Transitions in Computational Complexity of Continuous-Time Local Open Quantum Dynamics

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We analyze the complexity of classically simulating continuous-time dynamics of locally interacting quantum spin systems with a constant rate of entanglement breaking noise. We prove that a polynomial time classical algorithm can be used to sample from the state of the spins when the rate of noise is higher than a threshold determined by the strength of the local interactions. Furthermore, by encoding a 1D fault tolerant quantum computation into the dynamics of spin systems arranged on two or higher dimensional grids, we show that for several noise channels, the problem of weakly simulating the output state of both purely Hamiltonian and purely dissipative dynamics is expected to be hard in the low-noise regime.

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Introduction.—Locally interacting spin systems are of fundamental interest in many-body physics and, also, describe engineered quantum systems that underly quantum information technologies. Consequently, there is great interest in developing classical algorithms for simulating their dynamics [1–5]. It is recognized that simulating quantum spin system dynamics on classical computers is generically hard since they can encode quantum computations [6–8]. However, strong interaction with an external environment prevents significant entanglement of the individual spins [9–14]. Physical intuition then suggests that a transition occurs in the classical complexity of such dynamics on tuning the strength of the system-environment interaction, i.e., the spin system transitions from a classically tractable phase, whose dynamics can be simulated on a classical computer in time that scales, at most, polynomially with the number of spins, to a classically intractable phase.

This expectation has been made rigorous in the context of circuit model (or discrete-time model) of noisy quantum computation [15–21]. It was shown, very early on, that such a transition is expected for the circuit model of quantum computation on tuning the rate of noise. For a sufficiently high rate of noise, provably efficient classical algorithms to simulate quantum circuits [15,16] have been provided. Moreover, the threshold theorem for quantum computation [17,18] implied that, if the noise is below a certain threshold and fresh auxiliary qubits are available, then a quantum computation can be encoded into a noisy quantum circuit. The requirement of fresh auxiliary qubits was subsequently relaxed for quantum circuits in two or higher dimensions when the noise was not depolarizing [19].

Less attention has been paid to continuous-time dynamics. Not only does it underly the discrete-time circuit model, it is also physically more relevant for analyzing near term analog quantum simulators [22–29]. While some studies have focused on the classical complexity of bosonic systems as a function of evolution time [30–32], theoretical results on the classical complexity as a function of noise strength have, thus far, only been provided for fermionic systems [33].

In this Letter, we study computational complexity transition with noise rate in spatially local spin-systems. We consider an open system of n spins arranged on a d-dimensional lattice ($\mathbb{Z}^d$) that are initially in a product state. Within the Born-Markov approximation [34], the state of the spins $\rho(t)$ is governed by a quantum Lindblad equation $d\rho(t)/dt = \mathcal{L}(t)\rho(t)$, with the Lindbladian

$$\mathcal{L}(t) = -i[H(t), \rho(t)] + \sum_\alpha \left(L_\alpha(t)\rho(t)L_\alpha^\dagger(t) - \frac{1}{2}\{\rho(t), L_\alpha^\dagger(t)L_\alpha(t)\} \right), \tag{1}$$

where $H(t)$ is the (possibly time-dependent) Hamiltonian, and $L_\alpha(t)$ are (possibly time-dependent) jump operators. Here, we study a restricted class of master equations with generators $\mathcal{L}(t)$ of the form

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$$\mathcal{L}(t) = \mathcal{L}_0(t) + \kappa \sum_{i=1}^{n} (\mathcal{N}_i - \mathrm{id}), \tag{2}$$

where "id" is the identity channel. The generator of this master equation has two terms—the first term, $\mathcal{L}_0(t)$, is a Lindbladian [i.e., of the form of Eq. (1)] that models interactions between different spins and the second term captures noise, modeled by a channel $\mathcal{N}_i$ on the ith spin, acting at a constant rate κ. We do not restrict ourselves to $\mathcal{L}_0(t)$ being described by only a Hamiltonian since even dynamics described by Lindbladians with only jump operators (albeit acting simultaneously on multiple spins) can be classically intractable [35].

We constrain $\mathcal{L}_0(t)$ to be geometrically local with interaction range R and with a uniformly bounded interaction strength J, i.e., $\mathcal{L}_0(t)$ permits a representation

$$\mathcal{L}_0(t) = \sum_{\Lambda \subset \mathbb{Z}^d} \mathcal{L}_0^{\Lambda}(t), \tag{3}$$

where $\mathcal{L}_0^{\Lambda}(t)$ is a Lindbladian which is identity on spins outside Λ with $\mathrm{diam}(\Lambda) \leq R$ and $\|\mathcal{L}_0^{\Lambda}(t)\|_{1\to 1} \leq J$. The noise channel $\mathcal{N}_i$ is assumed to be entanglement breaking [36]—examples of such noise channels could include local depolarizing noise [$\mathcal{N}_i(\rho) = \mathrm{tr}_i(\rho) I/2$], dephasing noise [$\mathcal{N}_i(\rho) = (\rho + Z_i \rho Z_i)/2$], and amplitude damping noise [$\mathcal{N}_i(\rho) = \mathrm{tr}_i(\rho)|0\rangle\langle 0|$]. Throughout this Letter, we consider evolution times t that scale, at most, as $\mathrm{poly}(n)$.

In the high noise regime, we show that this problem is classically tractable. Our proof strategy, inspired by previous results for quantum circuits [15,37], is to identify a map between the quantum dynamics and a percolation problem. However, unlike the discrete-time setting, where the dynamics respect a strict light cone, thus, making this mapping direct, the continuous-time dynamics for local Lindbladians only has an approximate light cone [38]. Our key technical contribution is to show that an approximation of the continuous-time dynamics can be mapped to a correlated percolation problem, which we prove percolates at a sufficiently high rate of noise.

Next, we consider the complementary low noise regime and study the worst-case hardness of this problem. The threshold theorem for quantum computation [17,18] already suggests that local Lindbladians are classically intractable below a noise threshold. This is so because a local Lindbladian can be chosen to fault-tolerantly encode any given quantum computation [17,18], including those that cannot be efficiently classically simulated. However, it is also of interest to study two subclasses of local Lindbladians. First, where $\mathcal{L}_0(t)$ models Hamiltonian interactions, i.e., where $\mathcal{L}_0(t)\rho = -i[H(t), \rho]$ for some Hamiltonian $H(t)$—these models arise in out of equilibrium many-body systems [39–41]. Second, interactions described entirely by many-body traceless jump operators and no Hamiltonian, i.e., where $\mathcal{L}_0(t)\rho = \sum_k L_k(t)\rho L_k^{\dagger}(t) - \{L_k^{\dagger}(t)L_k(t), \rho\}/2$ for some jump operators $L_k(t)$ with $\mathrm{Tr}[L_k(t)] = 0$. Such interactions are often referred to as purely dissipative [42], and arise in many-body quantum optics [43–45]. While both of these classes of systems are known to be hard to classically simulate when $\kappa = 0$ [35,46,47], their worst-case hardness in the low noise regime does not follow from a direct application of the threshold theorem.

We show that, for both of these classes and for noise rates below a threshold, it is unlikely that an efficient classical algorithm can simulate Eq. (2) in two or higher dimensions for arbitrary noise channels $\mathcal{N}_i$. More specifically, by an adaptation of Ref. [19] to continuous time, we identify a class of noise channels (which includes, e.g., the amplitude damping channel) such that Eq. (2) with purely Hamiltonian $\mathcal{L}_0(t)$ is classically intractable below a noise threshold. Then, we consider $\mathcal{L}_0(t)$ to be purely dissipative, which is classically intractable without noise [35]. We show that, for amplitude damping or dephasing noise and when κ is below a threshold, the dissipative dynamics can encode a postselected quantum computation and, hence, is expected to be classically intractable.

Results.—Our first result considers the high-noise regime of Eq. (2), and shows its classical tractability.

Theorem 1.—For $\kappa > \kappa_{\mathrm{th}}$, where κ_{th} depends on the lattice dimension d, interaction range R and interaction strength J, there is a $\mathrm{poly}(n, 1/\varepsilon)$ randomized classical polynomial-time algorithm to sample within ε total variation distance of $\rho(t)$ obtained on evolving Eq. (2) for t scaling at most as $\mathrm{poly}(n)$.

Our general strategy for the classical algorithm is to map Eq. (2) to a percolation problem. This has previously been done for the local unitary circuits [15,37], where, at sufficiently high noise rate, the effective percolation problem is subcritical [48] and the circuit can be exactly broken into small noninteracting clusters which permit individual contraction. However, unlike local unitary circuits, the continuous-time dynamics does not respect an exact light cone [38], and consequently, this mapping is not direct. We circumvent this issue by mapping a Trotterized approximation of this dynamics to a correlated percolation problem that is then shown to percolate at sufficiently high noise rates κ.

Proof sketch for Theorem 1.—The steps in the proof are depicted in Fig. 1—first, we Trotterize the evolution with time-step $\delta t = O[\varepsilon/\mathrm{poly}(n)]$ chosen to incur a total variation error $\leq O(\varepsilon)$ [Figs. 1(a) and 1(b)]. Next, we approximate the channels resulting from the Trotterization of $\mathcal{L}_0(t)$, as a convex combination of identity, applied with probability $1 - O(J)\delta t$ and another channel, applied with probability $O(J)\delta t$. The probability distribution at the output of the Trotterized circuit, $p(\mathbf{x})$, is then expressed as

$$p(\mathbf{x}) = \sum_{\mathcal{C}} p(\mathcal{C}) p(\mathbf{x}|\mathcal{C}),$$

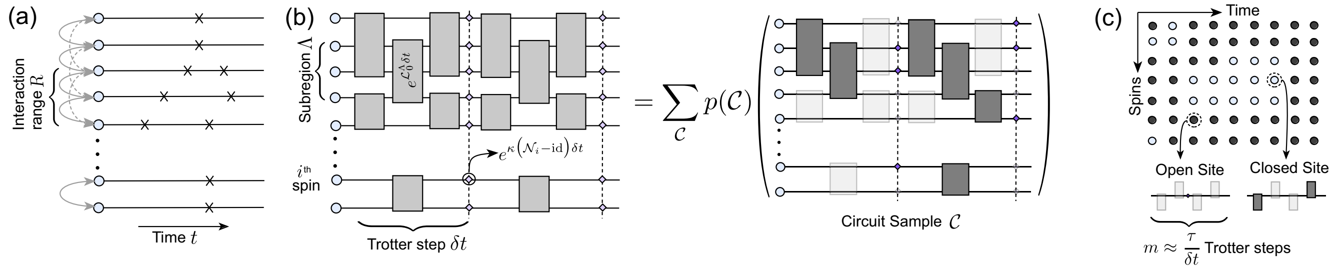

FIG. 1. Schematic depiction of the mapping of a continuous time model, to an equivalent percolation problem. For simplicity, we only depict a 1D setting. (a) The continuous-time model, where the spins interact with each through a local Lindbladian and with the crosses depicting the entanglement breaking noise occurring at a rate κ. (b) Trotterization of the continuous-time evolution, with the grey rectangles representing the channels resulting from $\mathcal{L}_0(t)$ and the purple rhombi being the Trotterized single-qubit noise channel. The Trotterized channels are then sampled from, with the faded channels being sampled to identity, (c) blocking $m \approx \tau/\delta t$ time-steps together for each qubit and identifying this block of time-steps with a site on the percolating lattice. The site is declared open if the qubit associated with the site experiences entanglement breaking noise at least once and does not couple to any of the neighboring qubits via the channels obtained on Trotterization of the Lindbladian $\mathcal{L}_0(t)$.

where the summation is over circuits $\mathcal{C}$ [Fig. 1(b)] obtained by randomly choosing between (i) identity or otherwise, instead of the Trotterization of local Lindbladian, and (ii) identity or $\mathcal{N}_i$ instead of the noise channel. $p(\mathcal{C})$ is the probability of choosing the circuit instance $\mathcal{C}$, thus, obtained and $p(\mathbf{x}|\mathcal{C})$ is the probability of obtaining $\mathbf{x}$ at the output of $\mathcal{C}$.

Next, we use percolation theory to show that, with high probability over $\mathcal{C}$, $p(\mathbf{x}|\mathcal{C})$ can be efficiently sampled from on a classical computer. First, we map sampling from $p(\mathcal{C})$ to a percolation problem on $\mathbb{Z}^{d+1}$ [Fig. 1(c)]—a site in this equivalent percolation problem is associated with a qubit and a block of m Trotterized time steps, where $m \approx \tau/\delta t$ for some $\tau > 0$. For a sampled circuit $\mathcal{C}$, the site is declared open if the associated qubit experiences noise at least once in the m associated time steps, and all channels arising from $\mathcal{L}_0(t)$ acting on it are replaced with identity, or else it is declared closed. Note that this percolation problem is correlated, i.e., the state of each site depends on its neighborhood. However, for sufficiently large κ, τ can be chosen such that the percolation problem is subcritical. Similar to discrete time [15], the sizes of the clusters in the subcritically percolated lattice are almost surely $O(\log n)$ [48], which allows us to classically compute $p(\mathbf{x}|\mathcal{C})$ and its marginals in polynomial time and, thus, sample from the output of $\mathcal{C}$. A detailed proof is provided in [49]. ■

Our next two results deal with the low-noise regime. First, we consider local Hamiltonians, i.e., $\mathcal{L}_0^\Lambda(t)$ in Eq. (3) satisfies $\mathcal{L}_0^\Lambda(t)\rho = -i[H^\Lambda(t), \rho]$ for some $H^\Lambda(t)$, and shows its low-noise intractability. We restrict ourselves to entanglement breaking noise channels $\mathcal{N}_i$ of the form

$$\mathcal{N}_i(\rho) = \text{Tr}_i(P\rho) \otimes |\alpha\rangle\langle\alpha| + \text{Tr}_i(Q\rho) \otimes |\beta\rangle\langle\beta|, \quad (4)$$

where $\{P, Q\}$ is a single-qubit positive operator-valued measurement (POVM), with $P - I/2$ being positive definite and $\{|\alpha\rangle, |\beta\rangle\}$ is an orthonormal basis for the qubit Hilbert space. The channel $\mathcal{N}_i$ thus, maps any initial state of the ith qubit to a mixture of $|\alpha\rangle$, $|\beta\rangle$ with a higher probability of being in $|\alpha\rangle$. An example of such a channel would be an amplitude damping channel. The proof of this result is a straightforward adaption of the discrete-time fault-tolerant construction previously used in Ref. [19], the only additional ingredient needed being the analysis of how faults in the unitary gates (as opposed to before or after them) do not impact the threshold theorem.

Theorem 2.—If $\mathcal{N}_i$ is of the form described in Eq. (4), then for qubits arranged on two or higher dimensional lattices and for κ below a threshold, there are instances of Eq. (2) with $\mathcal{L}_0(t)$ being a local Hamiltonian that cannot be weakly simulated on a classical computer within a small total variation error unless BQP = BPP.

A family of n-qubit quantum circuits is said to be weakly simulable within ε-total variation error if a classical computer can be used to sample in poly(n) time within a probability distribution p_{cl} such that $\|p - p_{\text{cl}}\|_1 \leq \varepsilon$, where p is the probability distribution at the output of the quantum circuit.

Proof sketch for Theorem 2.—We will restrict ourselves to two-dimensional lattices. First, we briefly review the discrete-time construction of Ref. [19]—the key idea is to fault-tolerantly encode 1D local quantum circuits, which can perform arbitrary quantum computations [6,7], in the continuous-time model. It has been previously established that fault tolerance can be achieved with just nearest neighbor unitary gates in 1D [17] if a RESTART operation (i.e., a quantum channel which replaces a qubit with a known pure state, say $|0\rangle$) is accessible. Reference [19] proposed to exploit the noise channel to implement the RESTART gate. Given a 2D grid of qubits [Fig. 2(a)], qubits in one column of the lattice are used as the computational qubits, comprising of data qubits (on which the quantum computation is performed) and ancilla qubits (which are used to perform error correction and need to be restarted). To restart an ancilla qubit, they utilize the qubits,

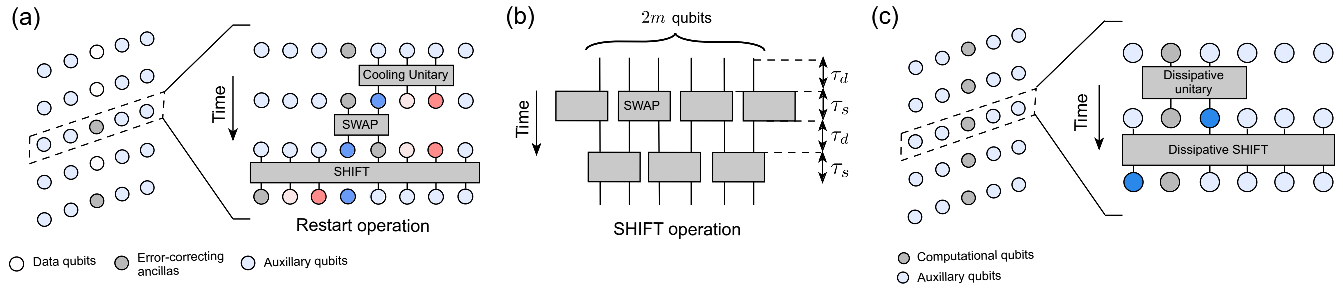

FIG. 2. (a) Construction of worst-case example of Eq. (2) when $\mathcal{L}_0(t)$ is a local Hamiltonian in 2D—one column is used for encoding a 1D fault-tolerant quantum computation and RESTART operation is implemented using the auxiliary qubits in the same row. The steps in the RESTART operation, also schematically depicted, include cooling the auxiliary qubits, swap with ancilla and shift for the next restart operation. (b) The SHIFT operation implemented by layers of SWAP gates (which take time τ_s) followed by allowing the noise to act on the individual qubits for time τ_d. The SWAP gates are faulty due to the noise, but the subsequent time interval τ_d is used to drive the swapped qubits to the fixed point of the noise channel. (c) Construction of worst-case example of Eq. (2) when $\mathcal{L}_0(t)$ is a local on a 2D lattice and purely dissipative. One column of qubits is again used to encode a 1D fault-tolerant quantum computation, with the remaining qubits used as the clock qubits to implement the involved unitaries dissipatively.

henceforth called the auxiliary qubits, in the row containing the ancilla. These qubits are initialized in the fixed point of $\mathcal{N}_i$ (and, hence, remain in it at all times), and when the ancillas need to be restarted, a (constant) number of auxiliary qubits are algorithmically cooled to a pure state [53,54], which is then swapped with ancilla. We point out that, since the noise channel (Eq. (4)) always maps to a state which has a higher probability of being in $|\alpha\rangle$, it does not drive the auxiliary qubits to the maximally mixed state, and hence, this cooling step is possible. The used auxiliary qubits are then shifted to bring unused auxiliary qubits next to the ancilla so that another RESTART gate can be performed when required.

If the noise is assumed to act only before or after the unitary gates, then this shift operation can be performed without any errors. However, in the continuous-time setting, the noise can act while the shift operation is being performed. Furthermore, since we could possibly need RESTART operations at $\Theta[\text{poly}(n)]$ time, which would need $\Theta[\text{poly}(n)]$ shift operations—thus, there is a possibility of accumulating a large error in the overall SHIFT operation at any, no matter how small, nonzero κ. To resolve this issue, we propose to perform an imperfect shift operation, followed by allowing the noise to act on the shifted qubits for time τ_d to drive them to its fixed point [Fig. 2(b)]. Clearly, if τ_d is chosen to be large enough, then the qubits would be in a state which can be subsequently cooled. However, increasing τ_d also increases the effective noise on the computational qubits since error correction is paused while the qubits are being restarted. A close analysis of this operation (provided in [49]) reveals that, to replenish m auxiliary qubits with the shift operation, τ_d can be chosen to be $\Theta(1/\kappa^{1-1/m})$, and hence, the error sustained in the computational qubits while error correction is paused for this shifting, which is proportional to $\kappa\tau_d$, can be made smaller than the error correction threshold for sufficiently small κ. ■

Our next result considers purely dissipative dynamics—for dephasing or amplitude damping channels, we provide theoretical evidence of the master equation remaining classically intractable at low noise rates. Our proof relies on using the Feynman clock construction [35], to encode a fault-tolerant quantum computation in a local dissipative master equation with postselected clock qubits. Since postselected quantum circuits that can encode (postselected) quantum computations are unlikely to be classically tractable [55–57], we obtain the low-noise intractability of the noisy dissipative master equation.

Theorem 3.—If $\mathcal{N}_i$ is the dephasing or amplitude damping channel, then for qubits arranged on two or higher dimensional lattices and for κ below a threshold, there are instances of Eq. (2) with $\mathcal{L}_0(t)$ being purely dissipative that cannot be weakly simulated on a classical computer within a small multiplicative error unless the polynomial hierarchy collapses to the third level.

A family of n-qubit quantum circuits is said to be weakly simulable within multiplicative error c if a classical computer can be used to sample in $\text{poly}(n)$ time within a probability distribution p_{cl} such that

$$\frac{1}{c}p(x) \leq p_{\text{cl}}(x) \leq c\,p(x) \quad \forall\ x \in \{0,1\}^n,$$

where p is the probability distribution at the output of the quantum circuit.

Proof sketch for Theorem 3.—To dissipatively apply a unitary U on ρ_0, we use an additional qubit, called the clock qubit, and $L = |1\rangle\langle 0| \otimes U$—with the initial state $|0\rangle\langle 0| \otimes \rho_0$ and postselecting on the clock qubit being in $|1\rangle$, the remaining qubits will be in $U\rho_0 U^\dagger$. Consider again $d = 2$—the computational qubits are laid out in one column, and the corresponding clock qubits are laid out in the rows [Fig. 2(c)]. A fault-tolerant quantum circuit can now be encoded in the dissipative master equation with the unitaries encoded as shown above, and the RESTART operations encoded with just an amplitude damping channel. We show, in [49], that errors in both the computational qubits and the clock qubits participating in a unitary can be translated to independent faults in the unitary gates being applied, and thus, the threshold theorem still holds. Finally, the clock qubits are replenished with a dissipative SHIFT to prepare for the next time step in the circuit (the SHIFT operation is performed again with two layers of SWAP, with SWAP being implemented dissipatively using the jump operators $|0,1\rangle\langle 1,0|$, $|1,0\rangle\langle 0,1|$)—we show, in [49], that if the noise channel under consideration is dephasing or amplitude damping, then the errors in the SHIFT operation do not impact the state of the computational qubits when postselected on the clock qubits being in $|1\rangle$. ∎

We remark that, in Theorem 3, we assumed the ability to implement a purely dissipative Lindbladian for a chosen jump operator. Physically, due to Lamb shift and nonzero environment temperatures, a Lindbladian with jump operator $L(t)$ is accompanied with two corrections [58,59]—a Hamiltonian $\propto L^\dagger(t)L(t)$ (the Lamb shift) and a Lindbladian with jump operator $L^\dagger(t)$ (the re-excitation). However, it can be shown that, for the specific choice of the jump operators used above and with postselection on the clock qubits, these corrections do not impact the encoded quantum circuit. We provide a detailed analysis of these corrections in [49].

Conclusion.—We studied noisy dynamics of many-body open quantum spin systems with local interactions. Our Letter provides rigorous evidence of transitions in the classical complexity of their continuous-time dynamics. As specific technical problems, we leave open the extensions of Theorems 2 and 3 to one-dimensional systems as well as to a larger class of noise channels. Furthermore, while we have exclusively focused on Markovian spin systems, future directions could include studying such transitions in other experimentally relevant models of many-body quantum systems. These could include non-Gaussian bosonic systems, which would be a model for many quantum optics experiments, and non-Markovian quantum systems.

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