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Modeling fast-and-frugal heuristics

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Abstract

Heuristics are simple rules that experts and laypeople rely on to make decisions under uncertainty as opposed to situations with calculable risk. The research program on fast-and-frugal heuristics studies formal models of heuristics and is motivated by Herbert Simon's seminal work on bounded rationality and satisficing. In this article, we first introduce the major theoretical principles (e.g., ecological rationality) and research approaches (e.g., competitive testing) that have been adopted in this research program, and then illustrate these principles and approaches with two heuristics: take-the-best and fast-and-frugal trees. We describe conditions under which simple heuristics predict as accurately as or better than more complex models, despite requiring less effort. We close by pointing out several issues that need to be further studied and better understood in the research on fast-and-frugal heuristics.

KEYWORDS

adaptive toolbox, artificial intelligence, computer simulation, ecological rationality, inference

INTRODUCTION

Making accurate predictions can be difficult. Before the 2016 presidential election in the United States, almost all forecasting pundits and agencies predicted, wrongly but with great confidence, that Hillary Clinton would win. This makes Allan Lichtman's prediction of Donald Trump's win stand out. In contrast to many sophisticated forecasting models that consider a host of factors and crunch a large amount of data, Lichtman's forecasting model is surprisingly simple. It is called "Keys to the White House" and consists of 13 true/false questions that Lichtman considers key to the incumbent party's victory (Lichtman, 2020). These "keys" are mainly about the achievements of the incumbent party or its candidate, such as the growth of the economy, lack of scandals, and major foreign or military success. If the incumbent party scores eight or more points out of 13, the model predicts the party to win the election; otherwise, not. The keys model made its first prediction in 1984, and has so far correctly picked nine winners out of 10.

The keys model is an example of the "tallying" heuristic. In deciding whether a target should be classified as belonging to one category (A) or the other (B) on the basis of n binary cues (or features, attributes, or dimensions), the heuristic defines a number k positive cue values that the target must have in order to be classified as A. In the keys model, the total number of cues n is 13, the cutoff criterion k is 8, and A is winning the election. Tallying is simple, because it ignores cue weight and only counts numbers. However, as demonstrated by the keys model, the heuristic can also be highly accurate.

Besides tallying, people use many other heuristics routinely to solve problems, draw inferences, and make decisions (e.g., Gigerenzer & Gaissmaier, 2011; Newell & Simon, 1972; Tversky & Kahneman, 1974). The purpose of this article is to introduce the systematic study of "fast-and-frugal heuristics," which are mental tools that help people decide in the face of uncertainty, on the basis of limited information, without spending much effort, but still with satisfactory outcomes. Over the past 20 or so years, there has been much research on fast-and-frugal heuristics, greatly improving our understanding of human psychology and decision-making (e.g., Gigerenzer et al., 2011; Gigerenzer & Selten, 2001; Gigerenzer et al., 1999; Hertwig et al., 2013b; Todd et al., 2012). The theoretical and methodological principles of this research program are highly innovative, but remain unfamiliar to many psychologists. In this article, we shall outline these principles and demonstrate how they are implemented in the studies of two specific heuristics.

THEORETICAL PRINCIPLES

The study of fast-and-frugal heuristics is driven by five theoretical principles. In this section, we explain each principle, together with counter-examples.

Ecological rationality

“A heuristic is ecologically rational to the degree that it is adapted to the structure of the environment” (Gigerenzer et al., 1999, p. 13). *Structure* means any systematic property or characteristic of the environment, such as high information redundancy among cues (see the section on “take-the-best”). The concept of ecological rationality follows directly from Herbert Simon’s scissors analogy: “Human rational behavior ... is shaped by a scissors whose two blades are the structure of task environments and the computational capabilities of the actor” (1990, p. 7). If one only looks at the side of cognition, as many psychologists have done, it is difficult to see why and when a heuristic succeeds or fails. Studies in ecological rationality analyze the fit between a heuristic and the environment in which it is applied, and require both an analysis of the environment and a theory of error. A test of ecological rationality typically involves comparing a heuristic’s performance in multiple environments and against other heuristics or nonheuristic models.

Opposite to: Context-free views on rationality, such as axiomatic theories of rationality, and logical coherence norms for reasoning (e.g., Arkes et al., 2016).

Adaptive toolbox

Ecological rationality implies that the mind must have a collection of tools at its disposal to deal with a multitude of tasks across different environments. The tools are not restricted to heuristics but include any mental mechanisms evolved to help people achieve their goals. Hence, in addition to understanding the performance of heuristics, the research program also aims to uncover the content of the mental toolbox and examine whether people can apply the tools adaptively (Gigerenzer & Selten, 2001).

Opposite to: One-tool-fits-all approaches to cognition and behavior, such as universal Bayesian and expected utility maximization theories of mind.

Process models

A fast-and-frugal heuristic is a process model that not only depicts the input–output relation between information and decision but also specifies the transitional processes in between, providing enough details to allow computer programs to simulate the heuristic. The transitional processes are built on empirical evidence, and are essentially hypotheses of human cognition. They allow strong tests of whether and how well fast-and-frugal heuristics are descriptive models of human cognition and behavior (e.g., Jarecki et al., 2020).

Opposite to: As-if models in psychology and behavioural economics that do not model the process of decision-making but aim only to predict behavior (e.g., Berg & Gigerenzer, 2010).

Building blocks

Fast-and-frugal heuristics can be described by three building blocks:

1. Search, which specifies how information search is conducted in the search space.
2. Stopping, which specifies when the search is stopped.
3. Decision, which specifies how a decision/prediction is made.

Opposite to: One-word heuristics, such as “availability” (Tversky & Kahneman, 1974), that do not specify one or more of the building blocks but consist of only general and ambiguous verbal labels (e.g., Gigerenzer, 2010).

Bias and variance

When a model is applied for prediction in a certain task, prediction errors come from three sources (e.g., Geman et al., 1992):

Total error = bias² + variance + irreducible error.

Bias refers to the systematic difference between a model’s average prediction and the true state of an event and represents how closely the model captures the true state or function. *Variance* refers to the sensitivity of a model to sampling error when it needs to learn parameter values of a model in one sample and apply the model for prediction in another sample. In general, models with fewer free parameters tend to have smaller variances but suffer from larger biases than models with more free parameters, and this “bias–variance dilemma” exists as long as the learning sample size is limited. Therefore, by being simple and with no or fewer free parameters, a heuristic can lead to high prediction accuracy because of its small variance, and be ecologically rational by having the least sum of bias and variance (e.g., Brighton & Gigerenzer, 2015).

Opposite to: Error theories that do not consider the variance component and its contribution to prediction errors, and the research practice of merely fitting models to known data instead of predicting new data.

RESEARCH METHODS

The principles outlined in the previous section are the theoretical foundations of the research program on fast-and-frugal heuristics. In this section, we introduce the main research methods that have been applied in the program.

Mathematical analysis

Mathematical analysis is indispensable in rigorous studies of heuristics. It allows the deduction of normative statements regarding heuristics’ ecological rationality, specifying how well heuristics and other strategies are *expected* to perform relative

to each other in well-defined environments. It can also lead to the discovery of counter-intuitive phenomena, such as the “less-is-more” effect in the use of the recognition heuristic (Goldstein & Gigerenzer, 2002). Furthermore, mathematical analysis points out necessary consequences of certain heuristics that may provide alternative explanations of known phenomena. For example, Katsikopoulos and Gigerenzer (2008) showed that the “priority heuristic,” a simple lexicographic model for choice, logically implies the fourfold pattern of risk attitudes, the reflection effect, and other major violations of expected utility theory. These violations are typically explained by modifying expected utility theory with added free parameters (e.g., Kahneman & Tversky, 1979).

Computer simulations

Because many real-world environments cannot be well described statistically owing to both their inherent complexity and limited observations, computer simulations are widely used in the study of fast-and-frugal heuristics (e.g., Luan et al., 2019; Todd et al., 2012). They have been applied to (a) estimate heuristics’ performance in well-defined environments when it cannot be derived analytically; (b) estimate heuristics’ performance in real-world environments; and (c) simulate environments in which a heuristic is subsequently tested, in order to explore the properties of environments a heuristic can exploit.

Competitive testing

The study of ecological rationality is about competition between models. Thus, most studies of fast-and-frugal heuristics involve comparing performances of at least two models, including at least one heuristic and one nonheuristic model. This approach is in the spirit of competitive hypothesis testing in scientific investigations (Cohen, 1994). Moreover, it is typical that models are evaluated and compared on more than one performance criterion. For example, besides accuracy, frugality—that is, how many cues are searched before the search is stopped and a decision is made—is often assessed as well (e.g., Gigerenzer & Goldstein, 1996; Luan et al., 2019; Payne et al., 1993). Competitive testing differs from the widespread practice of testing only one’s own model as well as from null hypothesis testing.

Prediction

In model testing, the research program emphasizes models’ performance in prediction, instead of fitting. Predictions are usually derived through the method of cross-validation, which estimates model parameters in one sample and applies the model to predict data in another sample (e.g., Czerlinski et al., 1999). Fitting, on the other hand, refers to estimating model parameters from one sample without testing how well the fitted model actually predicts new data. Prediction is not only more practically useful (e.g., consider the value of foresight over that of hindsight) but also better at capturing a model’s true performance in an uncertain world, in which observations are limited, true parameter values may change, and random noise is abundant. Furthermore, data-fitting by itself is not a good model testing practice because it often leads to overfitting.

Real-world tasks

Simulated tasks are useful for understanding the theoretical properties of heuristics, but real-world tasks are crucial for evaluating how well heuristics perform in environments that matter; thus, such tasks have been the subject of investigation in many studies of fast-and-frugal heuristics (e.g., Luan et al., 2014; Todd et al., 2012). This approach is in direct contrast to research that employs largely “toy” problems, such as the Wason selection task (Wason, 1968), the Linda problem (Tversky & Kahneman, 1983), and choices between monetary gambles, which may be intellectually interesting but fail to reflect many facets of real problems. The contrast not only is a practical one, but more importantly concerns the ultimate goal of the mind: The function of the adaptive toolbox is to cope with naturally occurring tasks and to be evolutionarily successful—not to adhere to content-free principles of logic and coherence (e.g., Arkes et al., 2016).

Human experiment

Experiments are necessary to help researchers of fast-and-frugal heuristics find out whether and in what task condition people use heuristics. The experiments in the fast-and-frugal heuristics paradigm often have three characteristics: First, many of them focus on the role of expertise in strategy choice, with the assumption that experts may decide in more ecologically rational manners and represent better human wisdom in decision-making (e.g., Dhimi, 2003; Luan et al., 2019; Pachur & Marinello, 2013). Second, they are usually guided by predictions from analyses of ecological rationality and computer simulations, and aim to test whether human participants would behave in accordance with these predictions (e.g., Goldstein & Gigerenzer, 2002; Luan & Reb, 2017). And third, they apply various model testing and comparison techniques to examine which type of models, heuristics or nonheuristics, people are more likely to use, instead of fixing attention on one model type (e.g., Fang et al., in press; Garcia-Retamero & Dhimi, 2009; Luan et al., 2019).

TAKE-THE-BEST

Among all fast-and-frugal heuristics, take-the-best is probably the most studied one. Here, we use the research on take-the-best to demonstrate how some of the theoretical principles and research methods introduced above have been implemented to understand the nature of this heuristic.

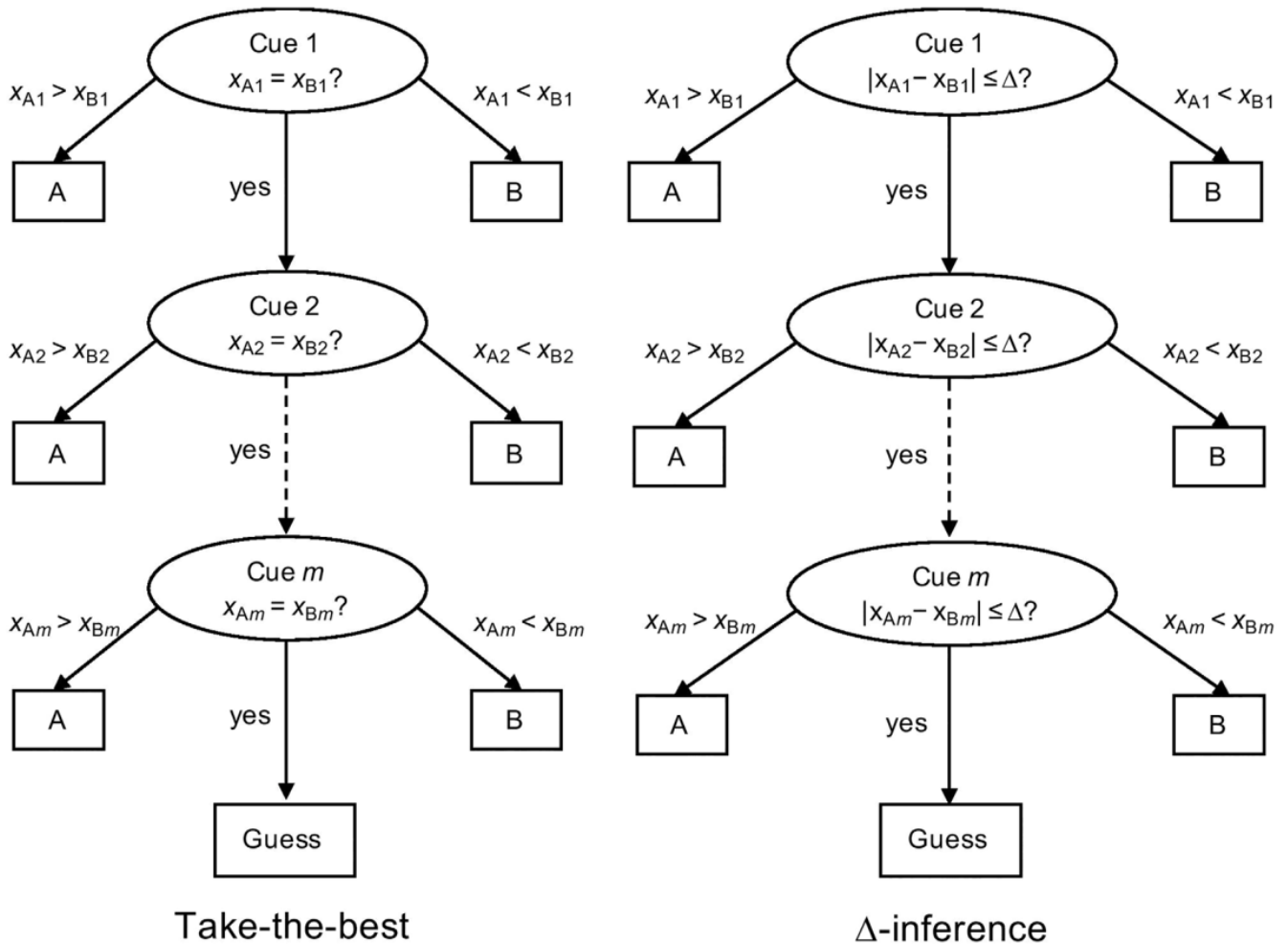


FIGURE 1 Schematic illustrations of take-the-best (left) and Δ -inference (right). A and B are two objects in comparison, and x_{Ai} and x_{Bi} are their values on the i th cue. Take-the-best is a heuristic for making inferences based on binary cues, while Δ -inference is a generalization of take-the-best to nonbinary cues.

Definition

Take-the-best is a heuristic for paired-comparison tasks in which the goal is to infer which object of two, A or B, has a larger value on a criterion variable (e.g., which team has a better chance of winning an upcoming match, Manchester United or Arsenal? Or, which city has a larger population, Xi'an or Wuhan?). The left panel of Figure 1 shows how take-the-best works schematically. It has three building blocks:

1. Search rule: Search through cues in order of their *validity*.
2. Stopping rule: Stop on the first cue in which the two objects have different values.
3. Decision rule: If a cue is positively related to the criterion, infer that the object with a larger cue value has a larger criterion value; otherwise, do the opposite. If search does not stop after all cues are searched, choose randomly.

A cue's validity v is a measure of the cue's inference quality and is given by

$$v = C/(C + W),$$

in which C and W are the numbers of correct and wrong inferences, respectively, when a cue discriminates (i.e., $x_{Ai} \neq x_{Bi}$).

Take-the-best is a fast-and-frugal heuristic because it is computationally simple, usually makes decisions without searching for all cues, requires only limited knowledge or estimation (i.e., of the cue order and cues' inference directions), and more importantly can be highly accurate in situations where it is ecologically rational (see below); in these situations, empirical evidence indicates that many people do use the heuristic (e.g., Bröder, 2012). Take-the-best can also be categorized as a noncompensatory, lexicographic decision model, for cues lower in the search hierarchy cannot over-turn the decisions made by cues higher in the hierarchy. In decision research, lexicographic models have been used mainly to explain *preferences*, in which choices are a matter of taste and accuracy is difficult to define. Take-the-best, however, has been studied largely as a heuristic for *inferences*, in which correct decisions can be defined and assessed. Research on take-the-best is influenced by the lens model framework (Brunswik, 1952; Hammond & Stewart, 2001), which has provided many useful concepts in inference studies, including conceptualizing distinct units of information as cues, measuring cue-criterion relations, and emphasizing the influence of environmental structures on performance.

Inferences in one task

Gigerenzer and Goldstein (1996) conducted the first study of take-the-best in the German city population task, in which a

person judges which of two German cities has a larger population. This study exemplifies several principles and research methods in studies of fast-and-frugal heuristics. First, it applied computer simulations to competitively test the accuracy of take-the-best, some other heuristics, and weighting-and-adding linear models; second, it showed that simple heuristics can result in better decision accuracy than linear models, challenging the notion that heuristics are always “second best”; third, it used both accuracy and frugality as criteria of model performance; and fourth, it investigated a real-world task with a clearly defined reference class, natural cues, and natural cue values. This approach is in stark contrast to many psychological studies that exclusively use artificial materials and risk the sacrifice of external validity for the sake of internal validity.

Inferences in many tasks

Results in one task are demonstrative but with limited generalizability. Czerlinski et al. (1999) tested take-the-best and other models in 20 natural tasks, calculating the average performance of each model and examining the environmental properties that may explain the performance of take-the-best relative to other models. They found that take-the-best was on average more accurate than multiple linear regression in prediction and that the more sparse the data, indicated by a lower object-to-cue ratio in a task data set, the larger the accuracy advantage of take-the-best over regression. These findings can be explained by the bias–variance analysis of error: When data are sparse, the variance component of error is typically larger for regression than take-the-best because the former needs to estimate more parameters, including the variance–covariance matrix, than the latter. More data can lead to better estimation of regression parameters, reducing its prediction variance and making regression more accurate than take-the-best in prediction eventually (e.g., Katsikopoulos et al., 2010; Şimşek & Buckmann, 2015).

The ecological rationality of take-the-best

It may appear unthinkable that take-the-best can make inferences as accurately as linear regression; after all, it relies on only one cue to make each inference and ignores data that linear models carefully weight and add. In fact, ignoring relevant information has been considered a sign of irrationality in the heuristics-and-biases program (Tversky & Kahneman, 1974). The bias–variance decomposition of error, however, implies that this evaluation is incorrect. Moreover, we know by now of three environmental conditions under which no linear model can outperform take-the-best in accuracy because both have the same bias, in term of bias and variance. These three conditions are noncompensatoriness, dominance, and cumulative dominance (e.g., Gigerenzer, 2016; Şimşek, 2013).

Noncompensatoriness. Martignon and Hoffrage (2002) proved that take-the-best leads to the same inferences as linear models in environments where the weights of binary-valued cues are *noncompensatory*; that is, for a set of m cues,

$$w_j > \sum_{k>j} w_k,$$

where w_j is the weight of the j th cue and $1 \leq j \leq m$. In words, an environment is noncompensatory if after cues are ordered by weight, each weight is larger than the sum of all weights to come; an example is the weight set $\{1, 1/2, 1/4, 1/8, 1/16\}$.

Dominance. If Object A's values are higher than Object B's in at least one cue and not worse than B's in all other cues, then A dominates B. For any pair where dominance holds, take-the-best will lead to the same decision as any linear model, as well as any reasonable nonlinear model.

Cumulative Dominance. The cumulative profile of an object consists of m values, in which the i^{th} value is the sum of the first i cues' values. Object A cumulatively dominates B if its cumulative profile exceeds or equals that of B in every term and exceeds it in at least one term (e.g., Baucells et al., 2008). If cumulative dominance holds, then a linear rule with the same order of cues predicts the cumulatively dominant object, just as a lexicographic rule does. Thus, for any pair where cumulative dominance holds, take-the-best will lead to the same decision as any linear model. Note that dominance implies cumulative dominance, but not vice versa.

To summarize, if one of these three conditions holds, take-the-best has the same bias as a linear model and will make predictions at least as accurately as a linear model. How often does this occur in the real world? Şimşek (2013) analyzed 51 natural data sets that cover diverse fields, such as biology, business, economics, and medicine. Among these data sets, the median frequency that at least one of the three conditions holds is 90%; that is, in half of the data sets, more than 90% of decisions are such that take-the-best will yield the same inferences as a linear model. When the cues are dichotomized at the median, this frequency increases to 97%. Together with the generally smaller prediction variances of take-the-best, these results explain why the heuristic often performs as well as or better than linear models in real-world tasks.

Furthermore, the finding about noncompensatory environments suggests that a lexicographic model such as take-the-best should do well when the quality or information value of cues after the deciding one is limited, so that ignoring these cues will cause little harm to the model's accuracy. Besides cues' linear weights, other measures have also been applied to assess cue quality, including validity (Katsikopoulos et al., 2010), the odds ratio of validity (Katsikopoulos & Martignon, 2006), and cues' bivariate Pearson correlations with the criterion (Hogarth & Karelaia, 2007). These studies show that take-the-best performs better relative to other models when the dispersion or variability of cue quality is high, which approximates the noncompensatoriness condition. Another property related to cues' information value is redundancy, that is, how much cues' information overlaps. The most popular measure of redundancy is the average intercue correlation (e.g., Dieckmann & Rieskamp, 2007; Hogarth & Karelaia, 2007; Luan et al., 2014), although more sophisticated measures have also been applied (e.g., Lee & Zhang, 2012). In general, the more redundant the cues, the better take-the-best performs relative to other models.

In sum, research on ecological rationality has identified environmental conditions that take-the-best can exploit to

make inferences as accurately as or better than linear models, despite searching for fewer cues and requiring less computation. When data are sparse, take-the-best is also likely to out-perform nonlinear models, such as neural networks, support vector machines, and other complex models (e.g., Brighton & Gigerenzer, 2015).

Variations and generalizations of take-the-best

Alternative search and stopping rules have been proposed and tested for take-the-best. With respect to the search rule, one could search cues randomly, start with the cue that led to a decision last time, or search according to other measures of cue quality (e.g., Gigerenzer et al., 2012; Martignon & Hoffrage, 2002). In general, these alternative search rules rarely improve take-the-best's prediction accuracy. With respect to the stopping rule, a much-discussed alternative is to stop searching when two cues, instead of one, favor the same object (e.g., Hogarth & Karelaia, 2007). This strategy tends to perform better than take-the-best in environments where cues are close to each other in quality.

Take-the-best was originally formulated to process binary cues only. Most of the later studies followed suit and converted nonbinary cues to binary before calculating take-the-best's performance. Thinking that this approach may not fully use information contained in nonbinary cues, Luan et al. (2014) proposed a version of take-the-best that is able to process the original cue values directly, calling the heuristic Δ -inference. A schematic illustration of how Δ -inference works is shown on the right side of Figure 1. In a nutshell, a threshold Δ is added to the stopping rule: If the difference between two objects on a cue exceeds Δ , search stops; otherwise, search continues until a larger-than- Δ difference is found in another cue.

Luan et al. (2014) showed that in simulated task environments, the accuracy of Δ -inference is a single-peaked function of Δ ; that is, the highest accuracy is reached when Δ is at an intermediate value. However, when tested in 39 real-world environments, the best Δ was often very small, and a Δ of zero—that is, deciding if there is any difference between two objects in a cue—had the best overall prediction accuracy compared to other values of Δ . Moreover, in comparison to more complex models, Δ -inference performed very well in the real-world environments, and it outperformed the binary-cue version of take-the-best in both prediction accuracy and frugality no matter how sparse or abundant the data are. In general, installing a threshold component in take-the-best improves the heuristic's flexibility and performance. It also makes it easier to generalize take-the-best to tasks where more than two objects are being compared.

In another approach to generalizing take-the-best, Lee and Cummins (2004) argued that both take-the-best and linear models are special cases of a general diffusion model in which information is sampled until the accumulated evidence surpasses a threshold. Take-the-best and linear models differ in how the stopping threshold is set and how information from the cues is accumulated. While theoretically sound, this "general-tool" approach is inconsistent with the principle of the adaptive toolbox and has not helped answer questions regarding the ecological rationality of take-the-best and linear models. Furthermore, showing that a model is a special case of a more general model is progress only if the general model is highly specified, but not if one merely replaces the specific parameters of a model with free parameters. Every model can be said to be a specific case of a more general one (e.g., Einstein's $e = mc^2$ is nothing but a polynomial of second degree). Progress, however, lies in a useful level of specificity, and with too much abstraction one can lose the important insights coming from specificity.

FAST-AND-FRUGAL TREES

Fast-and-frugal trees (FFTs) are a class of heuristics for binary classifications or decisions, as when one needs to decide to which of two categories an object belongs or which of two courses of action to take. Although such tasks may be more prevalent in real life than paired comparisons, FFTs are relatively new in heuristic research and there is still much to explore and understand about why they are so widely used in practice.

Definition and examples

Given m cues, an FFT is a decision tree that has $m + 1$ exits, with each of the first $m - 1$ cues having one exit and the last cue having two (Martignon et al., 2003). Like take-the-best, FFTs can be described with three building blocks:

1. Search rule: Search through cues in a predetermined order.
2. Stopping rule: Stop search as soon as the decision condition in a cue is satisfied.
3. Decision rule: Classify the object according to the decision rule of the cue.

Using tree-like representations to categorize entities and knowledge has a long history in human civilization (Wiley, 1988). In modern days, decision trees are popular machine-learning algorithms for classifications. Compared to other types of decision trees, FFTs are very simple in terms of the number of exits and the exit structure; and similar to take-the-best, FFTs are also lexicographic models that strive to make a decision as soon as there is a single piece of evidence or reason for it.

Figure 2 shows four three-cue FFTs applied in different domains. The first one (Figure 2A) describes how some people decide whether to forgive another person who committed a hurtful deed (Tan et al., 2017). In essence, people using this FFT decide to forgive if there is a *no* answer to any of the three questions. The second FFT (Figure 2B) is one that Green and Mehr (1997) developed to help emergency room doctors assign patients to either the coronary care unit or a regular nursing bed. They reported that this FFT achieved better prediction

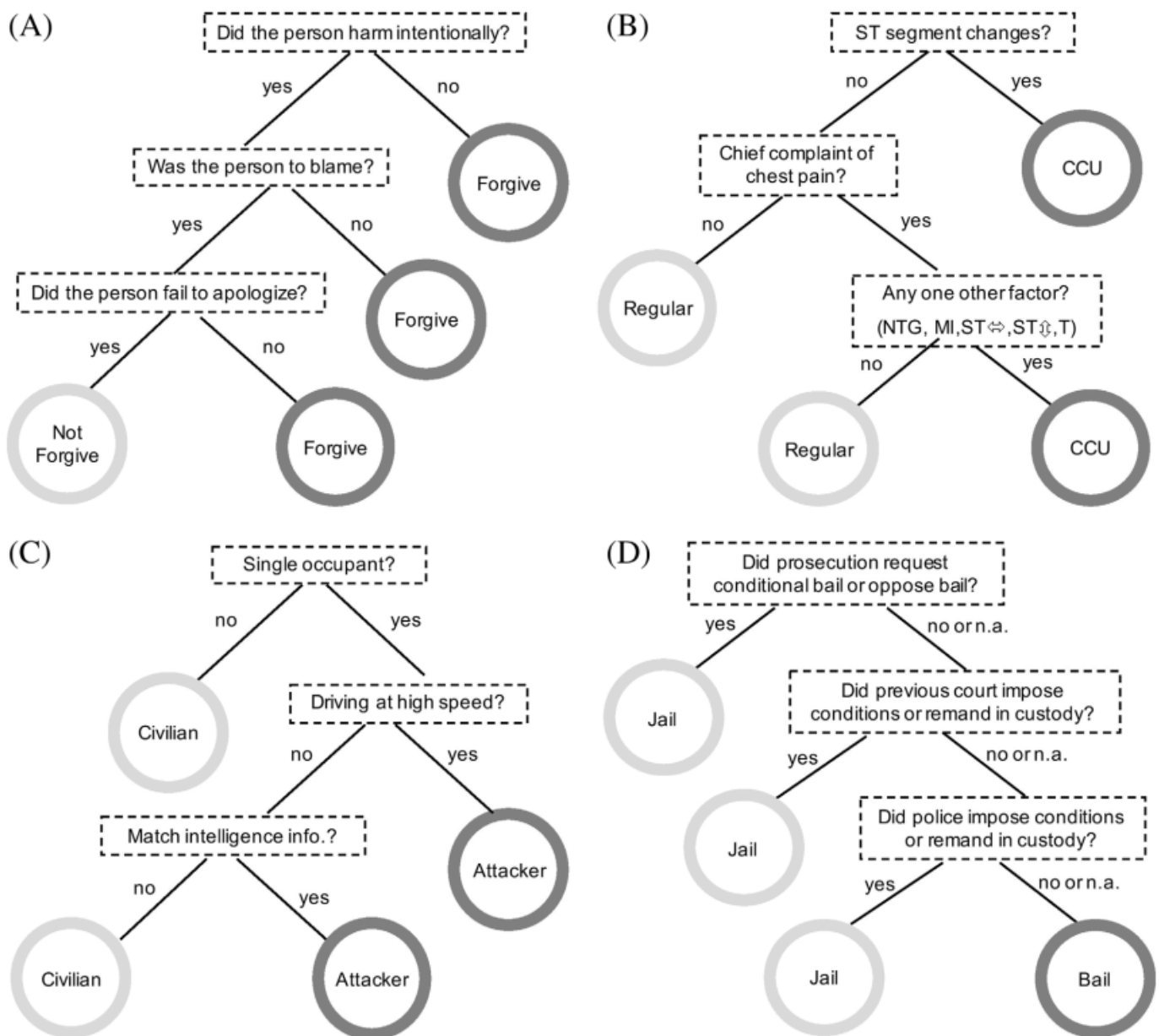


FIGURE 2 Four examples of fast-and-frugal trees (FFTs). (A) Moral decision-making: A descriptive model of how people decide whether to forgive a person who committed a hurtful deed. (B) Medical decision-making: A prescriptive FFT used in Michigan hospitals to assign patients with symptoms of a heart attack to either the coronary care unit (CCU) or a regular nursing bed (regular). Factors considered at the last stage (e.g., NTG) refer to positive outcomes of five relevant medical tests. (C) Military decision-making: A prescriptive FFT to judge whether a vehicle approaching a checkpoint carries a civilian or a suicide attacker. (D) Legal decision-making: A descriptive model of London magistrates' decisions whether to grant bail to a defendant or impose a punitive decision, such as jail.

accuracy (as measured by actual heart attacks) than doctors' clinical judgments and a logistic regression model. The third FFT (Figure 2C) is one that can help soldiers at checkpoints decide quickly whether an approaching vehicle carries a suicide attacker or just a civilian (Keller & Katsikopoulos, 2016). Using reports of 1060 force protection incidents in Afghanistan, it was estimated that applying this FFT could have reduced the actual 204 civilian casualties by more than 60%. The last FFT (Figure 2D) is a model describing how magistrates in London courts decide whether to grant bail to a defendant or impose a punitive decision, such as curfew or jail (Dhami, 2003). Following this FFT, the magistrates will not grant bail unless all three cues are checked and none of them are negative.

There are other applications of FFTs of either prescriptive or descriptive natures (e.g., Hertwig et al., 2013a; Katsikopoulos et al., 2020). FFTs are highly useful because of their transparent structures, easy implementation, fast execution, and good accuracy in prescriptive contexts. That being said, the three building blocks of FFTs require further specification.

Cue orders and decision cutoffs

There are three elements in an FFT, corresponding to the three building blocks, that can be changed depending on context: the cue order, the exit structure, and for nonbinary cues (e.g., speed of a vehicle), the decision cutoff set on each cue. With regard to cue order, it is recommended that cues should be ordered by unconditional measures of their qualities, such as their positive and negative predictive validities (e.g., Martignon et al., 2008), to reduce the prediction variance caused by estimating all conditional dependencies among cues. Using unconditional as opposed to conditional measures is also a feature of the search rules of take-the-best and Δ -inference.

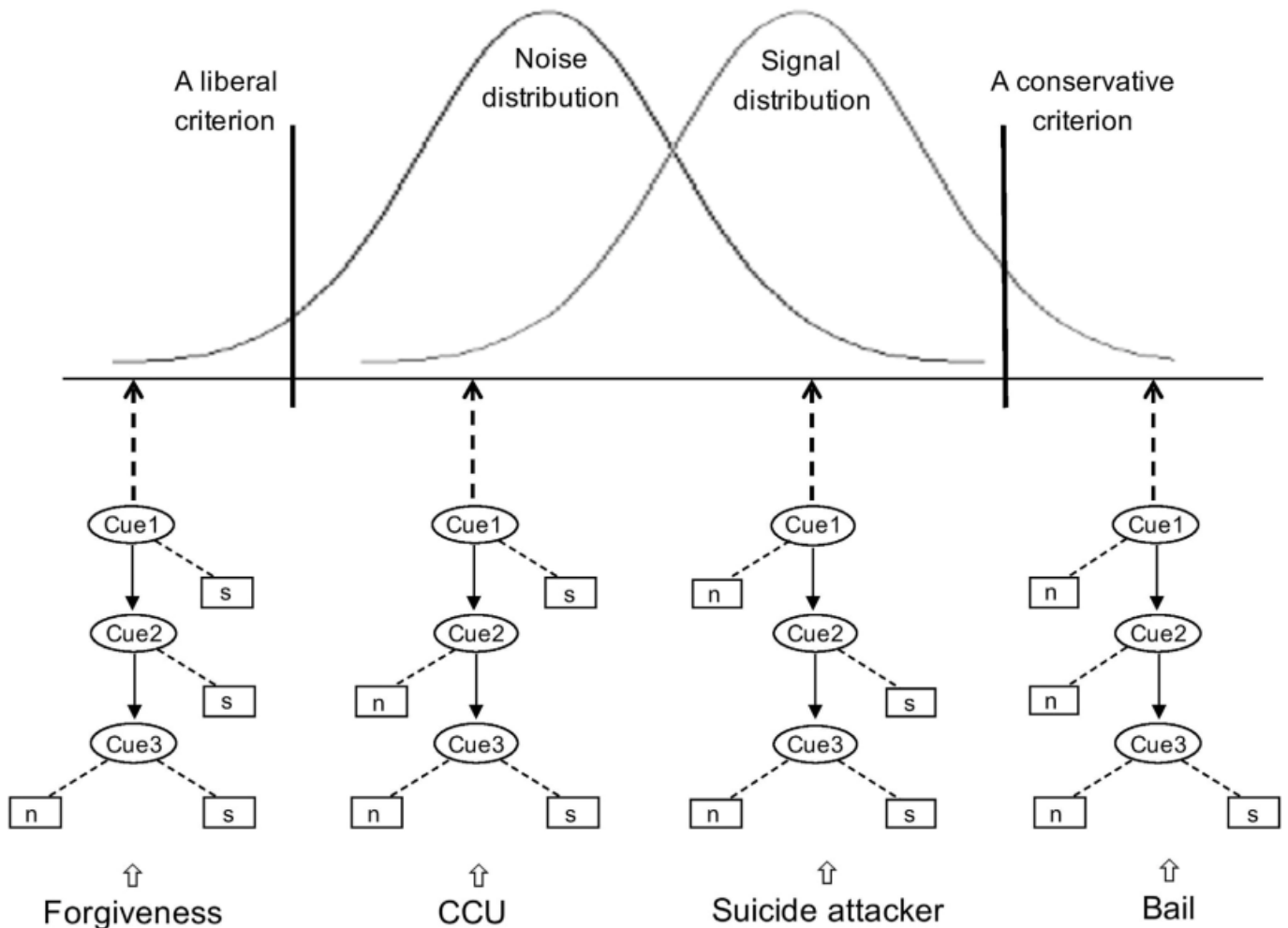


FIGURE 3 Connections between signal detection theory (SDT) and fast-and-frugal trees (FFTs). Top: Key concepts of SDT. Bottom: How FFTs with different exit structures correspond conceptually to the different decision criteria in SDT. Each of the four FFTs shown in Figure 2, labeled concisely at the bottom of this figure, has a different exit structure and embodies a certain type of decision criterion. Exits in an FFT that point to a signal and a noise decision are denoted “s” and “n,” respectively.

Moreover, studies have shown that different ordering measures, if theoretically justified, usually result in similar cue orders and lead to similar performances (e.g., Buckmann & Şimşek, 2017; Luan et al., 2011). Research on the decision cutoffs for non-binary cues is scarce, partly because cutoff selection is often idiosyncratic and bounded by conventions and experience (i.e., what speed qualifies as high). Nonetheless, Buckmann and Şimşek (2017) suggested that choosing a cutoff that maximizes the Gini coefficient in a cue could be a good method.

FFTs from the perspective of signal detection theory

With m cues and a fixed cue order, there are 2^{m-1} possible exit structures for an FFT. Which one should be applied in a task? Luan et al. (2011) showed that the exit structure of an FFT is functionally equivalent to the decision criterion in signal detection theory (SDT; Green & Swets, 1966). The upper part of Figure 3 illustrates some key concepts in SDT: (a) There are distributions of two events, referred to abstractly as “noise” and “signal,” in an evidence strength scale; (b) to make a decision, one needs to apply a criterion above which an observation will lead to the signal decision and below which, noise; (c) because the two possible decision errors, miss (false negative) and false alarm (false positive), often differ in cost, a biased decision criterion needs to be adopted to reduce the total expected cost; and (d) a “liberal” criterion will reduce the probability of misses at the cost of an increased probability of false alarms, while a “conservative” criterion will have the opposite effect. Among all the contributions of SDT to psychological research, the idea that an adaptive mind is often a biased one, in terms of the adopted decision criterion, is probably the most insightful one.

Luan et al. (2011) proved that *ceteris paribus*, an FFT with its “s” exits, the exits that lead to a signal decision (e.g., suicide attacker), occurring in earlier cues is always more liberally biased than another FFT with its “s” exits occurring in later cues. The lower part of Figure 3 illustrates the rough correspondences of four three-cue FFTs, each with a unique exit structure, to the locations of four different decision criteria in SDT; from left to right, the FFTs become less and less liberal. The four exit structures are also those of the four FFTs shown in Figure 2.

Having shown that different FFT exit structures imply different decision criteria, Luan et al. (2011) conducted a series of computer simulations to demonstrate the importance of choosing the right exit structure in a task. Specifically, they showed that in tasks where the cost of misses (e.g., not forgiving a deserving person or sending a seriously ill patient to a regular nursing bed) is higher than the cost of false alarms

(e.g., forgiving an undeserving person or sending a mildly ill patient to the coronary care unit), FFTs with liberal exit structures will result in higher net benefits than FFTs with conservative exit structures, and vice versa. Therefore, to be ecologically rational, an FFT user must consider the error costs in a task and choose an exit structure accordingly. Furthermore, they showed in simulated tasks that if the right exit structures are chosen, FFTs can outperform optimization-oriented SDT models unless those models are trained on quite a large amount of data (i.e., around 500 learning trials). Similar results were also obtained in a study using real-world instead of simulated tasks (Katsikopoulos et al., 2020).

The ecological rationality of FFTs

Research on FFTs has so far focused on specifying the content of the three building blocks and demonstrating FFTs' viability as good prescriptive and descriptive models. The ecological rationality of FFTs has not been understood as well as that of take-the-best. The situation is compounded by two practical difficulties: the multitude of measures that have been used to evaluate classification performance and the lack of a "go-to" benchmark model to which FFTs should be compared (e.g., Wu et al., 2008). Despite these difficulties, future research needs to address FFTs' ecological rationality. We speculate that environmental properties that define take-the-best's ecological rationality, such as noncompensatoriness and cue redundancy, would also be relevant for FFTs.

FUTURE RESEARCH DIRECTIONS

In this article, we reviewed the theoretical principles and methodological approaches that characterize the research on fast-and-frugal heuristics. We also used studies on two heuristics, take-the-best and fast-and-frugal trees, to demonstrate how these principles and approaches have been realized. Despite a substantial body of research on fast-and-frugal heuristics, there are still many issues that need to be further studied and better understood. In this last section, we discuss four of them.

Artificial intelligence and heuristics

Research and applications of artificial intelligence (AI) have made great progress in recent years, facilitated by the availability of big data and the rapid development of machine-learning algorithms. However, the current AI is also drifting away from the "psychological AI" originally envisioned by Herbert Simon and his contemporaries (Gigerenzer, 2022). In Simon's vision, AI should mimic the human mind and closely follow the rules of thinking, problem-solving, and decision-making used by human experts, making itself interpretable, flexible, and "wise." Unfortunately, these are the traits largely missing in today's AI.

Fast-and-frugal heuristics, meanwhile, are representations of human wisdom, and work particularly well in conditions of high uncertainty and small data, characters of many real-world tasks we face (e.g., Gigerenzer et al., 2022; Luan et al., 2019), and they are transparent. The lack of transparency in AI algorithms, such as deep neural networks, has been long criticized because it leads to distrust. Take COMPAS (Correctional Offender Management Profiling for Alternative Sanctions) for example. COMPAS is an algorithm that predicts whether a person would commit a crime again in the next 2 years, and it has been used by U.S. courts to decide the sentencing of over a million defendants. However, no one, including the judges, knows how the algorithm makes its predictions, or whether it discriminates against minorities. This has caused strong objections for its usage in court. In contrast, a simple rule that considers only three features and is absolutely transparent is found to be as accurate as COMPAS, but can be scrutinized readily for discrimination (Katsikopoulos et al., 2020).

There are many other places where simple but smart rules and heuristics could be of help in creating an AI that is more psychological, acceptable, and trustworthy to human users. To work with AI researchers and encourage them to instill heuristics in AI would be a meaningful way to not only expand the utility of existing fast-and-frugal heuristics, but also make discoveries of new ones that are integral parts of the AI toolbox.

Learning and strategy selection

Studies on fast-and-frugal heuristics have relied heavily on mathematical analyses and computer simulations to compare strategies and test their performance. As a result, most of their claims about ecological rationality are prescriptive, proving or showing that *if* the mind uses a certain heuristic in a certain task environment, *then* it will obtain a good performance. How does the mind actually learn to be ecologically rational? So far, this question has been studied mainly in the context of individual learning, to which models such as reinforcement learning are applied and according to which people select strategies based on their own experience with the strategies' past probabilities of success (e.g., Rieskamp & Otto, 2006; Todd & Dieckmann, 2004). Beyond that, not much progress has been made.

One direction to go will be to study how people learn socially instead of individually. One benefit of social living is that we can learn from others who have more experience or perform better than we do in certain tasks. Facilitated by well-developed language, teaching, and communication systems, social learning is usually safer, faster, and more efficient than individual learning (e.g., Boyd & Richerson, 2005). For tasks that no one has experienced before, learning in groups can also be more effective for all individuals involved (e.g., Garcia-Retamero et al., 2009). Given the prevalence and benefits of social learning, we believe that this is the predominant way people learn to select strategies and their key parameters (e.g., cue orders in take-the-best). However, how exactly such learning takes place still needs to be investigated.

Another important aspect of strategy selection is the phenomenon of "flat maximum": Multiple strategies are often able to achieve very similar levels of accuracy in the same task, making it unclear which one is the best (Lovie & Lovie, 1986).

A flat maximum makes finding the best strategy difficult but also makes choice easy because one can select any one of the better strategies if accuracy is the only concern. That being said, as discussed before, strategies usually differ in other aspects, such as simplicity and frugality. When there is little difference in accuracy, a strategy that fares better in aspects other than accuracy could be preferred, a process consistent with both the Occam's razor principle and the Δ -inference heuristic (Figure 1). Whether this is what people are doing in learning and strategy selection, however, needs to be examined.

The evolution of heuristics

In environments that hardly change over time and provide plenty of opportunities for learning, the evolved mental processes appear more fine-tuned than heuristics (e.g., Geisler, 2003). The argument is that in such environments, fine-tuning can reduce bias and sufficient learning can reduce variance. How, then, should one think about heuristics in those highly stable environments?

One way is to consider heuristics not as static entities but evolving with learning. For example, when applying Δ -inference, the heuristic can be simple at the beginning, using a quick-and-dirty method to sort cues and applying a fixed Δ for each cue. With more data coming in, however, the cue orders may be updated with more sophisticated ordering methods and Δ s in cues may vary to reflect more precisely the relative information value of each cue. Even the decision rule may change: Instead of guessing at the end of the search, the search can go back to the first cue with a revised, smaller Δ (Luan et al., 2014). "Growing" a heuristic should be subject to constraints, because making it more complex does not always pay off (e.g., Martignon & Hoffrage, 2002), and even highly evolved abilities, such as our visual system, appear to rely on simple heuristics. That said, no study has systematically investigated the issue and provided a principled method to judge the right or wrong way to grow heuristics.

In the real world, there are some stable environments but probably more with inherently high degrees of uncertainty (e.g., those involving lots of personal interactions) or whose characteristics are constantly changing (e.g., stock markets). Heuristics are supposed to thrive in such environments (e.g., Gigerenzer, 2008; Katsikopoulos et al., 2020), but they can also fail with ill-suited building blocks and by ignoring the wrong type of information. Similar to the growth of a heuristic, how the mind should adjust the building blocks through feedback, if needed, is another issue that deserves more research attention.

Preferences and inferences

Lexicographic heuristics were originally proposed to understand human preferences (Georgescu-Roegen, 1968). In fast-and-frugal heuristics research, they were generalized from preferences to inferences, and were shown to perform well, and in many cases, better than weighting-and-adding models that are considered as the "gold standard" in preferential choices (e.g., Payne et al., 1993). Future research should test how other preference models, such as elimination-by-aspect (Tversky, 1972) and satisficing (Simon, 1955), perform in inference tasks, as well.

This preference-to-inference direction of generalization, though productive, is likely the opposite of what has occurred in the evolution of the human mind. From an evolutionary perspective, most critical choices, from the type of food to consume to the kind of partner to mate with, are inferences rather than preferences, because they can be judged as correct or wrong based on their impacts on survival and reproductive success. It is reasonable to assume that strategies that work well for these important inference problems emerged from learning and competition and were eventually generalized to preferences, in which choices may not carry obvious evolutionary values, but to which these evolved strategies are applied anyways because the problem structures are similar.

The implication of this argument is that to understand why the mind adopts a certain strategy, research should focus more on its performance in inference tasks than how its outcomes fit "rational" axioms of preference. Therefore, just as biologists and physiologists study anatomical features, we suggest studying mental tools—including fast-and-frugal heuristics—with the goal of finding out what functions these tools served in the course of evolution. This would allow a better understanding of how these tools might have evolved from the demands of the environment, and how a changing environment may turn successful tools or heuristics into maladaptive ones, and vice versa.

Indeed, viewing heuristics through the lens of functionalism and evolution has been one overarching theme in the research on fast-and-frugal heuristics, leading to the concept of the adaptive toolbox and the methodological imperative of competitive testing. Ecological rationality also originated from this theme, and its research has corrected an early belief that heuristics would always be second best because they ignore information. One general insight from the fast-and-frugal heuristics research is that complex problems do not always require complex solutions. Under uncertainty, complex solutions tend to be fragile, while simple solutions are likely to be more robust. Less can be more.

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CONFLICT OF INTEREST

The authors declare that there are no conflicts of interest.

ETHICS STATEMENT

This article does not report studies involving human or animal subjects. Therefore, no ethics approval is needed.

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