Supplemental Material for:
Controlling the magnetic state of the proximate quantum spin liquid α-RuCl$_3$ with an optical cavity

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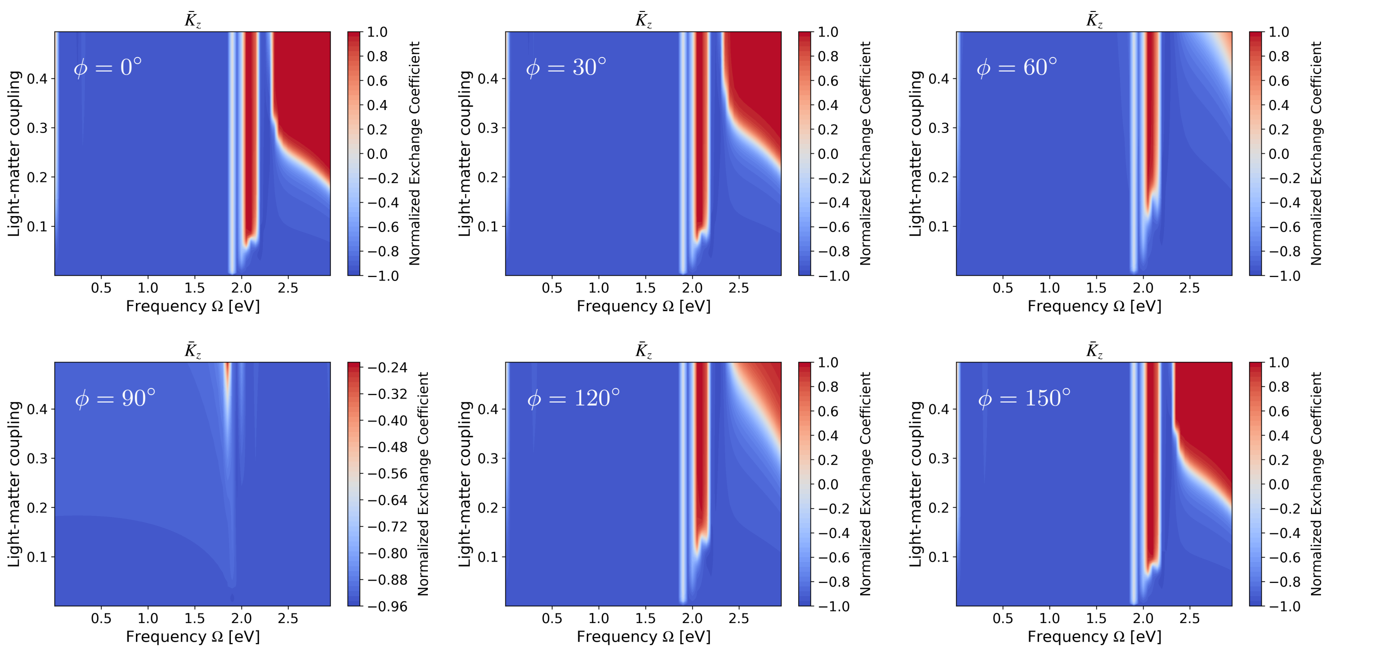

Supplementary Figure 1. **Polarization dependence of the Kitaev interaction for a linearly polarized cavity mode.** Effective Kitaev interaction $\bar{K}$ as a function of the effective light-matter coupling $\bar{g} = 2g\sqrt{n_{\rm av}}$ and frequency Ω for a cavity with $n_{\rm av} = 1$. The different panels correspond to polarization vectors making and angle ϕ with the x-axis. Due to the C_3 symmetry of the magnetic system, the polarization dependence shows a period of $\phi_0 = 2\pi/3$.

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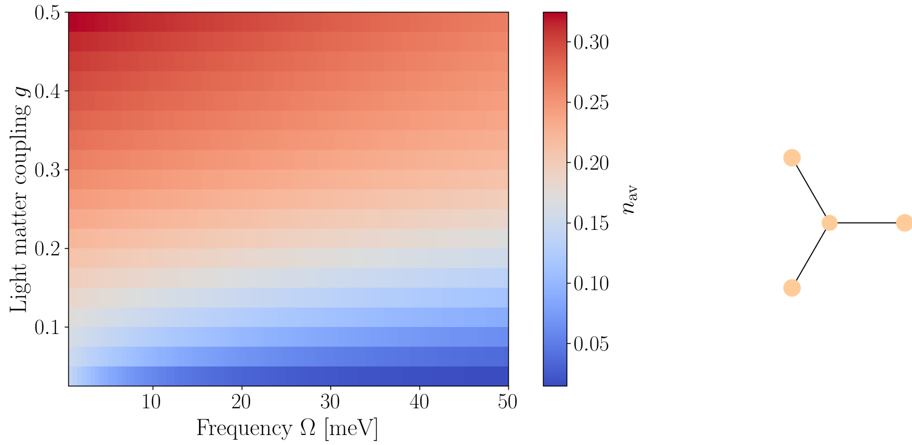
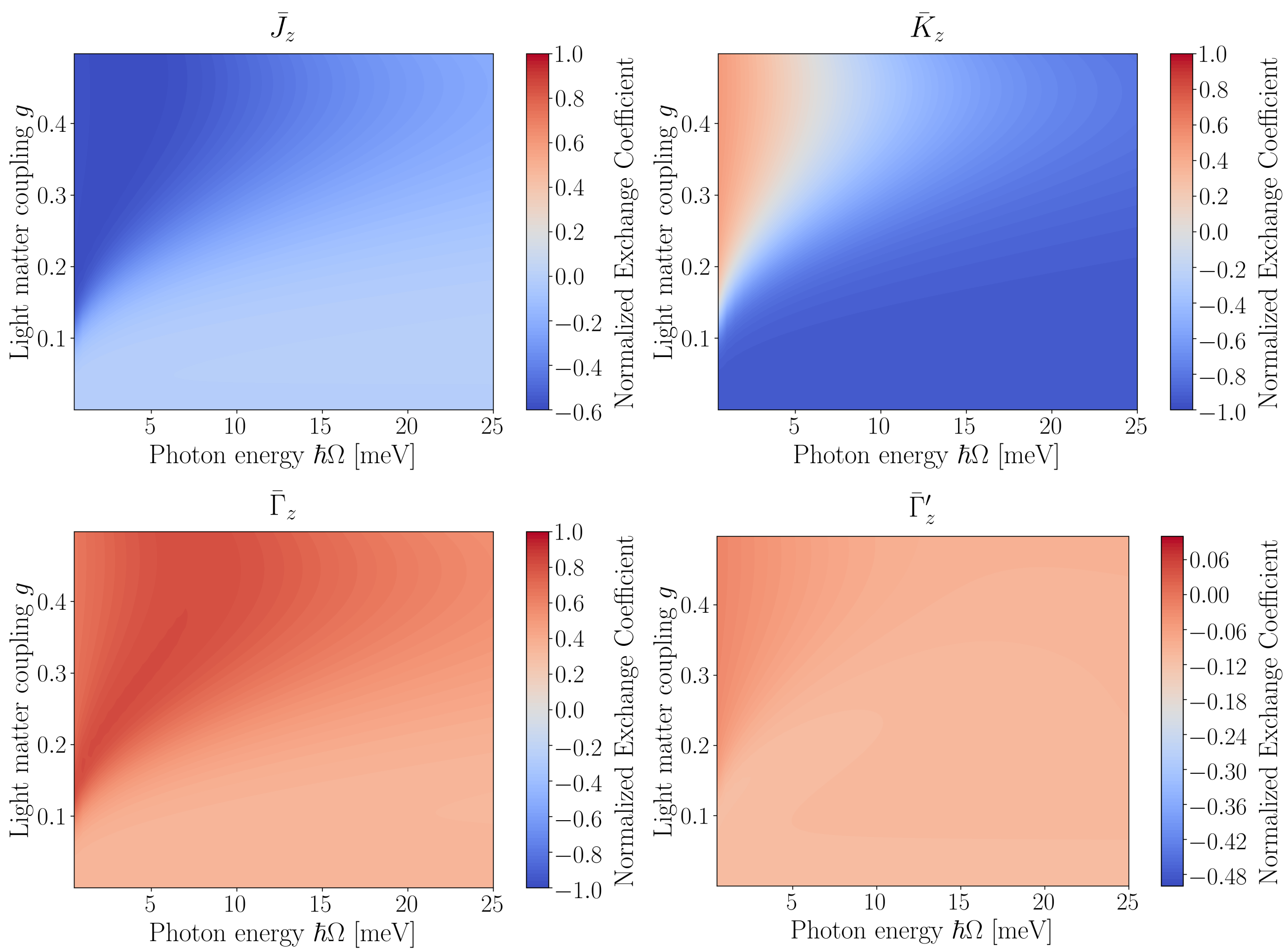
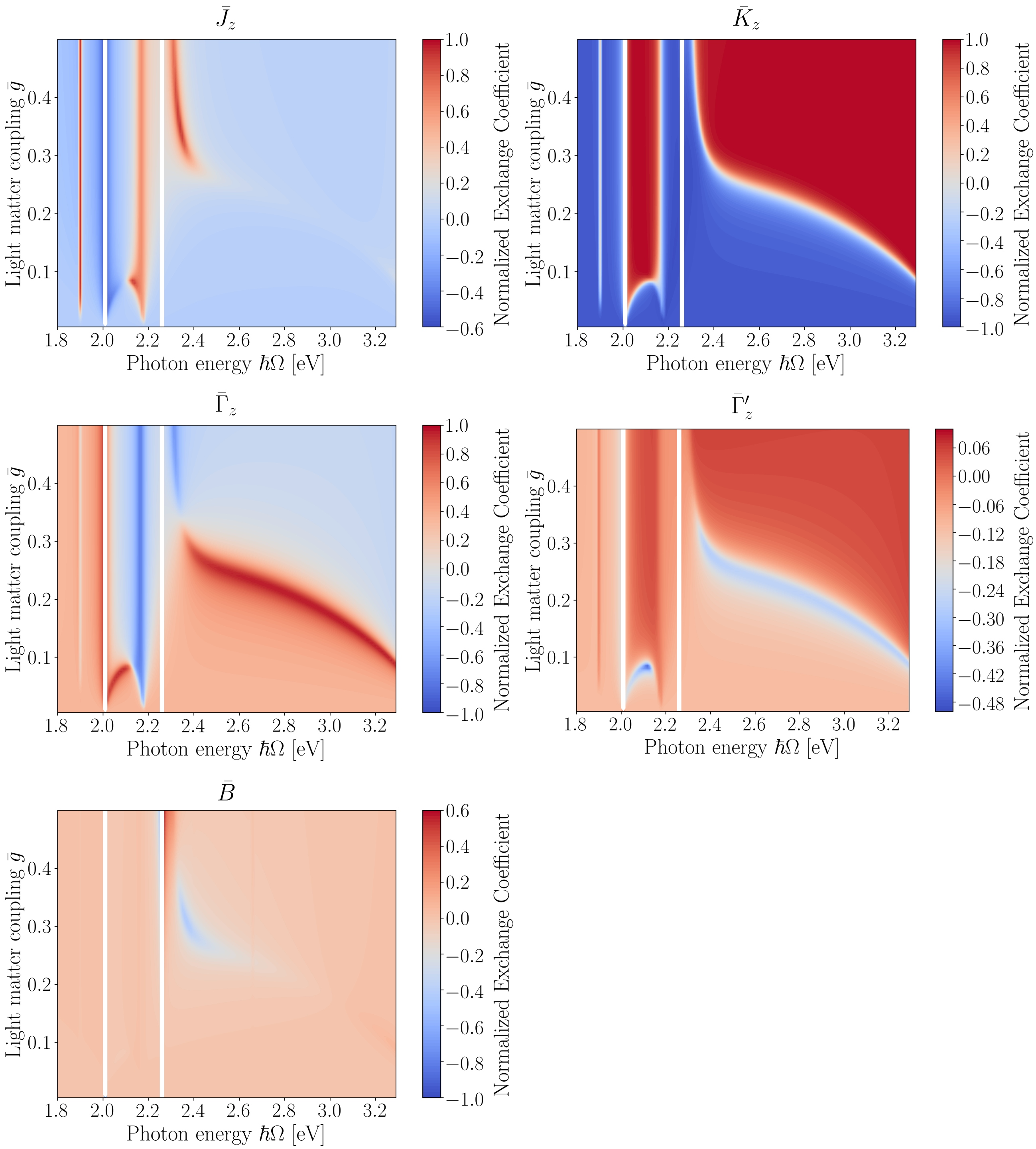


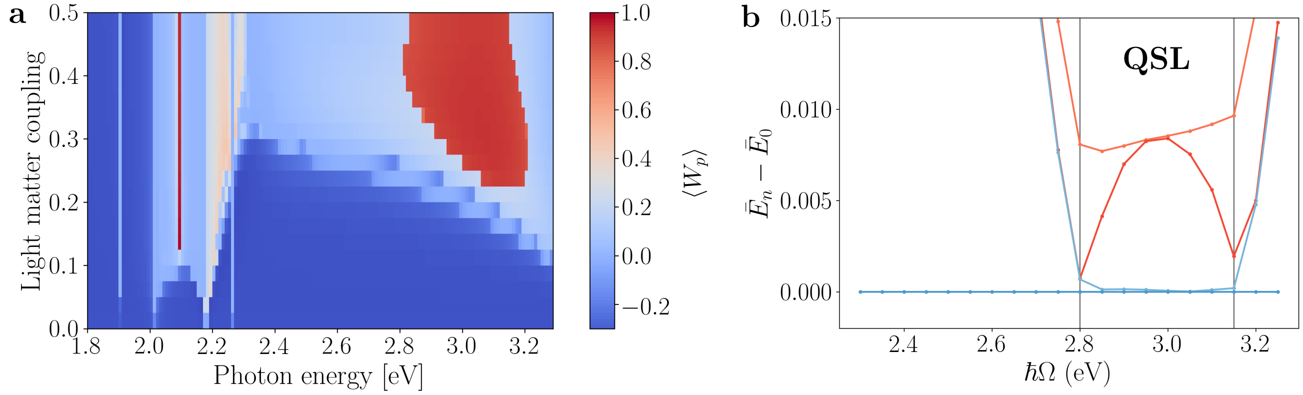
Supplementary Figure 2. **Photon occupation of an electronic cluster.** Photon occupation of the ground state of a four site Ru cluster described by the Hamiltonian in Eqs. 3 and 15 of the main text. The Ru cluster is schematically shown to the right.



Supplementary Figure 3. **Spin parameters of the dark cavity.** Normalized magnetic exchange interaction $\bar{J}$, Kitaev interaction $\bar{K}$, and anisotropy interactions $\bar{\Gamma}$ and $\bar{\Gamma}'$ as a function of the light-matter coupling g and the photon energy $\hbar\Omega$ in the zero photon sector $n_{\rm av} = 0$.



Supplementary Figure 4. **Spin parameters of the seeded cavity.** Normalized magnetic exchange interaction $\bar{J}$, Kitaev interaction $\bar{K}$, anisotropy interactions $\bar{\Gamma}$ and $\bar{\Gamma}'$ and induced magnetic field $\bar{B}$ as a function of the effective light-matter coupling $\bar{g}$ and the photon energy $\hbar\Omega$ in the zero photon sector $n_{\mathrm{av}} = 1$.



Supplementary Figure 5. **Validation of the Kitaev quantum spin liquid state. a,** Expectation value of the flux operator W_p as a function of photon energy $\hbar\Omega$ and light-matter coupling $\bar{g}_{\text{eff}} = 2g_{\text{eff}}\sqrt{n_{\text{av}}}$ for fixed average photon number $n_{\text{av}} = 1$. **b,** Lowest energy eigenstates of the effective photo-renormalized spin Hamiltonian as a function of photon energy $\hbar\Omega$ for $\bar{g}_{\text{eff}} = 0.4$ (corresponding to a horizontal slice through panel **a**). The blue lines show the energies of the ground state and the first excited state, while the red lines show the energies of the second and third excited state. For $\hbar\Omega = 2.8 - 3.2$ eV an excitation gap opens and a topological ground state degeneracy appears, indicating the appearance of a gapped Kitaev quantum spin liquid state.