected to happen here, it is merely a further example that the SM is reproduced in full. To start of, a word about which quantum numbers we put into the calculations: The isospin components of isospin doublets are given by $T_i = \frac{1}{2}\sigma_i\,,\ i = 1,2,3$ with the Pauli matrices σ_i. The weak hypercharge Y for each respective particle is taken to be as outlined in Table 2.1. No assumption about electric charge Q will be put into this calculation as it needs to be worked out according to Eq. (2.34).

[4]We keep disregarding W^2 because nothing interesting happens there.

particle	hypercharge Y LH chirality	RH chirality
ν_e, ν_μ, ν_τ	$-\frac{1}{2}$	0
e, μ, τ	$-\frac{1}{2}$	-1
u, c, t	$+\frac{1}{6}$	$+\frac{2}{3}$
d, s, b	$+\frac{1}{6}$	$-\frac{1}{3}$

Table 2.1: Weak hypercharge assignments

We start by looking at a Yukawa-term of a lepton generation l

$$\begin{aligned}\mathcal{L} &\supset -y_l \overline{l_L} \Psi l_R + h.c. \\ &= -y_l \left(\overline{\nu_l}\,,\, l_L^+\right) \frac{v}{\sqrt{2}} \begin{pmatrix} \sin\alpha \\ \cos\alpha \end{pmatrix} l_R^- \\ &= -y_l \frac{v}{\sqrt{2}} \left(\sin\alpha\, \overline{\nu_l} + \cos\alpha\, l_L^+\right) l_R^- \\ &\equiv -y_l \frac{v}{\sqrt{2}} \widetilde{l_L^+} l_R^-\end{aligned} \tag{2.36}$$

and find that this would introduce a mixing between the charged lepton and respective neutrino of each generation. By defining the mass eigenstates of the lepton $\widetilde{l_L^+}$ and the orthogonal mass eigenstate of the neutrino $\widetilde{\overline{\nu_l}}$

$$\begin{aligned}\widetilde{l_L^+} &= \sin\alpha\, \overline{\nu_l} + \cos\alpha\, l_L^+ \\ \widetilde{\overline{\nu_l}} &= \cos\alpha\, \overline{\nu_l} - \sin\alpha\, l_L^+\end{aligned} \tag{2.37}$$

one avoids this issue, but we will have to see whether these are also the interaction eigenstates.

The term for the fermion-photon interaction according to the covariant derivative in Eq. (2.34) is

$$\begin{aligned}\mathcal{L} &\supset e\gamma^\mu A_\mu \overline{l_L} Q_r l_L \\ &= e\gamma^\mu A_\mu \left(\overline{\nu_l}\,,\, l_L^+\right) \begin{pmatrix} \frac{1}{2}c_\alpha + Y & -\frac{1}{2}s_\alpha \\ -\frac{1}{2}s_\alpha & -\frac{1}{2}c_\alpha + Y \end{pmatrix} \begin{pmatrix} \nu_l \\ l_L^+ \end{pmatrix}.\end{aligned} \tag{2.38}$$

The eigenvalues of this matrix are $q_1 = Y + \frac{1}{2}$ and $q_2 = Y - \frac{1}{2}$, which is consistent with the SM, and the corresponding eigenstates are those already found in Eq. (2.37). Using Table 2.1 as input, we find that although the eigenstates are defined differently, the charge eigenvalues work out as zero for neutrinos and integers for the charged leptons.

For the interaction with the Z-boson we have

$$\begin{aligned}\mathcal{L} &\supset \frac{1}{\sqrt{g^2 + g'^2}} \gamma^\mu Z_\mu \overline{l_L} \left(c_\alpha g^2 T_3 - g'^2 Y - s_\alpha g^2 T_1\right) l_L \\ &= \frac{\gamma^\mu Z_\mu}{\sqrt{g^2 + g'^2}} \left(\overline{\nu_l}\,,\, l_L^+\right) \begin{pmatrix} \frac{g^2}{2}c_\alpha - g'^2 Y & -\frac{g^2}{2}s_\alpha \\ -\frac{g^2}{2}s_\alpha & -\frac{g^2}{2}c_\alpha - g'^2 Y \end{pmatrix} \begin{pmatrix} \nu_l \\ l_L^+ \end{pmatrix}.\end{aligned} \tag{2.39}$$

The eigenvalues for this interaction are $\lambda_{1,2} = -g'^2 Y \pm \frac{1}{2}g^2$ in which we again find the SM coupling to the Z-boson $\lambda = g^2 T_3 - g'^2 Y$. The corresponding eigenstates are again those of Eq. (2.37).

At this point we will leave it, satisfied that gauge invariance also holds up to explicit computation. One could argue that there is little to interpret here since we recovered exactly the textbook result we were bound to find by the very definition of the SM, but at the very least there are some lessons here that bear repeating:

The misconception that started this section was that the *rotated* VEV $\langle \Phi_r \rangle$ would result in a charged vacuum. The fact that it is neutral regardless of its concrete form is an expression of the central role that gauge invariance plays in the SM; the concrete form is irrelevant to the physical content of the theory, but the way in which it is relevant is of crucial importance. At the same time it is not so obvious why gauge invariance should play such a central role (e.g. in comparison to locality, whose importance as a concept for physics is directly plausible); there is philosophical literature that expands on this question further than is necessary for our purposes [10].

There is however a scenario in which this notion carries more meaning: In models with multiple particles with VEVs, most prominently 2-Higgs-Doublet-models (2HDM) [7], a charged VEV can occur. Most of these models assume that a SM Higgs particle H_1 has a VEV of the familiar SM form, which "fixes" the definition of electric charge and then there, among multiple others, the following possibilities:

1. The second VEV is EM-neutral and EM-charge is conserved in the model

$$\langle \Phi_{H_1} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} , \quad \langle \Phi_{H_2} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix} . \tag{2.40}$$

2. The second VEV is not EM-neutral and the conservation of EM-charge is violated in the resulting model

$$\langle \Phi_{H_1} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} , \quad \langle \Phi_{H_2} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 \\ 0 \end{pmatrix} . \tag{2.41}$$

While it is possible to determine whether the second VEV is neutral or not here, this determination of course still depends on the form of the first VEV. It is not the absolute form of the second VEV that is relevant but relative form in comparison to the first (again manifesting the gauge principle).

Chapter 3

Charge-breaking Higgs mutiplets

3.1 Theory setup

We will consider theories that break conservation of EM-charge through the introduction of an additional scalar Higgs field with non-zero charged VEV that breaks the $U(1)_Q$-symmetry. These theories will modify the scalar part of the SM Lagrangian as follows

$$\mathcal{L}_s = (D_\mu \Phi_h)^\dagger (D^\mu \Phi_h) + (D_\mu \Phi_i)^\dagger (D^\mu \Phi_i) - V(\Phi_h, \Phi_i)\,, \tag{3.1}$$

where D_μ is the covariant derivative with hypercharge Y and weak isospin T

$$D_\mu = \partial_\mu - ig'\, B_\mu Y - ig\, T^i W^i_\mu \tag{3.2}$$

and $V(\Phi, \Sigma)$ is the scalar potential

$$\begin{aligned} V(\Phi, \Sigma) = &- \mu_h^2 |\Phi_h|^2 + \frac{\lambda_h}{2} |\Phi_h|^4 \\ &- \mu_s^2 |\Phi_i|^2 + \frac{\lambda_s}{2} |\Phi_i|^4 + \lambda_P |\Phi_h|^2 |\Phi_i|^2\,. \end{aligned} \tag{3.3}$$

Φ_h is the SM Higgs doublet with $Y = 1/2$ and Φ_i with $i = s, d, t$ is the additional scalar Higgs field which is either an isospin singlet, doublet or triplet (respectively denoted by Φ_s, Φ_d or Φ_t) and which carries hypercharge Y_i (we will impose a constraint on Y_i on each realization respectively s.t. the VEV never ends up to be EM-neutral). The parameters of the potential are assumed to be $\mu_h^2, \lambda_h, \mu_i^2, \lambda_i > 0$ while there is no such constraint on λ_P. We demand both Higgs fields to have non-zero VEVs v_h, $v_i \neq 0$ however the values of v_h and v_i depend on each other due to the λ_P-term in Eq. 3.3. We find this by minimizing the potential

$$
\begin{aligned}
0 &= \left.\frac{\partial V}{\partial|\Phi_{h,i}|}\right|_{|\Phi_{h,i}|=\frac{v_{h,i}}{\sqrt{2}}} \\
&= \lambda_{h,i}\left(\frac{v_{h,i}}{\sqrt{2}}\right)^3 + \left(\lambda_P|\Phi_{i,h}|^2 - \mu_{h,i}^2\right)\frac{v_{h,i}}{\sqrt{2}} \\
&= \lambda_{h,i}\left(\frac{v_{h,i}}{\sqrt{2}}\right)^2 + \lambda_P|\Phi_{i,h}|^2 - \mu_{h,i}^2 \\
v_{h,i} &= \sqrt{\frac{2\mu_{h,i}^2 - \lambda_P|v_{i,h}|^2}{\lambda_{h,i}}}
\end{aligned}
$$

where we used $|\Phi_{i,h}|^2 = \frac{v_{i,h}^2}{2}$ in the last step. By inserting this expression for i into the one for h (and vice versa) we find an expression for the VEVs depending only on parameters of the Lagrangian

$$
v_{h,i} = \sqrt{\frac{2\lambda_{i,h}\mu_{h,i}^2 - 2\lambda_P\mu_{i,h}^2}{\lambda_{h,i}\lambda_{i,h} - \lambda_P^2}}\,. \tag{3.4}
$$

We also find that this can be rewritten as

$$
\begin{aligned}
&\frac{1}{2}\lambda_{h,i}v_{h,i}^2 - \mu_{h,i}^2 + \frac{1}{2}\lambda_P v_{i,h}^2 \\
&= \frac{\lambda_{h,i}\lambda_{i,h}\mu_{h,i}^2 - \lambda_{h,i}\lambda_P\mu_{i,h}^2 - \mu_{h,i}^2(\lambda_{h,i}\lambda_{i,h} - \lambda_P^2) - \lambda_P\lambda_{h,i}\mu_{i,h}^2 - \lambda_P^2\mu_{h,i}^2}{\lambda_{h,i}\lambda_{i,h} - \lambda_P^2} \\
&= 0
\end{aligned} \tag{3.5}
$$

which will turn out to be useful later on.

3.2 Higgs singlet

We consider a model containing the SM Higgs doublet Φ_h with $Y = 1/2$ and an additional complex singlet Φ_s with $Y = Y_s \neq 0$ with non-zero minimum

$$
\langle\Phi_s\rangle = \frac{1}{\sqrt{2}}\,v_s\,. \tag{3.6}
$$

We expand the two Higgs fields Φ_i around their respective VEVs v_i

$$
\Phi_h = \begin{pmatrix} \phi_h^+ \\ \frac{1}{\sqrt{2}}(v_h + \rho_h + i\eta_h) \end{pmatrix}, \Phi_s = \frac{1}{\sqrt{2}}(v_s + \rho_s + i\eta_s)\,. \tag{3.7}
$$

where $\phi_h^\pm$ is a complex charged field with $\left(\phi_h^\pm\right)^* = \phi_h^\mp$, $v_{h,s} = const. \in \mathbb{R} \neq 0$ and $\rho_{h,s}$ and $\eta_{h,s}$ are real fields.

3.2.1 Gauge bosons

First, we consider gauge boson masses. The term $(D_\mu \Phi_h)^\dagger (D^\mu \Phi_h)$ gives the same contributions as in the SM but new terms arise from

$$(D_\mu \Phi_s)^\dagger (D^\mu \Phi_s) = \frac{1}{2}\partial_\mu \rho_s \partial^\mu \rho_s - \frac{1}{2}\partial_\mu \eta_s \partial^\mu \eta_s + g'^2 Y_s^2 B_\mu B^\mu (v_s^2 + \rho_s^2 + \eta_s^2) \\ - 2\left(i\partial_\mu \rho_s \eta_s + \partial_\mu \eta_s (v_s + \rho_s)\right) Y_s B^\mu \,. \quad (3.8)$$

There are no modifications to $W^\pm$-boson mass, which is plausible since they are associated with isospin and the new Higgs field is an isospin singlet. There is however an additional B-boson mass term and due to the mixture with W^3 we need to consider a modified mass matrix

$$\begin{pmatrix} \frac{v^2}{4} g^2 & -\frac{v^2}{4} g g' \\ -\frac{v^2}{4} g g' & \left(\frac{v^2}{4} + v_s^2 Y_s^2\right) g'^2 \end{pmatrix} \quad (3.9)$$

which has the following eigenvalues

$$m_{Z,\gamma}^2 = \frac{1}{2}\left(\frac{v_h^2}{4}(g^2 + g'^2) + Y_s^2 v_s^2 g'^2 \pm \sqrt{\left(\frac{v_h^2}{4}(g^2 + g'^2) + Y_s^2 v_s^2 g'^2\right)^2 - g^2 g'^2 v_h^2 Y_s^2 v_s^2}\right) \quad (3.10)$$

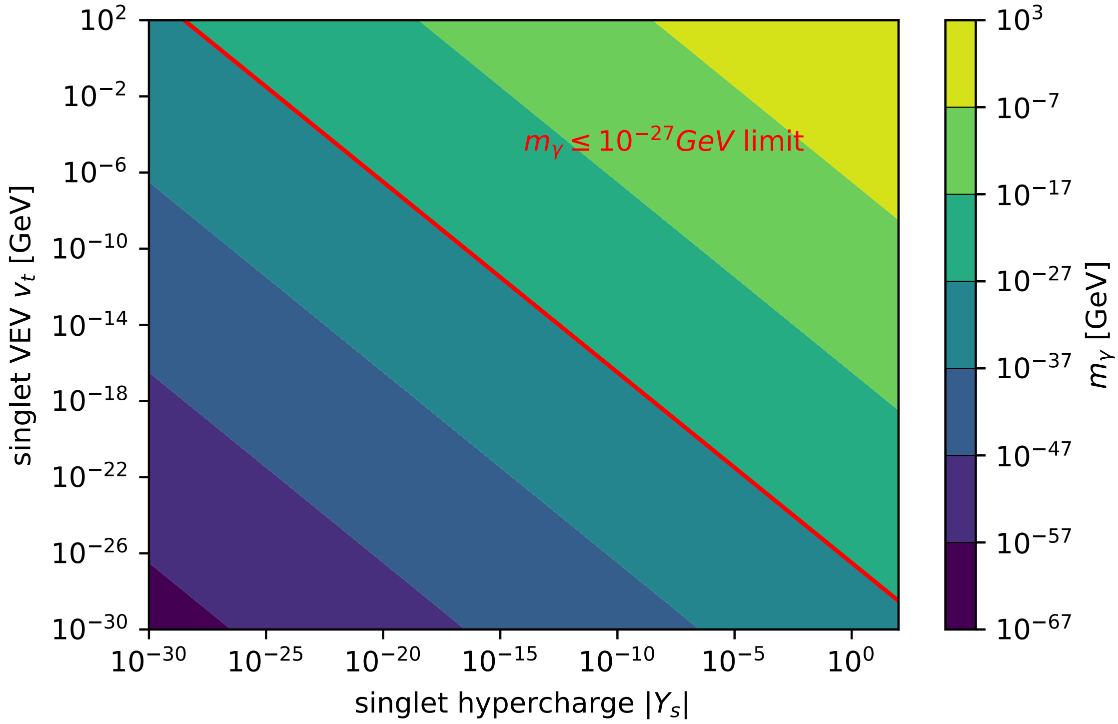


Figure 3.1: Photon mass m_γ in the singlet model, using Eq. 3.10 and assuming $v_h = v_{SM}$.

both of which are non-zero for $v_h, Y_s v_s \neq 0$. This means we have a photon with a non-zero rest mass upon which there is a stringent limit of $m_\gamma \leq 10^{-27} GeV$. Fig. 3.1

shows the photon mass as a function of v_s and Y_s and we see that it translates to a limit of $Y_s v_s \leq 10^{-27} GeV$ on our parameters[1]. Since the $W^{\pm}$-boson mass is not modified we know that $v_h = v_{SM} \approx 246\, GeV$ and so we can expand this result to first order in $\frac{Y_s^2 v_t^2}{v_h^2} \ll 1$

$$
\begin{aligned}
m_Z^2 &= \frac{v^2}{4}(g^2 + g'^2) + v_s^2 Y_s^2 g'^2 s_w^2 \\
m_A^2 &= v_s^2 Y_s^2 g'^2 c_w^2 \,.
\end{aligned}
\tag{3.11}
$$

where the effect of the newly introduced singlet becomes visible as a perturbation to the SM result.

3.2.2 Higgs sector

We write the potential

$$
\begin{aligned}
V(\Phi, \Sigma) = &- \mu_h^2 |\Phi_h|^2 + \frac{\lambda_h}{2} |\Phi_h|^4 \\
&- \mu_s^2 |\Phi_s|^2 + \frac{\lambda_s}{2} |\Phi_s|^4 + \lambda_P |\Phi_h|^2 |\Phi_s|^2 \,.
\end{aligned}
\tag{3.12}
$$

out in terms of the fields introduced in Eq. 3.7

$$
\begin{aligned}
V = &-\mu_h^2 [\phi_h^- \phi_h^+ + \frac{1}{2}(v_h^2 + \rho_h^2 + \eta_h^2 + 2 v_h \rho_h)] \\
&+ \frac{\lambda_h}{2} [\phi_h^{-2} \phi_h^{+2} + \phi_h^- \phi_h^+ (v_h^2 + \rho_h^2 + \eta_h^2 + 2 v_h \phi_h) \\
&+ \frac{1}{4}(v_h^4 + \rho_h^4 + \eta_h^4 + 4 v_h^2 \rho_h^2 + 2 v_h^2 \rho_h^2 + 2 v_h^2 \eta_h^2 + 4 v_h^3 \rho_h + 2 \rho_h^2 \eta_h^2 + 4 v_h \rho_h^3 + 4 \eta_h^2 v_h \rho_h)] \\
&- \mu_s^2 [\frac{1}{2}(v_s^2 + \rho_s^2 + \eta_s^2 + 2 v_s \rho_s)] \\
&+ \frac{\lambda_s}{2} \frac{1}{4} [v_s^4 + \rho_s^4 + \eta_s^4 + 4 v_s^2 \rho_s^2 + 2 v_s^2 \rho_s^2 + 2 v_s^2 \eta_s^2 + 4 v_s^3 \rho_s + 2 \rho_s^2 \eta_s^2 + 4 v_s \rho_s^3 + 4 \eta_s^2 v_s \rho_s] \\
&+ \lambda_P [\phi_h^- \phi_h^+ \frac{1}{2}(v_s^2 + \rho_s^2 + \eta_s^2 + 2 v_s \phi_s) \\
&+ \frac{1}{4}(v_h^2 v_s^2 + \rho_h^2 \rho_s^2 + \eta_h^2 \eta_s^2 + 4 v_h v_s \rho_h \rho_s + v_h^2 \rho_s^2 + v_s^2 \rho_h^2 + 2 v_h^2 v_s \rho_s + 2 v_s^2 v_h \rho_h \\
&\quad + v_h^2 \eta_s^2 + v_s^2 \eta_h^2 + 2 \rho_h^2 v_s \rho_s + 2 \rho_s^2 v_h \rho_h + \rho_h^2 \eta_s^2 + \rho_s^2 \eta_h^2 + 2 \rho_h v_h \eta_s^2 + 2 \rho_s v_s \eta_h^2)] \,.
\end{aligned}
$$

From this we drop all constant terms (i.e. terms containing only v) because they are irrelevant in a potential and we also drop all terms which are linear in any field (ϕ, ρ, η), because they can be transformed away. We sort the second order terms and do not care about higher orders

[1]Note that while $v_s \geq 0$ by definition, Y_s can in principle also be $Y_s < 0$. However since only Y_s^2 contributes to measurable quantities it is sufficient to consider $|Y_s| \geq 0$.

$$V = \frac{1}{2}\rho_h^2[\frac{3}{2}\lambda_h v_h^2 - \mu_h^2 + \frac{1}{2}\lambda_P v_s^2] + \frac{1}{2}\rho_s^2[\frac{3}{2}\lambda_s v_s^2 - \mu_s^2 + \frac{1}{2}\lambda_P v_h^2] + \frac{1}{2}\rho_h\rho_s\lambda_P v_h v_p$$
$$+ \frac{1}{2}\eta_h^2[\frac{1}{2}\lambda_h v_h^2 - \mu_h^2 + \frac{1}{2}\lambda_P v_s^2] + \frac{1}{2}\eta_s^2[\frac{1}{2}\lambda_s v_s^2 - \mu_s^2 + \frac{1}{2}\lambda_P v_h^2]$$
$$+ \phi_h^-\phi_h^+(\frac{1}{2}\lambda_h v_h^2 - \mu_h^2 + \frac{1}{2}\lambda_P v_d^2) + \mathcal{O}(fields^3)\,. \quad (3.13)$$

Here we can identify the mass terms of the fields which - through application of Eq. 3.5 - turn out to be

$$\begin{aligned} m^2_{\phi_h^\pm} &= 0 \\ m^2_{\eta_{h,s}} &= 0 \\ \frac{1}{2}\left(\rho_h\,,\rho_s\right) &\begin{pmatrix} \lambda_h v_h^2 & \lambda_P v_h v_s \\ \lambda_P v_h v_s & \lambda_s v_s^2 \end{pmatrix} \begin{pmatrix} \rho_h \\ \rho_s \end{pmatrix}. \end{aligned} \quad (3.14)$$

It is not surprising that we end up with four massless particle. This variant of the Higgs mechanism has given mass to four previous massless gauge bosons and according to the Goldstone theorem this necessitates the existence of four massless Goldstone bosons, which can commonly be thought to make up the longitudinal degrees of freedom (dofs) of the massive gauge bosons. We also find two massive real scalar fields ρ_h and ρ_s which mix into mass eigenstates we will label H_1 and H_2.

$$m^2_{H_1,H_2} = \frac{1}{2}\left(\lambda_h v_h^2 + \lambda_s v_s^2 \pm \sqrt{(\lambda_h v_h^2 + \lambda_s v_s^2)^2 - 4\lambda_P^2 v_h^2 v_s^2}\right) \quad (3.15)$$

3.3 Higgs doublet

Secondly, we consider a model containing the SM Higgs doublet Φ_h with $Y = 1/2$ and an additional complex doublet Φ_d with $Y = Y_s \neq -\frac{1}{2}$ with non-zero minimum

$$\langle\Phi_d\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_d \\ 0 \end{pmatrix}. \quad (3.16)$$

We expand the two Higgs fields around their respective VEVs

$$\Phi_h = \begin{pmatrix} \phi_h^+ \\ \frac{1}{\sqrt{2}}(v_h + \rho_h + i\eta_h) \end{pmatrix}, \Phi_d = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + \rho_d + i\eta_d) \\ \phi_d^- \end{pmatrix}. \quad (3.17)$$

where $\phi_{h,d}^\pm$ are complex charged fields with $\left(\phi_{h,d}^\pm\right)^* = \phi_{h,d}^\mp$, $v_{h,d} = const. \in \mathbb{R} \neq 0$ and $\rho_{h,d}$ and $\eta_{h,d}$ are real fields.

3.3.1 Gauge bosons

For our computation of gauge boson masses we only consider terms containing the gauge boson fields B or W^i and the VEVs v_h and v_d. There are many other terms governing the interaction with goldstone bosons which we omit for now.

$$
\begin{aligned}
&(D_\mu \Phi_h)^\dagger (D^\mu \Phi_h) + (D_\mu \Phi_d)^\dagger (D^\mu \Phi_d) \\
&\qquad = \frac{1}{2}\left(\frac{g^2 (v_h^2 + v_d^2)}{4} \left(W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu} \right) + \frac{g^2 \left(v_h^2 + v_d^2 \right)}{4} W_\mu^3 W^{3\mu} \right. \\
&\qquad \left. + \frac{g'^2}{4} \left(v_h^2 + 4 v_d^2 Y_d^2 \right) B_\mu B^\mu - \frac{1}{2} g g' \left(v_h^2 - 2 Y_d v_d^2 \right) B_\mu W^{3\mu} \right) .
\end{aligned} \tag{3.18}
$$

From this we can read the $W^\pm$-eigenstates and -masses as

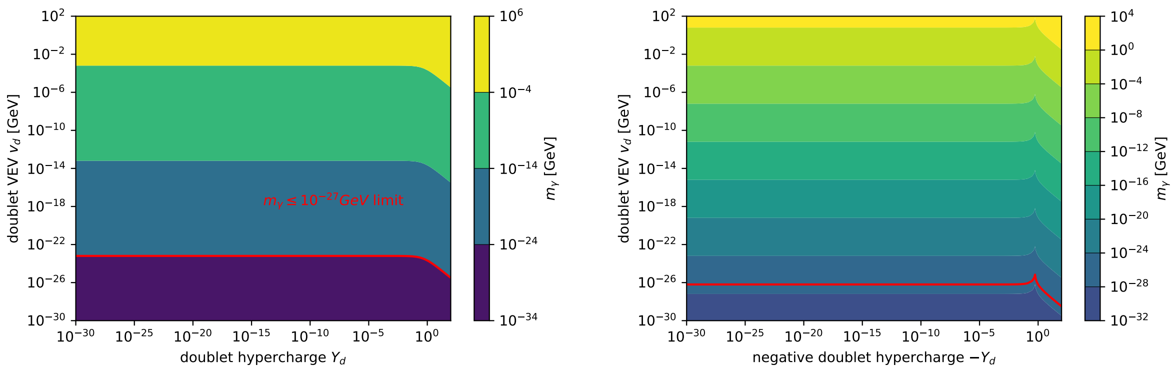
$$
\begin{aligned}
W_\mu^\pm &= \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp i W_\mu^2 \right) \\
m_{W^\pm} &= \frac{g}{2} \sqrt{v_h^2 + v_d^2}
\end{aligned} \tag{3.19}
$$

and we get a mixed mass matrix for W^3- and B-bosons

$$
\frac{1}{4} \begin{pmatrix} g^2 \left(v_h^2 + v_d^2 \right) & -g g' \left(v_h^2 - 2 Y_d v_d^2 \right) \\ -g g' \left(v_h^2 - 2 Y_d v_d^2 \right) & g'^2 \left(v_h^2 + 4 Y_d^2 v_d^2 \right) \end{pmatrix} \tag{3.20}
$$

with eigenvalues

$$
\begin{aligned}
m_{Z,\gamma}^2 = \frac{1}{2} &\left(\frac{v_h^2}{4} (g^2 + g'^2) + \frac{v_d^2}{4} (g^2 + 4 Y_d^2 g'^2) \right. \\
&\left. \pm \sqrt{ \left(\frac{v_h^2}{4} (g^2 + g'^2) + \frac{v_d^2}{4} (g^2 + 4 Y_d^2 g'^2) \right)^2 - 4 v_h^2 v_d^2 g^2 g'^2 (1 + 2 Y_d)^2 } \right) .
\end{aligned} \tag{3.21}
$$



(a) Hypercharge $Y_d > 0$

(b) Hypercharge $Y_d < 0$.

Figure 3.2: Photon mass m_γ in the doublet model, using Eq. 3.21 and setting $v_{SM}^2 = v_h^2 + v_d^2$. Note that the pole at $Y_d = \frac{1}{2}$ is not properly visible due to lack of resolution.

From this we can again consider the phenomenological constraints placed on our model. One important difference to the singlet model is that we have a dependence on $(1 + 2Y_d)^2$ in the radicand and not on Y_d^2. For one, we see that replicate the SM gauge boson masses for $Y_d = \frac{1}{2}$ - regardless of choice of v_d - which is why we excluded this from the beginning. But the behavior is also different for $Y_d < 0$ and $Y_d > 0$, meaning it is not sufficient to just consider $|Y_d|$ here.

For all our follow considerations we will consider the VEVs related by $v_{SM}^2 = v_h^2 + v_d^2$ due to Eq. 3.19. Due to the experimental uncertainty of the W-boson mass it is a simplification to take this relation as exact, but as we will see it would not make a relevant difference on the scales that are viable for this model. Assuming this, the photon mass according to Eq. 3.21 is plotted in Fig. 3.2, once for positive and once for negative Y_d. This allows us to limit $v_d \leq 10^{-23} GeV$ for almost any choice of Y_d except for a small region around $Y_d = -\frac{1}{2}$. While we excluded the pole itself from the beginning, there is nothing keeping us from choosing Y_d arbitrarily close to the pole, so we need another limit for this regime

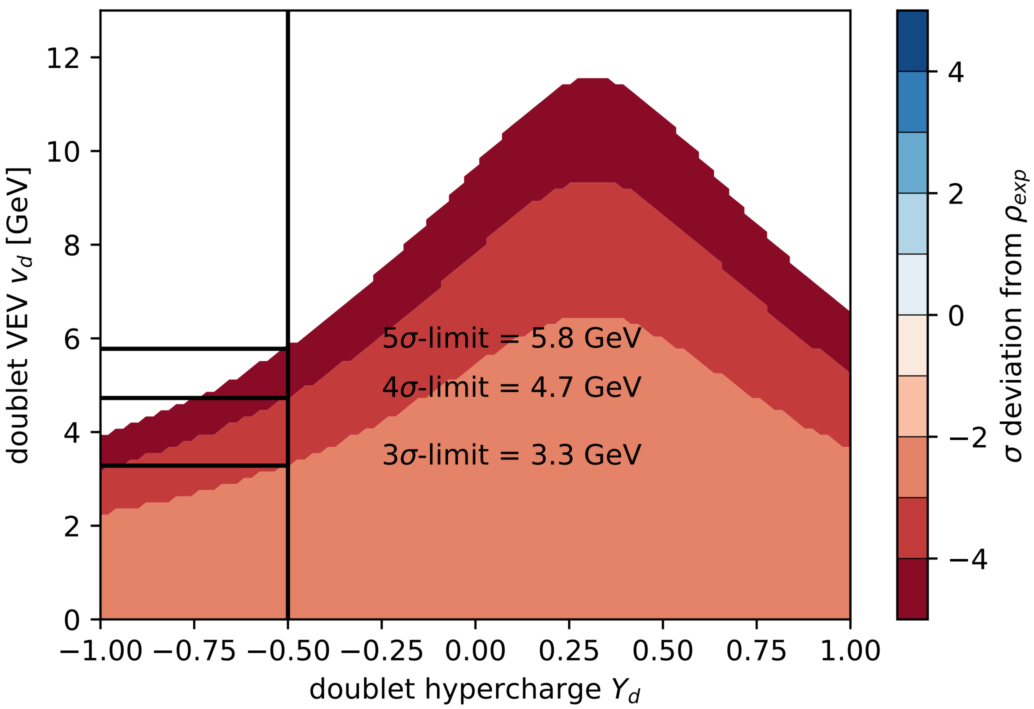


Figure 3.3: Limits from ρ-parameter in the doublet model.

But another, independent limit can be derived from the ρ-parameter

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} \tag{3.22}$$

which is measured to high accuracy $\rho_{exp} = 1.00039(19)$. Using Eq. 3.19 and Eq. 3.21 we can compute the ρ-factor of our model as a function of v_d and Y_d (again setting $v_{SM}^2 = v_h^2 + v_d^2$). In Fig. 3.3 we see that this approach in general delivers weaker limits (only $v_d \lesssim$ few GeV), however it does not have any divergent behavior for $Y_d = -\frac{1}{2}$.

Even using the weaker limit $v_d \leq 5.8 GeV$ we can expand Eq. 3.21 to first order in

$\frac{v_d^2}{v_h^2} \ll 1$. For clarity consider the general form

$$\begin{aligned}
&\sqrt{a+bx+cx^2} \rightarrow \sqrt{a} + \frac{b}{2\sqrt{a}}x + \mathcal{O}(x^2) \\
\Rightarrow \ &\frac{v_h^2}{4}(g^2+g'^2)[\dots]^{\frac{1}{2}} = \frac{v_h^2}{4}(g^2+g'^2) + \frac{v_d^2}{4}(g^2+4Y_d^2g'^2) - 2\frac{v_d^2}{4}\frac{g^2g'^2}{g^2+g'^2}(1+2Y_d)^2
\end{aligned} \tag{3.23}$$

and so we can identify a modified Z- and non-zero photon mass in a way that looks more familiar to the SM

$$\begin{aligned}
m_Z^2 &= \frac{v_h^2}{4}(g^2+g'^2) + \frac{v_d^2}{4}(g^2+4Y_d^2g'^2) - \frac{v_d^2}{4}\frac{g^2g'^2}{g^2+g'^2}(1+2Y_d)^2 \\
m_\gamma^2 &= \frac{v_d^2}{4}\frac{g^2g'^2}{g^2+g'^2}(1+2Y_d)^2 \,.
\end{aligned} \tag{3.24}$$

3.3.2 Higgs sector

We write out the potential again:

$$\begin{aligned}
V = &-\mu_h^2[\phi_h^-\phi_h^+ + \frac{1}{2}(v_h^2+\rho_h^2+\eta_h^2+2v_h\rho_h)] \\
&+ \frac{\lambda_h}{2}[\phi_h^{-2}\phi_h^{+2} + \phi_h^-\phi_h^+(v_h^2+\rho_h^2+\eta_h^2+2v_h\phi_h) \\
&+ \frac{1}{4}(v_h^4+\rho_h^4+\eta_h^4+4v_h^2\rho_h^2+2v_h^2\rho_h^2+2v_h^2\eta_h^2+4v_h^3\rho_h+2\rho_h^2\eta_h^2+4v_h\rho_h^3+4\eta_h^2v_h\rho_h)] \\
&- \mu_d^2[\phi_d^-\phi_d^+ + \frac{1}{2}(v_d^2+\rho_d^2+\eta_d^2+2v_d\rho_d)] \\
&+ \frac{\lambda_d}{2}[\phi_d^{-2}\phi_d^{+2} + \phi_d^-\phi_d^+(v_d^2+\rho_d^2+\eta_d^2+2v_d\phi_d) \\
&+ \frac{1}{4}(v_d^4+\rho_d^4+\eta_d^4+4v_d^2\rho_d^2+2v_d^2\rho_d^2+2v_d^2\eta_d^2+4v_d^3\rho_d+2\rho_d^2\eta_d^2+4v_h\rho_d^3+4\eta_d^2v_d\rho_d)] \\
&+ \lambda_P[\phi_h^-\phi_h^+\phi_d^-\phi_d^+ + \phi_h^-\phi_h^+\frac{1}{2}(v_d^2+\rho_d^2+\eta_d^2+2v_d\phi_d) + \phi_d^-\phi_d^+\frac{1}{2}(v_h^2+\rho_h^2+\eta_h^2+2v_h\phi_h) \\
&+ \frac{1}{4}(v_h^2v_d^2+\rho_h^2\rho_d^2+\eta_h^2\eta_d^2+4v_hv_d\rho_h\rho_d+v_h^2\rho_d^2+v_h^2\rho_d^2+2v_h^2v_d\rho_d+\rho_h^2v_d^2+\rho_h^2\eta_d^2 \\
&+ 2\rho_h^2v_d\rho_d+\rho_h^2v_d^2+\rho_h^2\rho_d^2+2\eta_h^2v_d\rho_d+2v_h\rho_h\rho_d^2+2v_h\rho_h\eta_d^2)]
\end{aligned} \tag{3.25}$$

As before we consider only the 2nd order terms of fields for contributions to the masses of scalar particles

$$\begin{aligned}
V = &\frac{1}{2}\rho_h^2[\frac{3}{2}\lambda_hv_h^2 - \mu_h^2 + \frac{1}{2}\lambda_Pv_d^2] + \frac{1}{2}\rho_d^2[\frac{3}{2}\lambda_dv_d^2 - \mu_d^2 + \frac{1}{2}\lambda_Pv_d^2] \\
&+ \frac{1}{2}\eta_h^2[\frac{1}{2}\lambda_hv_h^2 - \mu_h^2 + \frac{1}{2}\lambda_Pv_d^2] + \frac{1}{2}\eta_d^2[\frac{1}{2}\lambda_dv_d^2 - \mu_d^2 + \frac{1}{2}\lambda_Pv_d^2] \\
&+ \phi_h^-\phi_h^+(\frac{1}{2}\lambda_hv_h^2 - \mu_h^2 + \frac{1}{2}\lambda_Pv_d^2) + \phi_h^-\phi_h^+(\frac{1}{2}\lambda_hv_h^2 - \mu_h^2 + \frac{1}{2}\lambda_Pv_d^2) \\
&+ \frac{1}{2}2\lambda_Pv_hv_p\rho_h\rho_d + \mathcal{O}(fields^3)
\end{aligned} \tag{3.26}$$

Through the same procedure as for the singlet model, especially application of Eq. 3.5, we find the masses to be

$$\begin{aligned}
m^2_{\phi^{\pm}_{h,d}} &= 0 \\
m^2_{\eta_{h,d}} &= 0 \\
m^2_{H_1,H_2} &= \frac{1}{2}\left(\lambda_h v_h^2 + \lambda_d v_d^2 \pm \sqrt{(\lambda_h v_h^2 + \lambda_d v_d^2)^2 - 4\lambda_P^2 v_h^2 v_d^2}\right)
\end{aligned} \tag{3.27}$$

where H_1 and H_2 are again mass eigenstates of ρ_h and ρ_d. By setting $v_d = 0$ it becomes clear that H_1 is the SM Higgs boson. Since we have more constraints here than in the Singlet model, we can consider what range of mass for H_2 is phenomenologically viable here:

At a fundamental level we have five parameters in Eq. 3.27, which are, in principle, free: μ_h^2, μ_d^2 λ_h, λ_h and λ_P. Two of these, λ_h and λ_P, can be constrained through model-independent experimental measurements:

- $\lambda_P \leq 10^{-2}$ according to [4].
- $\kappa_{\lambda_h} \in [-1.0, 6.6]$ according to [5], where κ_{λ_h} is defined as $\lambda_h = \kappa_{\lambda_h}\lambda_{SM}$ with $\lambda_{SM} = \frac{m_H^2}{v_{SM}^2}$. Since we chose $\lambda_h > 0$ by construction this effectively reduces to $\kappa_{\lambda_h} \in [0, 6.6]$ for our purposes.

There are further constraints which affect products of several fundamental parameters: The mass of the physical Higgs boson must be reproduced, so $m_{H_1} = m_{H,SM} = 125.25(17)\, GeV$ [13] according to Eq. 3.27, as well as the previously derived limits on the VEVs, namely $v_d \leq 5.8\, GeV$ and $v_h = \sqrt{v_{SM}^2 - v_d^2} \approx 246\, GeV$ according to Eq. 3.4. For given values of λ_P and κ_{λ_h} we thus have a system of three non-linear equations with three unknowns

$$\begin{aligned}
\sqrt{\frac{2\lambda_d \mu_h^2 - 2(\lambda_P + \lambda_B)\mu_d^2}{\lambda_d \lambda_h - (\lambda_P + \lambda_B)^2}} - v_h &= 0 \\
\sqrt{\frac{2\lambda_h \mu_d^2 - 2(\lambda_P + \lambda_B)\mu_h^2}{\lambda_d \lambda_h - (\lambda_P + \lambda_B)^2}} - v_d &= 0 \\
\frac{1}{2}\left(\lambda_h v_h^2 + \lambda_d v_d^2 + \sqrt{(\lambda_h v_h^2 + \lambda_d v_d^2)^2 - \frac{1}{4}\lambda_P^2 v_h^2 v_d^2}\right) - m_H &= 0\,.
\end{aligned} \tag{3.28}$$

We are interested in what the highest possible mass of H_2 could be, so we consider the case of the highest possible doublet VEV. The numerical solution is displayed in Fig. 3.4 and we find that the highest possible mass is $m_{H_2} = 1.85$ GeV.

It is maybe somewhat surprising that all components of both Higgs fields except for H_1 and H_2 work out to be massless. We expect 4 of these 6 dofs to be massless in any case because through the SM and charge-breaking VEV we break all 4 generators of $SU(2) \times U(1)$ and so the Goldstone theorem tells us that we will have 4 massless Goldstone bosons (conventionally one would say that the $W^{\pm}$-bosons eat the $\phi_h^{\pm}$s, the Z-boson eats the η_h and the photon eats the η_d). However, we did not put anything into our theory which would force the $\phi_d^{\pm}$s to be massless (and we did not choose a gauge yet), so this looks suspicious.

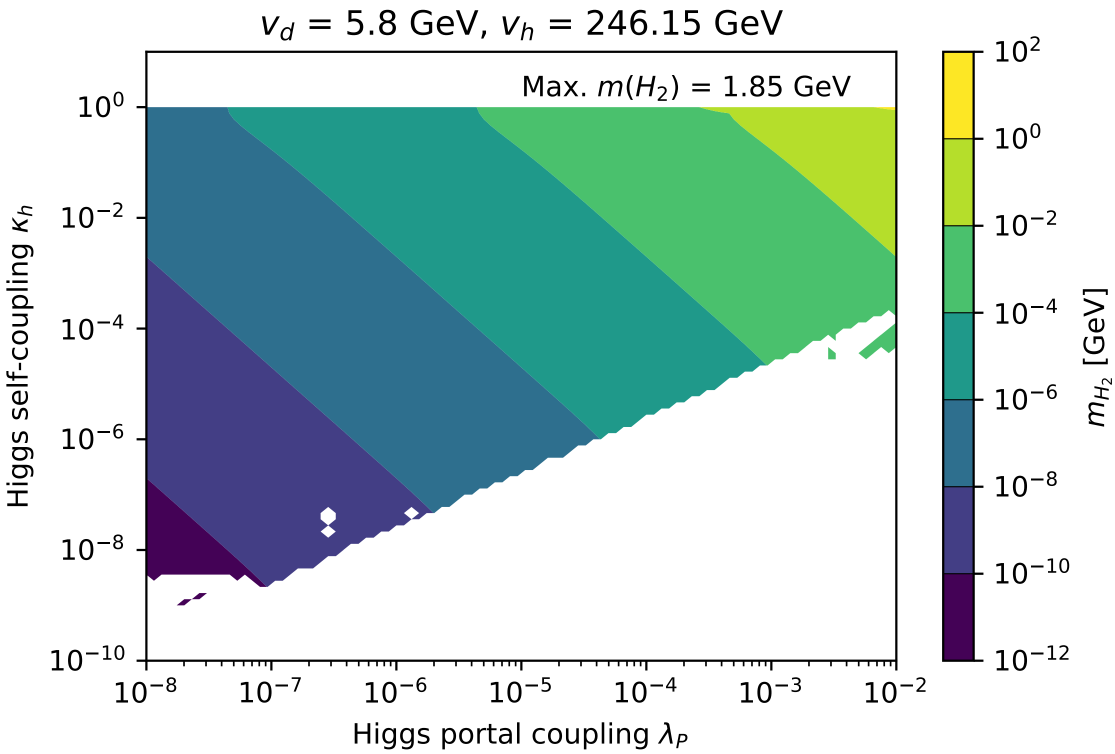


Figure 3.4: Mass of H_2 in the doublet model as a function of κ_h and λ_P for given values of v_d and v_h. For white spaces no solution of Eqs. 3.28 exists, meaning they are phenomenologically excluded.

3.4 Higgs triplet

Finally consider a model containing the SM Higgs doublet Φ_h with $Y = 1/2$ and an additional complex triplet Φ_t with $Y = Y_s \neq +1$ with non-zero minimum

$$\langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ v_t \end{pmatrix} . \tag{3.29}$$

Since the spin-1 representation of $SU(2)$ is not used often we give its generators $T^i = J^i$ explicitly here

$$J^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, J^2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, J^3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} . \tag{3.30}$$

We expand the two Higgs fields around their respective VEVs

$$\Phi_h = \begin{pmatrix} \phi_h^+ \\ \frac{1}{\sqrt{2}}(v_h + \rho_h + i\eta_h) \end{pmatrix}, \Phi_t = \begin{pmatrix} \phi_t^{++} \\ \phi_t^+ \\ \frac{1}{\sqrt{2}}(v_t + \rho_t + i\eta_t) \end{pmatrix} .$$

where $\phi_{h,t}^{\pm}$ and $\phi_{h,t}^{\pm\pm}$ are complex charged fields with $\left(\phi_{h,t}^{\pm}\right)^* = \phi_{h,t}^{\mp}$ and $\left(\phi_{h,t}^{\pm\pm}\right)^* = \phi_{h,t}^{\mp\mp}$, $v_{h,t} = const. \in \mathbb{R} \neq 0$ and $\rho_{h,t}$ and $\eta_{h,t}$ are real fields.

There would have two other choices for the form of the vacuum and Higgs field Φ_t (s.t. the fields have definite charges) and each of them also breaks $U(1)_{EM}$ in general (i.e. except for the one specific choice of Y s.t. the vacuum is neutral). One finds that the two choices

$$\Phi_{t,I} = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_t + \rho_t + i\eta_t) \\ \phi_t^- \\ \phi_t^{--} \end{pmatrix}, \Phi_{t,II} = \begin{pmatrix} \phi_t^{++} \\ \frac{1}{\sqrt{2}}(v_t + \rho_t + i\eta_t) \\ \phi_t^- \end{pmatrix} \tag{3.31}$$

yield results that are essentially equivalent, so we will only consider one of them. For a choice of $\Phi_{t,I}$ we find the following mass terms for the $W^{1,2}$-bosons

$$\frac{1}{2}W_\mu^1 W^{1,\mu}\left(g^2\left(\frac{v_h^2}{4}+v_t^2\right)\right)+\frac{1}{2}W_\mu^2 W^{2,\mu}g^2\frac{v_h^2}{4} \tag{3.32}$$

and so we see that W^1 and W^2 are still mass eigenstates but with unequal masses. This means that while it is still possible to construct $W^\pm$ that are EM-charge eigenstates, they cannot also be mass eigenstates (as we find in the SM). Since this does not conform to the observed behavior of $W^\pm$-bosons we will not consider this further. The same happens in reverse for a choice of $\Phi_{t,II}$.

3.4.1 Gauge bosons

The choice of Eq. 3.4 leads to identical masses for W^1 and W^2

$$\frac{1}{2}W_\mu^1 W^{1,\mu}\left(g^2\left(\frac{v_h^2}{4}+v_t^2\right)\right)+\frac{1}{2}W_\mu^2 W^{2,\mu}\left(g^2\left(\frac{v_h^2}{4}+v_t^2\right)\right) \tag{3.33}$$

so that we can construct $W^\pm$, as usual, as simultaneous EM-charge and mass eigenstates. To be consistent with the SM we require

$$\frac{v_{SM}^2}{4} \approx \frac{v_h^2}{4} + v_t^2 \tag{3.34}$$

within the uncertainty of the measured W-boson mass. For the W^3- and B-bosons we get a mixed mass matrix

$$\begin{pmatrix} g^2\frac{v_h^2}{4} & -gg'\frac{v_h^2}{4} \\ -gg'\frac{v_h^2}{4} & g'^2\left(\frac{v_h^2}{4}+Y_t^2 v_t^2\right) \end{pmatrix} \tag{3.35}$$

which results in mass eigenvalues

$$m_{Z,\gamma}^2 = \frac{1}{2}\left((g^2+g'^2)\frac{v_h^2}{4}+g'^2Y_t^2v_t^2 \pm \sqrt{\left((g^2+g'^2)\frac{v_h^2}{4}+g'^2Y_t^2v_t^2\right)^2 - g^2g'^2Y_t^2v_h^2v_t^2}\right). \tag{3.36}$$

At this point v_h, v_t and Y_t are unknown free parameters whose numerical values need to be constrained by experiments. We simplify our phenomenological analysis somewhat by assuming that all parameters except v_h, v_t and Y_t are only negligibly influenced w.r.t. their SM values and so we take them as fixed parameters. We also impose Eq. 3.34 strictly s.t. v_h and v_t are no longer independent variables. We can the compute the mass

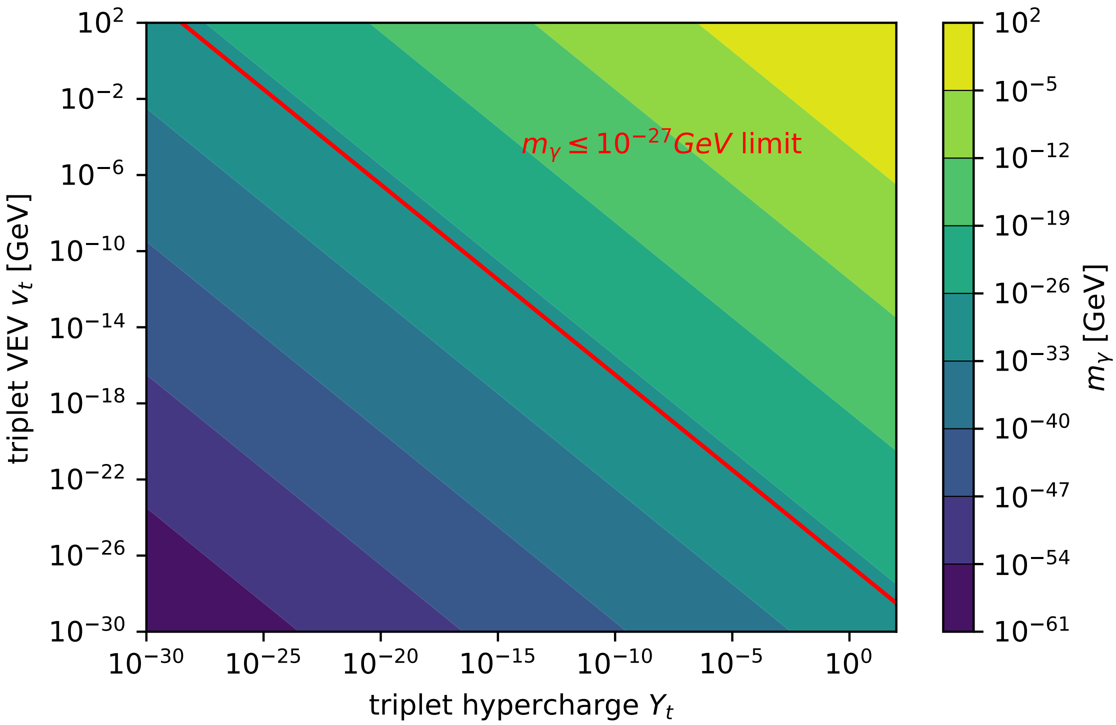


Figure 3.5: Photon mass m_γ in the triplet model

of the photon as a function of v_t and Y_t (Fig. 3.5) and compare this with the relevant limit of $m_\gamma \leq 10^{-27} GeV$.

From this a limit can be imposed on the product $v_t Y_t \lesssim 10^{-24} GeV$ but this degeneracy cannot be resolved without considering further aspects. Another limit can be derived from the ρ-parameter

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} \tag{3.37}$$

which is measured to high accuracy $\rho_{exp} = 1.00039(19)$. Fig. 3.6 shows which parts of the parameter space are compatible with ρ_{exp} at different levels of uncertainty.

The structure is more interesting here. We see that any triplet VEV $v_t \lesssim$ few GeV is compatible for arbitrary Y_t, but there are also very specific combinations for $Y_t \approx 4.5$ where VEVs up to $v_T \leq \frac{v_{SM}}{2}$ are allowed. However, we can get pretty stringent limits by combining the two.

This is done in Fig. 3.7 and finally allows some conclusions: While it is still true that the product $v_t Y_t \lesssim 10^{-27} GeV$ we also find that there is no case in which v_t can be more than a few GeV and so we can also see that in general $v_t \ll v_h \approx v_{SM}$.

Using the fact that $Y_t v_t \ll v_h$ allows us to simplify the expression in Eq. 3.36 by expanding it to first order

$$\begin{aligned} m_Z^2 &= (g^2 + g'^2) \left(\frac{v_h^2}{4} + 2 \sin^4 \theta_w Y_t^2 v_t^2 \right) \\ m_\gamma^2 &= (g^2 + g'^2) 2 \sin^2 \theta_w \cos^2 \theta_w Y_t^2 v_t^2 \end{aligned} \tag{3.38}$$

where the similarity to the SM result becomes clearer.

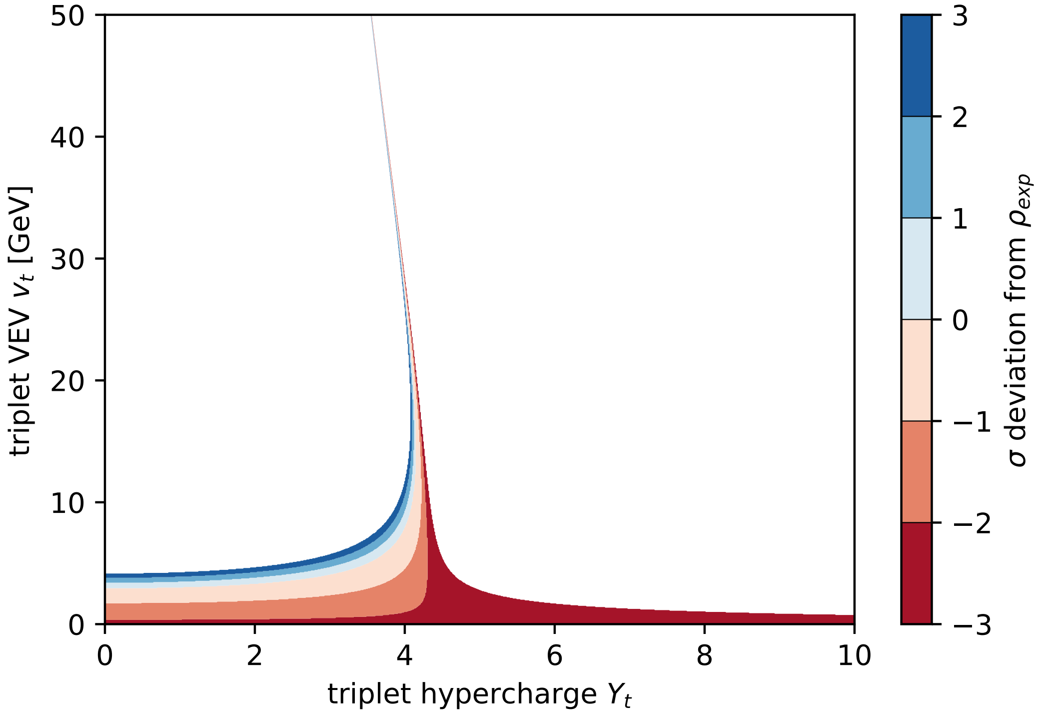


Figure 3.6: Limits from ρ-parameter in the triplet model.

3.4.2 Higgs sector

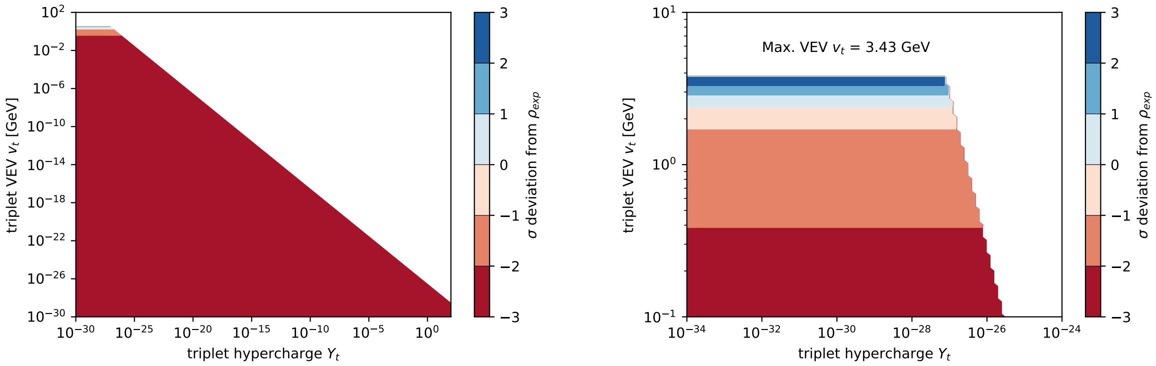
The full potential contains all of the terms of Eq. 3.25 (substituting subscripts of t for d) and the additional terms of

$$\begin{aligned}\mathcal{L} \supset -\mu_t^2 \phi_t^{--}\phi_t^{++} &+ \frac{\lambda_t}{2}\left[\phi_t^{--2}\phi_t^{++2} + \phi_t^{--}\phi_t^{++}(v_t^2+\rho_t^2+\eta_t^2+2v_t\rho_t)\right] \\ &+ \lambda_P\left[\phi_h^-\phi_h^+\phi_t^{--}\phi_t^{++} + \phi_t^{--}\phi_t^{++}\frac{1}{2}(v_h^2+\rho_h^2+\eta_h^2+2v_h\phi_h)\right]\end{aligned} \tag{3.39}$$

so, much as before, we find through use of Eq. 3.5

$$\begin{aligned} m^2_{\phi^{\pm}_{h,d}} &= 0 \\ m^2_{\phi^{\pm\pm}_t} &= 0 \\ m^2_{\eta_{h,d}} &= 0 \\ m^2_{H_1,H_2} &= \frac{1}{2}\left(\lambda_h v_h^2 + \lambda_d v_d^2 \pm \sqrt{(\lambda_h v_h^2+\lambda_d v_d^2)^2 - 4\lambda_P^2 v_h^2 v_d^2}\right) . \end{aligned} \tag{3.40}$$

We use the same numerical approach as in the doublet case to determine the maximum mass of H_2. The constraints on κ_{λ_h} and λ_P are the same as before, we take the maximum triplet VEV from Fig. 3.7b as v_t = 3.43 GeV and the according v_h from Eq. 3.34. From this we find that the maximum mass H_2 can have in this model is m_{H_2} = 0.93 GeV.



(a) Full parameter space

(b) Zoom-in on interesting region

Figure 3.7: Combined limits from m_γ and ρ-parameter. The color signifies by how many σ the resulting ρ differs from ρ_{exp}, white spaces either differ by more than 3σ or violate the photon mass limit.

3.5 Additional terms

Later on we noticed that it is possible to add more terms to the potential in Eq. 3.3 for the doublet and triplet case. These additional terms would all massless particles in Eq. 3.27 and Eq. 3.40, which are not Goldstone bosons, to become massive. The omission of these terms effectively placed an artificial constraint on our theory leading to aforementioned "suspicious" massless particles.

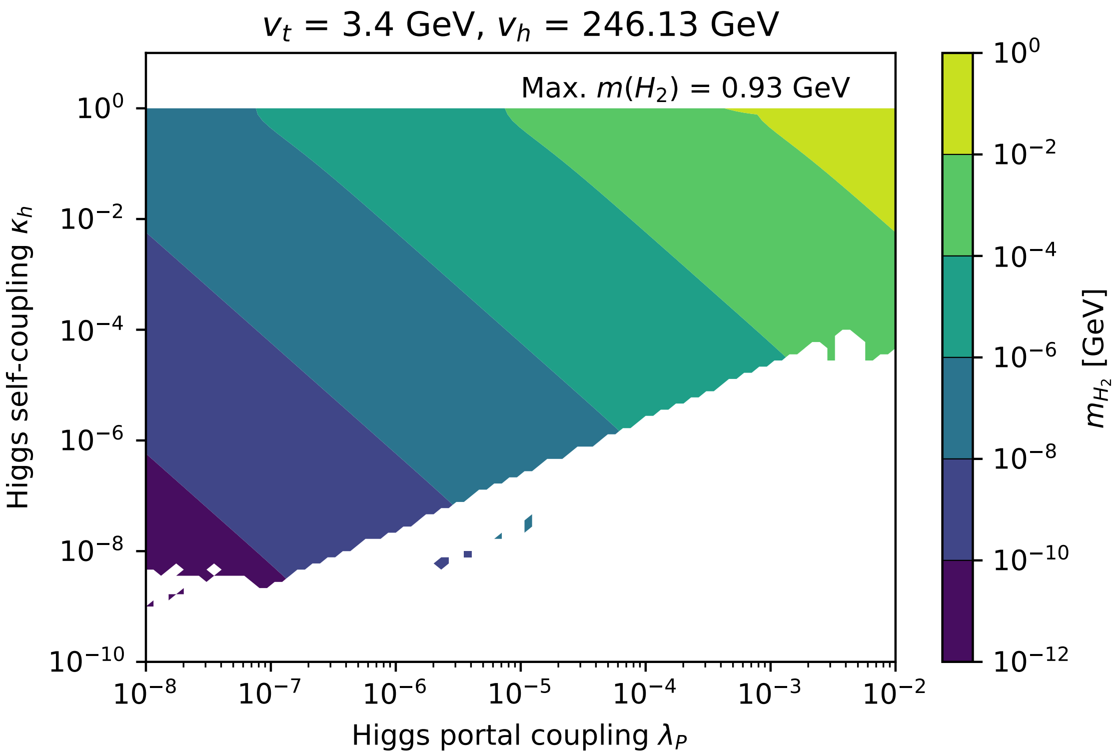


Figure 3.8: Mass of H_2 in the triplet model as a function of κ_h and λ_P for given values of v_t and v_h. For white spaces no solution of the equation system exists.

Chapter 4

Summary

In the previous chapter we have seen that for the doublet and triplet case it is possible to limit the parameter space in such a way that the physical Higgs boson of the newly introduced field cannot be heavier than a few GeV. Apart from making it difficult to justify why a particle in this mass-range would not already have been found in accelerator searches, it makes it impossible to reasonably describe such theories through EFT methods since the Higgs boson is too light to be integrated out of low-energy physics. We have not studied higher order multiplets but it stands to reason that the problems would be similar there, since they are mostly related to the way the isospin couples to SM particles.

For the singlet case it is not possible to arrive at such a limit since there is too much freedom to fine-tune its hypercharge and there is no constraint through isospin interactions. So while it would be possible to find an EFT description here, the benefit of finding a common description for all or several orders of multiplets is not given any more.

Further study into such theories would seem to encounter many problems, the most pressing being the very stringent limit placed on the value of the charged VEV by the photon rest mass. If some mechanism could be found to suppress or prevent the emergence of such a rest mass it would probably go a long way towards making further study phenomenologically viable. At the moment however, the conclusion would seem to be that there is practically no leeway for theories as described in this work.

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Erklärung:

Ich versichere, dass ich diese Arbeit selbstständig verfasst habe und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Heidelberg, den 19.09.2022 ....