Constraints on pseudo-Dirac neutrinos using high-energy neutrinos from NGC 1068

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Neutrinos can be pseudo-Dirac in Nature - they can be Majorana fermions while behaving effectively as Dirac fermions. Such scenarios predict active-sterile neutrino oscillation driven by a tiny mass-squared difference (δm^2), which is an outcome of soft-lepton number violation. Oscillations due to tiny δm^2 can take place only over astrophysical baselines and hence are not accessible in terrestrial neutrino oscillation experiments. This implies that high-energy neutrinos coming from large distances can naturally be used to test this scenario. We use the recent observation of high-energy neutrinos from the active galactic nuclei NGC 1068 by the IceCube collaboration to constrain $\delta m^2 \leq 1.1 \times 10^{-18}\mathrm{eV}^2$ at more than 90% confidence level - one of the strongest limits to date on the values of δm^2.

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Introduction – The advent of multi-messenger astronomy in the last decade has been one of the biggest success stories of particle physics [1]. The observation of high-energy neutrino events, in direct correlation with the gamma-ray flares from the blazer TXS 0506 + 056, identified blazars as powerful sources of these neutrinos [2, 3]. Since then, additional searches have been performed to locate more of these sources. The observations of these high-energy neutrinos, which can point back to their sources, indicate the presence of some of the most powerful cosmic accelerators in the Universe.

Very recently, the IceCube collaboration released a search result for neutrino emission from a list of 110 gamma-ray sources during the period 2011-2020 [4]. An excess of 79^{+22}_{-20} neutrino events, with energies of few TeV, was reported to be coming from the direction of a nearby active galactic nucleus (AGN) – the NGC 1068 – with a global significance of 4.2σ, which was a major improvement over a previous result of 2.9σ. The neutrino spectra can be fit using a power-law, $\Phi_{\nu_\mu+\bar{\nu}_\mu} = \Phi_0\,(E_\nu/1\,\mathrm{TeV})^{-\gamma}$, with Φ_0 being the overall flux normalization at neutrino energy $E_\nu = 1\,\mathrm{TeV}$ and γ the spectral index. Using this, the collaboration reported $\Phi_0^{1\mathrm{TeV}} = (5.0 \pm 1.5_{\mathrm{stat.}} \pm 0.6_{\mathrm{sys.}}) \times 10^{-11}\,\mathrm{TeV}^{-1}\mathrm{cm}^{-2}\mathrm{s}^{-1}$, with $\widehat{\gamma} = 3.2 \pm 0.2_{\mathrm{stat.}} \pm 0.07_{\mathrm{sys.}}$. The distance to the AGN is estimated to be $d = 14.4\,\mathrm{Mpc}$, although it has been quoted previously in the literature to lie between $10.3 \pm 3\,\mathrm{Mpc}$ [5] and $d = 16.5\,\mathrm{Mpc}$ [6].

The discovery of these high-energy neutrinos has opened up new avenues of testing exotic physics, which was otherwise inaccessible in terrestrial laboratories. The arrival of these neutrinos clearly indicates that the Universe is not opaque to neutrinos at this energy range. This can be used to put tight constraints on different kinds of non-standard physics in the new sector (see [7] and references therein for a detailed discussion).

The naturally long baseline offered by these neutrinos allows us to test the violation of lepton number in the Standard Model (SM). If lepton number is not considered a symmetry of the SM, and we know that it is already broken by quantum effects even within the SM, neutrinos can be Majorana fermions. The extent of lepton number violation in the SM is quantified by the Majorana mass term of neutrinos in the Lagrangian. However, if lepton number is violated *softly* - measurable through a tiny Majorana mass term - neutrinos can be pseudo-Dirac, where they behave effectively as Dirac neutrinos [8–14]. In this case, the neutrino mass-eigenstates corresponding to the active and sterile states develop a *tiny* mass-squared difference, proportional to the extent of lepton number violation. Oscillations driven by this tiny mass-squared difference (δm^2) will only be relevant for long baselines, inversely proportional to δm^2.

Clearly, for testing these oscillations due to tiny δm^2, one needs access to long baselines. Hence, constraints on the pseudo-Dirac hypothesis from terrestrial experiments are weak [15–17]. Stronger constraints are available from atmospheric neutrinos $\delta m^2 \lesssim 10^{-4}\mathrm{eV}^2$ [18], solar neutrinos $\delta m^2 \lesssim 10^{-12}\mathrm{eV}^2$ [13], as well as supernova neutrinos $\delta m^2 \lesssim 10^{-20}\mathrm{eV}^2$ [19, 20], which naturally provide a longer baselines for neutrinos. This naturally opens up the possibility of using high-energy neutrinos coming from the far reaches of the Universe to probe even smaller values of δm^2. Using models of gamma-ray bursts as sources for the astrophysical neutrino flux observed by IceCube, limits have been placed on values of $10^{-18}\mathrm{eV}^2 \lesssim \delta m^2 \lesssim 10^{-12}\mathrm{eV}^2$ [18, 21–25].

In this *letter*, we utilize, for the first time, the high-energy neutrino events observed from NGC 1068 to probe neutrino oscillations due to a small δm^2. The advantages are two-fold: firstly, there exists a precise measurement of the location of the source $d \sim 14.4\,\mathrm{Mpc}$, RA (right ascension) and DEC (declination) = $(40.667, -0.0069)$; secondly, the neutrino energies are well measured to be around a few TeV to tens of TeV. The combination of these allows us to precisely pin down the oscillation length, and study the sensitivity of neutrino oscillations to tiny mass-squared differences. This allows us to rule out $\delta m^2 \geq 1.1 \times 10^{-18}\mathrm{eV}^2$ at more than 90% confidence level. We emphasize that this is the smallest value of δm^2 constrained with such a high significance. An earlier work by one of the authors used data from SN1987A to probe even tinier values of $\delta m^2 \lesssim 10^{-20}\mathrm{eV}^2$ [20], however, the significance was lower due to the small number of events observed. This observation by IceCube, along with the data from SN1987A, provide two of the tightest constraints on the strength of lepton number violation in the SM.

Pseudo-Dirac neutrinos – To explain the masses of the neutrinos, the SM can be expanded to include three additional sterile neutrinos with at least two of them having Majorana masses M_N, which characterize the extent of lepton number violation. Apart from the Majorana mass term, the neutrino mass matrix also consists of Dirac-type masses given by $M_D = Yv$, where Y is the Yukawa coupling and v denotes the vacuum expectation value of the Higgs field. In the limit of soft lepton number violation, $|M_N| \ll |Yv|$, the generic neutrino mass matrix can be diagonalized by a 6×6 block-diagonal unitary matrix U, consisting of the PMNS matrix, and another 3×3 unitary matrix [11]. A generalization using the spectral density function was recently worked out in [26].

For a tiny $|M_N|$, the mixing between the active and sterile states becomes maximal. In this case, a neutrino flavor state can be written as a superposition of two almost degenerate mass states,

$$\nu_{\alpha L} = U_{\alpha k} \frac{(\nu_k^+ + i\,\nu_k^-)}{\sqrt{2}}\,, \tag{1}$$

with masses $m_{k,\pm}^2 = m_k^2 \pm \delta m_k^2/2$. Here $\nu_k^\pm$ correspond to the mass eigenstates associated with the flavor state. This scenario leads to active-sterile oscillations driven by $\delta m_k^2/2$, in addition to oscillations due to solar ($\Delta m^2_{\mathrm{sol}}$) and atmospheric ($\Delta m^2_{\mathrm{atm}}$) mass-squared differences [27]. For simplicity, we will assume that δm^2 is the same for all states, and hence drop the index k.

For neutrinos travelling over astrophysical baselines, flavor oscillations induced by ($\Delta m^2_{\mathrm{sol,atm}}$) average out due to rapid oscillations, causing decoherence [27]. However, for small enough δm^2, active-sterile oscillations can persist. The corresponding active neutrino survival probability can be computed as [19]

$$P_{\alpha\alpha}(E_\nu) = \frac{1}{2}\left(1 + e^{-\left(\frac{L}{L_{\mathrm{coh}}}\right)^2} \cos\left(\frac{2\pi L}{L_{\mathrm{osc}}}\right)\right)\,. \tag{2}$$

Here L_{osc} denotes the oscillation length and can be computed for a given neutrino energy E and δm^2 as,

$$L_{\mathrm{osc}} = \frac{4\pi E_\nu}{\delta m^2} \approx 15\,\mathrm{Mpc}\left(\frac{E_\nu}{1\,\mathrm{TeV}}\right)\left(\frac{5\times 10^{-18}\,\mathrm{eV}^2}{\delta m^2}\right)\,. \tag{3}$$

However, even these oscillations driven by δm^2 can get washed out due to the separation of wave packets over long distances, leading to decoherence. This is measured through the coherence length L_{coh}, which gives the length scale up to which flavor coherence is expected to be maintained. This depends also on the width of the neutrino wave-packet σ_x, which is usually determined from the process of neutrino production [28]. The coherence length is estimated as,

$$\begin{aligned} L_{\mathrm{coh}} &= \frac{4\sqrt{2}E_\nu^2}{|\delta m^2|}\sigma_x \\ &\approx 10^6\,\mathrm{Gpc}\left(\frac{E_\nu}{1\,\mathrm{TeV}}\right)^2\left(\frac{5\times 10^{-18}\,\mathrm{eV}^2}{\delta m^2}\right)\left(\frac{\sigma_x}{10^{-10}\ \mathrm{m}}\right)\,. \end{aligned} \tag{4}$$

Taking into account the active sterile oscillation probability, and the flavor averaging due to the solar and atmospheric terms, the probability of obtaining a neutrino flavor ν_β, starting from ν_α is given by

$$P_{\alpha\beta} = P_{\alpha\alpha}(E_\nu; L, \delta m^2)\sum_k |U_{\alpha k}|^2\,|U_{\beta k}|^2\,. \tag{5}$$

This is the new physics that we will constrain through our analysis in the next section. The long baseline ($d = 14.4\,\mathrm{Mpc}$) offered by the high-energy neutrinos observed by IceCube can be used to constrain tiny values of $\delta m^2 \sim 10^{-18}\mathrm{eV}^2$, which are otherwise difficult to access in other sources.

Analysis – The investigation performed in this work closely follows the procedure applied in Ref. [4]. Thus, we use an unbinned likelihood ratio method with a corresponding likelihood function defined as [29]

$$\log\mathcal{L} = \sum_i \log\left(\frac{n_s}{N}\mathcal{S}_i + \left(1 - \frac{n_s}{N}\mathcal{B}_i\right)\right)\,, \tag{6}$$

with $\mathcal{S}_i$ and $\mathcal{B}_i$ being the probability density functions for signal and background neutrino events, respectively. Further, n_s denotes the number of signal events and N is the total number of recorded events. The analysis routines provided by the IceCube collaboration [30] have been modified in order to incorporate the oscillation probability of pseudo-Dirac neutrinos, cf. Eq. 5. For the latter, we took the latest global fit results [31]. We set $\sigma_x \sim 10^{-10}\,\mathrm{m}$ - this number is just representative of the width of the neutrino wave packet. From Eq. 4, it is clear that this choice does not lead to additional decoherence due to wave-package separation on the Mpc-scale.

The neutrino flux of NGC 1068 in muon and the antimuon flavor is assumed to follow a power-law of the form

$$\Phi_{\nu_\mu+\bar\nu_\mu} = \Phi_0\left(\frac{E_\nu}{1\,\mathrm{TeV}}\right)^{-\gamma}\,, \tag{7}$$

with Φ_0 being the overall flux normalization associated with an energy of $1\,\mathrm{TeV}$ and γ is the spectral index. The corresponding number of signal events is then given by [32]

$$n_s = t \int d\Omega \int_0^\infty dE_\nu \; A_{\rm eff}(E_\nu, \Omega) \\ P(E_\nu; \delta m^2) \Phi_\nu(E_\nu; \Phi_0, \gamma)\,, \tag{8}$$

with the detector's lifetime t and its effective area $A_{\rm eff}$ being a function of the neutrino energy E_ν and the solid angle Ω.

Hence, in our analysis the number of signal events depends on the three parameters: $n_s \equiv n_s(\delta m^2, \Phi_0, \gamma)$. Existing correlations between Φ_0 and γ are accounted for by reproducing the covariance matrix of the original IceCube analysis and incorporating it with a two-dimensional Gaussian pull term to the likelihood function in Eq. 6. In doing so, we also ensure that the fit preserves the original best-fit values of Φ_0 and γ, taking into account a normalization correction due to the introduced oscillation probability in Eq. 5. Note that for tiny mass-squared differences $\delta m^2 \lesssim 10^{-22}$eV^2, the effects of active-sterile mixing become negligible for the energy region of interest, such that we coincide with the usual no additional oscillation case. In order to determine a limit on the mass-squared difference of pseudo-Dirac neutrinos, we determine a profile likelihood ratio as [33]

$$q_{\delta m^2} = -2 \log \left(\frac{\mathcal{L}(\delta m^2, \hat{\hat{\Phi}}_0, \hat{\hat{\gamma}})}{\mathcal{L}(\widehat{\delta m^2}, \hat{\Phi}_0, \hat{\gamma})} \right)\,, \tag{9}$$

where the numerator represents the likelihood function with a fixed δm^2, while in the denominator all three quantities are tuned in the minimization procedure. In particular, the parameters $\hat{\hat{\Phi}}_0$ and $\hat{\hat{\gamma}}$ are the conditional (maximum likelihood) estimators that depend on the chosen value of δm^2, whereas the parameters occurring in the denominator are the estimators that maximize the unconstrained likelihood function.

For each δm^2 value under study, we determine the likelihood ratio in Eq. 6 and with asymptotic sampling distributions given in Ref. [33] we can assign each so-determined ratio a corresponding p-value, i.e. $p_{\delta m^2} = \int_{q_{\delta m^2,{\rm obs}}}^\infty dq_{\delta m^2} f(q_{\delta m^2}|\delta m^2)$, which we use as an exclusion criterion for the tested δm^2 values. Since the introduced oscillation probability in Eq. 5 affects the expected signal only in one direction, i.e. a decrease of the expected neutrino flux, we have to perform a one-sided statistical test. Following the IceCube collaboration, we assume that the test statistic $f(q_{\delta m^2}|\delta m^2)$ used has approximately χ^2-like behavior [30], except for modifications due to the one-sided test case. For details about the modified profile likelihood ratio and the corresponding sampling distribution, we refer to Ref. [33]. In this work, we are interested in a limit on δm^2 at 90% confidence level (C.L.) $(1 - p_{\delta m^2})$. Thus, our results is determined by finding the value of δm^2, which yields $p_{\delta m^2} \leq 0.1$. The whole analysis has been performed within the Anaconda/SciPy framework [34, 35], while the iminuit package [36] was used to minimize the likelihood function in Eq. 6.

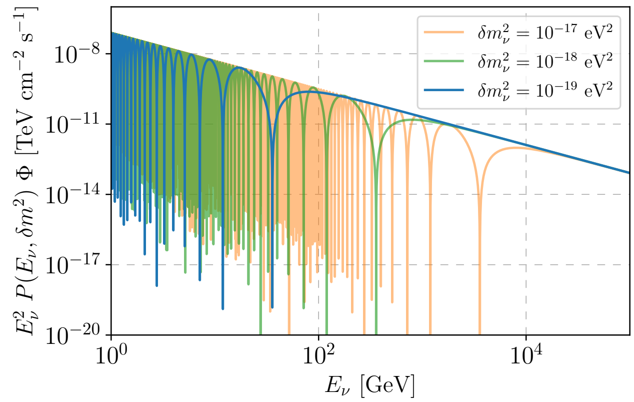

FIG. 1. Neutrino flux variations due to oscillations driven by δm^2. We fixed the neutrino flux parameters to the values determined by IceCube Collaboration [4], i.e. $\hat{\Phi}_0 \sim 5.0 \times 10^{-11}$ TeV^{-1}cm^{-2}s^{-1} and $\hat{\gamma} = 3.2$, and varied the pseudo-Dirac mass splitting δm^2 in a region that is relevant for this analysis.

Results – The fact that IceCube observed these neutrinos from NGC1068 implies that these did not oscillate into their sterile counterparts. In the presence of active-sterile oscillations, the expected power-law flux will undergo spectral distortions, as shown in Fig. 1 for different values of $\delta m^2 = \{10^{-17}, 10^{-18}, 10^{-19}\}$ eV^2. As a result, a profile likelihood analysis can be used to set constraints on possible values of δm^2. In our analysis, we assume that only muon (anti)neutrinos produced at the source are detected as muon (anti)neutrinos i.e., we set $\alpha = \beta = \mu$ in Eq. 5. This implies that we only detect $\sim 41\%$ of all neutrinos. This leads to a slightly different flux normalization from the original bf values observed by IceCube: $\hat{\Phi}_0 \sim 1.2 \times 10^{-10}$ TeV^{-1}cm^{-2}s^{-1}, while γ basically remains the same. In principle, the contribution of a ν_e produced at source and detected as ν_μ is expected to be few tens of percentage of that of the original ν_μ contribution. However, our results do not change much if we include this contribution.

Our main result, depicted in Fig. 2, shows that the data from IceCube can be used to rule out $\delta m^2 \geq 1.1 \times 10^{-18}$eV^2 at more than 90% confidence level ($p_{\delta m^2} \leq 0.1$). There is some sensitivity to even lower values of δm^2, albeit at lower confidence. This agrees with the analytical estimate in Eq. 3, which shows the mass-squared difference sensitive to this oscillation length for TeV energy neutrinos. To the best of our knowledge, this is the *strongest* constraint to date on the smallness of δm^2 with such a high significance. For smaller values of δm^2, the oscillation length becomes larger than the distance to the

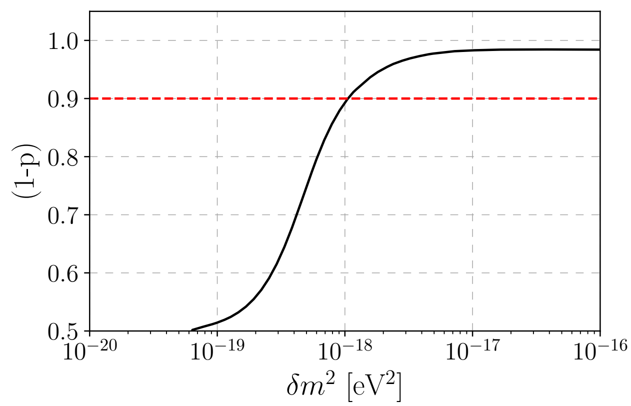

FIG. 2. Exclusion for the mass-squared difference δm^2 of pseudo-Dirac neutrinos. Confidence levels are shown for different δm^2 values. With the observation of neutrinos from NGC 1068, we can exclude $\delta m^2 > 1.1 \times 10^{-18}\,\mathrm{eV}^2$ (@ 90% C.L.).

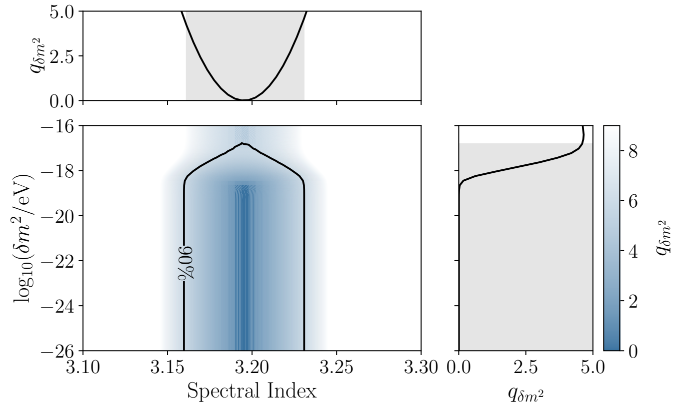

FIG. 3. Parameter correlation between δm^2 and the spectral index for a profiled flux normalization Φ_0. The black lines denote the 90% CL contours for these parameters.

source, hence sensitivity is lost.

Fig. 3 shows the correlation between δm^2 and γ for a profiled flux normalization Φ_0. Analogous to our determined limits, we indicate the 90% C.L. contour for both parameters. Correlations between Φ_0 and γ are considered via an appropriate pull-term in the applied likelihood function. The figure indicates that the data is clearly consistent with the "no-oscillation" hypothesis, which corresponds physically to oscillations with longer baselines. This shows that the addition of these oscillations does not help improve the fit, rather the data can be used to put constraints on these oscillations.

A similar argument can be used to put bounds on δm^2 from observations of a neutrino event of energy $E_\nu \sim 290\,\mathrm{TeV}$ from the blazar TXS 0506 + 056 at a redshift of $z = 0.33 \pm 0.0010$, corresponding to a distance $d = 1.3\,\mathrm{Gpc}$ [2, 3]. A naive estimate yields

$$L_{\mathrm{osc}} \approx 1.3\,\mathrm{Gpc}\left(\frac{E_\nu}{290\,\mathrm{TeV}}\right)\left(\frac{10^{-17}\,\mathrm{eV}^2}{\delta m^2}\right), \tag{10}$$

which is stronger due to the larger energy of the neutrino. Additionally, IceCube later reported a series of 13 ± 5 events from the same direction. These observations can be used, in principle, for a similar study to constrain values of δm^2.

Conclusion – Detection of high-energy neutrino events by IceCube has opened up new frontiers in multi-messenger astronomy. In particular, the naturally long baseline available to the neutrinos, coupled with their high energies, allows them to be the harbingers of new exotic physics, otherwise inaccessible to mankind. In this work, we used the IceCube observation of ~ 79 TeV-sh neutrino events coming from the direction of the active galactic nuclei NGC 1068 to probe the extent of lepton-number violation in the Standard Model.

A soft lepton-number violation can be manifested through active-sterile neutrino oscillations over astronomical baselines inversely proportional to the mass-squared difference (δm^2) between active and sterile neutrinos. These oscillations, if present, will lead to a distortion of the event spectra, and a reduction in the number of neutrinos observed.

This simple yet powerful idea can be utilized to probe the strength of lepton number violation, characterised by the magnitude of δm^2. The identification of the source of these neutrinos, and precise measurement of their energies, allowed us to precisely calculate the oscillation lengths associated with δm^2, and the corresponding spectral modifications. We used the analysis of the IceCube collaboration, outlined in Ref. [4], to rule out $\delta m^2 \geq 1.1 \times 10^{-18}\mathrm{eV}^2$ at more than 90% confidence level. The result obtained is one of the strongest bounds on the extent of lepton-number violation from experiments.

These extreme new physics scenarios are clearly inaccessible to any terrestrial experiments. The observations of these point-sources of astronomical neutrinos like NGC 1068 and TXS 0506+056 clearly pave way for new laboratories for neutrino physics.

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