Supplementary information:

The effect of environmental information on evolution of cooperation in stochastic games

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Supplementary Note 1: Games with deterministic transitions and weak selection

In this section, we summarize our results in the weak selection limit for games with deterministic transitions. Exact statements and all proofs are in Supplementary Note 4. To start with, consider a deterministic transition vector $\mathbf{q} = (q_{CC}^1, q_{DD}^1, q_{DD}^1, q_{CC}^2, q_{CD}^2, q_{DD}^2) \in \{0, 1\}^6$. In the limit of vanishing selection $\beta = 0$, we can exploit some symmetry properties of the system, because payoffs become irrelevant for the evolutionary process. To describe these symmetries formally, it is useful to introduce some notation. For each stochastic game \mathbf{q} , we can define an associated *twin* $\chi_q(\mathbf{q})$ by relabelling the states,

$$\chi_q(\mathbf{q}) = (1 - q_{CC}^2, 1 - q_{CD}^2, 1 - q_{DD}^2, 1 - q_{CC}^1, 1 - q_{CD}^1, 1 - q_{DD}^1).$$
(1)

Similarly, we can define an associated *mirror* game $\psi_q(\mathbf{q})$ by flipping the meaning of C and D,

$$\psi_q(\mathbf{q}) = (q_{DD}^1, q_{CD}^1, q_{CC}^1, q_{DD}^2, q_{CD}^2, q_{CC}^2).$$
(2)

We can also consecutively perform both transformations, yielding a mirror-twin,

$$\chi_q \circ \psi_q(\mathbf{q}) = (1 - q_{DD}^2, 1 - q_{CD}^2, 1 - q_{CC}^2, 1 - q_{DD}^1, 1 - q_{CD}^1, 1 - q_{CC}^1).$$
(3)

We define analogous transformations for the players' memory-one strategies,

$$\chi_{p}(\mathbf{p}) = (p_{CC}^{2}, p_{CD}^{2}, p_{DC}^{2}, p_{DD}^{2}, p_{CC}^{1}, p_{CD}^{1}, p_{DC}^{1}, p_{DD}^{1})$$

$$\psi_{p}(\mathbf{p}) = (1 - p_{DD}^{1}, 1 - p_{DC}^{1}, 1 - p_{CD}^{1}, 1 - p_{CC}^{1}, 1 - p_{DD}^{2}, 1 - p_{DC}^{2}, 1 - p_{CD}^{2}, 1 - p_{CC}^{2})$$
(4)

and for the invariant distributions,

$$\chi_{v}(\mathbf{v}) = (v_{CC}^{2}, v_{CD}^{2}, v_{DC}^{2}, v_{DD}^{2}, v_{CC}^{1}, v_{CD}^{1}, v_{DC}^{1}, v_{DD}^{1}),$$
(5)

$$\psi_v(\mathbf{v}) = (v_{DD}^1, v_{DC}^1, v_{CD}^1, v_{CC}^1, v_{DD}^2, v_{DC}^2, v_{CD}^2, v_{CC}^2).$$
(6)

With this notation, we can formulate a few useful relationships between a game with transition vector \mathbf{q} and its associated twin, mirror, and mirror-twin. To this end, let $\mathbf{v}(\mathbf{p}|\mathbf{q})$ denote the stationary distribution among two players with strategy \mathbf{p} interacting in the stochastic game with transition vector \mathbf{q} . We show the following relations (Supplementary Note 4, Lemma 1),

$$\mathbf{v}(\mathbf{p}|\mathbf{q}) = \chi_v \Big(\mathbf{v} \big(\chi_p(\mathbf{p}) \big| \chi_q(\mathbf{q}) \big) \Big), \tag{7}$$

$$\mathbf{v}(\mathbf{p}|\mathbf{q}) = \psi_v \Big(\mathbf{v} \big(\psi_p(\mathbf{p}) | \psi_q(\mathbf{q}) \big) \Big).$$
(8)

That is, suppose we know the invariant distribution v(p|q) of the game q with respect to strategy p.

Then we can directly infer the invariant distribution of the respective twin interaction – the one with transition vector $\chi_q(\mathbf{q})$ and strategies $\chi_p(\mathbf{p})$. Similarly, we can directly infer the invariant distribution of the respective mirror interaction, with $\psi_q(\mathbf{q})$ and $\psi_p(\mathbf{p})$. Now, if $\gamma(\mathbf{p}|\mathbf{q})$ denotes the average cooperation rate in a game with transition function \mathbf{q} among two players with strategy \mathbf{p} , we obtain as a consequence (Supplementary Note 4, Corollary 1),

$$\gamma(\mathbf{p}|\mathbf{q}) = \gamma(\chi_p(\mathbf{p})|\chi_q(\mathbf{q})), \qquad (9)$$

$$\gamma(\mathbf{p}|\mathbf{q}) = 1 - \gamma \left(\psi_p(\mathbf{p}) \mid \psi_q(\mathbf{q}) \right). \tag{10}$$

This result depends on the particular strategy \mathbf{p} used by the two players. However, as β becomes vanishingly small, we can derive an analogous statement independent of \mathbf{p} . To this end, for a given \mathbf{q} , let $\hat{\gamma}_0^F(\mathbf{q})$ be the average cooperation rate according to the invariant distribution of the full information game when individuals use deterministic strategies, in the limit of rare errors and $\beta = 0$. Then we show (Supplementary Note 4, Proposition 1) that

$$\hat{\gamma}_0^F(\mathbf{q}) = \hat{\gamma}_0^F(\chi_q(\mathbf{q})), \tag{11}$$

$$\hat{\gamma}_0^F(\mathbf{q}) = 1 - \hat{\gamma}_0^F(\psi_q(\mathbf{q})). \tag{12}$$

There are two reasons why the relationships in (11) are useful. First, with each stochastic game \mathbf{q} that we understand, we immediately understand three other stochastic games, $\chi_q(\mathbf{q})$, $\psi_q(\mathbf{q})$, and $\chi_q \circ \psi_q(\mathbf{q})$. This means that there are fewer distinct cases that need to be analyzed.

Second, in the special case that a transition vector \mathbf{q} is its own mirror, $\psi_q(\mathbf{q}) = \mathbf{q}$, it follows directly from the second equation in (11) that $\hat{\gamma}_0^F(\mathbf{q}) = 1/2$. As we prove in Proposition 2 in Supplementary Note 4, in the no-information setting the respective average cooperation rates always satisfy $\hat{\gamma}_0^N(\mathbf{q}) =$ 1/2, for all \mathbf{q} . The two results imply that for games with $\psi_q(\mathbf{q}) = \mathbf{q}$, the value of information is $V_0(\mathbf{q}) =$ $\hat{\gamma}_0^F(\mathbf{q}) - \hat{\gamma}_0^N(\mathbf{q}) = 0$. Similarly, if $\hat{\gamma}_0^F(\mathbf{q}) < 1/2$ for some transition vector \mathbf{q} , then its mirror necessarily has $\hat{\gamma}_0^F(\psi_q(\mathbf{q})) > 1/2$. It follows that for each case \mathbf{q} with a benefit of information we immediately obtain another case $\psi_q(\mathbf{q})$ in which there is a benefit of ignorance (of the same magnitude).

In general, we find that among the 64 deterministic games, exactly half of them is neutral. All these cases fall within four possible categories (see Supplementary Note 4, Proposition 3):

- (1) The transition vector has an absorbing state: $q_{ij}^1 = 1$ or $q_{ij}^2 = 0$ for all $i, j \in \{C, D\}$.
- (2) The transition vector is its own mirror: $\psi_q(\mathbf{q}) = \mathbf{q}$.
- (3) The transition vector is its own mirror-twin: $\chi_q \circ \psi_q(\mathbf{q}) = \mathbf{q}$.
- (4) The transition vector is state-independent: $q_{ij}^1 = q_{ij}^2$ for all $i, j \in \{C, D\}$.

As described in the main text, we can also define a simple proxy variable X that can be used to characterize whether information is beneficial, detrimental, or neutral in the limit of weak selection,

$$X = \left(\mathbb{1}_{q_{CC}^1 = 1} + \mathbb{1}_{q_{CC}^2 = 0}\right) - \left(\mathbb{1}_{q_{DD}^1 = 1} + \mathbb{1}_{q_{DD}^2 = 0}\right).$$
(13)

The rule is as follows: If one of the above four conditions (1) - (4) is satisfied, the game \mathbf{q} is neutral. Moreover, in most of these cases, we have X = 0 (the only exception occurs if the transition vector has an absorbing state, in which case X = -1 and X = 1 is also possible). Otherwise, if none of the conditions (1) - (4) are satisfied, there is a benefit of information if X > 0 and a benefit of ignorance if X < 0. We illustrate these relationships in Fig. 4**a**. In that figure, blue bars indicate a benefit of information, and red bars indicate a benefit of ignorance. Moreover, Fig. 4**b**,**c** shows how the respective case numbers change as we vary the benefit of cooperation b_1 in state 1, and as we vary the strength of selection β .

Supplementary Note 2: Single-stochastic games

As described in the main text, a game is called single-stochastic if all entries in \mathbf{q} but one are either zero or one (we refer to the remaining entry as q). It follows that there are $6 \cdot 2^5 = 192$ families of single-stochastic games. Out of those, we find that there are 24 transition vectors that have an absorbing state. By the same argument as before, information is neutral for those games, $V_{\beta}(\mathbf{q}) = 0$ for all β and q (see Proposition 3 in Supplementary Note 4). The other games can be analyzed numerically. Fig. S7– Fig. S10 show the respective results for no, weak, intermediate, and strong selection, respectively. In addition, Fig. S11 provides an overview of our numerical results across all $q \in [0, 1]$ and $\beta \in [10^{-3}, 10^1]$.

For the limit of no selection we find that the proxy variable X defined by (13) continues to make correct predictions in most cases. However, for 24 single-stochastic games we obtain X = 0 although the game does not exhibit neutral behavior for all $q \in [0, 1]$. Given the vector transformations (1)-(3), we only need to understand the behavior of 12 unique cases, where the stochastic transition q occurs only in state 1 of the game. We group these cases based on the transitions in state 2 (color names refer to Fig. S7),

- (1) $\mathbf{q}_3 = (q00; 010)$ [yellow] and $\mathbf{q}_{19} = (q10; 010)$ [yellow];
- (2) $\mathbf{q}_6 = (q00; 101)$ [blue] and $\mathbf{q}_{22} = (q10; 101)$ [blue];
- (3) $\mathbf{q}_8 = (q00; 111)$ [blue] and $\mathbf{q}_{24} = (q10; 111)$ [blue];
- (4) $\mathbf{q}_{10} = (q01; 001)$ [blue], $\mathbf{q}_{26} = (q11; 001)$ [blue], and $\mathbf{q}_{42} = (0q1; 001)$ [yellow];
- (5) $\mathbf{q}_{12} = (q01; 011)$ [yellow], $\mathbf{q}_{28} = (q11; 011)$ [red], and $\mathbf{q}_{44} = (0q1; 011)$ [blue].

These cases can be classified into three different qualitative classes. (*i*) The first class includes the five cases \mathbf{q}_6 , \mathbf{q}_8 , \mathbf{q}_{10} , \mathbf{q}_{22} , \mathbf{q}_{24} . In all these cases, the respective game is neutral for q = 0, shows a benefit of information for q = 1, and also shows a benefit of information for all intermediate q values. (*ii*) The second class includes the four cases \mathbf{q}_{26} , \mathbf{q}_{28} , \mathbf{q}_{42} , \mathbf{q}_{44} . In these cases, the stochastic game is neutral for both q = 0 and q = 1, but for intermediate q they either exhibit a benefit of information, a benefit of ignorance, or both. (*iii*) Finally, the last class includes the three cases \mathbf{q}_3 , \mathbf{q}_{12} , \mathbf{q}_{19} . In these cases, the stochastic game is neutral for q = 0, shows a benefit of information for q = 1, but shows a benefit of ignorance for some intermediate values of q.

Supplementary Note 3: A detailed analysis of some main text examples

After having provided some general information for games with deterministic or single-stochastic transitions, in the following we describe in more detail some of the specific examples that we have considered in the main text. In particular, we first discuss two the timeout game (Fig. 2**a**–**d**,Fig. 3**a**,**b**) and the timeout game with conditional return (Fig. 2**e**–**h**,Fig. 3**c**,**d**). In addition, we briefly discuss games with deterministic transitions that show a benefit of ignorance even for strong selection. Finally, we analyze the single-stochastic game highlighted in Fig. 5.

3.1 The timeout game

The first example we consider is the deterministic game with transition structure $\mathbf{q} = (1, 0, 0; 1, 1, 0)$. In the absence of selection, this game has a cooperation rate of 1/2 for both games with and without information. However, increasing selection results in higher cooperation rates in the full-information setting. In order to understand why cooperation rates are different in games with and without information, we explore the stability of the strategies that are most abundant in each case.

Stability of $\mathbf{p} = (1, 0, 0, 0; x, 0, 0, 1)$.

In the game with full information, for strong selection and parameter values as in Fig. 2, the strategy $\mathbf{p} = (1, 0, 0, 0; x, 0, 0, 1)$ is most successful (Fig. 2c), with $x \in \{0, 1\}$. This strategy can therefore be implemented in two ways: either as *Grim-WSLS* $\mathbf{p} = (1, 0, 0, 0; 1, 0, 0, 1)$, or as *Grim-Risker* $\mathbf{p} = (1, 0, 0, 0; 0, 0, 0, 1)$ analyzed in Ref. 1. To explore if this strategy is a subgame perfect Nash equilibrium, we employ the one-shot deviation principle². For this, we calculate continuation payoffs for each of the cases when players either use this strategy or deviate in one round and then return to using this strategy again. For this analysis, we assume that future payoffs may be discounted by a factor δ .

Let us first consider payoffs of non-deviating players. Given the nature of memory-1 strategies and state dependency of the game, we need to consider six possible cases, depending on the players' possible actions and environmental state.

1. CC in state 1. Then, the game transitions to state 1 and both players cooperate in all subsequent rounds, that is,

$$\pi_{CC,S1} = b_1 - c. \tag{14}$$

2. CD/DC in state 1. Then, the game transitions to state 2, where both players will defect. After that, the game returns to state 1 and both players still defect. Thereafter, the players return to state 2 in which they both cooperate. After this, the game returns to state 1, and players cooperate in all subsequent rounds. Therefore, the continuation payoff is given by

$$\pi_{CD,S1} = \delta^2 (1-\delta)(b_2 - c) + \delta^3 (b_1 - c).$$
(15)

3. DD in state 1. Here, the game transitions to state 2, where both players cooperate and, after returning to state 1, they cooperate in all subsequent rounds, that is,

$$\pi_{DD,S1} = (1 - \delta)(b_2 - c) + \delta(b_1 - c).$$
(16)

4. CC in state 2. This case is similar to CC in state 1 since the game transitions to state 1 and both players cooperate in all subsequent rounds, that is,

$$\pi_{CC,S2} = b_1 - c. \tag{17}$$

5. CD/DC in state 2. Then, the game transitions to state 1, where both players defect. After, the game returns to state 2 and both players cooperate, which recovers cooperation in all subsequent rounds as the game returns to state 1. Then, the payoff is given by

$$\pi_{CD,S2} = \delta(1-\delta)(b_2 - c) + \delta^2(b_1 - c).$$
(18)

6. DD in state 2. Here, the game transitions to state 1, where both players defect, and, via cooperating in state 2, as before, players recover mutual cooperation in all subsequent rounds, that is,

$$\pi_{DD,S2} = \delta(1-\delta)(b_2-c) + \delta^2(b_1-c).$$
(19)

Now, let us derive the payoffs of the deviating players.

 CC in state 1. In this case a deviating player defects for one round and then returns to the original strategy. After the mutant's defection, the game transitions to state 2 where both players defect. After the following mutual defection in state 1, players recover cooperation by cooperating in state 2 and cooperate in all subsequent rounds in state 1. That is,

$$\tilde{\pi}_{CC,S1} = (1-\delta)b_1 + \delta^3(1-\delta)(b_2 - c) + \delta^4(b_1 - c).$$
(20)

2. CD/DC in state 1. Here, the deviation requires the mutant to cooperate in state 2. After transitioning to state 1, both players defect, which again recovers mutual cooperation through transitioning to state 2. The payoff is then given by

$$\tilde{\pi}_{CD,S1} = -(1-\delta)c + \delta^2(1-\delta)(b_2 - c) + \delta^3(b_1 - c).$$
(21)

3. DD in state 1. As the game transitions to state 2, it requires that mutant defects while the second player cooperates. They still recover cooperation after mutual defection in state 1 as before. The

payoff of a deviating player is given by

$$\tilde{\pi}_{DD,S1} = (1-\delta)b_2 + \delta^2(1-\delta)(b_2 - c) + \delta^3(b_1 - c).$$
(22)

4. CC in state 2. This case is similar to CC in state 1 as yields the same payoff to the deviating player

$$\tilde{\pi}_{CC,S2} = (1-\delta)b_1 + \delta^3(1-\delta)(b_2 - c) + \delta^4(b_1 - c).$$
(23)

5. CD/DC in state 2. The deviating player will cooperate as game transitions to state 1 and the second player defects. This leads to the mutual defection in state 2 and a subsequent state 1. After this, both players cooperate in state 2 and all subsequent rounds in state 1. The payoff for this case is given by

$$\tilde{\pi}_{CD,S2} = -(1-\delta)c + \delta^3(1-\delta)(b_2 - c) + \delta^4(b_1 - c).$$
(24)

6. DD in state 2. Here, the game transitions to state 1, where the deviating player is required to cooperate. They again recover mutual cooperation as in the previous case after two rounds of mutual defection. The payoff is then given by

$$\tilde{\pi}_{DD,S2} = -(1-\delta)c + \delta^3(1-\delta)(b_2 - c) + \delta^4(b_1 - c).$$
(25)

In order for this strategy to be a subgame perfect Nash equilibrium, we need to check if it always yields higher payoffs than a one-shot deviation. For this, we require that

$$\pi_{CC,S1} \ge \tilde{\pi}_{CC,S1} \quad \text{and} \quad \pi_{CC,S2} \ge \tilde{\pi}_{CC,S2}$$

$$(26)$$

$$\pi_{CD,S1} \ge \tilde{\pi}_{CD,S1}$$
 and $\pi_{CD,S2} \ge \tilde{\pi}_{CD,S2}$ (27)

$$\pi_{DD,S1} \ge \tilde{\pi}_{DD,S1} \quad \text{and} \quad \pi_{DD,S2} \ge \tilde{\pi}_{DD,S2}$$

$$(28)$$

The respective inequalities for the payoffs after CD are always satisfied. The same is true for the inequality $\pi_{DD,S2} \ge \tilde{\pi}_{DD,S2}$. As payoffs in both states after mutual cooperation are identical, we only need to consider two cases.

1. We require $\pi_{CC} \geq \tilde{\pi}_{CC}$, which reduces to the following condition

$$(1+\delta+\delta^2)c \le \delta(1+\delta+\delta^2)b_1 - \delta^3 b_2.$$
⁽²⁹⁾

For $\delta \rightarrow 1$ this inequality simplifies to $3c \leq 3b_1 - b_2$, which is satisfied for the parameters in Fig. 2.

2. We require $\pi_{DD,S1} \geq \tilde{\pi}_{DD,S1}$, which can be written as

$$(1+\delta)c \le \delta(1+\delta)b_1 - \delta^2 b_2. \tag{30}$$

For $\delta \to 1$ this condition simplifies to $2c \le 2b_1 - b_2$, which is also satisfied for the set of parameters we chose.

Stability of *WSLS*, $\mathbf{p} = (1, 0, 0, 1; 1, 0, 0, 1)$.

Let us now apply a similar line of argument to WSLS. As can be seen in Fig. 2c, this strategy does neither evolve in the full-information nor in the no-information setting. First, we construct the payoffs for the non-deviating players.

1. CC in state 1. All players cooperate in all rounds, that is,

$$\pi_{CC,S1} = b_1 - c. \tag{31}$$

2. CD/DC in state 1. Then, players defect in state 2 and recover cooperation in all subsequent rounds in state 1, that is,

$$\pi_{CD,S1} = \delta(b_1 - c). \tag{32}$$

3. DD in state 1. The game transitions to state 2, where players cooperate, and then they cooperate in all subsequent rounds in state 1,

$$\pi_{DD,S1} = (1-\delta)(b_2 - c) + \delta(b_1 - c).$$
(33)

4. CC in state 2. Same as for CC in state 1,

$$\pi_{CC,S2} = b_1 - c. \tag{34}$$

5. CD/DC in state 2. Then, players defect in state 1 and recover cooperation in state 2 and cooperate in all subsequent rounds in state 1, that is,

$$\pi_{CD,S2} = \delta(1-\delta)(b_2-c) + \delta^2(b_1-c).$$
(35)

6. DD in state 2. The game transitions to state 1, where players cooperate in all subsequent rounds.

$$\pi_{DD,S2} = b_1 - c. \tag{36}$$

Payoffs of the deviating player then can be calculated in the following way.

1. CC in state 1. The deviating player defects in state 1 and after mutual defection in state 2, players recover mutual cooperation in state 1,

$$\tilde{\pi}_{CC,S1} = (1-\delta)b_1 + \delta^2(b_1 - c).$$
(37)

2. CD/DC in state 1. The deviating player cooperates in state 2, while the second player defects. This leads to mutual defection in state 1, followed by mutual cooperation in state 2 and subsequent cooperation in all rounds in state 1,

$$\tilde{\pi}_{CD,S1} = -(1-\delta)c + \delta^2(1-\delta)(b_2 - c) + \delta^3(b_1 - c).$$
(38)

3. DD in state 1. The deviating player defects in state 2, which leads to mutual defection in state 1 and recovery of mutual cooperation via state 2,

$$\tilde{\pi}_{DD,S1} = (1-\delta)b_2 + \delta^2(1-\delta)(b_2 - c) + \delta^3(b_1 - c).$$
(39)

4. CC in state 2. Same as for CC in state 1,

$$\tilde{\pi}_{CC,S2} = (1-\delta)b_1 + \delta^2(b_1 - c).$$
(40)

5. CD/DC in state 2. The deviating player cooperates in state 1, while the second player defects. This leads to mutual defection in state 2, followed by mutual cooperation in all rounds in state 1,

$$\tilde{\pi}_{CD,S2} = -(1-\delta)c + \delta^2(b_1 - c).$$
(41)

6. DD in state 2. The deviating player defects in state 1, which leads to mutual defection in state 2 and recovery of mutual cooperation,

$$\tilde{\pi}_{DD,S2} = (1-\delta)b_1 + \delta^2(b_1 - c).$$
(42)

As before, in each case we compare the payoffs for deviating and non-deviating players. We see that nondeviating players are better off whenever different actions were played in the previous round independent of the state. Conditions $\pi_{CC} \ge \tilde{\pi}_{CC}$, $\pi_{DD,S1} \ge \tilde{\pi}_{DD,S1}$ and $\pi_{DD,S2} \ge \tilde{\pi}_{DD,S2}$ simplify to

$$(1+\delta)c \le \delta b_1,\tag{43}$$

$$(1+\delta)c \le \delta(1+\delta)b_1 - \delta^2 b_2 \tag{44}$$

While the second condition can be rewritten as $2c \le 2b_1 - b_2$ for $\delta \to 1$ and is satisfied, the first condition challenges the stability of *WSLS*. As δ approaches 1, this condition can be written as $2c \le b_1$. For our set of parameters, this condition is not satisfied, which explains why we rarely observe players adopting this strategy.

In order to analyze strategies for the game without information, we can consider two cases. First, when players can deduct the current state based on the previous round outcome (assuming they can take into account payoffs they achieve and the actions) but cannot condition their strategy on the current state

as such and use this information only for computing the payoffs. This case will be similar in the analysis as the one-shot deviation principle considered for the game with full information. Since WSLS is not an equilibrium strategy, it explains why we mostly observe players using Grim and ALLD. Second, if we assume that players cannot take into account information about previous payoffs and, hence, cannot predict the current state, we analyse the ability of other strategies to invade a population of WSLS players by comparing payoffs mutants can achieve. Note that it is sufficient to consider only strategies like Grim, ALLD and Risker. The corresponding payoffs $\pi(x, y)$ of a player adopting strategy x against a player adopting strategy y are then given by

$$\pi(WSLS, WSLS) = b_1 - c + \mathcal{O}(\epsilon) \tag{45}$$

$$\pi(Grim, WSLS) = \frac{1}{35}(15b_1 + 6b_2 - 7c) + \mathcal{O}(\epsilon)$$
(46)

$$\pi(ALLD, WSLS) = \frac{2}{7}b_1 + \frac{3}{14}b_2 + \mathcal{O}(\epsilon)$$
(47)

$$\pi(Risker, WSLS) = \frac{2}{3}b_1 - \frac{1}{3}c + \mathcal{O}(\epsilon).$$
(48)

In particular, WSLS can only withstand an invasion by Risker if $2c < b_1$.

3.2 The timeout game with conditional return

This game (deterministic case 38) has the transition vector $\mathbf{q} = (1, 0, 0; 1, 1, 0)$. Similarly to the timeout game, in the absence of selection this game has a cooperation rate of 1/2 in both settings, with and without information. However, for larger selection strengths, these results can change. In particular for strong selection, we observe a considerable benefit of ignorance when b_1 is sufficiently large (Fig. 2h). To explain this observation, we again characterize the stability of the most abundant strategies.

For full information, there are three most abundant strategies for the parameter set we chose (Fig. 2g): (1, 0, 0, x; y, 0, 0, 1), (1, 0, 1, x; y, 0, 0, 1) and (1, 1, 1, x; y, 0, 1, 0), where $x, y \in \{0, 1\}$. All three strategies are self-cooperating and cooperate among each other. However, according to a one-shot deviation analysis, only the first strategy (1, 0, 0, x; y, 0, 0, 1) is subgame perfect. One key difference from the game with transition vector $\mathbf{q}_{39} = (1, 0, 0; 1, 1, 1)$ that makes *WSLS* a subgame perfect Nash equilibrium is that mutual defection in state 2 leads to the game remaining in the worse state 2. Then, deviations from *WSLS* necessarily yield lower payoff than the payoff of non-deviating players.

Without information, the payoffs that residents and mutants achieve for the most common non-selfcooperating strategies are given by

$$\pi(WSLS, WSLS) = b_1 - c + \mathcal{O}(\epsilon) \tag{49}$$

$$\pi(Grim, WSLS) = \frac{1}{5}(b_1 + 2b_2 - c) + \mathcal{O}(\epsilon)$$
(50)

$$\pi(ALLD, WSLS) = \frac{b_2}{2} + \mathcal{O}(\epsilon) \tag{51}$$

$$\pi(Risker, WSLS) = \frac{1}{3}(b_1 + b_2 - c) + \mathcal{O}(\epsilon)$$
(52)

A population of WSLS players is stable against any of these other strategies if $b_2 < 2(b_1 - c)$.

3.3 On other deterministic transitions with a benefit of ignorance

In Table S1 we summarize all games that show some benefit of ignorance for sufficiently large selection strength β and sufficiently large benefit b_1 in state 1.

Como numbor	Transition waston	Maximum value of the				
Game number	Transition vector	benefit of ignorance				
38	(100; 110)	0.0904				
44	(101;100)	0.0006				
46	(101; 110)	0.1288				
52	(110; 100)	0.0014				
53	(110; 101)	0.0188				
55	(110; 111)	0.0191				

Table S1: Games with some benefit of ignorance

The table suggests that games with a benefit of ignorance are rare. Moreover, for only two games this benefit is substantial, for game $q_{38} = (100; 110)$ considered in Fig. 2e-h and game $q_{46} = (101; 110)$. The transition function of game 46 differs from game 38 only in one entry (if both players defect in state 1, they remain in that state). As a general pattern, we observe that all transitions in Table S1 have the property that individuals move to state 1 if they both cooperated. Moreover, according to all of these games, some forms of defection in state 1 are punished with a transition to state 2. However, these two patterns cannot be used to fully characterize the games in Table S1; there are games that satisfy both properties without exhibiting a benefit of ignorance in the limit of strong selection (e.g. games 37 and 53 in Fig. S4). A consistent classification becomes even more complicated once we consider single-stochastic transition function (Fig. S7-Fig. S11).

3.4 A single-stochastic game with conditional return

Next, we consider the stochastic game considered in Fig. 5, with transition vector $\mathbf{q} = (1, 0, 0, q, 0, 0)$. According to this game, players find themselves in the less profitable state 2 if one or both players defected in the previous round. Otherwise, if both players cooperated, they remain in state 1 with certainty if they are already there, or they move towards state 1 with probability *q* if they start out in state 2.

Because the game transitions for sure to the second state after any defection, it follows that for strategies in the full-information setting the entries p_{CD}^1 , p_{DC}^1 , p_{DD}^1 are irrelevant (players never are to make any decision in state 1 after some player defected previously). This implies that all strategies of the form $\mathbf{p} = (p_{CC}^1, x, y, z; p_{CC}^2, p_{CD}^2, p_{DC}^2, p_{DD}^2)$ are behaviorally equivalent, for all $x, y, z \in [0, 1]$.

Hence, it suffices to consider a simplified 5-dimensional strategy space containing all strategies of the form $\mathbf{p} = (p_{CC}^1; p_{CC}^2, p_{CD}^2, p_{DC}^2, p_{DD}^2)$. These are the strategies that we depict in Fig. 5h.

The numerical simulations depicted in Fig. 5 show that there is a benefit of ignorance for a wide range of transition probabilities q and selection strengths β . We can further support these numerical results by considering the limits of weak and strong selection, respectively. In Supplementary Note 4, we derive the following two results for weak selection,

- No selection (Propositions 2 and 4): For β = 0, the no-information setting yields an average cooperation rate of γ^N(**q**) = 1/2. In contrast, full information yields an average cooperation rate of γ^F(**q**) = 1/2 ^{3q(1-q)}/_{64(1+q)}. In particular, there is a benefit of ignorance V₀(**q**) < 0 for all q∈(0,1).
- 2. Weak selection: The above result generalizes to positive but sufficiently small selection strengths. That is, for any given $q \in (0, 1)$ there is a threshold $\hat{\beta}_q$ such that for all $\beta < \hat{\beta}_q$ there is a benefit of ignorance $V_{\beta}(\mathbf{q}) < 0$.

To gain some intuition for dynamics under strong selection, we characterize the game's Nash equilibria among the pure memory-1 strategies, both for full information and no information. For no information, there are three Nash equilibria for the parameters used in Fig. 5. These equilibria are ALLD = (0, 0, 0, 0), Grim = (1, 0, 0, 0) and WSLS = (1, 0, 0, 1). Out of those, only WSLS is able to sustain cooperation in the presence of rare errors – that is, only for WSLS we have $\lim_{\varepsilon \to 0} \gamma(\mathbf{p}, \mathbf{p}) = 1$. For full information, we find six distinct equilibria that correspond to

$$\begin{aligned} ALLD &= (0; 0, 0, 0, 0), & Grim &= (1; 1, 0, 0, 0), \\ ALLD-Grim &= (0; 1, 0, 0, 0), & Grim-ALLD &= (1; 0, 0, 0, 0), \\ WSLS &= (1; 1, 0, 0, 1), & AWSLS &= (1; 0, 0, 0, 1). \end{aligned}$$

Out of those, only WSLS and AWSLS sustain cooperation in the presence of rare errors.

Supplementary Note 4: Proofs and mathematical derivations

In the following, we provide the proofs of our analytical statements. To remind the reader of the meaning of our mathematical notations, we provide a summary in Table S2.

Symbol	Description
р	memory-one strategy of a player
\mathbf{q}	environmental transition vector
$\mathcal{S}_N,~\mathcal{S}_F$	set of all memory-1 strategies in the no-information and full-information setting
$\mathcal{P}_N, \ \mathcal{P}_F$	set of all deterministic memory-1 strategies
$s_i \in \{s_1, s_2\}$	environmental state
b_1, b_2, c	parameters of the game (benefits in state 1 and 2 and cost)
eta	selection strength
ε	error rate
δ	continuation probability
$M(\mathbf{p} \mathbf{q})$	transition matrix when two players with strategy \mathbf{p} interact in game with vector \mathbf{q}
$\mathbf{v}(\mathbf{p} \mathbf{q})$	stationary distribution of $M(\mathbf{p} \mathbf{q})$
$\gamma(\mathbf{p} \mathbf{q})$	Resulting average cooperation rate, given strategies \mathbf{p} and transition vector \mathbf{q}
$\hat{\gamma}^N,\;\hat{\gamma}^F$	cooperation rates for populations with no information and with full information
$\pi(\mathbf{p},\mathbf{q})$	payoff of strategy \mathbf{p} given transition vector \mathbf{q}
ho	probability to switch to a different strategy
$V_eta({f q})$	value of information in a game \mathbf{q}
X	proxy-variable measuring the value of information
$\chi({f q})$	twin transformation of \mathbf{q} , $\chi_q(\mathbf{q}) = (1 - q_{CC}^2, 1 - q_{CD}^2, 1 - q_{DD}^2, 1 - q_{CC}^1, 1 - q_{CD}^1, 1 - q_{DD}^1)$
$\psi(\mathbf{q})$	mirror transformation of q , $\psi_q(\mathbf{q}) = (q_{DD}^1, q_{CD}^1, q_{CC}^1, q_{DD}^2, q_{CD}^2, q_{CC}^2)$

 Table S2: Table of notation

4.1 The effect of game transformations and strategy transformations

In the following, we derive the results summarized in Supplementary Note 1. We begin by introducing some additional notation. First, for a given memory-1 strategy \mathbf{p} we write

$$\mathbf{p} = (p_{CC}^1, p_{CD}^1, p_{DC}^1, p_{DD}^1, p_{CC}^2, p_{CD}^2, p_{DC}^2, p_{DD}^2) = (\mathbf{p}^1, \mathbf{p}^2).$$
(53)

Similarly, and slightly abusing our notation, we can write a 6-dimensional transition vector \mathbf{q} as an 8-dimensional vector

$$\mathbf{q} = (q_{CC}^1, q_{DD}^1, q_{DD}^1, q_{CC}^2, q_{CD}^2, q_{DD}^2, q_{DD}^2) = (\mathbf{q}^1, \mathbf{q}^2),$$
(54)

with the restriction that transitions are required to be symmetric, $q_{CD}^i = q_{DC}^i$ for $i \in \{1, 2\}$. Now consider two players with strategy **p** who interact in a stochastic game with transition vector **q**. Using the above

notation, we can write the transition matrix of the resulting Markov chain as

$$M(\mathbf{p}|\mathbf{q}) = \left(\begin{array}{c|c} ((\mathbf{q}^1)^{\mathsf{T}} \mathbf{1}) \otimes A(\mathbf{p}^1) & ((\mathbf{1} - \mathbf{q}^1)^{\mathsf{T}} \mathbf{1}) \otimes A(\mathbf{p}^2) \\ \hline ((\mathbf{q}^2)^{\mathsf{T}} \mathbf{1}) \otimes A(\mathbf{p}^1) & (((\mathbf{1} - \mathbf{q}^2)^{\mathsf{T}} \mathbf{1}) \otimes A(\mathbf{p}^2) \end{array} \right).$$
(55)

Here, $(q^i)^\intercal$ is the transpose of q^i , 1 is a vector of ones, \otimes is a Hadamard product (entry-wise multiplication), and

$$A(\mathbf{p}^{\mathbf{i}}) = \begin{pmatrix} p_{CC}^{i} p_{CC}^{i} & p_{CC}^{i} (1 - p_{CC}^{i}) & (1 - p_{CC}^{i}) p_{CC}^{i} & (1 - p_{CC}^{i}) (1 - p_{CC}^{i}) \\ p_{CD}^{i} p_{DC}^{i} & p_{CD}^{i} (1 - p_{DC}^{i}) & (1 - p_{CD}^{i}) p_{DC}^{i} & (1 - p_{CD}^{i}) (1 - p_{DC}^{i}) \\ p_{DC}^{i} p_{CD}^{i} p_{DC}^{i} & p_{DC}^{i} (1 - p_{CD}^{i}) & (1 - p_{DC}^{i}) p_{CD}^{i} & (1 - p_{DC}^{i}) (1 - p_{DC}^{i}) \\ p_{DD}^{i} p_{DD}^{i} p_{DD}^{i} & p_{DD}^{i} (1 - p_{DD}^{i}) & (1 - p_{DD}^{i}) p_{DD}^{i} & (1 - p_{DD}^{i}) (1 - p_{DD}^{i}) \end{pmatrix}$$
(56)

Using this notation, we can prove the two relationships formulated in Eq. (7).

Lemma 1. Consider an effective memory-one strategy $\mathbf{p} \in S_N$ and a transition vector \mathbf{q} such that the invariant distribution $\mathbf{v}(\mathbf{p}|\mathbf{q})$ is well-defined (i.e., the stationary distribution is unique). Then also $\mathbf{v}(\psi_p(\mathbf{p})|\psi_q(\mathbf{q}))$ and $\mathbf{v}(\psi_p(\mathbf{p})|\psi_q(\mathbf{q}))$ are well-defined. Moreover, the following relationships hold,

- (i) $\mathbf{v}(\mathbf{p}|\mathbf{q}) = \chi_v \left(\mathbf{v} \left(\chi_p(\mathbf{p}) \middle| \chi_q(\mathbf{q}) \right) \right),$
- (ii) $\mathbf{v}(\mathbf{p}|\mathbf{q}) = \psi_v \left(\mathbf{v} \left(\psi_p(\mathbf{p}) | \psi_q(\mathbf{q}) \right) \right).$

Proof. For the proof, we use permutation matrices.

(*i*) We note that if $M := M(\mathbf{p}|\mathbf{q})$ is the transition matrix with respect to the original vectors \mathbf{p} and \mathbf{q} , then the transition matrix $M_{\chi} := M(\chi_p(\mathbf{p})|\chi_q(\mathbf{q}))$ satisfies

$$E_{\chi}M_{\chi}E_{\chi} = M. \tag{57}$$

In this identity, E_{χ} is the permutation matrix

$$E_{\chi} = \left(\begin{array}{c|c} \mathbf{0} & \tilde{E}_{\chi} \\ \hline \tilde{E}_{\chi} & \mathbf{0} \end{array} \right), \tag{58}$$

where

$$\tilde{E}_{\chi} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$
(59)

Now, if $\mathbf{v} := \mathbf{v}(\mathbf{p}, \mathbf{q})$ is a stationary distribution of M, we have

$$\mathbf{v} = \mathbf{v}M = \mathbf{v}E_{\chi}M_{\chi}E_{\chi}.$$
 (60)

By multiplying E_{χ} from the right and noting that E_{χ}^2 is the identity matrix, we conclude

$$(\mathbf{v}E_{\chi}) = (\mathbf{v}E_{\chi})M_{\chi}.$$
(61)

It follows that if \mathbf{v} is the unique invariant distribution of M, then $\mathbf{v}E_{\chi}$ is the unique invariant distribution of M_{χ} . A straightforward calculation confirms that $\chi_v(\mathbf{v}E_{\chi}) = \mathbf{v}$.

(ii) The proof of the second identity is analogous; we only need to replace the permutation matrix by

$$E_{\psi} = \left(\begin{array}{c|c} \tilde{E}_{\psi} & \mathbf{0} \\ \hline \mathbf{0} & \tilde{E}_{\psi} \end{array}\right),\tag{62}$$

where

$$\tilde{E}_{\psi} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$
(63)

As an immediate consequence of the above lemma, we can also derive formulas for the respective average cooperation rates, as summarized in (9). The precise statement is as follows.

Corollary 1. Consider an effective memory-one strategy $\mathbf{p} \in S_N$ and a transition vector \mathbf{q} such that the resulting average cooperation rate $\gamma(\mathbf{p}|\mathbf{q})$ is well-defined. Then

$$\gamma(\mathbf{p}|\mathbf{q}) = \gamma\left(\chi_p(\mathbf{p})|\chi_q(\mathbf{q})\right) \quad and \quad \gamma(\mathbf{p}|\mathbf{q}) = 1 - \gamma\left(\psi_p(\mathbf{p})|\psi_q(\mathbf{q})\right). \tag{64}$$

Proof. In the main text, we have defined the respective average cooperation rate as for $\mathbf{v} := \mathbf{v}(\mathbf{p}, \mathbf{q})$ as

$$\gamma(\mathbf{p}|\mathbf{q}) = v_{CC}^1 + \frac{v_{CD}^1 + v_{DC}^1}{2} + v_{CC}^2 + \frac{v_{CD}^2 + v_{DC}^2}{2}.$$
(65)

By Lemma 1(i), we have

$$\mathbf{v}(\chi_p(\mathbf{p})|\chi_q(\mathbf{q})) = (v_{CC}^2, v_{CD}^2, v_{DC}^2, v_{DD}^2, v_{CC}^1, v_{CD}^1, v_{DC}^1, v_{DD}^1).$$
(66)

Therefore,

$$\gamma \left(\chi_p(\mathbf{p}) \middle| \chi_q(\mathbf{q}) \right) = v_{CC}^2 + \frac{v_{CD}^2 + v_{DC}^2}{2} + v_{CC}^1 + \frac{v_{CD}^1 + v_{DC}^1}{2} = \gamma(\mathbf{p}|\mathbf{q}).$$
(67)

Analogously, because of Lemma 1(ii), we have

$$\mathbf{v}(\psi_p(\mathbf{p})|\psi_q(\mathbf{q})) = (v_{DD}^1, v_{DC}^1, v_{CD}^1, v_{CC}^1, v_{DD}^2, v_{DC}^2, v_{CD}^2, v_{CC}^2).$$
(68)

It follows that

$$\gamma(\psi_{p}(\mathbf{p})|\psi_{q}(\mathbf{q})) = v_{DD}^{1} + \frac{v_{DC}^{1} + v_{CD}^{1}}{2} + v_{DD}^{2} + \frac{v_{DC}^{2} + v_{CD}^{2}}{2}$$

$$= 1 - \left(v_{CC}^{1} + \frac{v_{CD}^{1} + v_{DC}^{1}}{2} + v_{CC}^{2} + \frac{v_{CD}^{2} + v_{DC}^{2}}{2}\right) = 1 - \gamma(\mathbf{p}|\mathbf{q}).$$
(69)

The above results hold for any given strategy \mathbf{p} and transition vector \mathbf{q} (provided that the invariant distribution of the resulting game dynamics is unique). In the limit of rare mutations and weak selection, we can use these results to compute the average cooperation rate in evolving populations (see also Eq. (11)),

Proposition 1. Consider a stochastic game with transition vector $\mathbf{q} \neq (1, 1, 1, 0, 0, 0)$. Moreover, suppose players have full information and they can choose among all deterministic memory-one strategies. Let $\hat{\gamma}_0^F(\mathbf{q})$ denote the average cooperation rate across time, if the population evolves according to a process with rare mutations and no selection (for an arbitrary error rate $\varepsilon > 0$). Then

$$\hat{\gamma}_0^F(\mathbf{q}) = \hat{\gamma}_0^F\left(\chi_q(\mathbf{q})\right) \quad and \quad \hat{\gamma}_0^F(\mathbf{q}) = 1 - \hat{\gamma}_0^F\left(\psi_q(\mathbf{q})\right). \tag{70}$$

In particular, the relationships remain true in the limit of rare errors $\varepsilon \rightarrow 0$.

Proof. Because we consider the limit of rare mutations and no selection, all possible resident strategies $\mathbf{p} \in \mathcal{P}_F$ are equally likely to be played. Because of Corollary 1, and because the map $\chi_p : \mathcal{P}_F \to \mathcal{P}_F$ is a bijection, we obtain

$$\hat{\gamma}_{0}^{F}(\mathbf{q}) = \sum_{\mathbf{p}\in\mathcal{P}_{F}} \frac{\gamma(\mathbf{p}|\mathbf{q})}{|\mathcal{P}_{F}|} = \sum_{\mathbf{p}\in\mathcal{P}_{F}} \frac{\gamma(\chi_{p}(\mathbf{p})|\chi_{q}(\mathbf{q}))}{|\mathcal{P}_{F}|} = \sum_{\mathbf{p}\in\mathcal{P}_{F}} \frac{\gamma(\mathbf{p}|\chi_{q}(\mathbf{q}))}{|\mathcal{P}_{F}|} = \hat{\gamma}_{0}^{F}(\chi_{q}(\mathbf{q})).$$
(71)

Similarly, we obtain

$$\hat{\gamma}_{0}^{F}(\mathbf{q}) = \sum_{\mathbf{p}\in\mathcal{P}_{F}} \frac{\gamma(\mathbf{p}|\mathbf{q})}{|\mathcal{P}_{F}|} = \sum_{\mathbf{p}\in\mathcal{P}_{F}} \frac{1 - \gamma\left(\psi_{p}(\mathbf{p})\big|\psi_{q}(\mathbf{q})\right)}{|\mathcal{P}_{F}|} = \sum_{\mathbf{p}\in\mathcal{P}_{F}} \frac{1 - \gamma\left(\mathbf{p}\big|\psi_{q}(\mathbf{q})\right)}{|\mathcal{P}_{F}|} = 1 - \hat{\gamma}_{0}^{F}\left(\psi_{q}(\mathbf{q})\right).$$
(72)

In the no-information setup, the corresponding average cooperation rate takes an even simpler form, as the following result shows.

Proposition 2. Consider a transition vector $\mathbf{q} \neq (1, 1, 1, 0, 0, 0)$ and suppose players can choose among all deterministic memory-one no-information strategies. Let $\hat{\gamma}_0^N(\mathbf{q})$ denote the respective average cooperation rate under the evolutionary process with rare mutations and no selection (for an arbitrary error rate ε). Then $\hat{\gamma}_0^N(\mathbf{q}) = 1/2$.

Proof. Because players do not condition their behavior on the environmental state, it follows that their average cooperation rate does not depend on the game's transition vector \mathbf{q} . In particular, for $\mathbf{p} \in \mathcal{P}_N$ and $\mathbf{q} \neq (1, 1, 1, 0, 0, 0)$ we have

$$\gamma(\mathbf{p}|\psi_q(\mathbf{q})) = \gamma(\mathbf{p}|\mathbf{q}). \tag{73}$$

Because $\psi_p : \mathcal{P}_N \to \mathcal{P}_N$ is bijective and $\psi_p^2 = \text{Id}$, we can therefore use Corollary 1(*ii*) to conclude

$$\hat{\gamma}_{0}^{N}(\mathbf{q}) = \sum_{\mathbf{p}\in\mathcal{P}_{N}} \frac{\gamma(\mathbf{p}|\mathbf{q})}{|\mathcal{P}_{N}|} = \sum_{\mathbf{p}\in\mathcal{P}_{N}} \frac{\gamma(\mathbf{p}|\mathbf{q}) + \gamma(\mathbf{p}|\mathbf{q})}{2|\mathcal{P}_{N}|} = \sum_{\mathbf{p}\in\mathcal{P}_{N}} \frac{\gamma(\mathbf{p}|\mathbf{q}) + \gamma(\psi_{p}(\mathbf{p})|\mathbf{q})}{2|\mathcal{P}_{N}|}$$

$$= \sum_{\mathbf{p}\in\mathcal{P}_{N}} \frac{\gamma(\mathbf{p}|\mathbf{q}) + 1 - \gamma(\psi_{p}^{2}(\mathbf{p})|\psi_{q}(\mathbf{q}))}{2|\mathcal{P}_{N}|} = \sum_{\mathbf{p}\in\mathcal{P}_{N}} \frac{\gamma(\mathbf{p}|\mathbf{q}) + 1 - \gamma(\mathbf{p}|\psi_{q}(\mathbf{q}))}{2|\mathcal{P}_{N}|}$$
(74)
$$= \sum_{\mathbf{p}\in\mathcal{P}_{N}} \frac{\gamma(\mathbf{p}|\mathbf{q}) + 1 - \gamma(\mathbf{p}|\mathbf{q})}{2|\mathcal{P}_{N}|} = \frac{1}{2}.$$

By combining Propositions 1 and 2, we can describe for which games information is neutral.

Proposition 3. Consider a stochastic game with transition vector $\mathbf{q} \neq (1, 1, 1, 0, 0, 0)$ and suppose one of the following four conditions is satisfied.

- (1) The vector has an absorbing state: $q_{ij}^1 = 1$ or $q_{ij}^2 = 0$ for all $i, j \in \{C, D\}$.
- (2) The vector is its own mirror: $\psi_q(\mathbf{q}) = \mathbf{q}$.
- (3) The vector is its own mirror-twin: $\chi_q \circ \psi_q(\mathbf{q}) = \mathbf{q}$.
- (4) The vector is deterministic and state-independent, $q_{ij}^1 = q_{ij}^2 \in \{0, 1\}$ for all $i, j \in \{C, D\}$.

Then, in the limiting case of rare mutations and no selection, information is neutral, $V_0(\mathbf{q}) = 0$. Moreover, in case (1) and (4), the same result holds for positive selection strengths $V_\beta(\mathbf{q}) = 0$ for all $\beta \ge 0$.

Proof. (1) Suppose without loss of generality that the unique absorbing state is state 1. Then, for any given full-information strategy $\mathbf{p} = (p_{CC}^1, p_{DD}^1, p_{DC}^1, p_{DD}^1, p_{CC}^2, p_{CD}^2, p_{DD}^2, p_{DD}^2) \in \mathcal{P}_F$, we define

a corresponding no-information strategy $\tilde{\mathbf{p}} \in \mathcal{P}_N$ by

$$\tilde{\mathbf{p}} = (p_{CC}^1, p_{CD}^1, p_{DC}^1, p_{DD}^1, p_{CC}^1, p_{CD}^1, p_{DC}^1, p_{DD}^1, p_{DD}^1).$$
(75)

Because the first state is absorbing, eventually only the first half of the memory-1 strategy is relevant for a player's decision making. Therefore, $\gamma(\tilde{\mathbf{p}}|\mathbf{q}) = \gamma(\mathbf{p}|\mathbf{q})$ for all $\mathbf{p} \in \mathcal{P}_F$. Moreover, since each $\tilde{\mathbf{p}} \in \mathbf{p}$ has the same number of pre-images under this mapping, it follows that the average cooperation rate under full information coincides with the average cooperation rate under no information. We note that this argument does not require selection strength to be zero; it is merely based on the insight that for any full-information strategy there is a unique no-information strategy that gives rise to exactly the same behavior.

(2) If $\psi_q(\mathbf{q}) = \mathbf{q}$, it follows from Proposition 1 that

$$\hat{\gamma}_{0}^{F}(\mathbf{q}) = 1 - \hat{\gamma}_{0}^{F}(\psi_{q}(\mathbf{q})) = 1 - \hat{\gamma}_{0}^{F}(\mathbf{q}).$$
(76)

Therefore, $\hat{\gamma}_0^F(\mathbf{q}) = 1/2 = \hat{\gamma}_0^N(\mathbf{q}).$

(3) The case $\chi_q \circ \psi_q(\mathbf{q}) = \mathbf{q}$ follows analogously. Again by Proposition 1,

$$\hat{\gamma}_{0}^{F}(\mathbf{q}) = 1 - \hat{\gamma}_{0}^{F}(\psi_{q}(\mathbf{q})) = 1 - \hat{\gamma}_{0}^{F}(\chi_{q} \circ \psi_{q}(\mathbf{q})) = 1 - \hat{\gamma}_{0}^{F}(\mathbf{q}).$$
(77)

(4) The case of deterministic and state-independent transitions is similar to case (1). For each previous history (a, ã) ∈ {C, D}², let s_{a,ã} ∈ {s₁, s₂} denote the unique environmental state that is reached after that history. Then, for each p ∈ P_F we can define a behaviorally equivalent no-memory strategy p̃ ∈ P by

$$\tilde{\mathbf{p}} = (p_{CC}^{s_{CC}}, p_{CD}^{s_{DD}}, p_{DC}^{s_{DD}}, p_{DD}^{s_{CC}}, p_{CD}^{s_{CD}}, p_{DC}^{s_{DC}}, p_{DD}^{s_{DD}}).$$
(78)

Again, the statement follows because of $\gamma(\mathbf{\tilde{p}}|\mathbf{q}) = \gamma(\mathbf{p}|\mathbf{q})$ for all $\mathbf{p} \in \mathcal{P}_F$.

4.2 Analytical results for the transition vector $\mathbf{q} = (1, 0, 0; q, 0, 0)$

In the following, we describe in more detail the analytical results obtained for the single-stochastic game with transition vector $\mathbf{q} = (1, 0, 0; q, 0, 0)$. First, we again consider the case of weak selection, $\beta \rightarrow 0$. By Proposition 2, we know that without information, the game leads to a long-run average cooperation rate of $\hat{\gamma}_0^N(\mathbf{q}) = 1/2$. In the following, we discuss the case of full information. To this end, we note that for the given transition vector \mathbf{q} , the space of full-information memory-1 strategies is 5-dimensional, consisting of vectors $\mathbf{p} = (p_{CC}^1, p_{CC}^2, p_{CD}^2, p_{DC}^2, p_{DD}^2)$, see also Supplementary Note 3.

Proposition 4. When players have full information on the current state, the long-run cooperation rate of the population in the limit of rare errors is

$$\hat{\gamma}_0^F = 1/2 - \frac{3q(1-q)}{64(1+q)}.$$
(79)

In particular, $\hat{\gamma}_0^F \leq 1/2$, with equality if and only if q = 0 or q = 1. Moreover, the function $\hat{\gamma}_0^F$ is convex for $0 \leq q \leq 1$ and has a unique minimum at $q^* = \sqrt{2} - 1 \approx 0.41$.

Proof. The proof is by explicitly computing the cooperation rate $\gamma(\mathbf{p}|\mathbf{q})$ for all 32 possible resident strategies. As an example, when both players adopt the resident strategy $\mathbf{p} = (0, 1, 0, 0, 1)$ in the stochastic game with transition vector \mathbf{q} , the Markov chain according to Eq. (55) takes the following form,

The respective invariant distribution is

$$\mathbf{v}(\mathbf{p}|\mathbf{q}) = \left(0, 0, 0, \frac{q}{1+q}, \frac{1}{1+q}, 0, 0, 0\right).$$
(81)

It follows that the resulting average cooperation rate is

$$\gamma(\mathbf{p}|\mathbf{q}) = \frac{1}{1+q}.$$
(82)

By repeating the same computation for all other strategies $\mathbf{p} = (p_{CC}^1, p_{CC}^2, p_{CD}^2, p_{DC}^2, p_{DD}^2)$, we obtain

$$\begin{split} &\gamma\big((0,0,0,0,0)\big|\mathbf{q}\big) = 0, \quad \gamma\big((0,1,0,0,0)\big|\mathbf{q}\big) = 0, \qquad \gamma\big((1,0,0,0,0)\big|\mathbf{q}\big) = 0, \qquad \gamma\big((1,1,0,0,0)\big|\mathbf{q}\big) = 0, \\ &\gamma\big((0,0,0,0,1)\big|\mathbf{q}\big) = \frac{1}{2}, \quad \gamma\big((0,1,0,0,1)\big|\mathbf{q}\big) = \frac{1}{1+q}, \qquad \gamma\big((1,0,0,0,1)\big|\mathbf{q}\big) = 1, \qquad \gamma\big((1,1,0,0,1)\big|\mathbf{q}\big) = 1, \\ &\gamma\big((0,0,0,1,0)\big|\mathbf{q}\big) = \frac{1}{4}, \quad \gamma\big((0,1,0,1,0)\big|\mathbf{q}\big) = \frac{1}{4}, \qquad \gamma\big((1,0,0,1,0)\big|\mathbf{q}\big) = \frac{1+q}{4}, \quad \gamma\big((1,1,0,1,0)\big|\mathbf{q}\big) = \frac{1}{2}, \\ &\gamma\big((0,0,1,0,0)\big|\mathbf{q}\big) = \frac{1}{2}, \quad \gamma\big((0,1,0,1,1)\big|\mathbf{q}\big) = \frac{3+q}{4+4q}, \quad \gamma\big((1,0,0,1,1)\big|\mathbf{q}\big) = \frac{3}{4}, \qquad \gamma\big((1,1,0,1,1)\big|\mathbf{q}\big) = \frac{3}{4}, \\ &\gamma\big((0,0,1,0,0)\big|\mathbf{q}\big) = \frac{1}{4}, \quad \gamma\big((0,1,1,0,0)\big|\mathbf{q}\big) = \frac{1}{4}, \qquad \gamma\big((1,0,1,0,0)\big|\mathbf{q}\big) = \frac{1+q}{4}, \quad \gamma\big((1,1,1,0,0)\big|\mathbf{q}\big) = \frac{1}{2}, \\ &\gamma\big((0,0,1,0,1)\big|\mathbf{q}\big) = \frac{1}{2}, \quad \gamma\big((0,1,1,0,1)\big|\mathbf{q}\big) = \frac{3+q}{4+4q}, \quad \gamma\big((1,0,1,0,1)\big|\mathbf{q}\big) = \frac{3}{4}, \qquad \gamma\big((1,1,1,0,0)\big|\mathbf{q}\big) = \frac{1}{2}, \\ &\gamma\big((0,0,1,1,0)\big|\mathbf{q}\big) = 0, \quad \gamma\big((0,1,1,1,0)\big|\mathbf{q}\big) = \frac{3+q}{4+4q}, \quad \gamma\big((1,0,1,1,0,1)\big|\mathbf{q}\big) = \frac{3}{4}, \qquad \gamma\big((1,1,1,1,0,1)\big|\mathbf{q}\big) = \frac{3}{4}, \\ &\gamma\big((0,0,1,1,1,0)\big|\mathbf{q}\big) = 0, \quad \gamma\big((0,1,1,1,1,0)\big|\mathbf{q}\big) = 0, \qquad \gamma\big((1,0,1,1,1,0)\big|\mathbf{q}\big) = 1, \qquad \gamma\big((1,1,1,1,1,0)\big|\mathbf{q}\big) = 1, \\ &\gamma\big((0,0,1,1,1,1,0\big|\mathbf{q}\big) = \frac{1}{2}, \quad \gamma\big((0,1,1,1,1,1,0\big|\mathbf{q}\big) = \frac{1}{1+q}, \qquad \gamma\big((1,0,1,1,1,1,0\big|\mathbf{q}\big) = 1, \qquad \gamma\big((1,1,1,1,1,1,0\big|\mathbf{q}\big) = 1, \\ &\gamma\big((1,0,1,1,1,1,0\big|\mathbf{q}\big) = \frac{1}{2}, \qquad \gamma\big((1,1,1,1,1,1,0\big|\mathbf{q}\big) = \frac{1}{1+q}, \qquad \gamma\big((1,0,1,1,1,1,0\big|\mathbf{q}\big) = 1, \qquad \gamma\big((1,1,1,1,1,1,0\big|\mathbf{q}\big) = 1, \\ &\gamma\big((1,0,1,1,1,1,0\big|\mathbf{q}\big) = \frac{1}{2}, \qquad \gamma\big((1,1,1,1,1,1,0\big|\mathbf{q}\big) = \frac{1}{1+q}, \qquad \gamma\big((1,0,1,1,1,1,0\big|\mathbf{q}\big) = 1, \qquad \gamma\big((1,1,1,1,1,1,0\big|\mathbf{q}\big) = 1, \end{aligned}$$

To obtain the population's long-run average cooperation rate in the limit of rare mutations and vanishing selection, we compute the average of these 32 cooperation probabilities. This yields formula (79). \Box

Based on our previous results, we can also draw some conclusions for positive selection strengths. First, we note that for q=0, the transition vector takes the form $\mathbf{q} = (1, 0, 0; 0, 0, 0)$. This transition vector has an absorbing state (state 2). Therefore, it follows from the first case in Proposition 3 that $V_{\beta}(\mathbf{q}) = 0$ for all selection strengths $\beta \ge 0$.

Second, we note that a similar conclusion holds for q = 1. In that case, the transition vector takes the form $\mathbf{q} = (1, 0, 0; 1, 0, 0)$, and hence it is deterministic and state-independent. By the fourth case in Proposition 3 it again follows that $V_{\beta}(\mathbf{q}) = 0$ for all $\beta \ge 0$.

Finally, for a fixed $q \in (0, 1)$, Proposition 4 implies that $V_0(\mathbf{q}) < 0$. Because the map $\beta \mapsto V_\beta(\mathbf{q})$ is continuous in β , it follows that we can find some $\beta_q^* > 0$ such that for all $\beta < \beta_q^*$ we have $V_\beta(\mathbf{q}) < 0$. That is, for any q strictly between 0 and 1, we do not only observe a benefit of ignorance for no selection; there is also a benefit of ignorance for weak but positive selection (see also Fig. 5c). However, as our numerical computations suggest, this benefit of ignorance may turn turn into a benefit of information for sufficiently large q and appropriate selection strengths β (Fig. 5d-f).

Supplementary References

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Supplementary Figures



Figure S1: Robustness of our results with respect to parameter changes. To explore how different parameters affect our results, we revisit the two games introduced in Fig. 2. For both the timeout game and the timeout game with conditional return, we plot the evolving cooperation rates when there is no information (red) and full information (blue). a,b, While in the main text we have focussed on deterministic strategies, here we present simulations when players can choose among stochastic memory-1 strategies. To this end, we assume that mutating players randomly choose a new strategy whose entries are taken from an arcsine distribution, as in earlier work³. We find that in the game with timeout, information always provides advantage to cooperation. In the game with conditional return, ignorance is beneficial for sufficiently high values of b_1 . c-i, We then independently vary the error rate ε , the discount factor δ , and the mutation rate μ . For values of parameters sufficiently close to the values in Fig. 2, we see qualitatively similar behavior. As expected, the benefit of ignorance is not robust and only observed for specific values of the parameters (d,f,i). Solid lines indicate exact numerical results in the limit of rare mutations, whereas dashed lines represent simulations. If not specified otherwise, parameter values used for these plots are the same as in Fig. 2.



Figure S2: Robustness of our results with respect to the strategy choice. a,b, As a baseline to our results shown in Fig. 2, we explore how much cooperation evolves when players engage in a conventional repeated game (without state transitions). To this end we simulate the process with deterministic memory-1 strategies. Independent of whether we use the game in state 1 or state 2, we observe very little cooperation. We conclude that for the given parameter values, game transitions are crucial to establish cooperation. **c,d,** As another baseline, we explore whether cooperation evolves if individuals can only condition their behavior on the present state (but not on the players' actions in the last round). As one may expect, cooperation does not evolve. Parameters are the same as in Fig. 2.



Figure S3: Value of information under an alternative learning dynamics. Our previous evolutionary results are based on an imitation dynamics, using a pairwise comparison process⁴. Here, we compare these results to a different learning process, introspection dynamics⁵. Instead of considering a population of learners, this process only involves the two two players engaged in the respective game. At regular time intervals, one player is given the chance to update its strategy. To this end, the player compares its present payoff π with the hypothetical payoff $\tilde{\pi}$ that the player could have obtained by using a randomly determined alternative strategy. The player then switches to the alternative strategy with probability $(1 + \exp[-\beta(\tilde{\pi} - \pi)])^{-1}$. Here, $\beta \ge 0$ is again a parameter referred to as selection strength. For this figure, we have simulated this elementary updating process for 10^7 time steps, for the two games depicted in Fig. 2. To this end, again we assume that players can choose among all deterministic memory-1 strategies either with full information or with no information. In general, introspection dynamics is quite different from pairwise comparison⁵ (in particular, introspection dynamics requires larger values of β to approximate strong selection). As a consequence, the two processes lead to different absolute cooperation rates. However, as shown here, the two processes yield similar results on a qualitative level (compared to Fig. 2d,h). a, In the timeout game, we observe a benefit of information for all considered parameter values. b, In the game with conditional return, large benefits b_1 and large selection strengths β lead to a benefit of ignorance. Unless noted otherwise, we use the same parameters as in Fig. 2.



Figure S4: An analysis of all deterministic transition structures for different selection strengths. We plot cooperation rates for the games with full information (blue line) and no information (red line) as a function of the selection strength β . For each of the $2^6 = 64$ deterministic transition structures, we use colors to indicate the qualitative behavior. Cases with a consistent benefit of information are colored blue, whereas cases with a consistent benefit of ignorance are colored red. Yellow panels indicate that we observe both a benefit of information and of ignorance for different values of β . Finally, in the white panels, information is neutral. Unless noted otherwise, parameters are the same as in Fig. 2.



Figure S5: An analysis of all deterministic transition structures for different benefits in game 1. Again, we plot cooperation rates for games with full information (blue line) and no information (red line), but this time as a function of b_1 . For the panels, we use the same color code as before (blue – benefit of information, red – benefit of ignorance, yellow – inconsistent, white – neutral). Unless noted otherwise, parameters are the same as in Fig. 2.



Figure S6: An analysis of all deterministic transition structures for different error rates. As before, we plot cooperation rates for games with full information (blue line) and no information (red line), now as a function of the error rate ε . The panels use the same color code as before (blue – benefit of information, red – benefit of ignorance, yellow – inconsistent, white – neutral). Unless noted otherwise, parameters are the same as in Fig. 2.

0	q00000	q10000	0q0000	1q0000	00q000	10q000	000q00	100q00	0000q0	1000q0	00000q	10000q
~		17	33	49	65	81	97	113	129	145	161	177
0	a00001	a10001	0a0001	1a0001	00a001	10a001	000a01	100a01	0000a1	1000a1	00001a	10001a
0	.6		-									
ິ 0	.4 2	. 18	34	50	66	82	98	114	130	146	162	178
0	q00010	q10010	0q0010	1q0010	00q010	10q010	000q10	100q10	0001q0	1001q0	00010q	10010q
~		19	35	51	67	83	99	115	131	147	163	179
0	.4 a00011	a10011	0a0011	1a0011	00g011	10g011	000a11	100a11	0001a1	1001a1	00011a	10011a
0	.6								Ł		-	
0	.4 4	20	36	52	68	84	. 100	116	132	148	164	180
0	q00100	q10100	0q0100	1q0100	00q100	10q100	001q00	101q00	0010q0	1010q0	00100q	10100q
~		21	37	53	69	85	101	117	133	149	165	181
0	.4 a00101	a10101	0q0101	1a0101	00g101	10g101	001a01	101a01	0010a1	1010g1	00101g	10101a
0	.6		Ł						-		-	-
0	.4 6	. 22	_ 38	54	. 70	. 86	102	118	134	150	166	182
0	q00110	q10110	0q0110	1q0110	00q110	10q110	001q10	101q10	0011q0	1011q0	00110q	10110q
~	4 7	23	39	55	71	87	103	119	135	151	167	183
0	q00111	q10111	0q0111	1q0111	00q111	10q111	001q11	101q11	0011q1	1011q1	00111q	10111q
0	.6	·						-	-	·		
0	.4 8	24	40	56	72	88	104	120	136	152	168	184
0	q01000	q11000 F	0q1000 F	1q1000 F	01q000 F	11q000 F	010q00 -	110q00 F	0100q0 F	1100q0 F	01000q	11000q
~ ^	A 9	25	41	. 57	73	. 89	105	. 121	137	153	169	. 185
0	q01001	q11001	0q1001	1q1001	01q001	11q001	010q01	110q01	0100q1	1100q1	01001q	11001q
0 ح	.6		- 10					100	100			
0	.4 10	26	42	58	<u>14</u>	90	106	122	138	154	1/0	180
0	.6	411010				11q010	010010				01010q	
<u>ح</u> 0	4 11	27	43	59	75	. 91	107	123	139	155	171	187
-	q01011	q11011	0q1011	1q1011	01q011	11q011	010q11	110q11	0101q1	1101q1	01011q	11011q
0 ح	.0	20		60	76	02	109	124	140	156	172	100
0	.4	a11100	0g1100	101100	01a100	11a100	011000	111000	011000	111000	01100g	11100g
0	.6	411100								ł		ľ
ິ 0	.4 13	29	45	61	77	93	109	125	141	157	173	189
0	q01101	q11101	0q1101	1q1101	01q101	11q101	011q01	111q01	0110q1	1110q1	01101q	11101q
5		30	46	62	78	94	110	126	142	158	174	190
0	.4 a01110	a11110	0a1110	1a1110	01a110	11a110	011a10	111a10	0111a0	1111a0	01110g	11110a
_ 0	.6		-	-						Ŀ	-	
0	.4 15	_31	47	63	79	95	111	. 127	143	159	175	191
0	q01111 .6	q11111	0q1111	1q1111 ⊩	01q111	11q111	011q11	111q11 •	0111q1	1111q1 ⊩	01111q	11111q
~ ^	1 16	32	48	64	80	96	112	128	144	160	176	192
0	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1
	q	q	q	q	q	q	q	q	q	q	q	q

Figure S7: An analysis of all single-stochastic games for vanishing selection. As the most simple instantiation of games with an element of chance, we consider all $6 \cdot 2^5 = 192$ families of single-stochastic games. Here, exactly one transition is probabilistic (and we use q as the respective transition probability). For each family, we reproduce the cooperation rates for full information (blue curve) and no information (red curve). No information always yields 50% cooperation. Full information can yield the same cooperation rate (white), more cooperation (blue) or less cooperation (red). We use the same baseline parameters as in Fig. 2, but with a selection strength $\beta = 0$.

0	000000	q10000	0q0000	1q0000	00q000	10q000	000q00	100q00	0000q0	1000q0	00000q	10000q
~ 0	1	17	33	49	65	81	97	113	129	145	161	177
0	q00001	q10001	0q0001	1q0001	00q001	10q001	000q01	100q01	0000q1	1000q1	00001q	10001q
0 ~	.6								100			
0	.4 2	18	34	50	66	82	98	114	130	146	162	178
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ົ໐	.4 3	19	35	51	67	83	99	115	131	147	163	179
0	q00011	q10011	0q0011	1q0011	00q011	10q011	000q11	100q11	0001q1	1001q1	00011q	10011q
ں ح	.0	20	36	52	68	84	100	116	132	148	164	180
0	a00100	a10100	0q0100	1q0100	00g100	10g100	001a00	101a00	0010a0	1010a0	00100g	10100a
_ 0	.6	-	-		-		-	-			-	
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0	q00101	q10101	0q0101 F	1q0101	00q101	10q101	001q01	101q01	0010q1	1010q1 •	00101q	10101q
ر م	4 6	. 22	. 38	. 54	.70	. 86	102	118	134	150	166	182
	q00110	q10110	0q0110	1q0110	00q110	10q110	001q10	101q10	0011q0	1011q0	00110q	10110q
0 ~	.6	22	20	55	71	07	102	110	125	151	167	102
0	a00111	a10111	0q0111	1g0111	00g111	10g111	001a11	101011	0011a1	1011a1	00111g	101110
0	.6											
ິ 0	.4 8	24	40	56	72	88	104	120	136	152	168	184
0	q01000	q11000	0q1000 I	1q1000	01q000	11q000	010q00	110q00	0100q0	1100q0	01000q	11000q
~ 0		25	41	. 57	73	. 89	105	121	137	153	169	185
0	q01001	q11001	0q1001	1q1001	01q001	11q001	010q01	110q01	0100q1	1100q1	01001q	11001q
0 ~	0.6	26	40				100	400	420	454	170	400
0	0.4	a11010	0q1010	1g1010	010010	110010	010010	110010	010100	1101c0	01010g	110100
0	0.6							- India				
ິ 0	.4 11	27	43	. 59	75	91	107	123	139	155	171	187
0	q01011	q11011	0q1011	1q1011	01q011	11q011	010q11	110q11	0101q1	1101q1	01011q	11011q
~	4 12	28	44	. 60	76	.92	108	124	140	156	172	188
	q01100	q11100	0q1100	1q1100	01q100	11q100	011q00	111q00	0110q0	1110q0	01100q	11100q
0 ~	12	20	45	64	77	02	100	125	-	157	172	190
0	0.4 1	a11101	45 0g1101	1g1101	01a101	11g101	011001	111001	011001	1110g1	01101a	111010
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ິ 0	.4 14	_30	46	62	78	94	110	126	142	158	174	190
0	q01110	q11110 •	0q1110 •	1q1110	01q110 •	11q110	011q10	111q10 •	0111q0	1111q0 F	01110q	11110q
~ 0	4 15	31	47	63	79	.95	111	127	143	159	175	191
-	q01111	q11111	0q1111	1q1111	01q111	11q111	011q11	111q11	0111q1	1111q1	01111q	11111q
0 ~	16	22	19	64	20	06	412	120	144	160	176	102
0	0.4					0 1						
	q	q	q	q	q	q	q	q	q	q	q	q

Figure S8: An analysis of all single-stochastic games for weak selection. Same as Fig. S7, but for $\beta = 0.001$ instead of $\beta = 0$.

1	q00000	q10000	0q0000	1q0000	00q000	10q000	000q00	100q00	0000q0	1000q0	p00000	10000q
~	1	17	33	49	65	81	97	113	129	145	161	172
1	q00001	q10001	0q0001	1q0001	00q001	10q001	000q01	100q01	0000q1	1000q1	00001q	10001q
~	2	18	34	50	66	82	98	114	130	146	162	178
1	q00010	q10010	0q0010	1q0010	00q010	10q010	000q10	100q10	0001q0	1001q0	00010q	10010q
~	3	19	35	51_	67	83	99	115	131	147	63	179
1	q00011	q10011	0q0011	1q0011	00q011	10q011	000q11	100q11	0001q1	1001q1	00011q	10011q
~ (4	20	36	52	68	84	100	116	132	148	164	180
1	q00100	q10100	0q0100	1q0100	00q100	10q100	001q00	101q00	0010q0	1010q0	00100q	10100q
~	5	21	37	53	69	85	101	117	133	149	165	181
1	q00101	q10101	0q0101	1q0101	00q101	10q101	001q01	101q01	0010q1	1010q1	00101q	10101q
Y	6	22	38	54	70	86	102	118	134	150	166	182
C 1) q00110	q10110	0q0110	1q0110	00q110	10q110	001q10	101q10	0011q0	1011q0	00110q	10110q
Y	7	23	39	55	71	87	103	119	135	151	67	183
C 1	q00111	q10111	0q0111	1q0111	00q111	10q111	001q11	101q11	0011q1	1011q1	00111q	10111q
7	8 /	24	40	56	72	88	104	120	136	152	168	184
0	q01000	q11000	0q1000	1q1000	01q000	11q000	010q00	110q00	0100q0	1100q0	01000q	11000q
7	' 9	25	41	57	73	89	105	121-	137	153	169	185
0	q01001	q11001	0q1001	1q1001	01q001	11q001	010q01	110q01	0100q1	1100q1	01001q	11001q
۔ ح	- -		40	50	74 0		100		100		170	
C	01010	26	42	101010	010010	110010	010010	110010	010100	110100	010100	110100
1												
~ (11	27	43	59	75	91	107	123	139	155	171	187
1	q01011	q11011	0q1011	1q1011	01q011	11q011	010q11	110q11	0101q1	1101q1	01011q	11011q
~	12	28	44	60	76 🖊	92_/	108	124	140	156	172	188
1	q01100	q11100	0q1100	1q1100	01q100	11q100	011q00	111q00	0110q0	1110q0	01100q	11100q
×	13	29	45	61	77	93 1	109	125	141	157	173	189
C 1	q01101	q11101	0q1101	1q1101	01q101	11q101	011q01	111q01	0110q1	1110q1	01101q	11101q
~	14	30	46	62	78	94	110	126	142	158	174	190
1	q01110	q11110	0q1110	1q1110	01q110	11q110	011q10	111q10	0111q0	1111q0	01110q	11110q
~	15	31	47	63	70	95	111	127	143	159	175	101
C	, q01111	q11111	0q1111	1q1111	01q111	11q111	011q11	111q11	0111q1	1111q1	01111q	11111q
1 ~	16	32	48	64	80	96	112	128	144	160	176	192
C	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1
	q	q	q	q	q	q	q	q	q	q	q	q

Figure S9: An analysis of all single-stochastic games for intermediate selection. Same as previous two figures, but using $\beta = 1$.

1 q00000	q10000	0q0000	1q0000	00q000	10q000	000q00	100q00	0000q0	1000q0	00000q	10000q
≻ 1	17	33	49	65	81	97	113	129	145	161	177
0 1 q00001	q10001	0q0001	1q0001	00q001	10q001	000q01	100q01	0000q1	1000q1	00001q	10001q
≻ 2	18	34	50	66	82	98	114	130	146	162	178
0 1 q00010	q10010	0q0010	1q0010	00q010	10q010	000q10	100q10	0001q0	1001q0	00010q	10010q
~ 3	19	35	51	67	83	99	115	131	147	163	179
0 1 q00011	q10011	0q0011	1q0011	00q011	10q011	000q11	100q11	0001q1	1001q1	00011q	10011q
≻ 4	20	36	52	68	84	100	116	132	148	164	180
0 1 q00100	q10100	0q0100	1q0100	00q100	10q100	001q00	101q00	0010q0	1010q0	00100q	10100q
≻ 5	21	37	53	69	85	101	117	133	149	165	181
0 q00101	q10101	0q0101	1q0101	00q101	10q101	001q01	101q01	0010q1	1010q1	00101q	10101q
~ 6	22	38	54	70	86	102	118	134	150	166	182
0 q00110	q10110	0q0110	1q0110	00q110	10q110	001q10	101q10	0011q0	1011q0	00110q	10110q
~ 7	23	39	55	71	87	103	119	135	151	167	183
0 q00111	q10111	0q0111	1q0111	00q111	10q111	001q11	101q11	0011q1	1011q1	00111q	10111q
≻ 8	24	40	56	72	88	104	120	136	152	168	184
0	44000		4 4 9 9 9	01 000		010.00	110-00	0100-0	1100~0	01000	11000~
1 q01000	q11000	0q1000	1q1000 I	01q000	11q000 I	01000	TUQUU	UTUUQU	l		TTUUUq
<pre>> 1 q01000 > 9</pre>	q11000 25	0q1000 41	1q1000 57	01q000 73	11q000 89	105	121	137	153	169	185
<pre>1 q01000 > 9 0 q01001 1 q01001</pre>	q11000 25 q11001	0q1000 41 0q1001	57	01q000 73 01q001	11q000 89 11q001	105	110q00 121	0100q0 137 0100q1	1100q0 153	01000q 01001q	185
<pre>1 q01000 > 0 q01001 > 1 q01001 > 1 q01001 > 1 q01001</pre>	q11000 25 q11001 26	0q1000 41 0q1001 42	1q1000 57 1q1001 58	01q000 73 01q001 74	11q000 89 11q001	105 010q01	110q00 121 110q01 122	0100q0 137 0100q1 138	153 1100q1 154	01000q \169 01001q \170	185 11001q 186
<pre></pre>	q11000 25 q11001 26 q11010	0q1000 41 0q1001 42 0q1010	1q1000 57 1q1001 58 1q1010	01q000 73 01q001 74	89 11q001 90 11q010	10000 105 010q01 106 010q10	121 110q01 122 110q10	0100q0 137 0100q1 138 0101q0	153 1100q1 154 1101q0	01000q 01001q 01001q 01010q	185 11001q 186 11010q
<pre>1 q01000</pre>	q11000 25 q11001 26 q11010 27	0q1000 41 0q1001 42 0q1010 43	1q1000 57 1q1001 58 1q1010 59	01q000 73 01q001 74	11q000 89 11q001 90 11q010 91	10000 105 010q01 106 010q10 107	110q00 121 110q01 122 110q10 123	0100q0 137 0100q1 138 0101q0 139	153 1100q1 154 1101q0 155	01000q 01001q 01001q 01010q 01010q 1111	185 11001q 186 11010q 187
1 9 0 q01001 1 10 0 q01010 1 11 10 11 10 10 10 10 10	q11000 25 q11001 26 q11010 27 q11011	0q1000 41 0q1001 42 0q1010 43 0q1011	1q1000 57 1q1001 58 1q1010 59 1q1011	01q000 73 01q001 74 01q010 75	11q000 89 11q001 90 11q010 91 11q011	10000 105 010q01 106 010q10 107 010q11	110q00 121 110q01 122 110q10 123 110q11	0100q0 137 0100q1 138 0101q0 139 0101q1	153 1100q1 154 1101q0 155 1101q1	01000q 01001q 01001q 01010q 1111 01011q	185 11001q 186 11010q 187 11011q
$\begin{array}{c} & & 1 \\ & &$	q11000 25 q11001 26 q11010 27 q11011 28	0q1000 41 0q1001 42 0q1010 43 0q1011	1q1000 57 1q1001 58 1q1010 59 1q1011 60	01q000 73 01q001 74 01q010 75 01q011 76	11q000 89 11q001 90 11q010 91 11q011 92	10000 10000 10001 106 010010 107 010011 108	110q00 121 110q01 122 110q10 123 110q11 124	0100q0 137 0100q1 138 0101q0 139 0101q1 140	153 1100q1 154 1101q0 155 1101q1 156	01000q 01001q 01001q 01010q 01010q 1171 01011q 172	185 11001q 186 11010q 187 11011q 188
<pre>1 q01000</pre>	q11000 25 q11001 26 q11010 27 q11011 28 q11100	0q1000 41 0q1001 42 0q1010 43 0q1010 44 0q1100	1q1000 57 1q1001 58 1q1010 59 1q1011 60 1q1100	01q000 73 01q001 74 01q010 75 01q011 76 01q100	11q000 89 11q001 90 11q010 91 11q011 92 11q100	10000 105 010q01 106 010q10 107 010q11 108 011q00	110q00 121 110q01 122 110q10 123 110q11 124 111q00	1137 0100q1 138 0101q0 139 0101q1 140 0110q0	153 1100q1 154 1101q0 155 1101q1 156 1110q0	01000q 169 01001q 170 01010q 171 01011q 172 01100q	185 11001q 186 11010q 187 11011q 188 11100q
1 9 0 q01001 1 1 1 1 0 q01001 1 1 1 1 1 1 1 1 1 1 1 1 1	q11000 25 q11001 26 q11010 27 q11011 28 q11100 29	0q1000 41 0q1001 42 0q1010 43 0q1010 44 0q1100 45	1q1000 57 1q1001 58 1q1010 59 1q1010 1q1100 1q1100	01q000 73 01q001 74 01q010 75 01q011 76 01q100 77	11q000 89 11q001 90 11q010 91 11q010 92 11q100 93	10000 105 010q01 106 010q10 107 010q11 108 011q00 109	110q00 121 110q01 122 110q10 123 110q11 124 111q00 125	0100q0 137 0100q1 138 0101q0 139 0101q1 140 0110q0 141	153 1100q1 154 1101q0 155 1101q1 156 1110q0 157	01000q 01001q 170 01010q 171 01011q 172 01100q 173	185 11001q 186 11010q 187 11011q 188 11100q 188 11100q 189
<pre>1 q01000</pre>	a11000 25 a11001 26 a11010 27 a11011 28 a11100 29 a11101	0q1000 41 0q1001 42 0q1010 43 0q1011 44 0q1100 45 0q1101	1q1000 57 1q1001 58 1q1010 59 1q1011 60 1q1100 1q1101	01q000 73 01q001 74 01q010 75 01q010 76 01q100 77 01q101	11q000 89 11q001 90 11q010 91 11q010 92 11q100 93 11q101	10000 105 010q01 106 010q10 107 010q11 108 011q00 109 011q01	110q00 121 110q01 122 110q10 123 110q11 124 111q00 125 111q01	0100q0 137 0100q1 138 0101q0 139 0101q1 140 0110q0 141 0110q1	153 1100q1 154 1101q0 155 1101q1 156 1110q0 157 1110q1	01000q 169 01001q 170 01010q 171 01011q 01011q 01100q 172 01100q 173 01101q	185 11001q 186 11010q 187 11011q 188 11100q 189 11101q
<pre>1 q01000 > 0 q01001 > 0 q01010 > 0 q01010 > 0 q01010 > 0 q01011 > 0 q01100 > 0 q01101 > 0 q01101 > 13 0 14</pre>	q11000 25 q11001 26 q11010 27 q11011 28 q11100 29 q11101 30	0q1000 41 0q1001 42 0q1010 43 0q1011 44 0q1100 45 0q1101 46	1q1000 57 1q1001 58 1q1010 59 1q1011 60 1q1100 1q1100 1q1101 62	01q000 73 01q001 74 01q010 75 01q010 76 01q100 77 01q101 78	11q000 89 11q001 90 11q010 91 11q010 92 11q100 93 11q101 94	10000 105 010q01 106 010q10 107 010q11 108 011q00 109 011q01 110	110q00 121 110q01 122 110q10 123 110q11 124 111q00 125 111q01 126	1137 0100q1 138 0101q0 139 0101q1 140 0110q0 110q1 0110q1 142	1100q0 153 1100q1 154 1101q0 155 1101q1 156 1110q0 157 1110q1 158	01000q 169 01001q 170 01010q 171 01011q 172 01100q 172 01100q 173 01101q 174	185 11001q 186 11010q 187 11011q 188 11100q 189 11101q 190
1 9 0 q01001 1 1 1 1 0 q01010 1 1 1 1 1 1 1 1 1 1 1 1 1	a11000 25 a11001 26 a11010 27 a11011 28 a11100 29 a11101 30 a11110	0q1000 41 0q1001 42 0q1010 43 0q1010 43 0q1010 45 0q1100 45 0q1101 46 0q1110	1q1000 57 1q1001 58 1q1010 59 1q1011 60 1q1100 1q1101 62 1q1110	01q000 73 01q001 74 01q010 75 01q011 76 01q100 77 01q101 78 01q101	11q000 89 11q001 90 11q010 91 11q010 92 11q100 93 11q100 94 11q110	10000 105 010q01 106 010q10 107 010q11 108 011q00 109 011q00 1100 1100 011q10	110q00 121 110q01 122 110q10 123 110q11 124 111q00 125 111q01 126 111q10	0100q0 137 0100q1 138 0101q0 139 0101q1 140 0110q0 141 0110q1 142 0111q0	153 1100q1 154 1101q0 155 1101q1 156 1110q0 157 1110q1 158 1111q0	01000q 169 01001q 170 01010q 171 01011q 172 01100q 173 01101q 174 01110q	185 11001q 186 11010q 187 11011q 188 11100q 189 11101q 190 11110q
<pre>1 q01000</pre>	q11000 25 q11001 26 q11010 27 q11011 28 q11100 29 q11101 30 q11110 31	0q1000 41 0q1001 42 0q1010 43 0q1010 43 0q1011 44 0q1100 45 0q1101 46 0q1110 47	1q1000 57 1q1001 58 1q1010 59 1q1010 1q100 1q1100 1q1100 62 1q1110 62	01q000 73 01q001 74 01q010 75 01q010 76 01q100 77 01q101 78 01q101 77 01q101 77	11q000 89 11q001 90 11q010 91 11q010 92 11q100 93 11q100 94 11q110 95	100000 105 010q01 106 010q10 107 010q11 108 011q00 109 011q01 110 011q10 111	110q00 121 110q01 122 110q10 123 110q11 124 111q00 125 111q01 126 111q10 127	0100q0 137 0100q1 138 0101q0 139 0101q1 140 0110q0 141 0110q1 142 0111q0 143	153 1100q1 154 1101q0 155 1101q1 156 1110q1 157 1110q1 158 1111q0 159	01000q 169 01001q 170 01010q 171 01011q 172 01100q 173 01101q 174 01110q 175	185 11001q 186 11010q 187 11011q 188 11100q 189 11101q 190 11110q 191
1 9 0 q01001 1 10 0 q01010 1 10 10 10 10 10 10 10 10	q11000 25 q11001 26 q11010 27 q11011 28 q11100 29 q11110 30 q11110 31 q11111	0q1000 41 0q1001 42 0q1010 43 0q1010 43 0q1011 44 0q1100 45 0q1101 46 0q1110 47 0q1111	1q1000 57 1q1001 58 1q1010 59 1q1011 60 1q1100 1q1101 62 1q1110 63 1q1111	01q000 73 01q001 74 01q010 75 01q010 76 01q100 77 01q101 78 01q110 79 01q111	11q000 89 11q001 90 11q010 91 11q010 91 11q100 93 11q101 94 11q110 95 11q111	10000 105 010q01 106 010q10 107 010q11 108 011q00 109 011q01 110 011q10 111 011q11	110q00 121 110q01 122 110q10 123 110q11 124 111q00 125 111q01 126 111q10 127 111q11	0100q0 137 0100q1 138 0101q0 139 0101q1 140 0110q1 141 0110q1 142 0111q0 143 0111q1	153 1100q1 154 1101q0 155 1101q1 156 1110q1 158 1111q0 158 1111q0 159 1111q1	01000q 01001q 01001q 01010q 171 01011q 172 01100q 173 01101q 174 01110q 175 01111q	185 11001q 186 11010q 187 11011q 188 11100q 189 11101q 190 11110q 191
1 9 0 q01001 1 1 1 0 q01010 1 1 1 1 1 1 1 1 1 1 1 1 1	a11000 25 a11001 26 a11010 27 a11011 28 a11100 29 a11110 30 a11110 31 32	0q1000 41 0q1001 42 0q1010 43 0q1010 43 0q1011 44 0q1100 45 0q1101 46 0q1110 47 0q1111 48	1q1000 57 1q1001 58 1q1010 59 1q1011 60 1q1101 62 1q1110 63 1q1111 64	01q000 73 01q001 74 01q010 75 01q010 76 01q100 77 01q100 77 01q101 78 01q110 79 01q111 80	11q000 89 11q001 90 11q010 91 11q010 92 11q100 93 11q100 94 11q110 95 11q111 96	100000 105 010q00 106 010q10 107 010q11 108 011q00 109 011q00 109 011q01 110 011q10 111 011q11 112	110q00 1121 110q01 122 110q10 123 110q11 124 111q00 125 111q01 126 111q10 127 111q11 128	0100q0 137 0100q1 138 0101q0 139 0101q1 140 0110q0 141 0110q1 142 0111q0 143 0111q1 144	153 1100q1 154 1101q0 155 1101q1 156 1110q0 157 1110q1 158 1111q0 159 1111q1 160	01000q 169 01001q 170 01010q 171 01011q 172 01101q 172 01101q 172 01101q 173 01101q 174 01110q 175 01111q 176	185 11001q 186 11010q 187 11011q 188 11100q 189 11101q 190 11110q 191 11111q 192

Figure S10: An analysis of all single-stochastic games for strong selection. Same as previous figures, but with $\beta = 10$.

	10 90000	<u>q10000</u>	0 <u>q0000</u>	1 <u>q0000</u>	<u>00q000</u>	<u>10q000</u>	000q00	100q00	0000q0	1000q0	00000q	10000q
8		47	22		er	04	07	442	420	445	AEA	477
	0.001	-10001	0-0004	49	00-004	10=004	97	100-04	129	145		10001-
	10 00001						000001		-	100001	000010	100010
8	0.001 2	18	34	50	66	82	98	114	130	146	162	178
	10 900010	q10010	0q0010	1q0010	00q010	10q010	000q10	100q10	<u>0001q0</u>	<u>1001q0</u>	00010q	<u>10010q</u>
8	3	19	35	51	67	83	99	115	131	147	163	179
	0.001	a10011	000011	100011	00a011	10:0011	000a11	100a11	0001a1	1001a1	00011a	10011a
_	10						-					
<u> </u>	0.001	20	36	52	68	84	100	116	132	148	164	180
	10 900100	q10100	0q0100	1q0100	00q100	10q100	001q00	101q00	0010q0	1010q0	00100q	10100q
θ	5	21	37	53	69	85	101	117	133	149	165	181
	0.001	a10101	000101	100101	00a101	10a101	001a01	101 <i>a</i> 01	0010a1	1010a1	00101a	10101a
~	10										-	
_	0.001 6	22	38	54	70	86	102	118	134	150	166	182
	10 900110	q10110	0q0110	1q0110	00q110	10q110	001q10	101q10	0011q0	1011q0	00110q	10110q
æ	7	23	39	55	71	87	103	119	135	151	167	183
	0.001	a10111	0a0111	100111	00g111	10a111	001a11	101a11	0011a1	1011a1	00111a	10111a
~	10											
_	0.001 8	24	40	56	72	88	104	120	136	152	168	184
	10 9 <u>01000</u>	q11000	0q1000	1q1000	01q000	11q000	010q00	110q00	0100q0	1100q0	01000q	11000q
θ	9	25	41	57	73	89	105	121	137	153	169	185
	0.001	a11001	0q1001	1a1001	01q001	11q001	010q01	110001	0100a1	1100a1	01001g	11001a
~							-					_
_	0.001 10	26	42	58	74	90	106	122	138	154	170	186
	10 q01010	q11010	0q1010	1q1010	01q010	11q010	010q10	110q10	0101q0	1101q0	01010q	11010q
θ	11	27	43	59	75	91	107	123	139	155	171	187
	0.001	q11011	0q1011	1q1011	01q011	11q011	010q11	110q11	0101q1	1101q1	01011g	11011g
g			-				-		-			
	0.001 12	28	44	60	76	92	108	124	140	156	172	188
	10 001100	q11100	0q1100	1q1100	01q100	11q100	011q00	111000	0110q0	1110q0	01100q	11100q
8	0.001 13	29	45	61	77	93	109	125	141	157	173	189
	10 901101	q11101	0q1101	1q1101	01q101	11q101	011q01	111q01	0110q1	1110q1	01101q	11101q
g							-					
	0.001 14	30	46	62	78	94	110	126	142	158	174	190
	10 001110	d11110	001110	101110		110110		111010	011100	1111q0	011100	111100
8	0.001 15	31	47	63	79	95	111	127	143	159	175	191
	10 q01111	q11111	0q1111	1q1111	01q111	<u>11q111</u>	011q11	111q11	0111q1	<u>1111q1</u>	01111q	11111q
g												
	0.001	32	48	64	80	96	112	128	144	160	176	192
	U 1	U 1	U 1	U 1	υ 1 -	ບ 1 -	U 1	U 1	U 1	U 1	U 1	U 1
	Ч	q	q	q	Ч	ч	ч	q	q	Ч	q	q

Figure S11: A systematic analysis of the single-stochastic transition structures for positive selection strength. The figure reproduces the contour plot of Fig. 5 for all 192 single-stochastic games. There are exactly 24 transition structures for which there is no difference between full and no information for any selection strength. In all these games, one of the two environmental states is absorbing. Parameter values are the same as in Fig. 5.



Figure S12: Value of information in games with stochastic transitions. a, We assume the entries of q are taken from a finite grid $q_{ij}^k \in \{0, 0.2, 0.4, 0.6, 0.8, 1.0\}$, giving rise to $6^6 = 46, 656$ different games. For each game, we calculate the average cooperation rates for full and no information. We plot the distribution of the resulting value of information for all considered stochastic games. b, We bin the games into three categories: games with positive (blue), negative (red) and neutral (white) value of information. Most games exhibit a benefit of information. Parameter values are the same as in Fig. 2.



Figure S13: Effect of information in two 3-player games. a,b, We analyze a three-player public good game where players move for one round to the worse state 2 if at least two players defect. We compute cooperation rates for full information and no information, in the limit of no selection $\beta = 0$. We find a benefit of information. c,d, We perform the same analysis for a different 3-player game. Here, we find a benefit of ignorance. Parameter values: $r_1 = 1.6$, $r_2 = 1$, c = 1, population size N = 100, error rate $\varepsilon = 0.01$.