

From Dyson Models to Many-Body Quantum Chaos

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Understanding the mechanisms underlying many-body quantum chaos is one of the big challenges in theoretical physics. We tackle this problem by considering a set of perturbed quadratic Sachdev-Ye-Kitaev (SYK) Hamiltonians defined on graphs. This allows to disambiguate between operator growth and *delocalization*, showing that the latter is the dominant process in the single-particle to many-body chaotic transition. Our results are verified numerically with state-of-the-art numerical techniques, capable of extracting eigenvalues in a desired energy window of very large Hamiltonians, in this case up to dimension $2^{19} \times 2^{19}$. Our approach essentially provides a new way of viewing many-body chaos from a single-particle perspective.

Introduction.— Chaotic behaviour is ubiquitous in nature. For *classical* dynamical systems it is, at least conceptually, straightforward to detect [1, 2]. Consequently, most diagnostic tools can afford to focus on the exponential divergence of flow lines in phase space, or some variant thereof [3]. The inherent noise in quantum systems, as manifest in the Heisenberg Uncertainty Principle, renders the notion of localised trajectories in phase space all but meaningless [4]. Consequently, diagnosing if a given Hamiltonian exhibits quantum chaos or not is a remarkably difficult task, doubly so when dealing with an interacting quantum many-body system (see, for example, Ref. 5 and references therein).

Historically, deciding whether a quantum system was chaotic or not hinged on the spectral properties of its Hamiltonian matrix [6]. This is because, with enough time, the dynamics of any chaotic quantum system will eventually resemble that generated by a random matrix whose spectrum then exhibits a characteristic level-repulsion [7, 8]. This approach has two immediate shortcomings: (i) spectral analysis typically relies on diagonalization of the Hamiltonian which, for large particle numbers is exceedingly difficult, even with the recent remarkable developments in computational hardware of the past few years and (ii) level repulsion and other random matrix theory tools are poor diagnostics for early-time chaos which is often relevant for quantum systems like the SYK model [9, 10] conjectured to be dual to the interior of a black hole [9, 11–13].

More recently, progress in the study of strongly correlated quantum systems has led to the development of a more modern set of diagnostic tools for quantum chaos. The most popular of these is the out-of-time-order 4-point correlation function (OTOC) whose growth encodes the quantum Lyapunov exponent, and is an excel-

lent diagnostic of early-time chaos [14, 15]. Other such modern tools include: the spectral form factor which, in a sense, interpolates between the OTOCs and more conventional RMT measures [16]; Nielsen circuit complexity [17], and Krylov-complexity [18]. The latter furnishes a quantitative measure of the growth of operators as the quantum system evolves in time [19].

Central to these new developments is the propagation and scrambling of quantum information in a local many-body quantum system (see Ref. 20 for a recent discussion on the quantum information aspects of scrambling). While the propagation of quantum information is at least intuitively clear, scrambling is more subtle and consensus has not yet been reached on a rigorous definition. Ostensibly, it is the process by which simple operators become complex through unitary time evolution [21]. One complicating feature of this simple-to-complex evolution is that it is really made up of two interrelated parts; operator *growth* and *delocalization*, sometimes also known as *operator entanglement* in the literature [22]. Understanding the role that each plays in quantum information scrambling is essentially the subject of this letter.

To sharpen our question, we will try to understand how and when a chaotic *single-particle* model becomes *many-body chaotic*. Toward this end, we need to control for the effects of growth and delocalization in the model. This is achieved by considering a quadratic SYK-like Hamiltonians defined on a family of graphs. By tuning the adjacency matrix of couplings defining the graph, we can control the localization properties of the system. We then perturb each of the models (e.g. for each graph) with the same growth-inducing term and study their evolution. Our results demonstrate that, in contrast to popular lore which holds that quantum scrambling is synonymous with operator growth, it is in fact operator delocal-

ization that appears to be the driving mechanism behind many-body quantum chaos. Given the critical role that chaos plays in diverse phenomena from the thermalization of quantum systems [23], to new phases of matter such as many-body localization [24, 25], quantum computing [26–28], and new quantum technology platforms [29–31], we anticipate that our results will be of importance to an equal number of diverse communities.

Scrambling and the Space of Many-Body Operators.— The notions of operator growth, delocalization and scrambling are built upon the definition of *fundamental operators* [14, 32]. These are quantum mechanical operators which constitute the building blocks for all the operators of the system under investigation. They satisfy the following properties: (i) they are of *size 1*, which means that they act non-trivially on a single site only [33], and (ii) they have simple mutual (anti)commutation relations. An example are fermionic creation and annihilation operators $\hat{c}_i^\dagger, \hat{c}_j$, which are obviously of size 1 and satisfy $\{\hat{c}_i^\dagger, \hat{c}_j\} = \delta_{ij}$. Generic many-body operators can be *expanded* as follows:

$$\hat{O} = \sum_i (\alpha_i \hat{c}_i + \text{h.c.}) + \sum_{i,j} \left(\alpha_{ij} \hat{c}_i \hat{c}_j + \beta_{ij} \hat{c}_i^\dagger \hat{c}_j + \text{h.c.} \right) + \dots, \quad (1)$$

where $1 \leq i, j \leq N$ label sites, N is the total number of sites, α, β are coefficients and h.c. denotes Hermitian conjugation. Expressing an arbitrary operator \hat{O} as in Eq. (1) shows its two-fold structure, consisting of a combination of products of fundamental operators of various sizes, including operators of extensive (in N) size, as well as (large) superpositions of operators of the same size. The former is captured by the sizes of individual operators in the sum while the latter is related to the number of the same sized operators in the sums.

For chaotic dynamics [15, 34–36], fundamental operators become generic under time evolution, i.e.

$$\hat{c}_i(t) = e^{i\hat{H}t} \hat{c}_i e^{-i\hat{H}t} \quad (2)$$

is of the form (1) for any nonzero t . The process by which the size of an operator increases with time is called *operator growth*, while the generation of superposition of operators is called *operator delocalization* [22, 31]. These are the two driving mechanisms underlying quantum scrambling and they typically happen simultaneously, while they are difficult to separate [22]. In this work, we consider setups where these two mechanisms can be clearly separated, such that the role of both can be disentangled, see Fig. 1 for a pictorial description which we now turn to describe.

Model.— Sachdev-Ye-Kitaev (SYK) models of disordered Majorana fermions have emerged in the last few years as a useful set of toy models to investigate many-body quantum chaos related questions [9, 13, 16, 37–39].

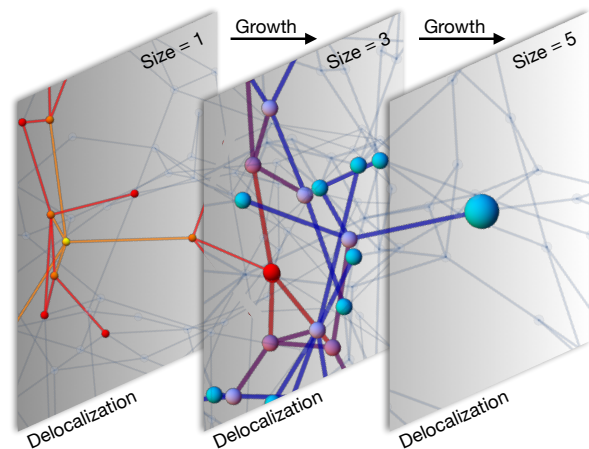


FIG. 1. In our model, delocalization and growth of operators can be clearly separated. A single Majorana (represented by a yellow dot in the left panel) becomes delocalized on the graph via the action of the SYK₂ Hamiltonian, but it does not grow. It begins to grow only once one of the Majoranas in the single 4-Majorana term $\hat{\gamma}^1 \hat{\gamma}^2 \hat{\gamma}^3 \hat{\gamma}^4$ is generated. For instance if $\hat{\gamma}_4$ is generated, the action of the 4-Majorana term will make the operator grow to include $\hat{\gamma}^1 \hat{\gamma}^2 \hat{\gamma}^3$ (represented by the larger red dot in the middle panel). Two steps of delocalization with SYK₂ are now required to arrive at e.g. $\hat{\gamma}^1 \hat{\gamma}^i \hat{\gamma}^j$, after which the 4-Majorana term will make the operator grow to include $\hat{\gamma}^2 \hat{\gamma}^3 \hat{\gamma}^4 \hat{\gamma}^i \hat{\gamma}^j$ (represented by an even larger blue dot in the right panel), and so on. Delocalization must take place at every step for growth to happen.

In particular, Ref. 31 has considered the delocalization properties (in *absence* of operator growth) of a set of quadratic SYK Hamiltonians defined on graphs.

Here, we want to study how models with different localization properties respond when a *minimal* growth-inducing term is turned on. Hence, we consider the Hamiltonian,

$$\hat{H} = -i \sum_{G(E,V)} J_{ij} \hat{\gamma}^i \hat{\gamma}^j + \hat{\gamma}^1 \hat{\gamma}^2 \hat{\gamma}^3 \hat{\gamma}^4. \quad (3)$$

The operators $\hat{\gamma}^i$, with $i = 1, \dots, N$, denote the Majorana fermions, satisfying $\{\hat{\gamma}^i, \hat{\gamma}^j\} = \delta^{ij}$. The graph $G(E, V)$ consists of a collection of N vertices V and edges $E \subseteq V \otimes V$. We assign a Majorana operator $\hat{\gamma}^i$ to each vertex $i \in V$, and a coupling J_{ij} to each edge (i, j) . The couplings J_{ij} are random variables sampled from a Gaussian distribution with vanishing mean and variance $\langle J_{ij}^2 \rangle = (N-1)/2n_E$, where n_E is the number of edges in $G(E, V)$.

Among all possible graphs G , we choose *small-world* graphs [40, 41] as it was shown in Ref. 31 that operator delocalization — for the quadratic part of the Hamiltonian defined in Eq. (3) — is sensitive to the parameters p, k defining this family of graphs. More precisely, these parameters are an integer k and a probability $p \in [0, 1]$. The procedure to generate graph samples is called the

Watts-Strogatz algorithm [40] and works as follows: for a given value of k , the algorithm starts by generating the regular circulant network in which each vertex is connected to its $2k$ nearest neighbors. Edges are then randomly rewired with probability p , keeping the graph connected and avoiding both self loops and edge duplications. Importantly, the number of edges in the graph depends on k only, $n_E = kN$, while its locality properties are controlled by p , interpolating between a regular lattice and a random Erdős-Renyi graph [42, 43]. In the following, we fix $k = 2$ and we work at different values of p . For a visual representation of this kind of graphs at small and large values of p we refer to Fig. 2.

The roles of the two different terms appearing in Eq. (3) are visually explained in Fig. 1. The space of many-body operators splits in a foliation-like structure, with each leaf containing operators of definite size. The first, quadratic terms, induce the delocalization of operators of a given size: they create superposition of operators within a leaf without inducing jumps between leaves. In contrast, the quartic term acts as a *leaf changing operator*: it only moves operators from one leaf to another, inducing operator growth without delocalization.

Let us discuss how the Hamiltonian in Eq. (3) connects with previous studies. Refs. 44–48 consider a *many-body integrable* Hamiltonian perturbed by a single impurity term, like the quartic term of Eq. (3). On the other hand, Ref. 49 considers a *single particle* Hamiltonian, similar to the quadratic terms of Eq. (3), perturbed by an *extensive* many-body Hamiltonian. Our setup differs conceptually from both: the extensive terms define a *single particle* problem, perturbed by a *single* many-body impurity. This is a clever setup to understand how single particle chaos can be embedded into and induces many-body chaos.

Numerical Results.— To investigate the many-body chaotic properties of the models we use the *r-ratios*, which provide a robust diagnostics of level repulsion [50]. They are defined via the formula

$$\langle r \rangle \equiv \langle r_i \rangle = \left\langle \frac{\text{Min}(s_i, s_{i+1})}{\text{Max}(s_i, s_{i+1})} \right\rangle, \quad (4)$$

where $s_i = E_{i+1} - E_i$ are the energy spacings between two neighboring eigenstates of the Hamiltonian and the average is taken over several levels in the same spectrum. $\langle r \rangle$ take specific values when the spectra show RMT (or integrable) distribution: $\langle r \rangle \approx 0.38$ for integrable (Poissonian) spectra, while for a random matrix ensemble (chaotic), depending on the symmetry class, we have $\langle r \rangle \approx 0.53, 0.60, 0.67$ for GOE, GUE, and GSE [50], respectively. In the case at hand, the quadratic terms in Eq. (3) enforce GUE symmetry class [39, 51].

In Eq. (3) there are two different sources of randomness. First, for $p \neq 0$ the particular graph realization is randomly determined via the Watts-Strogatz algorithm. Second, for a fixed graph, the non-vanishing values of J_{ij}

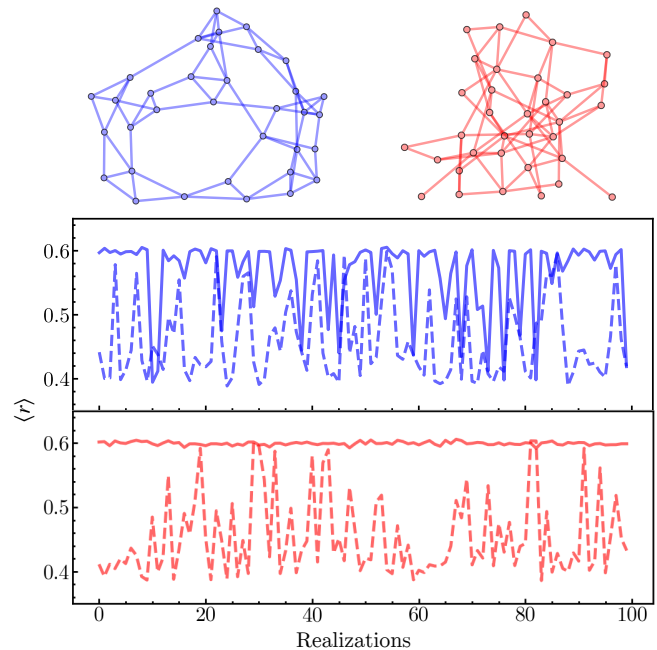


FIG. 2. The averaged r -ratios $\langle r \rangle$ for the Hamiltonians in Eq. (3), computed for $N = 34$ Majorana fermions. Here we show two specific graph realizations corresponding to rewiring probabilities $p = 0.1$ (solid blue), $p = 0.9$ (solid red) respectively. The dashed curves correspond to the base non-rewired circulant graph with next-to-nearest neighbors ($k = 2$) using the *same* coupling realizations. While there are strong fluctuations both for no rewiring and small rewiring $p = 0.1$, these disappear as p increases.

are randomly extracted. Our goal is to disentangle these two sources of randomness and understand how the non-locality properties of a graph, captured by p , affect the many-body chaotic properties of the model. Therefore we have generated, for a given G , many different realizations of the couplings J_{ij} and, for each of them, we have computed $\langle r \rangle$ for a bunch of eigenvalues lying at the center of the corresponding spectrum [52]. The results, for two representative graphs with low ($p = 0.1$) and large ($p = 0.9$) rewiring probabilities are reported in Fig. 2.

It is evident that when $p = 0.1$ the $\langle r \rangle$ are *highly* fluctuating, with values ranging from clearly chaotic to Poissonian depending on the particular J_{ij} realization. Interestingly, in absence of rewiring but with the same values of J_{ij} the results are qualitatively similar but quantitatively different, with large fluctuations of the $\langle r \rangle$ but between different values. This result signals that, when the rewiring is low or absent at all, the level correlations are mostly determined by the specific values of J_{ij} , while the role of the graph is very hard to detect. The situation becomes dramatically different when p is large and the resulting graph is highly non-local. In this case the $\langle r \rangle$ are very close to the GUE value and *independent* on the J_{ij} realization. These results, which we tested to

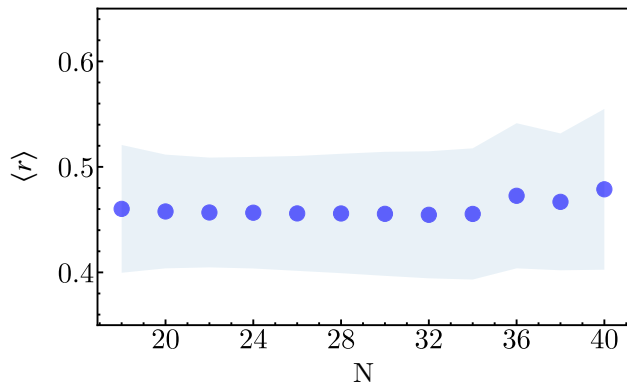


FIG. 3. $\langle r \rangle$ as a function of N for the case $p = 0$. The shaded regions are the standard deviation computed over coupling realizations and represent the magnitude of the fluctuations.

be robust against graph realizations, show that the graph topology is *crucial* in making the models in Eq. (3) many-body chaotic or not. As intrinsic in the Watts-Strogatz algorithm, all the graphs have exactly the same number of non-vanishing couplings, $2N$. Hence, the different physics they give rise to must be a result of the graph geometry only. Large values of p are also the values argued to be efficient for operator delocalization purposes in Ref. 31, thus confirming the prominence of operator delocalization in scrambling physics.

Without rewiring, the values of $\langle r \rangle$ are highly fluctuating. To see whether these fluctuations can be clearly deemed as finite-size artifacts, we have analyzed the statistical distributions of the $\langle r \rangle$, in absence of rewiring, up to $N = 40$ [53]. In Fig. 3, we report, as a function of N , the mean values and the variances of $\langle r \rangle$.

They appear to be N -independent and so they cannot be manifestly regarded as finite-size effects. Further insights can be gained by looking at the histograms of the $\langle r \rangle$ distributions themselves, as we discuss in the Supplemental Material.

The Underlying Dyson Model.— We can now work an heuristic argument to explain our findings. To start with, the quadratic part of the Hamiltonian can be always diagonalized, via a *canonical* Bogoliubov transformation \hat{U} , to the form

$$\hat{H}_2 = -i \sum_{i \in \text{odd}} \epsilon_i \hat{\chi}^i \hat{\chi}^{i+1}, \quad (5)$$

where ϵ_i are the single particle energy levels and, since \hat{U} is canonical, the new Majorana fermions $\hat{\chi}^i$ satisfy the standard anticommutation relations $\{\hat{\chi}^i, \hat{\chi}^j\} = \delta^{ij}$, i.e. they are *as fundamental as* the operators $\hat{\gamma}^i$. When expressed in terms of the new fermions $\hat{\chi}^i$, Eq. (5) does not show any kind of operator delocalization properties, with fundamental operators having a trivial evolution.

Hence, one is led to question whether there is any intrinsic meaning behind the notion of operator delocalization. The answer is affirmative and it becomes clear when considering the *full* Hamiltonian, \hat{H} , in Eq. (3). In terms of the fermions $\hat{\chi}^i$ it reads

$$\hat{H} = -i \sum_{i \in \text{odd}} \epsilon_i \hat{\chi}^i \hat{\chi}^{i+1} + \hat{H}_4(\hat{\chi}^i), \quad (6)$$

where the expression $\hat{H}_4(\hat{\chi}^i)$ denotes the quartic Hamiltonian obtained by acting with \hat{U} on the interacting term $\hat{\gamma}^1 \hat{\gamma}^2 \hat{\gamma}^3 \hat{\gamma}^4$. We are then led to conclude that, once rewritten as in Eq. (6), the model is many-body chaotic if $\hat{H}_4(\hat{\chi}^i)$ displays an *extensive* number of terms. This conclusion agrees with studies on sparse versions of SYK, which have shown that the details of the sparsification procedure are largely irrelevant as long as an extensive number of terms are preserved [54–56].

This prompts us to wonder under which conditions the Bogoliubov transformation \hat{U} generates an extensive number of quartic terms when acting on $\hat{\gamma}^1 \hat{\gamma}^2 \hat{\gamma}^3 \hat{\gamma}^4$. To answer, we observe that the columns of \hat{U} are given by the eigenvectors, $|\psi_i\rangle$, of the *single particle* problem defined by the coupling matrix iJ_{ij} , viewed as a random hopping Hamiltonian — i.e. as a Dyson-like problem [57–59] — defined over the Watts-Strogatz graph. Hence, we argue that \hat{H} is many-body chaotic when the eigenvectors $|\psi_i\rangle$ are *extended* over the Watts-Strogatz graph, since in this case \hat{U} have an extensive number of columns with non-vanishing components over all the vertices of the Watts-Strogatz graph, thereby creating an extensive number of terms when acting on $\hat{\gamma}^1 \hat{\gamma}^2 \hat{\gamma}^3 \hat{\gamma}^4$. Indeed, the Hamiltonian $\hat{h} \equiv iJ_{ij}$ shows an integrable/chaotic transition when p increases from 0 to 1, see Fig. 4, which will be studied elsewhere [60].

The single particle perspective explains the origin of the large fluctuations observed for vanishing or small rewiring. Such nearest-neighbor Dyson-like Hamiltonians on lattices have diverging density of states and localisation length at $E = 0$ as well as anomalously localised $E = 0$ states [61–63]. We conjecture that these large oscillations are due to the presence of these anomalously localised states. Therefore they are finite-size effects, which nevertheless are very strong and tend to disappear for very large sizes only, which cannot be studied via state-of-the-art exact diagonalization techniques.

Conclusions.— We propose a new framework to address the relation between single particle and many-body quantum chaos.

By embedding a single particle Hamiltonian into a many-body setup, together with a *single* impurity inducing operator growth, we have studied the conditions under which the resulting many-body system turns out to be chaotic. In this way, we have been able to show that operator delocalization, discovered and described in Ref. 31, is the driving mechanism behind operator

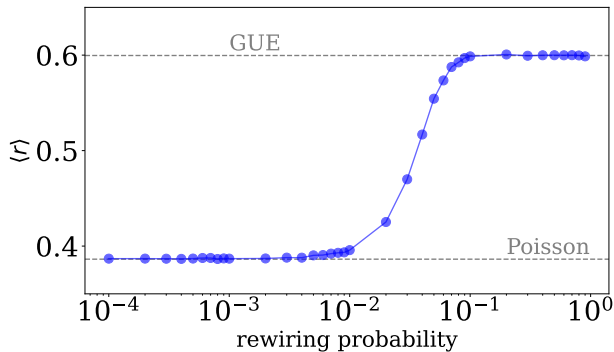


FIG. 4. Transition from Poissonian value of $\langle r \rangle$ to GUE value for the single particle part of the Hamiltonian (3) as a function of the rewiring probability of the underlying graph. Data is shown for $N = 6000$ Majorana fermions averaged over 30 realizations of the couplings J_{ij} for 5 different graphs for each rewiring probability.

scrambling and many-body quantum chaos. This study provides a new, single particle perspective on many-body quantum chaos: a single “many-body” impurity is enough to induce many-body quantum chaos, provided that the underlying single particle Hamiltonian exhibits *delocalized* eigenstates. In a sense, our results are very reminiscent of the celebrated Kondo effect [64].

Recently, there has been a growing interest in understanding the relation between single- and many-body quantum chaos [65, 66], and we believe that our results here will stimulate further study in this direction.

For instance, the single particle Dyson-like problem defined by the coupling matrix J_{ij} displays an interesting localized/delocalized transition triggered by the geometry of the small-world graph [60]. A detailed quantitative study of how such a transition is detected by the many-body system remains an interesting open question. To address this point, many-body systems of much larger size must be studied. Such large systems however, cannot be studied by exact diagonalization techniques. Methods such as the height function [67–69] used to study OTOCs at large system sizes will be very useful.

Further afield, we anticipate that the interpretation proposed in this paper will be of relevance in quantum computing due to the similarities it shares with the stabilizer formalism [26]. More specifically, operator growth relates to how the weight of Pauli string operators grow in circuits with Clifford elements whose dynamics are efficiently simulated classically. However, non-Clifford operations generate superposition of Pauli strings – the parallel with operator delocalization is evident – which is the element that makes quantum circuits transition into the *quantum advantage* regime [70, 71]. The problem considered in this article can be thought of as a close rel-

ative of the usual situation considered when dealing with many-body localization [24], i.e. to what extent are the localization properties of a single particle Hamiltonian preserved when turning on an interaction (in this case, an impurity similarly to Ref. 72). We hope that our results will be shed some much needed light in that context as well.

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- [1] S. H. Strogatz, *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry and Engineering* (Westview Press, 2000).
- [2] D. K. Arrowsmith and C. M. Place, *An introduction to dynamical systems* (Cambridge University Press, Cambridge, 1994).
- [3] C. Skokos, G. Gottwald, and J. Laskar, *Chaos Detection and Predictability*, Lecture Notes in Physics (Springer Berlin Heidelberg, 2016).
- [4] M. Gutzwiller, *Chaos in Classical and Quantum Mechanics*, Interdisciplinary Applied Mathematics (Springer New York, 1991).
- [5] F. Haake, *Quantum Signatures of Chaos*, Physics and astronomy online library (Springer, 2001).
- [6] O. Bohigas, M. J. Giannoni, and C. Schmit, Characterization of chaotic quantum spectra and universality of level fluctuation laws, *Phys. Rev. Lett.* **52**, 1 (1984).
- [7] O. Bohigas and M.-J. Giannoni, Chaotic motion and random matrix theories, in *Mathematical and Computational Methods in Nuclear Physics*, edited by J. S. Dehesa, J. M. G. Gomez, and A. Polls (Springer Berlin Heidelberg, Berlin, Heidelberg, 1984) pp. 1–99.
- [8] H. Stöckmann, *Quantum Chaos: An Introduction*, Cambridge nonlinear science series (Cambridge University Press, 1999).
- [9] J. Maldacena and D. Stanford, Remarks on the sachdev-ye-kitaev model, *Phys. Rev. D* **94**, 106002 (2016).
- [10] J. Polchinski and V. Rosenhaus, The spectrum in the sachdev-ye-kitaev model, *Journal of High Energy Physics* **2016**, 1 (2016).
- [11] J. Maldacena, D. Stanford, and Z. Yang, *Conformal symmetry and its breaking in two dimensional nearly anti-de-sitter space* (2016).

- [12] A. Almheiri and J. Polchinski, [Models of \$\text{ads}_2\$ backreaction and holography](#) (2014).
- [13] A. Kitaev and S. J. Suh, The soft mode in the sachdev-ye-kitaev model and its gravity dual, [Journal of High Energy Physics](#) **2018**, 183 (2018).
- [14] D. A. Roberts, D. Stanford, and A. Streicher, Operator growth in the syk model, [Journal of High Energy Physics](#) **2018**, 122 (2018).
- [15] C. W. von Keyserlingk, T. Rakovszky, F. Pollmann, and S. L. Sondhi, Operator hydrodynamics, otocs, and entanglement growth in systems without conservation laws, [Phys. Rev. X](#) **8**, 021013 (2018).
- [16] J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and M. Tezuka, Black holes and random matrices, [Journal of High Energy Physics](#) **2017**, 118 (2017).
- [17] R. A. Jefferson and R. C. Myers, Circuit complexity in quantum field theory, [Journal of High Energy Physics](#) **2017**, 107 (2017).
- [18] D. E. Parker, X. Cao, A. Avdoshkin, T. Scaffidi, and E. Altman, A universal operator growth hypothesis, [Phys. Rev. X](#) **9**, 041017 (2019).
- [19] E. Rabinovici, A. Sánchez-Garrido, R. Shir, and J. Sonner, Operator complexity: a journey to the edge of krylov space, [Journal of High Energy Physics](#) **2021**, 62 (2021).
- [20] S. Xu and B. Swingle, [Scrambling dynamics and out-of-time ordered correlators in quantum many-body systems: a tutorial](#) (2022).
- [21] Y. Sekino and L. Susskind, Fast scramblers, [Journal of High Energy Physics](#) **2008**, 065 (2008).
- [22] X. Mi, P. Roushan, C. Quintana, S. Mandrà, J. Marshall, C. Neill, F. Arute, K. Arya, J. Atalaya, R. Babush, J. C. Bardin, R. Barends, J. Basso, A. Bengtsson, S. Boixo, A. Bourassa, M. Broughton, B. B. Buckley, D. A. Buell, B. Burkett, N. Bushnell, Z. Chen, B. Chiaro, R. Collins, W. Courtney, S. Demura, A. R. Derk, A. Dunsworth, D. Eppens, C. Erickson, E. Farhi, A. G. Fowler, B. Foxen, C. Gidney, M. Giustina, J. A. Gross, M. P. Harrigan, S. D. Harrington, J. Hilton, A. Ho, S. Hong, T. Huang, W. J. Huggins, L. B. Ioffe, S. V. Isakov, E. Jeffrey, Z. Jiang, C. Jones, D. Kafri, J. Kelly, S. Kim, A. Kitaev, P. V. Klimov, A. N. Korotkov, F. Kostritsa, D. Landhuis, P. Laptev, E. Lucero, O. Martin, J. R. McClean, T. McCourt, M. McEwen, A. Megrant, K. C. Miao, M. Mohseni, S. Montazeri, W. Mruczkiewicz, J. Mutus, O. Naaman, M. Neeley, M. Newman, M. Y. Niu, T. E. O’Brien, A. Opremcak, E. Ostby, B. Pato, A. Petukhov, N. Redd, N. C. Rubin, D. Sank, K. J. Satzinger, V. Shvarts, D. Strain, M. Szalay, M. D. Trevithick, B. Villalonga, T. White, Z. J. Yao, P. Yeh, A. Zalcman, H. Neven, I. Aleiner, K. Kechedzhi, V. Smelyanskiy, and Y. Chen, Information scrambling in quantum circuits, [Science](#) **374**, 1479 (2021).
- [23] L. D’Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol, From quantum chaos and eigenstate thermalization to statistical mechanics and thermodynamics, [Advances in Physics](#) **65**, 239 (2016).
- [24] D. A. Abanin, E. Altman, I. Bloch, and M. Serbyn, Colloquium: Many-body localization, thermalization, and entanglement, [Rev. Mod. Phys.](#) **91**, 021001 (2019).
- [25] F. Alet and N. Laflorencie, Many-body localization: An introduction and selected topics, [Comptes Rendus Physique](#) **19**, 498 (2018), quantum simulation / Simulation quantique.
- [26] D. Gottesman, [Stabilizer codes and quantum error correction](#) (1997).
- [27] J. Kim, J. Kim, and D. Rosa, Universal effectiveness of high-depth circuits in variational eigenproblems, [Phys. Rev. Res.](#) **3**, 023203 (2021).
- [28] J. Kim, Y. Oz, and D. Rosa, [Quantum chaos and circuit parameter optimization](#) (2022).
- [29] S. Nimmrichter, J. Dai, A. Roulet, and V. Scarani, Quantum and classical dynamics of a three-mode absorption refrigerator, [Quantum](#) **1**, 37 (2017).
- [30] D. Rosa, D. Rossini, G. M. Andolina, M. Polini, and M. Carrega, Ultra-stable charging of fast-scrambling syk quantum batteries, [Journal of High Energy Physics](#) **2020**, 67 (2020).
- [31] J. Kim, J. Murugan, J. Olle, and D. Rosa, Operator delocalization in quantum networks, [Phys. Rev. A](#) **105**, L010201 (2022).
- [32] M. Carrega, J. Kim, and D. Rosa, Unveiling operator growth using spin correlation functions, [Entropy](#) **23**, 10.3390/e23050587 (2021).
- [33] We are considering quantum mechanical systems defined on lattices.
- [34] E. H. Lieb and D. W. Robinson, The finite group velocity of quantum spin systems, [Communications in Mathematical Physics](#) **28**, 251 (1972).
- [35] A. A. Patel, D. Chowdhury, S. Sachdev, and B. Swingle, Quantum butterfly effect in weakly interacting diffusive metals, [Phys. Rev. X](#) **7**, 031047 (2017).
- [36] A. Nahum, S. Vijay, and J. Haah, Operator spreading in random unitary circuits, [Phys. Rev. X](#) **8**, 021014 (2018).
- [37] D. Chowdhury, A. Georges, O. Parcollet, and S. Sachdev, Sachdev-ye-kitaev models and beyond: Window into non-fermi liquids, [Rev. Mod. Phys.](#) **94**, 035004 (2022).
- [38] A. M. García-García and J. J. M. Verbaarschot, Spectral and thermodynamic properties of the sachdev-ye-kitaev model, [Phys. Rev. D](#) **94**, 126010 (2016).
- [39] T. Nosaka, D. Rosa, and J. Yoon, The thouless time for mass-deformed syk, [Journal of High Energy Physics](#) **2018**, 41 (2018).
- [40] D. J. Watts and S. H. Strogatz, Collective dynamics of ‘small-world’ networks, [Nature](#) **393**, 440 (1998).
- [41] R. Albert and A.-L. Barabási, Statistical mechanics of complex networks, [Rev. Mod. Phys.](#) **74**, 47 (2002).
- [42] P. Erdős and A. Rényi, On random graphs i, [Publications Mathematicae Debrecen](#) **6**, 290 (1959).
- [43] E. N. Gilbert, Random Graphs, [The Annals of Mathematical Statistics](#) **30**, 1141 (1959).
- [44] L. F. Santos, Integrability of a disordered heisenberg spin-1/2 chain, [Journal of Physics A: Mathematical and General](#) **37**, 4723 (2004).
- [45] E. J. Torres-Herrera and L. F. Santos, Local quenches with global effects in interacting quantum systems, [Phys. Rev. E](#) **89**, 062110 (2014).
- [46] M. Brenes, E. Mascarenhas, M. Rigol, and J. Goold, High-temperature coherent transport in the xxz chain in the presence of an impurity, [Phys. Rev. B](#) **98**, 235128 (2018).
- [47] M. Brenes, T. LeBlond, J. Goold, and M. Rigol, Eigenstate thermalization in a locally perturbed integrable system, [Phys. Rev. Lett.](#) **125**, 070605 (2020).
- [48] A. Bastianello, Lack of thermalization for integrability-breaking impurities, [Europhysics Letters](#) **125**, 20001 (2019).
- [49] V. B. Bulchandani, D. A. Huse, and S. Gopalakrishnan,

- Onset of many-body quantum chaos due to breaking integrability, *Phys. Rev. B* **105**, 214308 (2022).
- [50] Y. Y. Atas, E. Bogomolny, O. Giraud, and G. Roux, Distribution of the ratio of consecutive level spacings in random matrix ensembles, *Phys. Rev. Lett.* **110**, 084101 (2013).
- [51] A. M. García-García, B. Loureiro, A. Romero-Bermúdez, and M. Tezuka, Chaotic-integrable transition in the sachdev-ye-kitaev model, *Phys. Rev. Lett.* **120**, 241603 (2018).
- [52] Such a procedure can be conveniently performed via full-diagonalization routines up to $N = 34$. For each realization of J_{ij} the average is taken over 20% of the center of the spectrum.
- [53] To reach such large value of N , we have implemented a Chebyshev filter diagonalization algorithm, based on Ref. 73, to collect an order of 700 eigenvalues in the central region of the spectrum for $N = 36, 38, 40$.
- [54] S. Xu, L. Susskind, Y. Su, and B. Swingle, *A sparse model of quantum holography* (2020).
- [55] A. M. García-García, Y. Jia, D. Rosa, and J. J. M. Verbaarschot, Sparse Sachdev-Ye-Kitaev model, quantum chaos, and gravity duals, *Phys. Rev. D* **103**, 106002 (2021).
- [56] E. Cáceres, A. Misobuchi, and R. Pimentel, Sparse syk and traversable wormholes, *Journal of High Energy Physics* **2021**, 15 (2021).
- [57] G. Theodorou and M. H. Cohen, Extended states in a one-dimensional system with off-diagonal disorder, *Phys. Rev. B* **13**, 4597 (1976).
- [58] L. Fleishman and D. C. Licciardello, Fluctuations and localization in one dimension, *Journal of Physics C: Solid State Physics* **10**, L125 (1977).
- [59] A. Krishna and R. Bhatt, Beyond the universal dyson singularity for 1-d chains with hopping disorder, *Annals of Physics* **435**, 168537 (2021), special Issue on Localisation 2020.
- [60] A. Andreanov, M. Carrega, J. Murugan, J. Olle, D. Rosa, and R. Shir, to appear (2023).
- [61] C. M. Soukoulis and E. N. Economou, Off-diagonal disorder in one-dimensional systems, *Phys. Rev. B* **24**, 5698 (1981).
- [62] M. Inui, S. A. Trugman, and E. Abrahams, Unusual properties of midband states in systems with off-diagonal disorder, *Phys. Rev. B* **49**, 3190 (1994).
- [63] G. De Tomasi, S. Roy, and S. Bera, Generalized dyson model: Nature of the zero mode and its implication in dynamics, *Phys. Rev. B* **94**, 144202 (2016).
- [64] J. Kondo, Resistance Minimum in Dilute Magnetic Alloys, *Progress of Theoretical Physics* **32**, 37 (1964).
- [65] M. Winer, S.-K. Jian, and B. Swingle, Exponential ramp in the quadratic sachdev-ye-kitaev model, *Phys. Rev. Lett.* **125**, 250602 (2020).
- [66] Y. Liao and V. Galitski, Universal dephasing mechanism of many-body quantum chaos, *Phys. Rev. Res.* **4**, L012037 (2022).
- [67] T. Zhou and X. Chen, Operator dynamics in a brownian quantum circuit, *Phys. Rev. E* **99**, 052212 (2019).
- [68] X. Chen and T. Zhou, Quantum chaos dynamics in long-range power law interaction systems, *Phys. Rev. B* **100**, 064305 (2019).
- [69] T. Zhou, A. Guo, S. Xu, X. Chen, and B. Swingle, Hydrodynamic theory of scrambling in chaotic long-range interacting systems, *Phys. Rev. B* **107**, 014201 (2023).
- [70] L. Leone, S. F. E. Oliviero, Y. Zhou, and A. Hamma, Quantum Chaos is Quantum, *Quantum* **5**, 453 (2021).
- [71] M. Hinsche, M. Ioannou, A. Nietner, J. Haferkamp, Y. Quek, D. Hangleiter, J.-P. Seifert, J. Eisert, and R. Sweke, *A single t -gate makes distribution learning hard* (2022).
- [72] P. Brighi, A. A. Michailidis, K. Kirova, D. A. Abanin, and M. Serbyn, Localization of a mobile impurity interacting with an anderson insulator, *Phys. Rev. B* **105**, 224208 (2022).
- [73] A. Pieper, M. Kreutzer, A. Alvermann, M. Galgon, H. Fehske, G. Hager, B. Lang, and G. Wellein, High-performance implementation of chebyshev filter diagonalization for interior eigenvalue computations, *Journal of Computational Physics* **325**, 226 (2016).

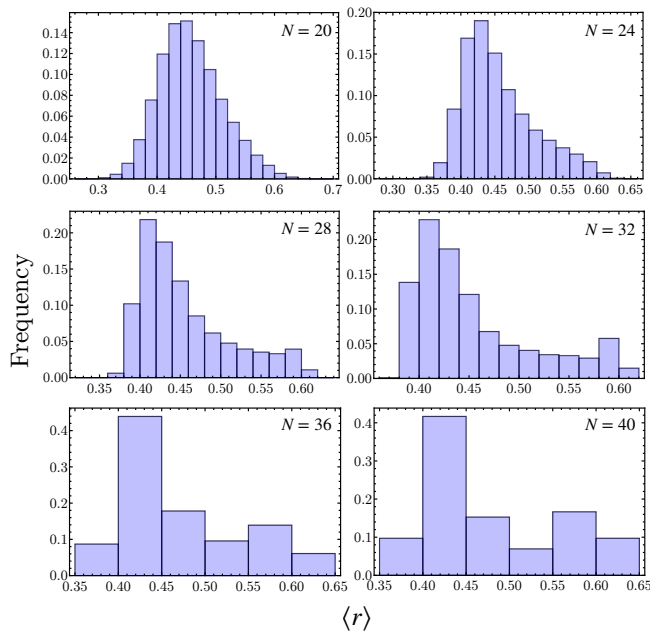


FIG. 5. Histogram of the data shown in Fig. 3. By increasing N we observe a gradual cross-over from a single peak to a double peak structure of the distribution of $\langle r \rangle$.

SUPPLEMENTAL MATERIAL

In this Supplemental Material, we discuss the statistical distributions of the r-ratios $\langle r \rangle$, computed at different

values of N .

THE DISTRIBUTIONS OF THE R-RATIOS

As discussed in the main text, and as shown in Fig. 3, in absence of rewiring the first two moments of the $\langle r \rangle$ distributions look to be completely N -independent. In other words, both the mean values and the variances of $\langle r \rangle$ do not show any significant differences by increasing N . To further understand whether this is a property of the first two moments only or it is a property of the distributions themselves, we have studied the histograms of the $\langle r \rangle$ distributions, for different values of N . Results are reported in Fig. 5.

We see that the distributions are *not* N -independent. At small N they are rather centered around the mean value, $\langle r \rangle \approx 0.45$ and they decay while moving from the peak. Instead, for large N the behavior is different: the peak splits into a major peak, which moves towards the Poisson value of $\langle r \rangle \approx 0.4$ while another, smaller, peak appears around the GUE value, $\langle r \rangle \approx 0.6$. This behavior suggests that, by increasing N and in absence of rewiring, the model turns out to be either manifestly chaotic or manifestly Poissonian depending on the specific realization of J_{ij} . In contrast, for sufficiently non-local graphs, chaoticity is robust and independent of J_{ij} .