# Eberhard Hopf's (1902-1983) Work in Astrophysics and Astronomy *

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**Abstract**

The text begins with a brief longitudinal cut through the history of mathematical astronomy from Johannes Kepler to today. Then we focus on Eberhard Hopf in Astronomy. Eberhard Hopf is mostly known for his mathematical work, but actually he also made very deep and influential contributions to astronomy and mathematical astrophysics. His two books and many of his papers deal with mathematical problems from astronomy and astrophysics and/or are influenced by astronomical literature, but they are also substantial contributions to mathematics. His book *Mathematical Problems of Radiative Equilibrium* (1934, repr. 1964) was praised in 1935 by three reviewers as an important contribution for a modern, mathematically oriented astrophysics. Some people judge that his book *Ergodentheorie* actually started the field, though there was earlier work by other authors. His famous paper on the Hopf bifurcation (1942 published in German, reprint/English translation published and discussed in 1976 and in 2002) was strongly influenced by Poincaré's work on celestial mechanics (ca 1895).

## 1 From Johannes Kepler (1571-1630) to Eberhard Hopf (1902-1983) to today

We begin our longitudinal cut through the history of mathematical astronomy with Johannes Kepler (1571-1630), because he revolutionized astronomy with his book *Astronomia Nova* in 1609 [Kep09, GoH05]. We note that last year was the 450th anniversary of his birth and it could not be celebrated adequately because of Covid 19.

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Today we say that Kepler's laws of planetary motion follow from the exact solution of *the mathematical 2-body problem*, i.e., from considering the orbits of two mass points in a gravitational field. This statement actually summarizes the scientific development from Kepler's *Astronomia nova* of 1609 to Newton's *Philosophiae naturalis principia mathematica* of 1687 [New87]. When Kepler derived his famous laws, he actually did consider *three* bodies in special constellations. His revolutionary approach was to abandon the then common concept of *orbs* and to replace it with the concept of *orbits*, i.e. the concept of paths of planets together with their physical cause. Thus he laid the seed for today's theory of dynamical systems. It is known that Newton did not read Kepler's publications, but he knew Kepler's work from other publications. Newton was clearly influenced by Kepler's work when he formulated his theory of gravitational forces. [GoH05]

> Newton thus extended the core idea of Kepler's celestial physics to all of physics – the same physical laws apply equally to celestial and terrestrial bodies. [GoH05, p. 104]

Celestial mechanics is a *multi*-body problem. To follow Newton et al. directly by solving *the mathematical three-body problem* turned out to be qualitatively completely different and very difficult: only very few special cases are exactly solvable, many solutions are chaotic (see for instance the book by Parker and Chua [PaC89] for properties of chaotic systems). For a history of the three-body problem see publications by June Barrow-Green, for instance [Bar96]. In addition to their attemps to solve the three-body problem directly, astronomers employed several other methods and thus developed mathematical methods which turned out later on to be very valuable, also for completely different applications. Among these methods are, for instance: solve a two-body system which is perturbed by a third small body ($\Rightarrow$ mathematical perturbation theory) or solve the n-body problem numerically. Many of these attempts of astronomers contributed to the development of the Qualitative Theory of Dynamical Systems, to Ergodic Theory and to Numerical Particle Simulations (like Particle in Cell (PIC) methods). Eberhard Hopf made many contributions to the theory of dynamical systems and to ergodic theory. Numerical Particle Simulations were successfully developed in Plasma Physics and Astrophysics [MSp18a] and involve nowadays several million particles. Their field of applications grew considerably. Today, they include such diverse fields as medicine, hydrodynamics and graphics for computer games.

## 2 Eberhard Hopf and Astrophysics/Astronomy

Eberhard Hopf was born in Salzburg in 1902. He finished school in Berlin in 1920 and studied mathemaics and physics at the universities of Berlin (7 semesters) and Tübingen (one semester). In 1925/26 he finished his dissertation in mathematics in Berlin, with advisors *Erhard Schmidt (1876-1959)* and *Issai Schur (1875-1941)*. In 1927 he became *wissenschaftlicher Assistent* at the *Astronomisches Recheninstitut (ARI)*[1] of

[1] Institute for Astronomical Calculations

Berlin University. In 1929 he finished his habilitation and obtained the *venia legendi* for mathematics and astronomy.[Tob06]

With a Rockefeller stipend Hopf became an *International Research Fellow* at *Harvard College Observatory* for 1930-1932. [RSS98, pp 38f], [Tob06]. This was very attractive to him for several reasons: In the US there was a strong tradition in celestial mechanics; because of the economic situation in 1929 it was very hard to find an adequately paid position in Germany, and *George D. Birkhoff (1884-1944)* was at the Harvard College Observatory and worked on the theory of dynamical systems and on ergodic theory. During his time at Harvard Hopf met *Norbert Wiener (1894-1964)* of the neighboring MIT.[RSS98, RSS09] Their famous joint paper appeared in 1931 [WiHo31]. When Hopf's stipend ended, he would have liked to go back to Germany, but letters from Berlin made clear that there were no positions available. So Norbert Wiener helped him in 1932 to get a position as assistent professor at the mathematics department of MIT. Thus Hopf moved in 1932 from astronomical institutes to a mathematical institute. Formally, this move turned out to be for good: From MIT he moved to mathematical chairs in Leipzig (1936), LMU Munich (1944) and Indiana University (1948). When the people in Leipzig discussed whom to offer the vacant mathematical chair in 1936, there were very active Nazis among the students and in the *Dozentenschaft* at U Leipzig. They strongly opposed an offer to any non-Nazi candidate, and thus also to Eberhard Hopf. But finally a group of professors won with their *Sondervotum* (dissenting opinion) for Hopf: they were the physicists *Werner Heisenberg (1901-1976)* and *Friedrich Hund (1896-1997)*, the mathematician *Bartel Leendert van der Waerden (1903-1996)* and the geophysicist *Ludwig Weickmann (1882-1961)* [Gir09, MSp18b]. They appreciated Hopf's astronomical work.

This move from astronomical institutes to mathematical institutes did not really change the character of Hopf's work: all his publications were mathematically well-founded, and much of his later work was motivated by astronomical problems and related publications, especially by *Henri Poincaré's (1854-1912)* work [Poi92-99].

During the years 1927 to 1934 Hopf focused on mathematical problems of radiative equilibrium of stars. He obviously was introduced to this topic by *Erwin Freundlich (1885-1964)*[2] and *U. Wegner* at the Einstein Institute at Potsdam. U. Wegner had published on a closely related mathematical problem before. In their joint paper they examined several recent papers by Russian authors who had 'attempted to bring forward a proof that the homogeneous integral equation of radiative equilibrium possesses no solution other than one identically equal to zero.' [FHW27]. Freundlich, Hopf and Wegner point out that this is in contradiction to all approximate solutions of the integral

[2] Erwin Finlay Freundlich (1885-1964) was a German astronomer with a British mother. He was a student of Felix Klein (1849-1925) and Karl Schwarzschild (1873-1916). After 1910 he worked at the Potsdam observatory. As a working associate of *Albert Einstein (1879-1955)* his main task became to verify the general theory of relativity by designing and performing experiments for testing the theory by astronomical observations. In 1920 the Einstein Institute was founded in Potsdam with an Astrophysical Observatory, in the Einstein Tower. Freundlich was appointed there as observer in 1921. He emigrated in 1933. Later on he became *Napier Professor of Astronomy* in St Andrews, Scotland. When retired, in 1957, he returned to his native town Wiesbaden and became honorary professor at U Mainz. [McTu]

equation which were obtained by Schwarzschild, Jeans and Milne by using several different numerical methods. Also, the testing of the theory by observation confirmed the theory. In their article, Freundlich, Hopf and Wegner discuss in detail which statements in the arguments of the Russian authors are wrong (some for mathematical, some for physical reasons), and Hopf announced a proof of the existence of a non-zero solution. In a series of three papers Hopf proved in a physics context a) the existence of a non-zero solution [Hopf28a] and b) the uniqueness of this solution [Hopf28b]; c) in a purely mathematical context in a much more general setting he then proved further mathematical properties [Hopf28c]. He continued to publish on related problems, sometimes in the astrophysics context, sometimes in purely mathematical context. Especially remarkable in this series is the article *Remarks on the Schwarzschild-Milne Model of the Outer Layers of a Star*, [Hopf30]. Among other results, he found an explicit expression for the ratio of the two characteristic temperatures in the problem, while previous authors (Jeans, Milne and Eddington) approximated that term numerically. Without a co-author with a British mother, Hopf sent his manuscript to Prof. Milne in German, and Milne[3] translated it himself (7 pages). Altogether Hopf published[4] about 15 articles on this topic during the years 1927 to 1932. Mixed into this series there are also papers on other topics, from 1929 on also articles on problems from the theory of dynamical systems and ergodic theory. Already at MIT, Hopf summarized his work on radiative equilibrium and published a monograph in 1934: *Mathematical Problems of Radiative Equilibrium* [Hopf34, 104 pages]. This book was praised in 1935 by the three reviewers *Harry Bateman (1882-1946)* (in the US), *Subrahmanyan Chandrasekhar (1910-1995)* (at that time still in England), and *Boris P. Gerasimovič (1889-1937)* (at the Poulkovo Observatory near Leningrad) as an important contribution for a modern, mathematically oriented astrophysics [Bat35, Cha35, Ger35]. Only Chandrasekhar criticized that the book did not cover the topic completely, that there were some open points not even mentioned. Thus Hopf wrote one more article on the topic, dealing with work both by Chandrasekhar and himself and discussing relations between their results. He submitted it such that it was communicated by Chandrasekhar [Hopf36]. For a 1994-discussion on the importance of Hopf's radiative-transfer-work see the article by Andrzej Icha [Ich94, p.77].

Hopf wrote a first article on the theory of dynamical systems in 1929. From 1932 on, he wrote several articles on dynamical systems and ergodic theory, certainly strongly influenced by the work of George Birkhoff at Harvard College Observatory. His book on ergodic theory appeared while he was in Leipzig, and thus in German. *Ergodentheorie*, Springer Verlag, 83 pages [Hopf37]. Some people judge that this book actually started the field, though there was earlier work by other authors, among them George Birkhoff.

Hopf's famous paper on the Hopf bifurcation [Hopf42] (1942 published in German, reprint/English translation published and discussed in 1976 and in 2002) was strongly influenced by Poincaré's work on celestial mechanics [Poi92-99], though Poincaré is only mentioned in a footnote and the applications discussed in Hopf's article were of recent interest in fluid dynamics.

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[3] *Edward Arthur Milne(1896-1950)*, British mathematician and astrophysicist

[4] according to the publication list in Morawetz, Serrin, Sinai 2002 [MSS02, pp. xvii-xxi]

During the years 1926 to 1971 Eberhard Hopf wrote[5] in total two books and approximately 79 journal articles, of which one book (1937) and 41 articles are in German and one book (1934) and 38 articles are in English. Both books were motivated by mathematical problems from astronomy and/or astrophysics. About 18 articles were closely related to radiative equilibria and integral equations, the most famous one in this group is the article with Norbert Wiener: *Über eine Klasse singulärer Integralgleichungen* [WiHo31]. At least 16 articles dealt with ergodic theory and the theory of dynamical systems. The most famous one in this group is the one on the *Hopf bifurcation*: *Abzweigung einer periodischen Lösung von einer stationären Lösung eines Differentialsystems* [Hopf42].

As was discussed in detail in my Miesenbach Talk in 2018, the curriculum vitae of Eberhard Hopf was not unique, but very unusual: He was one of the very few German scientists who moved from the US to Germany in 1936, and this was so though he had a secure position at MIT. Moreover he returned to the US shortly after 1945, with the help of *Richard Courant (1888-1972)*. The behavior of Hopf and also of Courant found dismay, disapproval and understanding in the math community. As a consequence, there are many falsified references to Hopf's work, but there is also enthusiastic praise of the high quality of his mathematical results. For details and a discussion see [MSp18b] and the references therein.

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