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NUNCIUS 38 (2023) 311–339



The Search for the Tang Royal Domain (*Wangji* 王畿)

*How Yixing (683–727) Used the Zhou-Era “Nine Domains” (Jiufu 九服)
to Map the Tang Dynasty’s New Terrestrial Realm*

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Abstract

This essay discusses how the traditional Chinese geographical concept Nine Domains (*Jiufu* 九服) was applied in Yixing’s 一行 (683–727) large-scale gnomonic survey of the Tang dynasty. Yixing’s surveying method is reconstructed by comparing textual records with schematic delineations. This comparison shows that the centralized and symmetrical pattern in the arrangement of survey sites conforms to the visual and schematic delineations of the concept of Nine Domains. The design was meant to show that the Earth was in harmony with heavenly patterns, thus confirming Emperor Xuanzong of Tang’s 唐玄宗 mandate of heaven to govern. In this essay, I lay out Yixing’s empirical scientific work and symbolic cultural activity regarding calendrical reform to re-establish the legitimacy of the Tang dynasty regime. The case demonstrates an understanding of how Chinese astronomers interrelated Heaven and Earth and indicates an ongoing tension in the history of science about concept-practice relations.

Keywords

Nine Domains (*Jiufu* 九服) – Yixing – Concept-Practice Relations

Published with license by Koninklijke Brill NV | DOI:10.1163/18253911-bja10059

© CHIA-YUN WU, 2023 | ISSN: 0394-7394 (print) 1825-3911 (online)

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1 Introduction

The Nine Domains (*Jiufu* 九服) first appeared during the Warring States period (403–221 BC).¹ In the *Rites of Zhou* (*Zhouli* 周禮, ca. fourth century BCE, hereinafter referred to as *Rites*), the term is a geographical concept, delineating tributary and tax regions (*fu* 服, lit. “tributary territory”).² *Wangji* 王畿, that is, the Royal Domain, formed the center of the tributary territory. The nine concentric squares surrounding the Royal Domain indicated different levels of tributary obligation determining the frequency of court visits as well as the appropriate tributes. The ideal distance between each concentric square was five hundred *li* 里 (Chinese kilometer).³ (See Fig. 1)

- 1 Since the Nine Domains (*Jiufu*) is an important schema depicting the ideal boundaries of administrative districts in the Zhou dynasty, many scholars have mentioned it while discussing geographical concepts in ancient China. The term *fu* is not easy to translate and, therefore, I have chosen to rely on the translation proposed by John B. Henderson, one of the leading authorities of the anglophone academic literature on Early Chinese cosmology. See John B. Henderson, *The Development and Decline of Chinese Cosmology* (New York: Columbia University Press, 1984), 66–68; John B. Henderson, “Chinese Cosmographical Thought: The High Intellectual Tradition,” in *Cartography in the Traditional East and Southeast Asian Societies* (The History of Cartography, vol. 11.2), ed. John B. Harley and David Woodward (Chicago: The University of Chicago Press, 1994), 203–227. For more discussions about the Nine Domains (*Jiufu*), see John S. Major, “The Five Phases, Magic Squares and Schematic Cosmography,” in *Explorations in Early Chinese Cosmology*, ed. Henry Rosemont (Chico, CA: Scholar Press, 1984), 133–166; Vera Dorofeeva-Lichtmann, “Political Concept Behind an Interplay of Spatial ‘Positions,’” *Extrême-Orient, Extrême-Occident*, no. 18 (1996): 9–33; Vera Dorofeeva-Lichtmann, “Ritual Practices for Constructing Terrestrial Space (Warring States—Early Han),” in *Early Chinese Religion, Part One: Shang through Han (1250 BC–220 AD)*, ed. John Lagerwey and Marc Kalinowski (Leiden: Brill, 2009), 605–607; Tang Xiaofeng 唐曉峰, *Cong hundun dao zhixu—zhongguo shanggu dili sixiang shi lunshu* 從混沌到秩序—中國上古地理思想史論述 [From chaos to order: a study of the history of ancient Chinese geographical thought] (Beijing: Zhonghua shuju, 2010), 228.
- 2 An earlier system of *fu* was the so-called Five Domains (*Wufu* 五服). Although both systems consisted of concentric squares, they differ as to their specific arrangements. For more discussions of the Five Domains, see Tang, *Cong hundun dao zhixu*, 227–233; John D. Wong, “The Shifting Concept of Space and Territory in China during the Warring States Era,” *Asia Major* 28, no. 1 (2015): 1–35.
- 3 According to the excerpt of *Rites* about the Nine Domains, the ten concentric zones are named as follows, from the center to the outermost zone: Royal Domain (*Wangji*), Noble’s Domain (*Houfu* 侯服), Sovereign Domain (*Dianfu* 甸服), Baronial Domain (*Nanfu* 男服), High Officials’ Domain (*Cai fu* 采服), Domain of Defense (*Weifu* 衛服), Domain of *man* 蠻 Uncivilized Tribes (*Manfu* 蠻服), Domain of *yi* 夷 Uncivilized Tribes (*Yifu* 夷服), Domain of Garrisons (*Zhenfu* 鎮服), and Military Domain (*Fanfu* 藩服). See *Zhouli zhushu* 周禮注疏 [Commentaries and subcommentaries to the *Rites of Zhou*]. In *Shisan jing zhushu* 十三

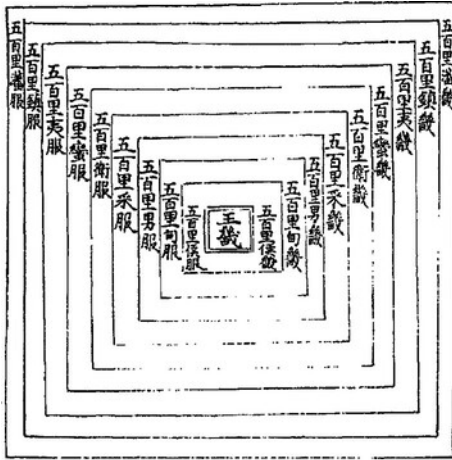


FIGURE 1

The diagram of the concept of Nine Domains in *Liu jingtu* 六經圖 [Illustrations of Six Classics], 1166. Yang Jia 楊甲, *Liu jingtu* 六經圖, SKQS ed., 4: 28a

In *Historical knowledge* (*Shilin zashi* 史林雜識), the Chinese historian Gu Jie-gang 顧頡剛 suggested that the concept of Nine Domains was purely schematic and primarily depicted an imperial social order unfolding in territorial space. In other words, whereas Gu acknowledged the symbolic functions of the Nine Domains, he denied any possibility that they were applied with any analytical purpose in geography, astrology/astronomy, or calendar calculation in China. However, the monk Yixing (683–727, whose original name was Zhang Sui 張遂) selected the term “Nine Domains” for his new calendrical method in 724. The decision was strongly related to the political context of the time.

Yixing was proficient in Chinese astronomy, mathematics and a well-known master of Tantrism.⁴ Due to his identity as a Tantric monk, Yixing had acquired Indian astronomical expertise through a number of channels,⁵ thus construct-

經注疏 [Commentaries and subcommentaries to the *Thirteen Classics*], ed. Ruan Yuan 阮元 (Beijing: Zhonghua shuju, 1980), 1864.

4 Tantrism is a form of esoteric Buddhism characterized by mantras, deity yoga, visualization-based meditations, etc. It has been developed and flourished among the Chinese people since the Tang dynasty. See Chou Yi-liang, “Tantrism in China,” *Harvard Journal of Asiatic Studies* 8, no. 3/4 (1945): 241–332.

5 For example, he had close contacts with two Tantric monks from India, Vajrabodhi 金剛智 (671–741) and Śubhakarasiṃha 善無畏 (637–735). Besides, he had learned Indian astronomy through the translated Indian *Jiu Zhi Calendar* 九執曆. Christopher Cullen, “An Eighth Century Chinese Table of Tangents,” *Chinese Science* 5 (June 1982): 19–25; Wu Hui 吳慧, “Seng Yixing shengping zai yanjiu” 僧一行生平再研究 [Further research on the biography of the Monk Yixing], *Yuan Kuang Journal of Buddhist Studies* 圓光佛學學報, no. 14 (2009): 93–107; Jeffrey Kotyk, “The Astronomical Innovations of Monk Yixing 一行 (673–727),” *Religions* 13, no. 6 (2022): 3.

ing his complex knowledge background. However, under the banner of Emperor Xuanzong of Tang's 唐玄宗 (personal name Li Longji 李隆基, 685–762, r. 712–756) ritual revival programs, Yixing chose to use the legacy of the Zhou dynasty—namely, the Nine Domains—to formulate this large-scale survey. This reveals the ongoing tension between technical knowledge and political discourse.

Upon his return from a pilgrimage to the Indian subcontinent, Yixing was chosen by Emperor Xuanzong of Tang because of his ancestors' exploits as well as his ability and talent. In Yixing's new calendar, the *Dayan Calendar* 大衍曆, he eschewed traditionally frequent terms such as the “Four Seas” (*sihai* 四海) and “All under Heaven” (*tianxia* 天下), which were used to refer to the entire territory of the Tang. Addressing a new “Method to measure the gnomon and clepsydra” (*Bu guilou shu* 步晷漏術), he repeatedly suggests that measurements have to be pursued “in [all locations within] the Nine Domains” (*Qiu jiu fu* 求九服).⁶ In ancient China, the legitimacy of rulership was based on accurate astronomical observations and calculation of time.⁷ This article explores the role of the Nine Domains in Yixing's famous attempt to map the Tang-era terrestrial realm in a large-scale gnomonic survey firstly aimed at calculating the calendar. Drawing on still extant sources from Yixing's survey, I bring together the textual (i.e., how calculations were actually done) and the visual (i.e., how the data were depicted) layers, in order to reconstruct the original design of the large-scale survey. I also consider why Yixing chose the geographical concept of Nine Domains and how he implemented his choice.

2 Background of Yixing's Large-Scale Gnomonic Survey

The Tang dynasty reign was interrupted by Wu Zetian 武則天 (624–705, r. 665–705), who took the throne as empress regnant and established a new Zhou dynasty. In 705, the Li family returned to the throne and the Tang dynasty was restored. At the beginning of Emperor Xuanzong of Tang's reign (712), the court was still in a state of chaos; the Emperor needed to bolster his power and re-establish the dominion of the Li family. Hence, in 717, the fifth year of the Kaiyuan reign-period (713–741), Emperor Xuanzong of Tang promulgated the following order: “If there are any descendants of ministers who made great

6 *Jiu Tangshu* 舊唐書 [Old history of the Tang] (Beijing: Zhonghua shuju, 1975), 1256; hereinafter referred to as *Old Tang History*.

7 Christopher Cullen, *Heavenly Numbers—Astronomy and Authority in Early Imperial China* (Oxford: Oxford University Press, 2017), 7–49.

contributions in the Wude 武德 (618–626) and Zhenguan 貞觀 (627–649) reign-periods and who are not officials now, [officials] should visit them and report to the Emperor. If there are any hermits who do not wish to serve the government, Regional Governors could recommend them on the basis of their reputation.”⁸ Yixing was known to have a good pedigree⁹ and be very talented, and so was invited to the capital Chang’an. Four years later (721), Yixing was assigned the task of making a new calendar. He was also tasked with discussing and correcting calendrical methods with the Learned Officers (*xueshi* 學士) and determining the center of the world,¹⁰ the most suitable location to situate the empire.¹¹ Yixing was directly appointed calendar maker on the recommendation of the powerful Prime Minister Zhang Yue 張說 (667–730),¹² despite the fact that most appointees were traditionally officials of the Directorate of Astrology (*Taishi ju* 太史局).¹³ A long held tradition in China is that celestial phenomena are viewed as a vehicle of meaning directed to the human world in general, but especially to the ruler of that world—the emperor himself.¹⁴ So, harmony between heaven and earth is associated with the legitimacy of

8 The original text reads: “武德、貞觀以來勛臣子孫無位者，訪求其後奏聞。有嘉遁幽棲養高不仕者，州牧各以名薦。” See *Jiu Tangshu*, 177.

9 Yixing’s great-grandfather Zhang Gongjin 張公謹 (584–632) was the commander-in-chief of Xiangzhou 襄州, also the Duke of Tanguo 鄴國 of the early Tang dynasty. See Chen Jinhua, “The Birth of a Polymath: The Genealogical Background of the Tang Monk-Scientist Yixing (673–727),” *Tang Studies*, no. 18/19 (2000): 15–16; Wu, “Seng Yixing shengping zai yanjiu,” 88.

10 Liu Su 劉肅 (fl. 806–820), *Datang xinyu* 大唐新語 [New stories from the Great Tang] (Taipei: Xinyu chubanshe, 1985), 194.

11 The *Rites* mentioned that “the center of the Earth is where heaven and Earth unite, where the four seasons intermingle, where winds and rains join together, and where *yin* and *yang* merge. Therefore, hundreds of creatures are well nurtured and settled there, and thus the empire may be established there. 地中，天地之所合也，四時之所交也，風雨之所會也，陰陽之所和也。然則百物阜安，乃建王國焉。” See *Zhouli zhushu*, 1517; cf. the translation in A. Beer, Ho Ping-Yü, Lu Gwei-Djen, J. Needham, E.G. Pulleyblank, and G. Thompson, “An 8th-Century Meridian Line: I-Hsing’s Chain of Gnomons and the Pre-History of the Metric System,” *Vistas in Astronomy* 4 (1961): 9.

12 Lu Yi-Ning 盧意寧, “Shengtang jixian yuan de chengli yu yiyi—jianlun xuanzong chao xueshu wenhua fengqi de fazhan” 盛唐集賢院的成立與意義—兼論玄宗朝學術文化風氣的發展 [A study of Jixian academy—with reference to intellectual changes during the Xuanzong reign of the Tang dynasty] (Master’s thesis, National Taiwan University, 2009), 38–46.

13 The Directorate of Astrology (*Taishi ju* 太史局) is the bureau of the Tang responsible for astro-calendrical practices, including observing astronomical phenomena, producing the official calendars, and writing and archiving reports of prognostication. See Du You 杜佑, *Tongdian* 通典 [Comprehensive Institutions] (Beijing: Zhonghua shuju, 1988), 738–739.

14 Cullen, *Heavenly Numbers*, 2.

the reign. From the moment the Li family resumed the throne, Emperor Xuanzong of Tang and Zhang Yue were determined to establish a ritual program to bring the Tang dynasty into harmony with heaven and earth and to show that the current Emperor was as virtuous as the Former Kings of antiquity who had “responded to the Way” (*Dao* 道).¹⁵

Yixing’s central task of determining the center of the Earth¹⁶ was preparatory to the calendar production¹⁷ but represented also a political aim in itself, because of its critical role in ancient Chinese astronomy and geography. This task has to be understood in the context of the Chinese Flat-Earth cosmology. Although there were several descriptions of Heaven (i.e., the cosmos), such as the Vaulted-Heaven Theory (*Gaitian shuo* 蓋天說), the Spherical-Heaven Theory (*Huntian shuo* 渾天說), and the Infinite-Empty-Space Theory (*Xuanye shuo* 宣夜說), the Chinese Flat-Earth cosmology remained unaltered until the seventeenth century.¹⁸ Since the Earth was believed to be flat and square, it was considered possible to determine its center

Some techniques of observation with gnomons depended considerably on the gnomon’s position relative to where the conceptual determination of the Earth’s center was.¹⁹ The pre-Han treatise *Rites* mentioned the problem of

15 Peter K. Bol, *This Culture of Ours*—*Intellectual Transitions in T’ang and Sung China* (Stanford: Stanford University Press, 1992), 109.

16 It should be noted that “the center of the Earth” here means the center of the Chinese known world during the reign of the Tang dynasty, not the modern definition of a spherical Earth.

17 The observational data at the center of the Earth, such as the shadow-lengths at noon, had been viewed as an essential reference for official calendars since the Eastern Han dynasty (25–220). Thus, determining the center of the Earth and having the observational data was a prerequisite for calendar production. See Qu Anjing 曲安京, Yuan Min 袁敏, and Wang Hui 王輝, “Zhongguo gudai lifa zhong de jiufu guiying suanfa” 中國古代曆法中的九服晷影算法 [Algorithm for calculating solar shadows in ancient Chinese calendars], *Studies in the History of Natural Sciences* 自然科學史研究 20, no. 1 (2001): 44–52; Guan Zengjian 關增建, “Zhongguo tianwenxue shi shang de dizhong gainian” 中國天文學史上的地中概念 [The concept of the center of the earth in Chinese astronomical history], *Studies in the History of Natural Sciences* 自然科學史研究 19, no. 3 (2000): 255.

18 The Vaulted-Heaven Theory described heaven as an umbrella covering the earth; the Spherical-Heaven Theory described heaven as a sphere surrounding earth; and the Infinite-Empty-Space Theory described heaven as being without substance while the heavenly bodies float freely. Though there are various descriptions of Heaven, the earth was always flat (perhaps with a bit of bulging). See Christopher Cullen, “A Chinese Eratosthenes of the Flat Earth: A Study of a Fragment of Cosmology in Huai Nan tzu 淮南子,” *Bulletin of the School of Oriental and African Studies, University of London* 39, no. 1 (1976): 107.

19 For more details on the role of the center of the Earth in Chinese astronomical traditions, see Alexei Volkov, “Zhao Youqin (1271–1335?) on the Center of the Earth,” in *Science and*

determining the center of the Earth with a gnomon, indicating the center of the Earth as the place on the Earth's surface where the shadow of a 8-*chi*—high (*chi* 尺 Chinese meter) gnomon is 1-*chi*-and-5-*cun* (寸 Chinese inch) long at the summer solstice.²⁰ Zheng Xuan's 鄭玄 (127–200) commentary on the *Rites* clarified how to obtain the aforementioned value of the shadow length:

From the place where the shadow is 1 *chi* 5 *cun* it would be 15,000 *li* 里 [Chinese kilometer] to the South to the place directly below the Sun. The Earth makes its four excursions and the stars rise and fall within a range of 30,000 *li*, so by taking one half of this one gets the center of the Earth.²¹

Many locations meet this condition. However, before the Tang dynasty, the center of the Earth usually referred to Luoyi 雒邑 (the ancient name of Luoyang) or Yangcheng (then considered the city at the center of the Earth).²² The *Classic Arithmetic of the Zhou Gnomon* (*Zhoubi suanjing* 周髀算經, compiled no later than the first century CE)²³ further used this rule to link gnomon measurements to distances on a cosmic scale. This reads as follows: “Method: the Zhou Gnomon (*zhoubi*) is eight-*chi* long, and the decrease or increase of the base (*gou* 句) is one-*cun* for a thousand *li*.”²⁴ This is also the theoretical basis to define the center of the Earth. Later on, many famous scholars and even astronomers accepted this shadow rule.²⁵ They not only quoted it but also pro-

Cultural Diversity: Proceedings of the XX1st International Congress of History of Science, Mexico City, July 7–14, 2001, ed. Juan José Saldaña, CD-ROM edition (Mexico City: Sociedad Mexicana de Historia de la Ciencia y de la Tecnología-Universidad Nacional Autónoma de México, 2003), 2303–2315; Guan, “Zhongguo tianwenxue shi shang de dizhong gainian,” 251–263.

- 20 The original text reads: “日至之景，尺有五寸，謂之地中。” See *Zhouli zhushu*, 1517.
- 21 The translation is quoted from Beer et al., “Meridian Line,” 9, with slight modifications. The original text reads: “景尺有五寸者，南戴日下萬五千里，地與星辰四游升降於三萬里之中，是以半之，得地之中焉。” See *Zhouli zhushu*, 1517.
- 22 Guan, “Zhongguo tianwenxue shi shang de dizhong gainian,” 253–257.
- 23 Date of compilation unknown; historians of astronomy suggested different dates. Recently Cullen conjectured that the final compilation took place in the early first century CE on the basis of older materials; see Christopher Cullen, *Astronomy and Mathematics in Ancient China: The Zhoubi suanjing* (Cambridge: Cambridge University Press, 1996).
- 24 The original text reads: “法曰：周髀長八尺，句之損益寸千里。” 句 (*gou*) is a variant Chinese script of 勾; it refers to the baseline of a triangle, which is the side opposite the vertex. See Zhao Junqing 趙君卿, commentator, *Zhoubi suanjing* 周髀算經 (Taipei: Zhonghua shuju, 1971), 22a. Cf. the translation in Cullen, *Astronomy and Mathematics*, 178.
- 25 Among them were Zhang Heng 張衡 (78–139), Zheng Xuan, Wang Fan 王藩 (active in the third century CE), and Jiang Ji 姜岌 (fl. late fourth–early fifth century CE). See Wang Xiaohu 汪小虎, “‘Riyang qianli cha yicun’ xueshuo de lishi yanbian” “日影千里差一寸”

vided explanations in their books. After the large-scale gnomonic survey of the Tang dynasty, Yixing refuted the commonly accepted relation between the shadow-length of a gnomon and the corresponding distance on Earth.²⁶ However, Yixing did not attempt to overturn the deep-rooted concept of flat Earth.²⁷ On the contrary, he criticized how gnomons were used for such observations and concluded that:

The original idea behind using a gnomon to measure shadow-length was the desire to synchronize harmonized *qi*²⁸ and support the nation's prosperity,²⁹ not the diameters of celestial orbits.³⁰ The idea behind granting calendar calculation an important role is the desire to respect farming cycles in response to seasonal change and to manifest that your majesty respects this as a celestial phenomenon, not because of theoretical debates about a vaulted or spherical heaven.³¹

學說的歷史演變 [The historical evolution of the shadow principle], *Journal of SJTU (Philosophy and Social Sciences)* 上海交通大學學報 (哲學社會科學版) 4, no. 16 (2008): 74–80.

- 26 The observational data in the southern dynasties (420–589) had already shown that the relationship between the shadow-length of a gnomon and the corresponding distance on Earth was unlike the old theory of “one *cun* increment.” Later, Liu Zhuo 劉焯 (544–610) and Li Chunfeng 李淳風 (602–670) both questioned this theory, but they did not propose a new approach to refute the old one. See Wang, “‘Riyang qianli cha yicun’ xueshuo de lishi yanbian,” 78.
- 27 Marc Kalinowski, “Le calcul du rayon céleste dans la cosmographie chinoise,” *Revue d'histoire des sciences* 43, no. 1 (1990): 29–34.
- 28 Here, *qi* 氣 refers to the twenty-four solar terms which can refer either to the twenty-four instants dividing the time interval between two successive winter solstices into equal periods or to the periods that begin with those instants. The most critical four solar terms are Spring Equinox (*Chunfen* 春分), Summer Solstice (*Xiazhi* 夏至), Autumn Equinox (*Qiufen* 秋分), and Winter Solstice (*Dongzhi* 冬至), which represent the mid-points of the four seasons. See Christopher Cullen, *The Foundations of Celestial Reckoning—Three Ancient Chinese Astronomical Systems* (London: Routledge, 2016), 12–13.
- 29 The twenty-four *qi* reflect the changes in climate, natural phenomena, agricultural production, and other aspects of human life, thus playing important roles in agricultural affairs and farming activities. Following the law of nature to harmonize the twenty-four *qi* can make the harvest plentiful.
- 30 *Chen* 辰 and *ci* 次 are two systems to divide the heavens into twelve equal sections, associated with the (approximately) twelve-year cycle of Jupiter round the heavens. See Cullen, *The Foundations of Celestial Reckoning*, 109. I suggest *chen* and *ci* here can be merged and refer to celestial orbits.
- 31 The original text reads: “原人所以步圭景之意，將欲節宣和氣，輔相物宜，而不在於辰次之周徑；其所以重曆數之意，將欲敬授人時，欽若乾象，而不在於渾、蓋之是非。” See *Jiu Tangshu*, 1307.

The statement reflects that, for Yixing, calendrical reform had a practical purpose: to help organize the country and society properly. His patron, Emperor Xuanzong of Tang, was eager to prove that he was a sage Emperor with the Mandate of Heaven³² so as to affirm the legitimacy of the Li family's regime.

The creation of a new calendar was one of a series of symbolic cultural activities designed to re-establish the legitimacy of the Tang dynasty regime.³³ The *New history of the Tang* (*Xin Tangshu* 新唐書, hereinafter referred to as *New Tang history*)³⁴ explains why Yixing launched a large-scale survey three years into his assignment:

To produce the calendar, [Li] Chunfeng, first of all, determined the gnomonic shadow-lengths at noon of each of the twenty-four solar terms (*qi* 氣), [and they] were quite different from the lengths given by Zu Chongzhi;³⁵ nobody knew which of these [data] was correct. As soon as Yixing produced the *Dayan Calendar*, the officials of the Directorate of Astrology were ordered to measure the shadow-lengths in all [parts of] China to seek the center of the Earth and determine the numerical constants of the calendar.³⁶

Since the records of shadow-lengths at noon of each of the twenty-four *qi* had been essential measurements allowing official calendars to indicate the exact

32 The "Mandate of Heaven" is an ancient Chinese philosophical concept dated to the early Zhou period (the reign of King Cheng 成王, ca. 1042–1021 BC). The Mandate was obtained due to the ruler's superb morality and proper political conduct. However, if the ruler does not fulfil his obligations as emperor, he will lose the Mandate and, thus, the right to be emperor. See Luo Xinhui 羅新慧, "Omens and Politics: The Zhou Concept of the Mandate of Heaven as Seen in the Chengwu 程寤 Manuscript," in *Ideology of Power and Power of Ideology in Early China*, ed. Yuri Pines, Paul Goldin, and Martin Kern (Leiden: Brill, 2015), 49–68.

33 Ge Zhaoguang 葛兆光, *An Intellectual History of China, Volume Two*, trans. Michael Duke and Josephine Chiu-Duke (Leiden: Brill, 2018), 5–6.

34 The *New Tang history* and the *Old Tang history* are two of the twenty-four official Chinese dynastic histories: these official dynastic histories were usually written by successors based on official court records of the former dynasty.

35 Zu Chongzhi 祖冲之 (429–500) was an official during the Liu Song and Southern Qi dynasties, and was also good at astronomy and mathematics. In 465, he produced the *Daming* calendar 大明曆. See *Zhongguo gudai tianwen xuejia* 中國古代天文學家 [Ancient Chinese astronomers], ed. Chen Jiujin 陳久金 (Beijing: Zhongguo kexue jishu chubanshe, 2008), 172–173.

36 The original text reads: "初，淳風造曆，定二十四氣中晷，與祖冲之短長頗異，然未知其孰是。及一行作大衍曆，詔太史測天下之晷，求其土中，以為定數。" See *Xin Tangshu* 新唐書 (Beijing: Zhonghua shuju, 1975), 812.

time of the twenty-four *qi* ever since the Eastern Han dynasty (25–220),³⁷ Yixing needed to provide a more accurate set of shadow-lengths than those in the past. Moreover, Yixing aimed to provide a new calendar that might be adapted to the expanding borders of the Tang dynasty. In order to do this, on the days of solstice and equinox he needed to collect the shadow-lengths of a standard gnomon at noon in different locations.³⁸ However, the relationship between the shadow-length of a gnomon and the corresponding distance on Earth had been refuted, and Yixing could not determine the center of the Earth based on the old rule (i.e., one *cun* of decrement or increment in the shadow corresponds to one thousand *li*). Thus, he followed the tradition of choosing Yangcheng as the center of the Earth and took it as the point of reference to make the new calendar.³⁹

3 New and Old Mathematics, Astronomy, and Geography in Yixing's Methods

The *Institutional history of the Tang* (*Tang huiyao* 唐會要, hereinafter referred to as *Tang institutional history*) was compiled by Wang Pu 王溥 in 961. It recorded the production of calendars during the Tang dynasty, including reports not mentioned in the *Old Tang history* and the *New Tang history*.⁴⁰ From such reports we learn that several teams were sent to measure shadow-lengths in some selected sites. One of the leaders of this project was a long-standing, experienced official of the Directorate of Astrology, Nangong Yue 南宮說.⁴¹ Yue had been Supervisor of the Directorate of Astrology (*Taishi jian* 太史監) dur-

37 The earliest calendar containing this kind of record was the *Sifen Calendar* 四分曆 (used in 85–236) of the Eastern Han dynasty. See Zhang Peiyu 張培瑜, Chen Meidong 陳美東, Bo Shuren 薄樹人, and Hu Tiezhu 胡鐵珠, *Zhongguo gudai lifa* 中國古代曆法 [Chinese ancient calendars] (Beijing: Zhongguo kexue jishu chubanshe, 2008), 34.

38 Previous official calendars only recorded shadow-lengths at noon of each of the twenty-four *qi* at the center of the Earth or the capital. Yixing's large-scale survey for observing the shadow-lengths in the Tang territory was probably the most ambitious program in the history of science up to that date. Cullen, "An Eighth Century Chinese Table of Tangents," 1.

39 Guan Zengjian deduced that selecting Yangcheng as the center of the Earth might be the Tang Emperor Xuanzong's idea. See Guan Zengjian 關增建, "Zhongguo dadi celiang de lishi yanbian" 中國大地測量的歷史演變 [The Historical Evolution of Chinese Astrogeodesy Measurement], *Studies in the History of Natural Sciences* 自然科學史研究 37, no. 4 (2018): 427–428.

40 Wang Bu 王溥, *Tang huiyao* 唐會要 (Taipei: Shijie shuju, 1963), 755.

41 *Zhongguo gudai tianwen xuejia*, 235–236.

ing the Kaiyuan reign and was put in charge of the measurements in Henan.⁴² Daxiang Yuantai 大相元太 was responsible for observations of the Southern part of the empire. There are no additional sources about this person.

These reports also mention two instruments used by Yixing's observational teams: a gnomon used for measuring shadow-lengths, and an Inverted Try square (*Fuju* 覆矩) for measuring polar altitudes.⁴³ There is no extant portable gnomon that might have been suitable for travelling groups. However, there is a fixed stone gnomon in Gaocheng which supposedly dates back to the Tang dynasty.⁴⁴ It may have been employed in a survey pursued over several years.

The historical records show that before the Tang period gnomons were designed as a combination of two structures: *biao* 表 and *gui* 圭. A *biao* was always eight *chi* high and vertical to the ground; a *gui* was horizontal and most likely used to measure the length of the shadow; it was therefore essential for the observers to ensure that the ground was flat.⁴⁵ The *Imperially endorsed illustrated Book of Documents* (*Qinding shujing tushuo* 欽定書經圖說) compiled in 1905 features an image that had been circulating at the Qing court and which fits into the structure of gnomons in the historical materials before or during the Tang dynasty (Fig. 2).⁴⁶

The second astronomic instrument, the inverted try square (*Fuju*), was used to measure polar altitudes in the large-scale survey. Unfortunately, no such artefacts have been found, so that we do not know what the inverted try square looked like. Modern scholars have hypothesized that it was an instrument comparable to a quadrant, combining a try square, a quarter circle, a plumb line, and a tubular speculum (see Fig. 3).⁴⁷ One side of the try square contains a tubular speculum; a scale drawn on the quarter circle connects the open ends of the try square; and the plumb line is attached to the vertex. To determine the

42 The text does not specify if he himself was able to pursue measurements.

43 The polar altitude above the horizon is equal to the observers' geographical latitude. In ancient Chinese astronomy, the unit for polar altitude was *du* and *fen* (Chinese minute 分). One *du* is constituted of ten *fen*. See *Jiu Tangshu*, 1304. It should be noted that 1 *du* = (360/365.25)°, i.e., 0°98563. See Beer et al., "Meridian Line," 1121.

44 Jing Rizhen 景日昉, *Shuosong* 說嵩 [On Mount Song] (Taipei: Wenhai chubanshe, 1983), 321–322; Dong Zuo-Bin 董作賓, Liu Dun-Zhen 劉敦楨, and Gao Ping-Zi 高平子, *Guoli zhongyang yanjiu yuan zhuankan—Zhongong cejingtai diaocha baogao* 國立中央研究院專刊—周公測景台調查報告 [Report of Zhongong's tower for the measurement of the shadow of the sun] (Changsha: Shangwu yinshu guan, 1939).

45 *Suishu* 隋書 [History of the Sui] (Beijing: Zhonghua shuju, 1973), 524.

46 *Qinding shujing tushuo* 欽定書經圖說 [Imperially endorsed illustrated Book of Documents], ed. Sun Jianai 孫家鼐 et al. (Beijing: Daxuetang bianshujū, 1905).

47 Liu Jinyi 劉金沂, "Fuju tukao" 覆矩圖考 [Study on the Fuju diagram], *Studies in the History of Natural Sciences* 自然科學史研究 7, no. 2 (1988): 112–118.



FIGURE 2
A gnomon depicted in *Qinding shujing tushuo*, 11a

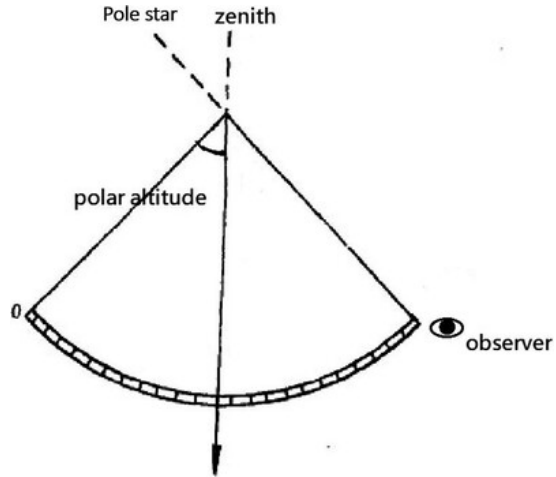


FIGURE 3
A reconstruction of the instrument *Fuju*. It is redrawn according to figure 2 on Liu Jinyi, “*Fuju tukao*,” 113

polar altitude, the inverted try square must be angled so that the light from the Pole star can be seen through the tubular speculum, then the plumb line will indicate a point on the scale, which corresponds to the degree of polar altitude. Guo Jinsong further hypothesized that the “Diagrams of inverted try square” (*Fuju tu* 覆矩圖), mentioned in the texts concerning Yixing’s survey, could be geometric diagrams based on the shape of the inverted try square, which can be helpful in converting polar altitudes and shadow lengths.⁴⁸

Yixing and his team developed several mathematical and astronomical methods for large-scale surveys. According to the calendrical treatises (*Lizhi* 曆志) of the *New Tang history*, Yixing applied the “method to measure the gnomon and clepsydra” (*Bu guilou shu*) to manage all of the shadow-length data and obtain constant values for the new calendar.⁴⁹ This data could then

48 Guo Jinsong 郭津嵩, “Yuanchu ‘sihai ceyan’ didian yu yitu bianzheng—jianji tang kaiyuan ceying 元初‘四海測驗’地點與意圖辨證—兼及唐開元測影” [Measurement across the four seas in the early Yuan: locations, intention and measuring sun shadow in the Kaiyuan Period of the Tang], *Wenshi* 文史, no. 2 (2021): 172–173. It should be noted that Cullen argued the *Fuju tu* means tangent table. Suppose the *tu* here implies the tangent table; in that case, he could not explain how the observers can “measure [the polar altitudes] with using the *tu* (*yi tu ce zhi* 以圖測之)” and “sight obliquely in accordance with using the *tu*” (*an tu xieshi* 按圖斜視). See Cullen, “An Eighth Century Chinese Table of Tangents,” 26.

49 Since the values of noon shadows at the commencement of each *qi* are not identical for all places in the world, Yixing had to use specific method to get these values in order to calcu-

be adapted for use at locations other than Yangcheng.⁵⁰ Historians of Chinese astronomy largely agree that, in order to obtain the “ideal” values recorded in the *Dayan Calendar*,⁵¹ Yixing composed a new tangent table to recalculate the observational data.⁵² However, this calculation could make the records of the shadow-lengths contradictory.

Yixing’s survey thereby refuted the old theory according to which “one *cun* of decrement or increment in the shadow corresponds to one thousand *li*.”⁵³ Based on the records of the gnomonic observation in Henan, Yixing argued that the distance of these two places was not correlated with the shadow-length and that the parameter associated with distance was the polar altitude. The brand-new value is as follows: the distance per *du* 度 (Chinese degree) of polar altitude equals 351 *li* plus 80 *bu* 步 (paces).⁵⁴ The old theory of “one *cun* increment” was

late the shadow-lengths at every place. The method was recorded in the *Dayan Calendar* and titled “On the search for the constant values of the gnomon [shadows measured] on the first day of each *qi* in [all locations within] the Nine Domains” (*Qiu jiufu suozai mei qi churi zhonggui changshu* 求九服所在每氣初日中晷常數). See *Jiu Tangshu*, 1256. For more discussions of this method, see Cullen, “An Eighth Century Chinese Table of Tangents,” 11–15.

50 *Xin Tangshu*, 661; Qu Anjing et al., “Zhongguo gudai lifa zhong de jiufu guiying suanfa,” 44–52.

51 Beer et al. argued that it seemed more elegant to offer up a set of “ideal values” computed by mathematical methods at that time. Beer et al., “Meridian Line,” 26. Qu Anjing even demonstrated that the astronomers of the Tang dynasty did not exclusively use observational data to make calendars, but also drew on calculated data. See Qu Anjing 曲安京, *Zhongguo lifa yu shuxue* 中國曆法與數學 [Chinese calendar and mathematics] (Beijing: Kexue chubanshe, 2005), 344–351.

52 For an eight-*chi*-long gnomon, the ratio of the length of the sun’s shadow, *l*, to the length of the gnomon, *h*, is exactly the value of tangent of the zenith angle, *x*:

$$l/h = \tan(x)$$

Therefore, if we know the zenith angle *x*, we can calculate its shadow length by checking the tangent table. See Qu Anjing 曲安京, “Zhengqie hanshubiao zai tangdai ziwuxian celiang zhong de yingyong” 正切函數表在唐代子午線測量中的應用 [Applications of the tangent table in Yixing’s meridian survey of the eighth century], *Chinese Studies* 漢學研究 16, no. 1 (1998): 93fn6.

53 Zhao, commentator, *Zhoubi suanjing*, 22a.

54 The corresponding text recorded in the *New Tang history* reads: “Roughly speaking, when the distance difference between two places is three hundred and fifty-one *li* and eighty *bu*, their polar latitudes will be one *du* different” (大率三百五十一里八十步，而極差一度。). See *Xin Tangshu*, 813. In the Tang dynasty, there were two systems, “long *li*” and “short *li*”, for measuring distances. Yixing adopted “short *li*” in the large-scale gnomonic survey. The value was “1 *li* = 300 *bu*.” See Hu Ji 胡戟, “Tangdai duliang heng yu muli zhidu 唐代度量衡與畝里制度” [System of measurement and Chinese units of *mu* and *li* in the Tang dynasty], *Journal of Northwest University (Philosophy and Social Sciences Edition)* 西北大學學報 (社會科學版) 4 (1980): 39–40.

abandoned and eventually disappeared after the Tang dynasty. Moreover, Yixing used the recorded polar altitudes and the obtained value corresponding to 1 *du* to calculate the distances between Yangcheng—the center of the Earth—and other sites. Modern-day scholars have recalculated these results and found that they either perfectly match or are very close to the current recorded distances.⁵⁵ We can conclude that except for the four sites in Henan, Yixing used the obtained value to detect the distances between Yangcheng and other observational sites. The recorded distances indicate that Yixing idealized all these sites to be distributed symmetrically around the center, Yangcheng. I argue that the symmetrical distribution of the observational sites with Yangcheng at the center turns out to be strongly related to the traditional concepts of the center of the Earth and the Nine Domains. This essay intends to demonstrate this point in the following pages.

In addition to disproving the theory of “one *cun* increment,” Yixing also reformed the system of “field allocation” astrology (*fenye* 分野), an astral-terrestrial correspondence that assigned segments of the sky to ancient territories of China.⁵⁶ In the textual description of the system, Yixing directly refers to *Wangji* (Royal Domain, see Fig. 1), the central zone in the Nine Domains framework. The *Rites* describe *Wangji*, the center of the Zhou world, as a fixed rectangular area of one thousand *li* at the center of the Zhou world with nine expanding zones of five hundred *li*. Yixing adjusted the concept to the different geopolitical reality he observed. I argue that he made Yangcheng, the center of the Earth, the new point of reference to rebuild the structure of concentric squares with less rigidly defined zone distances. Yixing’s observation is the basis of my reconstruction of *Wangji* in the large-scale survey. The above-mentioned methods of application provide a clue to help us imagine the blueprint of his envisioned large-scale survey.

55 For example, we can obtain the distance between Weizhou and Yangcheng through simple calculations as follows:

$$40 \text{ du (Weizhou's polar altitude)} - 34.7 \text{ du (Yangcheng's polar altitude)} = 5.3 \text{ du} \\ 5.3 \text{ du} \times 351 \text{ li } 80 \text{ bu} = 1861 \text{ li } 214 \text{ bu}$$

The calculated value perfectly matches the recorded distance in the historical sources. More discussions about the distance calculations, see Beer et al., “Meridian Line,” 16; Qu, *Zhongguo lifa yu shuxue*, 352–353.

56 See *Xin Tangshu*, 817–825.

4 Yixing's Rationale to Select Observational Sites of the Large-Scale Survey

Besides the four observational sites in the *Tang institutional history*, other contemporary sources make it possible to identify all fifteen sites of Yixing's survey.⁵⁷ Guo Jinsong conjectured that some listed sites of Yixing's large-scale survey might have been merely hypothetical, without exact observations.⁵⁸ Nonetheless, how and why Yixing decided to select these specific places (for a full list, see Appendix 1) remains an important question. To show more easily the rationale of Yixing's choice, I have divided the observational sites into three groups (see Fig. 4). The first group includes the four sites in Henan, namely, Huazhou baima (abbr. Huazhou), Bianzhou junyi (abbr. Bianzhou), Xuzhou fugou (abbr. Xuzhou), and Caizhou shangcai (abbr. Caizhou), marked with a triangle; the second group contains nine sites, Linyi guo,⁵⁹ Annan, Langzhou, Xiangzhou, Gaocheng, Yangcheng, Taiyuan fu, Weizhou and Tiele, marked with circles; the remaining two sites, Jingzhao and Luoyang, form the third group, marked by squares.

The reason for choosing the observational sites in Henan is that the difference between the polar altitudes of these four sites is close to 0.5 *du*⁶⁰ (see Appendix 1). This conformity would have been convenient for Yixing to examine if the shadow-length indeed increased by one *cun* every one thousand *li*. It is also possible that Yixing's team took Sui dynasty astronomer Liu Zhuo's 劉焯 (544–610) advice to conduct the gnomonic observations on the flatter ground in Henan.⁶¹

57 Four primary texts describe Yixing's large-scale survey; they are the *Tang institutional history*, the *six statutes of the Tang dynasty* (*Tang liudian* 唐六典), hereinafter referred to as *Six statutes* and *Tianwen zhi* 天文志 [Astronomical treatises] of the *Old Tang history* and of the *New Tang history*. The first text includes an entry titled "Measurement of Shadow-Lengths" (*cejing* 測景), mentioning eleven observational sites of the survey. The texts in astronomical treatises of the *Old Tang history* and of the *New Tang history* both mentioned ten sites. There is a text in the *Six statutes* providing a description of the Directorate of Astrology of the Tang dynasty; at the end of the text, the gnomonic survey is mentioned, and the list of the sites contains two additional sites which were not included in other records. See *Tang Huiyao*, 755; Li Linfu 李林甫, *Tang liudian* 唐六典 (Beijing: Zhonghua shuju, 1992), 303–304; *Jiu Tangshu*, 1303–1309; *Xin Tangshu*, 812–816.

58 Guo, "Yuanchu 'sihai ceyan' didian yu yitu bianzheng," 166–173.

59 Linyi guo 林邑國 (Linyi kingdom) was a Cham kingdom located in today's central Vietnam. Chinese historical texts sometimes shorten it as Linyi, hereinafter also referred to as Linyi.

60 Beer et al., "Meridian Line," 13–14.

61 According to the *Dili zhi* 地理志 [Geographical treatises] of the *New Tang history*, the four observational sites were already set in the Sui dynasty. See *Xin Tangshu*, 1432–1436.



FIGURE 4 Yixing's observational sites marked on the map of the Tang territory in the twenty-ninth year (741) of the Kaiyuan reign-period (713–741). Source: Tan Qixiang 譚其驤, *Zhongguo lishi ditu ji* 中國歷史地圖集 [The historical atlas of China] (Beijing: Zhongguo ditu chubanshe, 1982), vol. 5: 34–35

The second group of sites forms a longer vertical chain. Yangcheng had been considered the center of the Earth for a long time. Besides Yangcheng, Tiele, Linyi, and Annan (Jiaozhou) had been mentioned in earlier surveys.⁶² It is worth noting that the place called “Gaocheng” in the *Tang institutional history* seems to be identical to Yangcheng, which is mentioned in the *Old Tang history* and the *New Tang history*.⁶³ However, recent research has refuted that the *Tang institutional history* had simply changed the toponym “Yangcheng” to “Gaocheng.” Qu Anjing argued that the polar altitude of Gaocheng in the *Tang institutional history* might be a calculated result and that it was further used to

62 Guan, “Zhongguo tianwen xueshi shang de dizhong gainian,” 255–257.

63 The record of the second Shensong 神龍 reign-period (706) in the *Old Tang history* can prove it: “In the *yi si* day of the eleventh month ... [the name] ‘Henan’ was changed to ‘Hegong’; ‘Luoyang’ was changed to ‘Yongchang’; ‘Songyang’ was changed to ‘Defeng’; ‘Yangcheng’ was changed to ‘Gaocheng.’” The original text reads: “十一月乙巳 ... 改河南為合宮，洛陽為永昌，嵩陽為登封，陽城為告成。” See *Xin Tangshu*, 143. Relevant texts are found in *Jiu Tangshu*, 1423.

determine the shadow-length at Gaocheng on the day of the summer solstice.⁶⁴ Even though the *Old Tang history* and the *New Tang history* only mentioned Yangcheng in the list of observational sites, there are apparently two separate sets of data for Yangcheng: One set was identical to the data of Gaocheng in the *Tang institutional history*;⁶⁵ the other was not. Guo Jinsong indicated that the latter set of data (i.e., associated with Yangcheng) resulted from observations, testing the old theory of “one *cun* increment;” the former (i.e., associated with Gaocheng) was the ideal value through calculation and, therefore, the benchmark for further calculations as well as the composition of a comprehensive calendrical model for the whole Tang territory.⁶⁶ Furthermore, the calculated shadow-length on the summer solstice was closer to 1 *chi* 5 *cun*, which was believed to be the shadow-length at the center of the Earth. This rationalizes Yixing’s decision to choose Yangcheng (Gaocheng) as the benchmark to measure the distances between other sites.

Tiele was the northernmost site in the large-scale survey and was 6900 *li* to the North of the capital in Chang’an.⁶⁷ The vocabulary used in the description given in the *Old Tang history* underpins Guo Jinsong’s argument that Yixing’s team did not conduct gnomonic observations in Tiele, which therefore may have been used as a hypothetical site corresponding symmetrically with Linyi in the far south.⁶⁸

Let us suppose that going north from Yangcheng to the land of Tiele, the difference is also 17 *du* 4 *fen* [in polar elevation], being exactly the same as that between Yangcheng and Linyi. Then in the fifth month (at the Summer Solstice), the Sun will be 27 *du* 4 *fen* [to South of] the zenith, the polar elevation will be 52 *du*, and the circle of perpetual visibility 104 *du*.⁶⁹ The

64 Qu, *Zhongguo lifa yu shuxue*, 352–353.

65 One collation note concerning Yangcheng’s two sets of data in the *Old Tang history* reads: “If we recalculate the differences of the polar altitudes, distances, and shadow-lengths in Yangcheng between Wuling, Hengyejun, Annan and Linyi, Tiele, we will find that the polar altitude and shadow-length in Yangcheng were replaced by those in Henan fu [Gaocheng]. 核算下文所記陽城與武陵、橫野軍、安南、林邑、鐵勒等地極高差、距離和影差時，發現俱係用河南府（告成）極高和影長代替陽城極高和影長。” See *Jiu Tangshu*, 826.

66 Guo, “Yuanchu ‘sihai ceyan’ didian yu yitu bianzheng,” 169.

67 *Xin Tangshu*, 813.

68 Guo, “Yuanchu ‘sihai ceyan’ didian yu yitu bianzheng,” 170.

69 The “circle of perpetual visibility” means circumpolar circle, where circumpolar stars are always visible above the horizon at a specified locality on the earth’s surface. The angular measure of the radius of this circle equals the observer’s latitude.

shadow on the day of Summer Solstice will be 4 *chi* 1 *cun* 3 *fen*, and on the day of Winter Solstice, 29 *chi* 2 *cun* 6 *fen*.⁷⁰

We can see that the sentence has the structure “Let us suppose ... then.” This wording differs considerably from that of the other parts of the text describing the sites in Yixing’s survey. The subjunctive mood seems to indicate that the data of Tiele was not observational but rather hypothetical; this statement also implies that Yixing was able to use mathematical calculation to conjecture the position of Tiele. Another issue is that Tiele was not mentioned in the last paragraph of the *New Tang history* and the *Tang institutional history*, both of which contained lists of the shadow-lengths measured at different sites. If Chinese astronomers indeed went to Tiele to observe the shadows, why did they not add the results of their observations to the list? The two points above indicate that in all likelihood there were no gnomonic observations made at Tiele.

The observations in Linyi and Annan started during the Yuanjia 元嘉 reign (424–453) of the Liu Song 劉宋 dynasty; the relevant record in the *New Tang history* reads as follows:

In the midst of the Yuanjia reign-period of the [Liu] Song dynasty, there was an expedition to Linyi. In the fifth month they set up a gnomon, and when the observation was made, the Sun was to the North of the gnomon, the shadow of Jiaozhou was 3 *cun* south of the gnomon, and that of Linyi 9 *cun* 1 *fen*.⁷¹

Jiaozhou 交州 can be identified as the place called Annan duhu fu (Annan Protectorate, abbr. Annan)⁷² of the Tang dynasty. The Tang scholars considered Linyi symmetrical to Tiele,⁷³ and both sites were located in countries neighboring the territory of the Tang Empire.⁷⁴ The Linyi site was in the kingdom of

70 The translation is quoted from Beer et al., “Meridian Line,” 12, with modifications to the romanization of Chinese terms, here following the *pinyin* system. The original text reads: “假令距陽城而北，至鐵勒之地亦十七度四分，合與林邑正等，則五月日在天頂南二十七度四分，北極之高五十二度，周圍一百四度，常見不隱。北至之晷四尺一寸三分，南至之晷二丈九尺二寸六分。” See *Jiu Tangshu*, 1305.

71 The original text reads: “宋元嘉中，南征林邑，五月立表望之，日在表北，交州影在表南三寸，林邑九寸一分。” See *Xin Tangshu*, 812.

72 Duhu fu 都護府 was an administrative division controlling frontier regions during the Han and Tang dynasties.

73 See *Jiu Tangshu*, 1305.

74 A statement to this effect in *Dili zhi* 地理志 [Geographical treatises] of the *New Tang history* reads: “The Easternmost territory [of the Tang dynasty] was along the coast, the

Champa. The record depicting the observation in the Yuanjia reign-period indicates that the gnomonic observation was performed during an expedition, and there is a historical record of this expedition in the *Complete Annals of Great Viet* (*Đại Việt sử ký toàn thư* 大越史記全書, compiled in 1797) which reads as follows:

In the thirteenth year of the Yuanjia reign-period of the [Liu] Song dynasty (436), in the second month of spring, the Song emperor dispatched the prefectural governor of Jiaozhou, Tan Hezhi, to crusade Linyi By the fifth month, [Tan] Hezhe and others captured the area of Qusu city.⁷⁵

Also, a statement in *Commentary to the River Classic* (*Shuijing zhu* 水經注, compiled in ca. fifth century CE) mentioned that there was a gnomon in Qusu. It reads as follows: “The 8 *chi* gnomon was set in Qusu, the shadow-length was 8 *cun* to the South.”⁷⁶ On the basis of these two statements, one can argue that in Yixing’s large-scale survey the observational site mentioned as “Linyi” was located in Qusu city, a place that may have been near the modern-day Đông Hà 東河 (E107°5’50”, N16°49’49”) in the middle of Vietnam. Although there were some differences between the latitude of Đông Hà and the polar altitude of Linyi (17.4 *du*, which is equal to sexagesimal N17°9’),⁷⁷ this data supports the

Westernmost territory was Yanqi 焉耆, the Southernmost territory was to the Southern border of Linzhou, the Northernmost territory was adjacent to Xueyantuo 薛延陀. 其地東極海，西至焉耆，南盡林州南境，北接薛延陀。” See *Xin Tangshu*, 960.

75 The original text reads: “宋元嘉十三年。春，二月，宋帝遣交州刺史檀和之討林邑 ... 五月，和之等拔區粟城。” See Ngô Sĩ Liên 吳士連 et al., *Đại Việt sử ký toàn thư* 校合本大越史記全書 [Complete annals of Great Viet] (Tokyo: Institute for Advanced Studies on Asia, University of Tokyo, 1984–1986), 144–145.

76 The original text reads: “區粟建八尺表，日影度南八寸。” See Li Daoyuan 酈道元, *Shuijing zhu* 水經注 (Taipei: shijie shuju, 1965), 450.

77 Yan Gengwang 嚴耕望 hypothesized that the site of Yixing’s gnomonic survey, Linyi, was the capital of Champa. And he further argued that, based on the polar altitude of Linyi, the likely location might be modern-day Dong Hoi 洞海 (E106°35’, N17°27’). This conjecture has two weak points: first, archaeologists have since discovered that before 757 the capital, Linyi, was Tra Kieu 茶喬, not Dong Hoi. So the basis of his hypothesis was wrong. Second, according to historical sources, the site of Linyi was located in Qusu city (modern Đông Hà), which is not even close to Dong Hoi. See Yan Gengwang 嚴耕望, “Tangdai shengshi yu xinan linguo zhi jiangjie” 唐代盛時與西南鄰國之疆界 (The border between southwest neighboring countries and the Great Tang), *Bulletin of the Institute of History and Philology Academia Sinica* 中央研究院歷史語言研究所集刊 59, no. 4 (1988): 972–974; Huang Lan-Shiang 黃蘭翔, *Yuenan chuantong juluo, zongjiao jianzhu yu gongdian* 越南傳統聚落、宗教建築與宮殿 [Traditional villages, religious architecture, and palaces in Vietnam] (Taipei: Center for Asia-Pacific Area Studies, 2008), 127.

hypothesis that Yixing's intention was to show that the difference of the polar altitudes between the southernmost site in Linyi and Yangcheng (as the center of Earth) was approximately equal to that between Yangcheng and the northernmost Tiele. This was also a politically necessary step, since it situated the new Royal Domain (*Wangji*) of the Tang on a symmetrical North-South axis. The fact that Jiaozhou (Annan in the Far South) and Weizhou (in the Far North) "were the two sites at the Southern and Northern limits of China"⁷⁸ may well have been the reason why Yixing selected Annan and Weizhou as two of the test sites.

Only three capital districts (*fu* 府) existed in the Kaiyuan reign-period of the Tang dynasty: Jingzhao fu, Henan fu, and Taiyuan fu. They were called the "Three Capital Districts" (*Sandu fu* 三都府), also named the Western Capital (*Xijing* 西京), the Eastern Capital (*Dongjing* 東京), and the Northern Capital (*Beijing* 北京).⁷⁹ The *Six statutes* highlights Taiyuan fu in particular and mentions five other observational sites in Yixing's large-scale survey, namely, Jingzhao, Weizhou, Luocheng, Taiyuan, and Annan.⁸⁰ Since Jingzhao, Weizhou, Luocheng and Annan were previously used to observe shadow-lengths before the Tang dynasty, it appears reasonable to suggest that there were probably some observational activities in Taiyuan fu, just as in the other four sites. In the selections of the two remaining observational sites in the second group, Xiangzhou, and Langzhou, Yixing followed two principles. First, three sites in the second group were chosen as geographic reference spots: Yangcheng at the center, Weizhou at the Northern limits of the Tang territory, and Taiyuan as a central administrative unit of the dynasty. Then they were matched as follows: Tiele with Linyi, Weizhou with Langzhou, and Taiyuan fu with Xiangzhou.

Yixing's survey included two sites with observational facilities, Chang'an 長安 located in Jingzhao fu (Metropolitan Prefecture),⁸¹ which is where the Directorate of Astrology was located, and Luocheng, which is where earlier experts, such as Zhang Heng and Ma Rong 馬融 (79–166),⁸² had placed the center of the Earth.⁸³ As the *Jade Ocean* (*Yuhai* 玉海, comp. 13th century CE) mentioned,

78 The original text reads: "交州 蔚州 此二所為中土南北之極。" See *Tang Huiyao*, 755.

79 Xue Zuoyun 薛作雲, *Tangdai difang xingzheng zhidu yanjiu* 唐代地方行政制度研究 [A study on local administration system in the Tang dynasty] (Taipei: Taiwan shangwu yinshuguan, 1974), 12–13.

80 *Tang Liudian*, 303–304.

81 *Jiu Tangshu*, 1395.

82 Guan, "Zhongguo tianwenxue shi shang de dizhong gainian," 253.

83 Several records of shadow-lengths at solstices can be found in the astronomical treatises of the *History of the Sui*. See *Suishu*, 525.

Luocheng had had an astronomical observatory during the Tang [dynasty], that had been located in the same courtyard as Yixing's living quarters at the imperial palace.⁸⁴ It indicates Yixing and his coworkers had made astronomical observations in Chang'an and Luocheng.

5 Design of Yixing's Large-Scale Gnomonic Survey and Its Correlation with the "Nine Domains"

So, what is the correlation between Yixing's large-scale gnomonic survey and the traditional concept of Nine Domains? The theoretical framework of Chinese Flat-Earth cosmology entails that no reconstructed map, such as that of the Tang territory by Tan Qixiang (Fig. 4), can ever reflect the people's common geographical understanding in the Tang period, since latitude and longitude were not adopted until after the seventeenth century. Unfortunately, no maps of the Tang dynasty have so far been found. Some extant Song dynasty maps, such as the *Map of prefectures and counties of the Nine Districts* (*Jiuyu shouling tu* 九域守令圖, 1121) and the *Map of Yu's tracks* (*Yuji tu* 禹跡圖, 1136), include the geographical descriptions produced during the Tang dynasty. Still, the scales and directions on these maps do not conform to the extant descriptions of Yixing's survey.⁸⁵ Therefore, I provide a hypothetical scheme to show the distribution of these sites corresponding to the geographic picture that may well have been in Yixing's mind. (See Fig. 5)

The three groups of Yixing's observational sites formed a configuration in the shape of a cross with two vertical lines and one horizontal line; the short vertical line on the right corresponds to the four sites in Henan; the central horizontal line consists of Jingzhao, Luocheng, Yangcheng, and Henan; and the long vertical line on the left results from linking the other sites.⁸⁶ Yixing opted for a cross-shaped design for three reasons: first, he divided two groups along the two vertical lines of sites belonging to different systems. According to Qu Anjing's argument, the observational sites on the left side along the North-

84 Wang Yinglin 王應麟, *Yuhai* 玉海 [Jade ocean] (Taipei: Taiwan shangwu yinshuguan, 1983), 3112.

85 Chia-Yun Wu 吳佳芸, "Yi-Xing's Large-Scale Gnomonic Survey: A Revisit" (Master's thesis, National Tsing Hua University, 2012), 124–133.

86 The distances between Jingzhao and Luoyang, Luoyang and Bianzhou were based on Yan Gengwang's study, see Yan Gengwang, *Tangdai jiaotong tukao* 唐代交通圖考 [Atlas of the transport history of the Tang period] (Shanghai: Shanghai guji chubanshe, 2007), 20, 1795. The distances between other sites and Yangcheng were based on the records in the historical books.

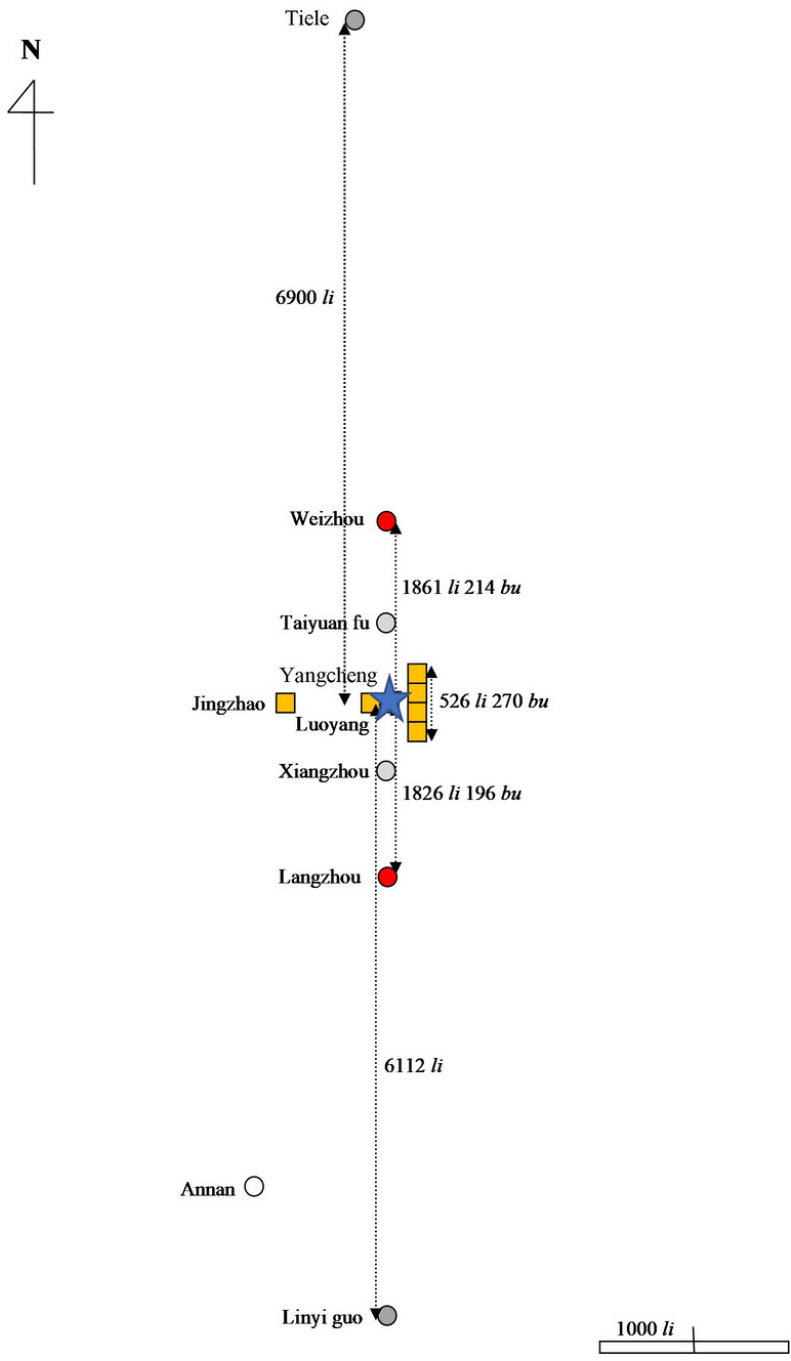


FIGURE 5 The hypothetical scheme used by Yixing

South vertical chain were used to examine the results obtained on the four sites based in Henan⁸⁷ (see Fig. 5). Yixing may have thought that the second vertical chain of sites could be useful, especially because it formed a straight line much longer than that in Henan, except for Annan which, however, constituted the Southern border in China. The other sites of the second group were distributed symmetrically around Yangcheng. Annan was the only site without a match.

Second, the horizontal line of sites made up by the four sites in Henan could be considered as one group, since they were relatively close to each other. Together with Luoyang and Jingzhao, they also formed one of the most important post roads in the Tang dynasty⁸⁸ crossing the center of the Tang Empire, including the Capital Chang'an.

Third, medieval Chinese astronomers could use polar altitudes to measure distances between various locations on the Earth's surface or to decide if two places were on the same latitude. However, they could not exactly know whether they were in the same longitude.⁸⁹ Tiele and Linyi were not on the true North-South straight line in Yixing's large-scale survey. This fact, however, was not known until the eleventh century, that is, until the discovery of the technique called "Flying Birds" (*feiniao* 飛鳥)⁹⁰—which consisted in calculating

87 Qu, *Zhongguo lifa yu shuxue*, 347.

88 Bianzhou, one of the four observational sites in Henan, was also a vital post station, just like Luoyang and Jingzhao, which were the two metropolises in the Tang dynasty. See Yan, *Tangdai jiaotong tukao*, 1793.

89 Determining if two or more places were on the same longitude was much harder than calculating their latitude. Although astronomers, theoretically, could use magnetic compasses and gnomonic observations to see whether the two places were on the same longitude, different landforms would block the path. In fact, the longitude problem was not solved until John Harrison's chronometer in the 1770s, for which he won the Longitude Prize. See Rupert T. Gould, *The Marine Chronometer: Its History and Development* (London: Holland Pr., 1978).

90 The passage regarding "Flying Birds" (*feiniao*) in Shen Gua's *Dream pool essays* (*Mengxi bitan* 夢溪筆談) reads: "The *Feiniao* system may be explained as follows. As roads and paths are sometimes winding and sometimes straight, without any definite rule, if a walker starts out in any one of the four directions from a given point along a path, his pacing will not help us get the direct distance. Therefore, what we call 'straight lines in the four directions' have to be measured by other methods, just as a bird can fly in a straight line unaffected by the convolutions of mountains and rivers. 所謂「飛鳥」者，謂雖有四至里數，皆是循路步之，道路迂直而不常，既列為圖，則里步無緣相應，故按圖別量徑直四至，如空中鳥飛直達，更無山川回屈之差。" See Shen Gua, *Mengxi bitan* 夢溪筆談 (Sichuan: Bashu shushe, 1996), 990–991. The translation is quoted from Joseph Needham, *Science and Civilisation in China Vol. 3: Mathematics and the Sciences of the Heavens and Earth* (Cambridge: Cambridge University Press, 1995), 576–577. More discussion about this method by modern scholars, see Zuo Ya, *Shen Gua's Empiricism* (Cambridge, MA: Harvard University Asia Center, 2018), 155.

the direct distance between two geographical points—applied by Shen Gua 沈括 (1031–1095) together with Pei Xiu's 裴秀 (224–271) method called “Six cartographical structures” (*zhitu liuti* 製圖六體), which allowed astronomers and cartographers to check if places were along the same longitude.

However, the cross-like configuration of the observational sites that I have reconstructed (Fig. 5) actually implies the concept of Nine Domains. In the model of the Nine Domains the center of the territory was the Royal Domain (*Wangji*); the other parts were based on hierarchical social relations to form nine areas. This kind of spatial classification was based on the theories of the special role of the supreme ruler (*wang*, or later, emperor), which considered central China as the most developed area.⁹¹ Since Yixing used Yangcheng as the benchmark to measure the distance between other sites, the central area of Yixing's large-scale survey was a square formed by Jingzhao, the four sites of Henan, Taiyuan fu, and Xiangzhou; other sites symmetrically distributed around Yangcheng thus determined the outer squares (see Fig. 6). I argue that Yixing noticed that the shadow-lengths were the same in the observational sites having identical polar altitudes and that, therefore, he mainly focused on the sites on the North-South straight line and not as much on sites of the East-West line. The underlying Nine-Domains logic makes it understandable why the distribution looks like a cross with two vertical lines and one horizontal line. But the distribution actually conforms with the model of concentric squares.

To sum up, Yixing selected the observational sites on the basis of their historical records, location, and astronomical calculations and then framed them within the Zhou-dynasty model of the Nine Domains—a schematic way of ordering space that, in my opinion, Yixing considered to be the best adapted to the Tang's geopolitical reality. During the reign of Emperor Xuanzong of Tang, the Tang territory stretched from the Tarim Basin to the Korean Peninsula as well as from Mongolia south to modern-day Vietnam.⁹² And its capital, Chang'an was in the middle of the empire, that is, *Wangji*. The coincidence of the center of the Earth with the *Wangji* highlighted the centrality of the Tang Empire within a hierarchized spatial structure. It also reaffirmed the authority of Emperor Xuanzong of Tang as emanating from the cosmo-geographic center, and thereby symbolically brought the Tang Empire into harmony within Heaven and Earth.

91 Tang, *Cong hundun dao zhixu*, 228.

92 Tan Qixiang 譚其驤, *Zhongguo lishi ditu ji* 中國歷史地圖集 [The historical atlas of China] (Beijing: Zhongguo ditu chuban she, 1982), vol. 5: 34–35.

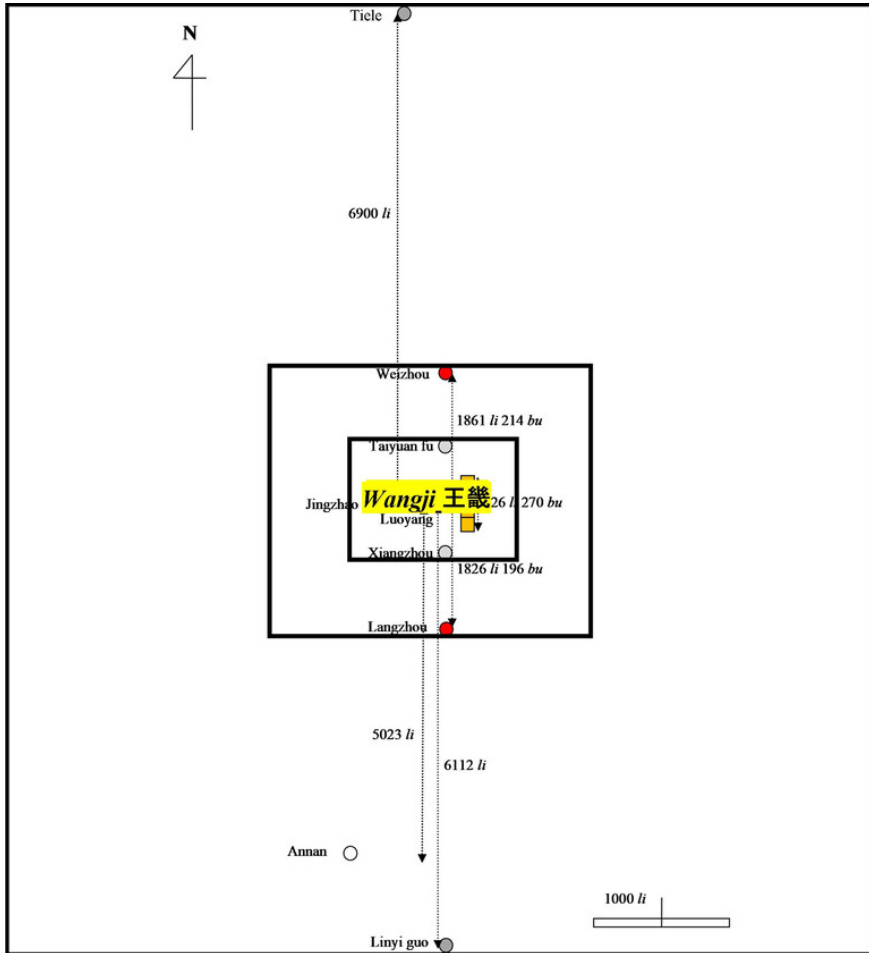


FIGURE 6 The hypothetical scheme used by Yixing and based on the concept of Nine Domains (*Jiufu*)

6 Conclusion

To conclude, the brand-new value proposed by Yixing—“the distance per *du* of polar altitude equals 351 *li* plus 80 *bu*”—can be best understood as a paradigm shift in Chinese astronomy. Yixing used the new value related to polar altitudes to calculate the data in the large-scale survey; this conveys much more about the scientific aspect than the political one.

One of Yixing’s assignments was to compose a new calendar that might be used everywhere in China within the borders of the Tang dynasty. To fulfill this aim, Yixing and his team had to launch a geographical survey to know the

influence of different positions in the north and south on the data, such as the polar altitudes, the shadow lengths, and the lengths of daytime and nighttime, to improve the new calendar's applicability for different geographical areas in China within the borders of the Tang dynasty. This is why creating the calendar was connected with the geographical survey.

The shadow-lengths at the twenty-four *qi* were essential for official calendars. However, the official calendars before Emperor Xuanzong of Tang only recorded this data at the center of the Earth or in the capital. Yixing intended to compose a calendar that might be applied everywhere in the Tang territory. He needed the data of the shadow-lengths at the twenty-four *qi* at every place, which was impossible to do merely by observation. Thus, he proposed a new method, including a tangent table to get the values of shadows at noon, at the commencement of each *qi*. To be more precise, once he knew the polar altitude of a place, Yixing could calculate its shadow length by checking the tangent table.

Combining the historical records and schematic delineations within the Nine Domains model makes it possible to reconstruct Yixing's original design for the large-scale survey, and provides us a better understanding thereof. The survey's "data" and the geographic picture they depict reflect Yixing's a-priori spatial conceptualization. This demonstrates that he used data which did not solely derive from empirical measurement. The case of the Nine Domains model exemplifies the constant tension in the history of science between concept and practice.

By analyzing and reconstructing the configuration of the observational sites in Yixing's large-scale survey, we find that the centralized and symmetrical pattern in the arrangement of the survey sites perfectly conforms to the schematic delineation of the zones of Nine Domains. In my opinion, Yixing made Yangcheng (Gaocheng)—the center of the Earth—a point of reference, in order to rebuild the structure of concentric squares with less rigidly defined zone distances. This model divided the world into a series of zones of declining order and degree of civilization. Since the Nine Domains was the ideal model for Emperor Xuanzong of Tang to re-establish the legitimacy of the Tang-dynasty regime, it is not surprising that this concept played an essential role in Yixing's large-scale gnomonic survey, which was part of the calendrical reform during the Kaiyuan reign-period. In fact, I argue that the design of Yixing's large-scale gnomonic survey, including the selection and distribution of observational sites, was highly relevant to the political and cultural milieu of the High Tang. This aspect shows the duality of Yixing's large-scale survey, which was not only conducted to improve quantitative accuracy, as emphasized by previous scholars, but also to exemplify the fact that quantitative methods were used

for political purposes. To be more precise, the Nine Domains of the Zhou era proved to be useful for Yixing's project to position Emperor Xuanzong of Tang's Royal Domain as the new center of Heaven and Earth.

Acknowledgments

This essay resulted from my Master Thesis, *Yi-Xing's Large-Scale Gnomonic Survey: A Revisit*. I would like to express my deep gratitude to my advisor, Prof. Alexei Volkov (NTHU), for supervising my thesis. Also, I am grateful to Prof. Dagmar Schäfer (MPIWG), Prof. Marta Hanson (MPIWG), and Dr. Vera Dorofeeva-Lichtmann (CNRS-MPIWG) for remarks and suggestions on this essay, to Prof. Guo Jinsong 郭津嵩 (PKU) and two anonymous reviewers for valuable comments on the article. All mistakes found in this paper are my own responsibility. I was preparing the manuscript for publication during my stay as a Visiting Pre-doctoral Fellow at the Max Planck Institute for the History of Science in Berlin (September–December 2021) and considerably benefited from its excellent research facilities and most helpful staff.

Appendix 1: The Records of Observational Sites in Yixing's Large-Scale Survey (from South to North)

Sites	Polar altitudes	Shadow-lengths (summer solstice)	Shadow-lengths (equinox)	Shadow-lengths (winter solstice)
Linyi guo 林邑國	17 du 4 fen	5 cun 7 fen (South of the gnomon)	2 chi 8 cun 5 fen (<i>New Tang history</i> , <i>Old Tang history</i>)/ 2 chi 6 cun 5 fen (<i>Tang institutional history</i>)	6 chi 9 cun
Annan duhu fu 安南都護府	20 du 4 fen (<i>New Tang history</i>)/ 26 du 6 fen (<i>Old Tang history</i>)/ 21 du 6 fen (<i>Tang institutional his- tory</i>)	3 cun 3 fen/ 3 cun (<i>Six statutes</i>) (South of the gnomon)	2 chi 9 cun 3 fen	7 chi 9 cun 4 fen
Langzhou Wuling 朗州武陵	29 du 5 fen	7 cun 7 fen	4 chi 3 cun 7.5 fen (<i>New Tang history</i> , <i>Old Tang history</i>)/ 4 chi 4 cun 7 fen (<i>Tang institutional history</i>)	1 zhang 丈 5 cun 3 fen
Xiangzhou 襄州	n/a	n/a	4 chi 8 cun	n/a
Caizhou Wujinguan 蔡州武津館	33 du 8 fen	1 chi 3 cun 6.5 fen (<i>New Tang history</i> , <i>Old Tang history</i>)/ 1 chi 3 cun 6 fen (<i>Tang institutional history</i>)	5 chi 2 cun 8 fen	1 zhang 2 chi 3 cun 8 fen
Xuzhou Fugou 許州扶溝	34 du 3 fen	1 chi 4 cun 4 fen	5 chi 3 cun 7 fen	1 zhang 2 chi 5 cun (<i>Tang institutional history</i>)/ 1 zhang 2 chi 5 cun 3 fen (<i>Old Tang his- tory</i>)/ 1 zhang 2 chi 5 cun 5 fen (<i>New Tang his- tory</i>)
Yangcheng 陽城	34 du 4 fen	1 chi 4 cun 7 fen 8 mao 毫	5 chi 4 cun 3 fen	1 zhang 2 chi 7 cun 1.5 fen
Henanfu Gaocheng 河南府告成	34 du 7 fen	1 chi 4 cun 9 fen	5 chi 4 cun 5 fen	1 zhang 2 chi 7 cun 1 fen

(cont.)

Sites	Polar altitudes	Shadow-lengths (summer solstice)	Shadow-lengths (equinox)	Shadow-lengths (winter solstice)
Luoyang (Luocheng) 洛陽/洛城	n/a	n/a	5 chi 2 cun 5 fen	n/a
Jingzhao 京兆	n/a	1 chi 4 cun 2 fen	5 chi 3 cun 4 fen	1 zhang 2 chi 9 cun 2 fen
Bianzhou Junyi 汴州浚儀	34 du 8 fen	1 chi 5 cun 3 fen	5 chi 5 cun	1 zhang 2 chi 8 cun 5 fen
Huazhou Baima 滑州白馬	35 du 3 fen	1 chi 5 cun 7 fen	5 chi 5 cun 6 fen (New Tang history, Tang institutional history)/ 5 chi 3 cun 6 fen (Old Tang history)	1 zhang 3 chi
Taiyuan fu 太原府	n/a	n/a	6 chi	n/a
Weizhou Hengyejun 蔚州橫野軍	40 du	2 chi 2 cun 9 fen	6 chi 4 cun 4.5 fen (New Tang history, Six statutes)/ 6 chi 4 cun 3 fen (Old Tang history)/ 6 chi 6 cun 2 fen (Tang institutional history)	1 zhang 5 chi 8 cun 9 fen
Tiele 鐵勒	52 du	4 chi 1 cun 3 fen	9 chi 8 cun 7 fen	2 zhang 9 chi 2 cun 6 fen