Flux surface mapping in non-nested topologies and its acceleration with deep neural networks on Wendelstein 7-X

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Plasma diagnostics in fusion research devices often observe distinct locations in the target plasma. To gain a consistent picture and separate geometric effects from 3D transport phenomena, these observations are commonly mapped into a radial coordinate. For the Wendelstein 7-X Stellarator (W7-X) [1, 2], this procedure is usually performed with the assistance of the Fourier-representation based MHD equilibrium code VMEC (see [3] and references therein). Such an approach, however, can not map out magnetic islands and other non-nested magnetic topologies, which severely hampers its extension onto the plasma edge. To analyze the edge-core relationship and define approximate profiles in the plasma edge, we would like to construct a general approach to magnetic surface mapping,, which does not require these surfaces to form simple nested topologies.

Mathematical construction of topology-aware mapping

We would like our surface label function to fulfill the following criteria:

- It should be formulated as a function from $\mathbb{R}^3$ to $\mathbb{R}$ to allow us to map arbitrary lines of sight into identical profile spaces.
- The label should be constant along magnetic field lines.
- In a perfect circular Tokamak, the mapping should ideally correspond to the minor radius.
- It should be monotonic not only in the plasma core, but also inside magnetic islands. We do, however, not require the O-point to reside at a minimum, a maximum is also acceptable.

Such a mapping can be constructed by the following procedure:

1. For a 3D magnetic field $\vec{B}: \mathbb{R}^3 \to \mathbb{R}^3$ and a piecewose differentiable path $\gamma \in C^{1,p}\left([0,1], \mathbb{R}^3\right)$ we define its **perpendicular length** as

$$l_\perp(\gamma) = \int_0^1 \left|\vec{B} \times \frac{d\gamma}{d\tau}(\tau)\right| d\tau$$

.

2. For a pair of magnetic surfaces $S_1, S_2 \subset \mathbb{R}^3$, we define their **perpendicular distance** as

$$d_\perp(S_1, S_2) = \min_{\substack{\gamma \in C^{1,p}\left([0,1],\mathbb{R}^3\right) \\ \gamma(0)\in S_1, \gamma(1)\in S_2}} l_\perp(\gamma)$$

3. Given a surface $S \subset \mathbb{R}^3$ and assuming a magnetic axis $A \subset \mathbb{R}^3$, we define its **perpendicular radius** as

$$r_\perp(S) = d_\perp(S, A)$$

For a single point $x \in \mathbb{R}^3$, we interpret its perpendicular radius $r_\perp(x)$ as $r_\perp(\{x\})$.

Numerical approximation of mapping

While the above-mentioned mapping fulfills the desired criteria, it can not be exactly evaluated. We therefore propose the following 3-step procedure to approximate it on a set of sample point:

1. Extend the sample points (including at least one known point on the magnetic axis) into approximate magnetic surfaces by tracing field lines from their start positions and recording intersections with preselected toroidal angle planes.

2. Estimate the geometric distance between magnetic surfaces by computing the pairwise distance between all recorded points per cross section, and then minimizing over all points in identical surfaces.

3. Record all mutual distances in a fully connected graph and compute the shortest path lengths from the magnetic axis (for example using Dijkstra's algorithm).

This yields a surface label for each traced surface, and therefore for each point recorded on this surface (including the original sample points). By taking advantage of GPU processing power, this bulk calculation can be performed in reasonable time for offline analysis (1 hour on NVidia 1080 TI, 500 surfaces traced for 100 turns across 20 cross sections).

Acceleration using deep learning techniques

While the above mentioned bulk computation is acceptable for the occasional precise offline analysis of specific experiment scenarios, it is prohibitively slow for near-real-time data analysis during or shortly after experiments. Furthermore, it requires a pre-calculated equilibrium to match the experiment conditions.

Therefore, we desired a procedure that would allow us to interpolate the computed mapping both in position and in configuration space. After initial investigations into Delaunay-interpolators (which gave poor results), we chose to train a deep neural network on the computed samples, which can identify and take advantage of the underlying structure of the magnetic configurations. The network was build and trained in TensorFlow [4], with hyperparameters for the training selected by Optuna [5]. As exemplarily demonstrated in figure 1, the resulting mapping function is able to capture the complicated magnetic topology of W7-X, particularly the structure of its island divertor.

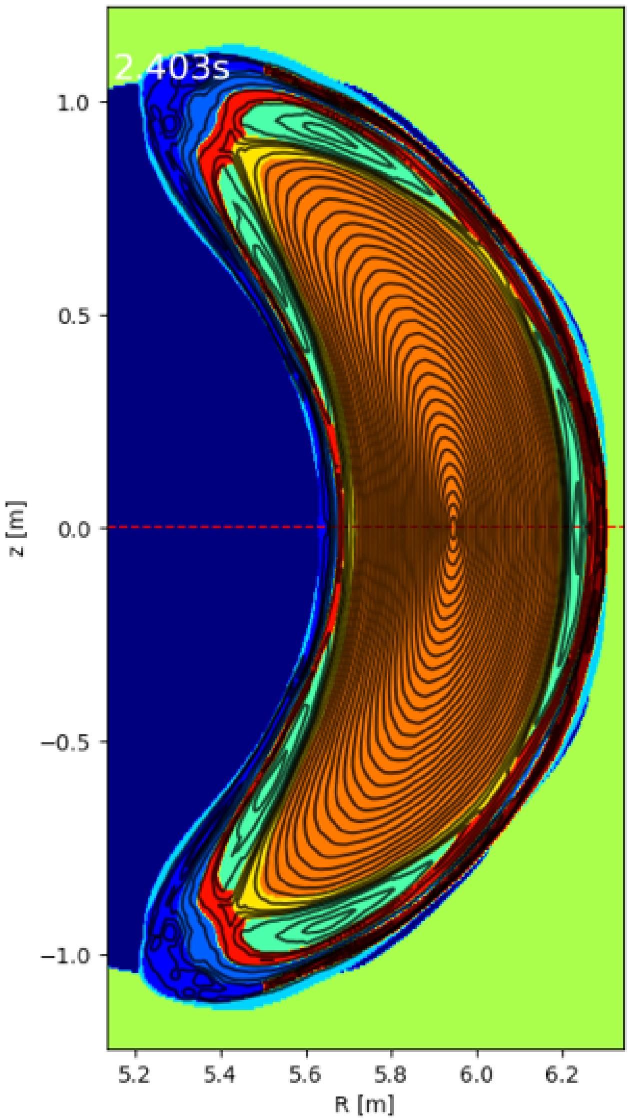

Figure 1: Contour plot of the interpolated mapping function in the W7-X magnetic standard configuration. The background color indicates an attempt by the trained network to partition the magnetic geometry into subdomains (such as magnetic islands).

Summary and outlook

The presented implementation of diagnostic mapping serves both as a proof of principle and a first practical implementation. Efforts to integrate it into the W7-X data processing pipeline, as well as to fully automate the training process in order to make it easier to apply to other devices, are already ongoing. Additionally, alternative training methods are investigated to potentially reduce the (multi-day) training time required.

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