Spectral Transfer of Turbulent Energy and Temperature Variance

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Abstract:

The equations for the three-dimensional kinetic energy spectrum and temperature variance spectrum of homogeneous turbulence have been solved numerically for the transfer in the wave number space and for the cospectra of vertical momentum and heat flux. ITie equations have been closed by relating the spectral transfer to the spectra of kinetic energy and temperature variance using the concept of a spectral eddy viscosity and thermal diffusivity. Instead of specifiing analytically the unknown eddy diffusivity and thermal viscosity the three-dimensional spectra for kinetic energy and temperature variance, which have been deduced from one-dimensional measured spectra, have been considered to be known. The spectral diffusivities, determined in this procedure, show a similar behaviour to Heisenberg's assumption characterized by a - 4/3 slope in the inertial subrange, but deviate from it in the ränge of the energy containing eddies, especially under unstable stratifications.

In consequence of the spectral diffusivities the cospectra of the vertical momentum flux reveal at small wave numbers for unstable thermal stratifications a ränge, where eddies transport momentum upward against the mean velocity gradient. Also, the resulting shift of the maximum spectral density to smaller wave numbers with decreasing stability is in agreement with experimental results.

Zusammenfassung: Spektraler Transport von turbulenter Energie und Temperaturvarianz

Die Gleichungen für das dreidimensionale Spektrum für kinetische Energie und Temperaturvarianz bei homogener Turbulenz wurden numerisch gelöst, um den Transport im Wellenzahlraum und die Kospektren des vertikalen Impuls-und Wärmeflusses zu bestimmen. Durch einen einfachen Ansatz für die spektralen Transporte von Energie und Temperaturvarianz mit Hilfe eines spektralen turbulenten Diffusionskoeffizienten sind die Gleichungen geschlossen worden. Statt die unbekannten Diffusionskoeffizienten analytisch in den Gleichungen vorzugeben, sind die dreidimensionalen Spektren für Energie und Temperaturvarianz, die aus gemessenen eindimensionalen Spektren konstruiert wurden, als bekannt vorausgesetzt worden.

Die auf diese Weise errechneten spektralen Diffusionskoeffizienten zeigen im Inertialbereich ein dem Heisenberg'schen Ansatz ähnliches Verhalten mit einem - 4/3 Abfall für größer werdende Wellenzahlen, weichen aber von diesem besonders bei instabiler Schichtung im Bereich der energiereichen Wirbel ab.

Als Folge davon zeigen die Kospektren für den Impulsfluß bei kleinen Wellenzahlen unter thermisch instabilen Verhältnissen ein Gebiet, in dem die Turbulenzwirbel Impuls gegen den mittleren Windgradienten transportieren. Die berechnete Verlagerung der maximalen spektralen Dichte zu kleineren Wellenzahlen bei abnehmender Stabilität ist in Üereinstimmung mit dem experimentellen Befund.

Resume': Transfer! spectral d'energie turbulente et variance de la temperature

Les equations pour le spectre tridimensionnel de l'energie cinetique et le spectre de la variance de la temperature en turbulence homogene sont resolues numeriquement afin de determiner le transfert dans l'espace des nombres d'onde et les cospectres pour le flux vertical de quantite de mouvement et de chaleur. On realise de fermeture des equations en liant le transfert spectral aux spectres de l'energie cinetique et de la variance de la temperature, ä l'aide de coefficients spectraux de diffusion turbulente.

Au lieu de specifier analytiquement dans les equations les coefficients de diffusion inconnus, on suppose connus les spectres tridimentionnels pour l'energie et la variance de la temperature, qui ont ete construits ä partir de spectres unidimensionnels mesures.

Les coefficients spectraux de diffusion obtenus de cette maniere suivent, dans le domaine inertiel, une loi analogue ä celle de Heisenberg, avec une decroissance en - 4/3 pour les nombres d'onde Croissants, mais s'en ecartent dans le domaine des tourbillons contenant le maximum d'energie, surtout si la stratification est instable.

Par suite, les cospectres pour le flux de la quantite de mouvement revelent, aux petits nombres d'onde et pour une stratification thermiqe instable, un domaine dans lequel les tourbillons de turbulence transportent de la quantite de mouvement vers le haut, contre le gradient moyen de la vitesse.

De plus, le glissement de la densite spctrale maximum vers les petits nombres d'onde quand la stabilite decroit est en accord avec les resultats experimentaux.

1. Introduction

Many studies on spectral quantities based on measurements of atmospheric turbulence have been published in the past, but only a comparatively small number of theoretical papers are concerned with the spectral dynamics. This reflects the closure difficulty of turbulence. Available closure assumptions for the spectral energy transfer, e.g. by HEISENBERG (1948) which gives realistic results for the spectrum except at very high wavenumbers (ORSZAG and RAILA, 1973), are limited to the case of Isotropie turbulence. In spite of these limitations Heisenberg's model has been used also as an approximation to the spectral energy transfer for atmospheric inhomogeneous and anisotropic turbulence. GISINA (1966) and ROTH (1972), for example, applied this approximation in Order to predict spectra of turbulent stress and vertical heat flux and the wind profiles along smooth and rough walls.

In this paper a different approach will be examined. Instead of using an analytical expression for the spectral energy transfer functions, observed and theoretical power spectra of turbulent kinetic energy and temperature variance will be used in order to deduce the unknown spectral energy transfer functions and to study their Variation with stability. Fürther, the spectra of turbulent stress and heat flux are deduced from the model computations and are compared with corresponding observed spectra.

2. Basic Equations

Spectral characteristics of atmospheric turbulence are generally interrelated to quantities of the mean flow, the vertical gradients of mean velocity and temperature. In the case of a stationary and homogeneous flow (wheh includes constant gradients in the vertical) the interaction between the spectrum of turbulent kinetic energy E_e (k) as function of wavenumber k, the spectrum of temperature fluctuations E_T (k) and the vertical mean flow gradient $d\bar{th}/öxa$ and vertical mean temperature gradient $d\bar{T}/dx_3$ may be expressed by the dynamic equations of the energy spectrum and the temperature variance spectrum. These equations may be derived from the equations of motion, continuity and heat conductivity. A detailed explanation of the procedure is given in the Appendix.

These spectral equations are of the following form (see Equation (A17) and (A18) in the Appendix).

$$e - 2 p \int_{o}^{k} k^2 E_e (k)\, dk = 2 p(k) \int_{o}^{k} k^2 E_e (k)\, dk + p(k) \left[\left(\frac{\partial \bar{U}_1}{\partial x_3} \right)^2 - \alpha(k) \frac{g}{T_0} \frac{d\bar{T}}{dx_3} \right] \qquad (1)$$

$$X - 2 X \int_{o}^{k} k^2 E_t (k)\, dk = 2 a (k)\, p(k) \int_{o}^{k} k^2 E_T (k)\, dk + a (k)\, p(k) \left(\frac{J\bar{T}}{dx_3} \right)^2 \qquad (2)$$

Here e and X are, respectively, the total dissipation of turbulent kinetic energy and of temperature variance, p is kinematic viscosity, X molecular temperature conductivity, n(k) is turbulent viscosity, a(k) specifies the difference of turbulent viscosity for kinetic energy transfer and temperature fluctuations from small to larger wave-numbers, g is gravitational acceleration, T_o is mean temperature of the flow.

The left side of Equation (1) and (2) represents the dissipation of kinetic energy in the wave-number ränge from k to infinity, i.e.

$$D_e(k) = e - 2\nu \int_0^k k^2 E_e(k)\,dk = 2\nu \int_k^\infty k^2 E_e(k)\,dk \qquad (3)$$

and the dissipation of temperature variance in the same interval, i.e.

$$D_T(k) = \chi - 2\chi \int_0^k k^2 E_T(k)\,dk = 2\chi \int_k^\infty k^2 E_T(k)\,dk \qquad (4)$$

The dissipation terms are given respectively as result of spectral kinetic energy production in the interval (k, 00)

$$P_E(k) = »\text{-}(k) \; [79ÜA^2 \; \text{-}«O0\text{-}\frac{g}{} \; QTI \; j \qquad (5)$$

and production of temperature variance

$$P_T(k) = a(k)\, p(k) \frac{\partial \overline{T}}{\partial x_3} = p_t(k) \frac{\partial \overline{T}}{\partial x_3} \qquad (6)$$

In Equation (5) the first term on the right side represents mechanical production and the second term is due to buoyant production, which may be positive or negative dependent on the sign of $9\overline{T}/9x_3$.

In deriving these expressions (5) and (6) for production we assumed the weak interaction concept (TCHEN 1953, 1954) between the mean flow and the turbulent eddies, which is consistent with our assumption of a homogeneous flow field. This includes a scale of Variation of the mean flow being greater than the scales of the turbulent eddies.

Finally, in Equation (1) and (2) the dissipation is dependent, respectively, on the amount of transfer of turbulent energy $T_E(k)$ and temperature variance $T_t(k)$ from the spectral ränge (0, k) into the ränge (k, 00) accross wave-number k:

$$T_E(k) = - \int_0^k F_E(k)\,dk = 2\,p(k) \int_0^k k^2 E_E(k)\,dk \qquad (7)$$

$$T_t(k) = - \int_0^k F_t(k)\,dk = 2\,a(k)\,p(k) \int_0^k k^2 E_T(k)\,dk \qquad (8)$$

$F_e(k)$ and $F_T(k)$ are the net-transfer rates at wave-number k. Equations (7) and (8) represent the transfer model originally proposed by HEISENBERG (1948).

According to our assumptions the unknown spectra of turbulent stress and heat flux, which contribute to the production of turbulent kinetic energy and temperature variance and the unknown transfer spectra have been related to the spectra of turbulent kinetic energy and temperature variance.

In previous work (e.g. GISINA (1966), ROTH (1972)) assumptions have been introduced for the unknown spectral turbulent viscosity to close the System. Our approacb differs from these papers in that observed spectra Eg. (k) and $E_T(k)$ will be used to find $\nu(k), \alpha($ and from our assumptions the transfer spectra $F_e(k)$ and $F_T(k)$ and the spectra of momentan! flux $0_{U)}\,u_3(k)$ and heat flux $_{ü3}(k)$ (see Equation

A13, A14, A15 and A16 in the Appendix). The spectral densities of transfer, production and dissipation as related to turbulent kinetic energy and temperature variance are found directly from Equation (1) and (2) by differentiation to k:

$$\frac{\partial T_E(k)}{\partial k} = -F_E(k) = -2\,\nu(k)\,k^2 E_e(k) + 2\,\frac{\partial \nu(k)}{\partial k}\int_0^k k^2 E_e\,dk \qquad (9)$$

$$\frac{\partial T_T(k)}{\partial k} = -F_T(k) = 2\,(k)\,k^2 E_T(k) + 2\,\frac{\partial \nu_T(k)}{\partial k}\int_0^k k^2 E_T(k)\,dk \qquad (10)$$

$$\frac{dP_E(k)}{dk} = \frac{d\bar{U}_1}{dx_3}\, 0U1U3(\;) \quad \frac{g}{T_0}\, 0\ddot{O}U3(\;) \quad \frac{\partial \nu(k)}{dk} \quad \frac{g}{T_0}\frac{\partial \bar T}{dx_3}\frac{M(k)}{dk} \qquad (11)$$

$$\frac{\partial P_t(k)}{dk} = \frac{dT}{dx_3}\, 00U3() \quad \left(\frac{dT}{dx_3}\right)? \quad \frac{dp(k)}{dk} \qquad (12)$$

$$\frac{dD_E}{\partial k} = -2\,\nu\,k^2 E_E(k) \qquad (13)$$

$$\frac{dD_T}{\partial k} = -2\,\chi\,k^2 E_T(k) \qquad (14)$$

Fürther, from Equation (1) and (2) we obtain both the eddy viscosity $\nu(k)$ and the turbulent thermal diffusivity $\nu_t(k)$

$$\nu(k) = \frac{\varepsilon - 2\nu\int_0^k k^2 E_e(k)\,dk}{2\int_0^k k^2 E_E(k)\,dk + \left(\frac{\partial \bar U_1}{\partial x_3}\right)^2} + \frac{\left[\chi - 2\chi\int_0^k k^2 E_T(k)\,dk\right]\frac{g}{T_0}\frac{\partial \bar T}{dx_3}}{\left[2\int_0^k k^2 E_E(k)\,dk + \left(\frac{\partial \bar U_1}{\partial x_3}\right)^2\right]\left[2\int_0^k k^2 E_T(k)\,dk + (fj)\right]} \qquad (15)$$

and

$$\nu_T(k) = \alpha(k)\,\nu(k) = \frac{\chi - 2\chi\int_0^k k^2 E_T(k)\,dk}{2\int_0^k k^2 E_T(k)\,dk + \left(\frac{\partial \bar T}{\partial x_3}\right)^2} \qquad (16)$$

Differentiation of equations (15) and (16) with respect to k yields also $\partial\nu(k)/\partial k$ and $\partial\nu_T(k)/\partial k$ in terms of $E_e(k)$ and $E_T(k)$.

Having derived the necessary equations, we may now formulate the problem that is to be solved in this paper: provided that both spectra, i.e. of energy $E_e(k)$ and of temperature variance $E_T(k)$ are known as empirical data, both covariance spectra $\Phi_{u_1u_3}(k)$, $\Phi_{\vartheta u_3}(k)$, the eddy viscosity $\nu(k)$ and the turbulent thermal diffusivity $\nu_T(k)$ may be gained as Solutions. In addition, all the individual terms in the equations of balance (1) and (2), i.e. corresponding terms of production $P_E(k)$, $P_T(k)$, transfer $T_E(k)$, and $T_t(k)$ and the dissipation $D_E(k)$, $D_T(k)$, together with their spectral densities, will be found from the given set of equations for all wave-numbers $0 < k < \infty$. As input variables for the System of equations we choose the energy dissipation ε and temperature dissipation χ and the gradients of the mean flow $\partial\overline{U}_1/\partial x_3$ and mean temperature $\partial\overline{T}/\partial x_3$.

According to the similarity theory of Monin-Obukhov these variables may be written in the form:

$$\frac{\partial\overline{U}_1}{\partial x_3} = \frac{u_*}{\kappa x_3}\,\varphi_M\left(\frac{x_3}{L_*}\right) \qquad \frac{\partial\overline{T}}{\partial x_3} = \frac{T_*}{\kappa x_3}\,\varphi_H\left(\frac{x_3}{L_*}\right)$$

$$\varepsilon = \frac{u_*^3}{\kappa x_3}\,\varphi_\varepsilon\left(\frac{x_3}{L_*}\right) \qquad \chi = \frac{\kappa u_* T_*^2}{x_3}\,\varphi_H\left(\frac{x_3}{L_*}\right) \tag{17}$$

where the Monin-Obukhov length L_*, the friction velocity u_*, and the scaling temperature T_* are defined as follows:

$$u_* = \sqrt{-\overline{u_1u_3}}\,; \qquad T_* = \frac{1}{\kappa u_*}\,\frac{H}{\rho c_p} \tag{18}$$

In the Equations (17) and (18) the quantity $H = \rho c_p\,\overline{\vartheta u_3}$ denotes the turbulent flux of sensible heat in the vertical direction, c_p the specific heat at constant pressure and $\kappa \approx 0.4$ is von Kärmän's constant. It should be noted that the assumption of homogeneity includes constant vertical gradients of wind and temperature, which contradicts equations (17). Therefore, we consider the relations only to be valid for one height with a continuation of the same gradients into upper and lower levels and study the effect of stability variations. Because of this restriction we also renounce to present the results in similarity Coordinates. We relate the results, however, always to the same amount of energy production which is identical to assuming the same energy dissipation ε under stationary conditions.

The following empirical formulae for the stability functions have been gained from the measurements in the surface boundary layer of the atmosphere (see KAIMAL et al. (1972) and GARRETT (1972)):

$$\varphi_M\left(\frac{x_3}{L_*}\right) = \begin{cases} 1 + 5\,\dfrac{x_3}{L_*} & \text{if } \dfrac{x_3}{L_*} > 0 \\[2ex] \left(1 - 15\,\dfrac{x_3}{L_*}\right)^{-1/4} & \text{if } \dfrac{x_3}{L_*} < 0 \end{cases}$$

$$\varphi_H\left(\frac{x_3}{L_*}\right) = \begin{cases} 0.74 + 4.7\,\dfrac{x_3}{L_*} & \text{if } \dfrac{x_3}{L_*} > 0 \\[2ex] 0.74\left(1 - 9\,\dfrac{x_3}{L_*}\right)^{-1/2} & \text{if } \dfrac{x_3}{L_*} < 0 \end{cases} \tag{19}$$

$$\varphi_\epsilon\left(\frac{x_3}{L_*}\right) = \begin{cases} \left(1 + 2.5\left|\frac{x_3}{L_*}\right|^{3/5}\right)^{3/2} & \text{if } \frac{x_3}{L_*} > 0 \\ \left(1 + \frac{1}{2}\left|\frac{x_3}{L_*}\right|^{2/3}\right)^{3/2} & \text{if } \frac{x_3}{L_*} \leqslant 0 \end{cases} \tag{20}$$

One-dimensional spectra of energy and temperature variance (i.e. $E_{1E}(k_1)$ and $E_{1T}(k_1)$, are usually presented in the form

$$\frac{k_1 E_{1E}(k_1)}{u_*^2} = F_E\left(f, \frac{x_3}{L_*}\right) \qquad \frac{k_1 E_{1T}(k_1)}{T_*^2} = F_T\left(f, \frac{x_3}{L_*}\right) \tag{21}$$

where

$$f = \frac{k_1 x_3}{2\pi}; \quad E_{1E}(k_1) = \frac{\bar{U}_1}{2\pi} E_{1E}(n); \quad E_{1T}(k_1) = \frac{\bar{U}_1}{2\pi} E_{1T}(n); \quad k_1 = \frac{2\pi n}{\bar{U}_1} \tag{22}$$

Here f is a dimensionless wavenumber and n is frequency. By Equation (22) frequency spectra may be transformed into wave-number spectra according to Taylor's frozen-wave hypothesis.

Since we use spectra averaged over spherical Shells (see Appendix), which depend then only on the magnitude of the wave-number k, as an approximation to the anisotropic atmospheric turbulence, we may also apply the corresponding relationship between one-dimensional spectra $E_{1E}(k_1)$ and $E_{1T}(k_1)$ and corresponding three-dimensional spectra $E_E(k)$ and $E_T(k)$ in Order to construct an approximated three-dimensional Spectrum from measured one-dimensional spectra:

$$E_E(k) = \left[\frac{1}{2}k^2 \frac{\partial^2 E_{1E}(k_1)}{\partial k_1^2} - \frac{1}{2}k \frac{\partial E_{1E}(k_1)}{\partial k_1}\right]_{k_1=k}; \quad E_T(k) = \left[-k \frac{\partial E_{1T}(k_1)}{\partial k_1}\right]_{k_1=k} \tag{23}$$

From Equations (17) to (23) it is evident that the parameters

$$x_3 = z, \quad \frac{z}{L_*}, \quad u_* \quad \text{and} \quad k \tag{24}$$

determine uniquely our input variables.

Thus our model of turbulence is completely defined, if the height z, the friction velocity u_*, (z/L_*) and k are specified. Since ϵ and u_* are uniquely related to each other by equation (17) we prefer to choose the energy dissipation ϵ as independent variable, because then all different cases with variable stability and a constant value of ϵ will have the same total production of energy and in the inertial subrange all energy spectra fall together.

Therefore we take

$$z, \quad \frac{z}{L_*}, \quad \epsilon \quad \text{and} \quad k \tag{25}$$

as independent variables and compute the friction velocity u_* from

$$u_* = \left[\frac{\kappa z \epsilon}{\phi_\epsilon\left(\frac{z}{L_*}\right)}\right]^{1/3} \tag{26}$$

as a dependent variable.

Thus, in Order to see the dependence of the turbulence field on the parameter of thermal stratification (z/L_*), we will vary in our computations only this parameter, while the height z and dissipation ϵ will be kept constant.

2.2. Spectrum of energy $E_e(k)$

From measurements of atmospheric turbulence only one-dimensional spectra are known for limited regions of wave numbers. What we need, however, in our model equations are three-dimensional spectra which are valid for the whole wave number ränge. Of course, no complete theory is available as yet which gives this three-dimensional spectrum in one analytical expression. What is known are derivations of the spectral density function only for the inertial subrange and the viscous ränge and some empirical descriptions of the one-dimensional spectra in the energy-containing ränge and inertial subrange. Therefore, our procedure will be to construct from that knowledge a three-dimensional spectrum which describes atmospheric turbulence as a function of stability and wave number.

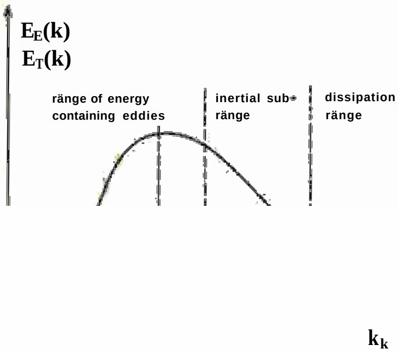


Figure 1

Schematic sketch of the overlapping langes of the kinetic energy and temperature variance spectra. For details see text

Bild 1

Schematische Darstellung der Überlappungsbereiche des Spektrum der kinetischen Energie und der Temperaturvarianz

At first, we will discuss a convenient formula for E_e (k) expressing the properties of the energy spectrum both in the inertial and dissipation ranges (see Figure 1), i.e. within those ranges where the energy production can be neglected. Then, we will consider a formula for E_e (k) which, on the contrary, will be applicable not only in the inertial ränge but also in the energy-containing ränge. Although different from each other, both formulae for E_e (k) will satisfy equally well the law of Kolmogorow in the inertial ränge, for having both the common limit

$$E_e(k) = a e^{2/3} k^{-5/3}.$$

Therefore, it will be easy to match the two spectra at a point inside of the inertial ränge, in order to find the spectral density E_e (k), which can be used in the wave number ränge $0 < k < +\infty$.

According to LIN (1972) the three-dimensional energy spectrum in the inertial and dissipation ränge (= ID) may be expressed by

$$E_E(k) = E_E^{ID}(k) = «e^M \; k^{-s,3} (1 + e^{-1/6} »^{1,2} k^{2,3}) \exp[-a(|e^{-1,3} x k^{4/3} + e^{-1,2} K^{3,2} k^2)]l \quad (27)$$

for $k > k_m$, where k_m is the wave number of the energy containing eddies. As it is seen, for $k_K > k > k_m$, E_e^5 (k) is identical to E_e (k), the 5/3 law". At wave numbers k of the order of k_E the energy density decreases exponentially.

Let us now consider the extension of the spectrum to the ränge of the energy containing eddies. As theoretical analyses for this ränge are not available we refer to empirical results and make use of some hypotheses which are in accordance to our previous assumptions.

One-dimensional spectra are rather well-known from experimental studies, but only for the spectra of the vertical velocity component an empirical relation is available, which describes energy densities also as a function of stability. We will use this relation, convert this spectrum to a longitudinal spectrum Eie (k) by

$$E_{3E}(k) = i \left[E,e(k) - k \frac{\partial E_{E}(k)}{\partial k} \right] \quad (28)$$

whichis strictly correct, however, only for Isotropie turbulence. Since we use spectra averaged over spherical Shells, this may be considered as a crude approximation for the whole Spectrum. In the same way, we apply the transformation (23) from the Isotropie turbulence theory to derive a three-dimensional turbulence spectrum.

According to BUSCH and PANOFSKY (1968) the one-dimensional spectrum of the vertical velocity component can be described by

$$\frac{k_1 E_{3E}(k_1)}{u_*} = \frac{1.075\, f}{1 + 1.5\left(\frac{f}{f_m}\right)^{5/3}} \tag{29}$$

Here $f = \frac{nz}{U_1}$ and f_m is the dimensionless wave number where E_{3E} shows its maximum. Converting this spectrum to a spectrum E_{1e} (k) of the longitudinal component we obtain from Eqation (23) the three-dimensional spectrum valid within the energy containing and the inertial ränge (= EI), henceforth be denoted by $E_e^{I}(k)$.

Now, the position of the maximum of the energy spectrum $E_e^{I}(k)$, as given through the dimensionless wave number f_m (or through the wave number $k_m = \frac{2\pi f_m}{z}$, which defines the ränge of the energy-containing eddies)

$$f_m = 0.32 \quad \varphi_e\left(\frac{z}{L_*}\right) \tag{30}$$

depends on z/L*, since $\varphi_e\left(\frac{z}{L_*}\right)$ is given by (20).

In a region far enough from the ränge of energy-containing eddies, i.e. inside of the inertial ränge

$$k_1 > k_m \tag{31}$$

we find from (29)

$$E_{1E}(k_1) = 0.46\ e^{2/3}\ k_1^{-5/3} \tag{32}$$

Substituting (32) into (23), we obtain the corresponding three-dimensional energy spectrum $E_E(k)$, for the inertial ränge

$$E_e(k) = E_e^{I}(k) = 1.42\ e^{2/3}\ k^{-5/3} \quad \text{for } k > k_m \tag{33}$$

From the spectrum in Equation (27) we find for the same ränge

$$E_e(k) = E_E^{ID}(k) = a\, e^{2/3}\, k^{-5/3} \tag{34}$$

The constant *a* is known quite well from the literature, e.g. *a* = 1.44 is given by ROTTA (1972). These two independently determined values for *a* agree quite well and *a* = 1.44 will be used in the computations in this paper.

The two branches of the spectrum, i.e. $E_e^{I}(k)$ and $E_E^{ID}(k)$ are matched at a wave number k representing the middle of the inertial subrange ($k_m < k^* < k_K$).

With values $e = 20\ cm^2\ s^{-3}$ and $\nu = 0.148\ cm^2\ s^{-1}$ that will be used later in our computations, we obtain for the Kolmogorov wave number $k_K = (e\,\nu^{-3})^{1/4} \approx 9\ cm^{-1}$.

As the maxima of E_e (k) appear in the ränge of wave numbers from 10^{-3} to $10^{-2}\ cm^{-1}$ the choice $k^* = 0.09\ cm^{-1}$ seems quite adequate. The energy dissipation e is one of the input parameters (25) and moreover, it occurs as a parameter in (27).

If k* has been chosen correctly, this value of e should agree well with

$$6int = 2p \int_0^{OO} k^2 E_e (k)\ dk = 2 \int_0^{k*} k^2 E*^1 (k)\ dk + 2\nu \int_{k*}^{oo} k^2 E^D (k)\ dk \qquad (35)$$

Therefore, the difference of both values of dissipation Ae = e - will serve as a measure of accuracy. Numerical experiments have shown that |Ae| < $4 \cdot 10^{-2}\ cm^2 s^{-3}$ if k* is in the ränge $0.13\ cm^{-1} < k^* <$ $< 0.06\ cm^{-1}$ and $\frac{z}{L_*}$ varies from - 1 to + 0.5.

2.3. Spectrum of Temperature Variance Ej (k)

The temperature fluctuions are carried by the wind fluctuations, since heat is a quasi passive contaminant of the atmosphere. In consequence, it is quite reasonable to treat the temperature variance Spectrum E_T (k) in a similar way to that of turbulent kinetic energy.

Let us consider first the temperature variance spectrum inside of the inertial and dissipation ranges. In both ranges, the following relations can be assumed

$$\left| \frac{3Py(k)}{3k} \right| \ll \left| \frac{dT_T (k)}{9k} \right| \text{ and } \left| \frac{dP_T (k)}{9k} \right| \ll \left| \frac{9D_t (k)}{9k} \right| \qquad (36)$$

As has been shown by STRAKA et al. (1978), the three-dimensional spectral density of temperature variance within the inertial and dissipation ränge (= ID), i.e. for k > k_m , may be expressed by

$$E_t (k) = E^*P(k) = ßxe^{-1/3} k^{-5/3} (1 + a^{1/2} e^{-1/6} X^{1/2} k^{2/3}) \cdot$$
$$\cdot \exp\left[-ß /3 e^{1/3} Xk^{4/3} + ct^{1/2} e^{1/2} X^{3/2} k^2 \right] \qquad (37)$$

where $a = \frac{\nu}{X}$ is the Prandtl number and

$$X - 2X \int_0^{\infty} k^2 E_T (k)\ dk \qquad (38)$$

is the dissipation of temperature variance by molecular heat conduction. The spectrum (37) results from Equation (4) by an analogous treatment to the derivation of the turbulent energy spectrum in these ranges. Under the condition $k_m < k <$ ky where $k_T = (eX^{3})^{1/4}$ Equation (37) leads to

$$E_T (k) = E^*P(k) = /?xe^{-1/3} k^{-5/3} \qquad (39)$$

For the temperature-variance spectrum in the inertial and production ränge PANOFSKY (1969) has shown that the one-dimensional spectrum E_{1T} (kJ obeys Monin-Obukhov similarity theory i.e.

$$\frac{k_1 E_{1T}(k_1)}{T_*^2} = \frac{4.6\, Y_{TM} (e?) £}{\left[1 + 1.5 \frac{f}{f_m}\right]^{5/3}} \qquad \text{if } \frac{z}{L_*} < 0 \qquad (40)$$

and

$$\frac{k_1 E_{1T}(k_1)}{T_*^2} = \frac{2.5\, Y_m \left(\frac{z}{L_*}\right) \frac{f}{f_m}}{1 + 1.5 \left(\frac{f}{f_m}\right)^{5/3}} \qquad \text{if } \frac{z}{L_*} > 0 \qquad (41)$$

The corresponding three-dimensional spectrum of temperature variance E? (k) can again be obtained by means of the relation

$$E \quad k > 4- \quad r \; k \; \frac{dE_1T(k_1)'|}{--iki-J} \Big|_{ki=k} \qquad (42)$$

which is correct only for isotropic turbulence and will be used here as an approximation.
From (40) and (41) we find

$$E?'(k) = -11.5 Ti Y m \left(\frac{r}{L*} \right) k \left[\frac{\_}{LZ7Tl_m} \right]^2 \left[1 + 1.5 \left(\frac{z}{Z7Tl_m} \right) k \right]^{n"8/3} \quad if \; p \frac{-}{L*} < 0 \qquad (43)$$

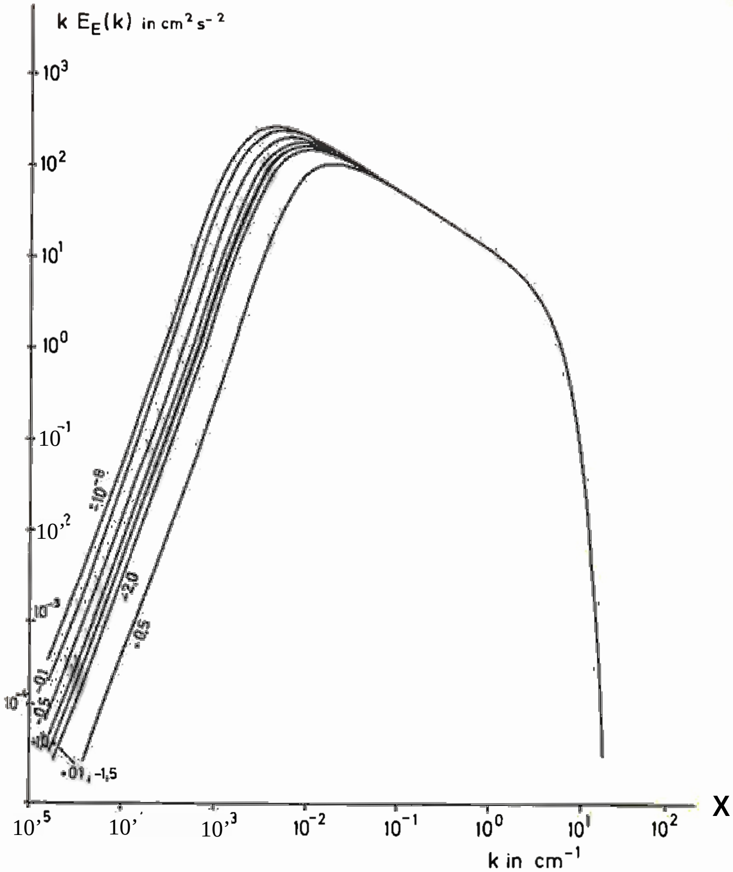
and

$$E?^{I}(k) = -6.25 Ti Y m \left(\frac{p}{E*} \right) \left[\frac{z}{ImJ} \right]^{.8/3} \left[1 + 1.5 \left(\frac{z}{X- Im} \right)^{S/3} k^{s,3} \right]^{-2} k^{sz3} \quad if \; L* > 0 \qquad (44)$$

Both, the dimensionless wave number f_m at the maximum of E_{1T} (kJ and the Ordinate Y_m (z/L*) at that maximum, are functions of z/L*. In this study empirical relationships for f_m and Y_m were used as obtained earlier by PANOFSKY (1969).
From (42) it is evident that E_{1T} (kJ and E_T (k) are similar in the inertial subrange, i.e. from (39) it follows

$$E_{1T}(kJ = \beta_T \, x \, e"^{1/3} k?^{s/3}$$

$$for \; ki > k[\qquad (45)$$



- Figure 2
  Three-dimensional energy spectrum for z/L* values from 0.5 to - 2.0. Parameter is z/L*

- Bild 2
  Dreidimensionales Energiespektrum für z/L* Werte von 0.5 bis - 2.0. Parameter is z/L*

This spectrum overlaps with Equations (40) and (41), respectively in the inertial ränge. From (40) and (41) we find with

$$k_i > k_{1m} \tag{46}$$

(k_{1m} is the characterisic wave number of the one-dimensional spectrum, where it shows its maximum) the one-dimensional spectra in the inertial ränge

$$E_{1T}(k_1) = T_*^2\, 2.3\, Y_m \left(\frac{z}{L_*}\right) [2\pi f_m]^{2/3} z^{-2/3} k_1^{-5/3} \quad \text{for} \quad \frac{z}{L_*} < 0 \tag{47}$$

and

$$E_{1T}(k_1) = T_*^2\, 1.7\, Y_m \left(\frac{z}{L_*}\right) [2\pi f_m]^{2/3} z^{-2/3} k_1^{-5/3} \quad \text{for} \quad \frac{z}{L_*} \geq 0 \tag{48}$$

Hence, Equations (47) and (48) have to be identical with (45).
Thus Equations (45) and (46) yield

$$\beta_T = \kappa^{-2/3}\, 2.3\, Y_m \left(\frac{z}{L_*}\right) (2\pi f_m)^{2/3} \left[L_c\left(\frac{z}{L_*}\right)\right]^{1/3} \left[\Phi_H\left(\frac{z}{L_*}\right)\right]^{-1} \quad \text{for} \quad \frac{z}{L_*} < 0. \tag{49}$$

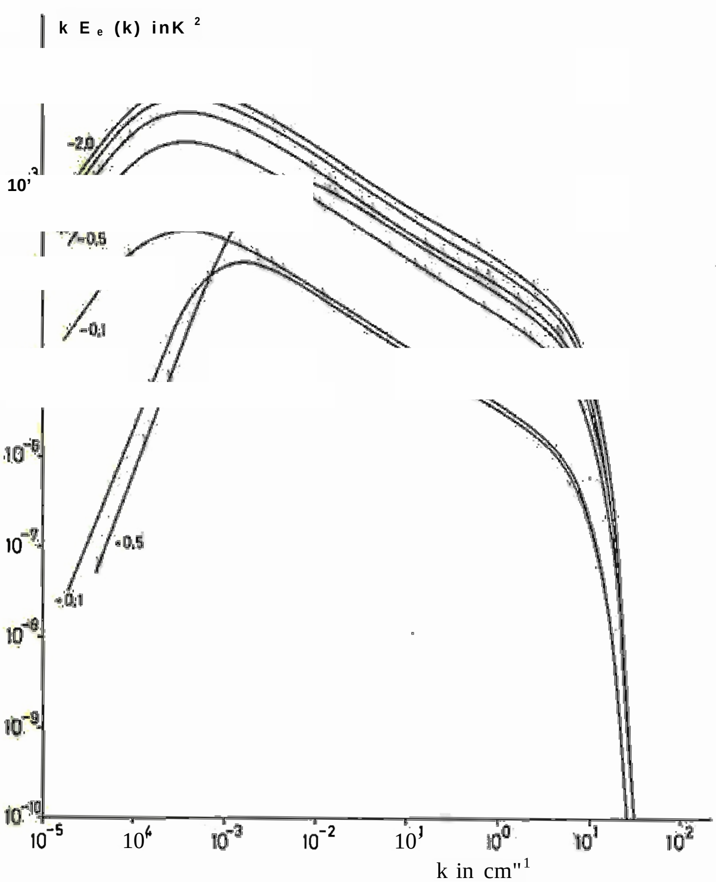


Figure 3

Threedimensional temperature variance spectrum for z/L* values from 0.5 to - 2.0. Parameter is z/L*

Bild 3

Dreidimensionales Spektrum der Temperaturvarianz für z/L* Werte von 0.5 bis - 2.0. Parameter ist z/L*

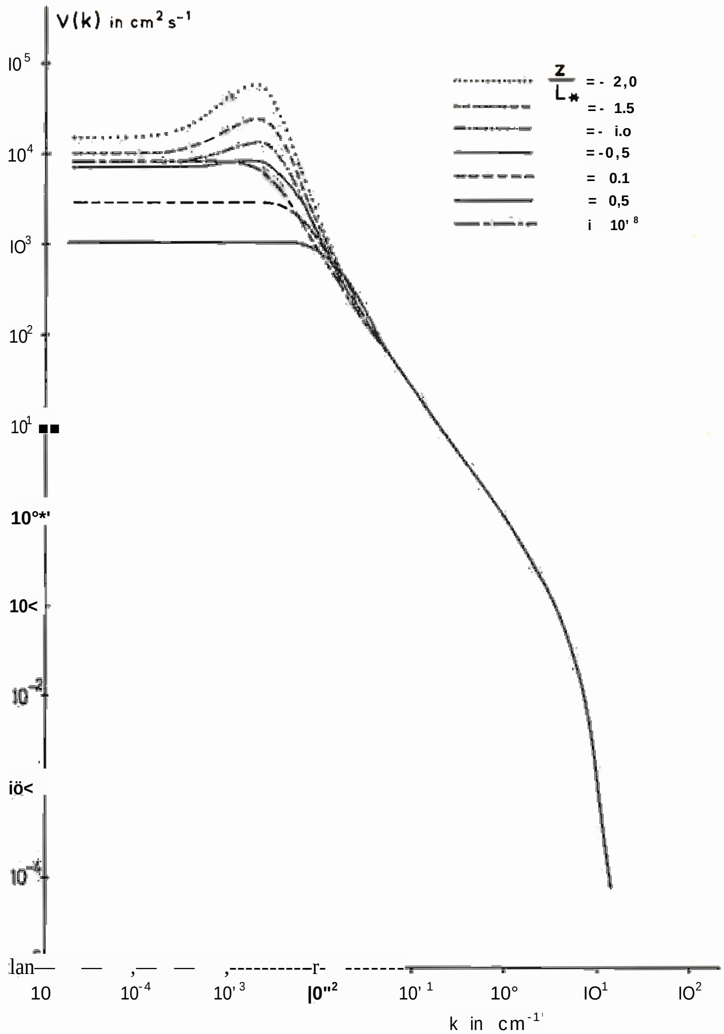
Here e and x have been expressed by Equation (17). From (45) and (47) we find for ßT in the case of stable conditions (z/L* > 0)

$$ß_r = _K -^{4,3} \; 1.7 \; Y_m \; (jM \quad (2nf_m]^M \; Mr.)]^{1/2} \; [-(*)]^{-1} \quad \text{for } \frac{7}{L_*} > 0 \qquad (50)$$

Since Equation (45) expresses the Kolmogorow law for the one-dimensional spectrum Ej_T (kJ of temperature variance, the quantity $ß_T$ should be constant in analogy with the law for the energy spectrum. However, it is obvious from (49) and (50), that, if the used empirical relationships are not completely consisteot to eädi ötitär, the values of *ßr* wfll changp slightly with stability. This is indeed the case as can. be seen from die values given in Table l ; disrcrepancies appear near $\frac{z}{L_*}$ = 0 what should be expected because of experimental uncerlaintjes, e.g. of Y_m (z/L*).

By averaging $ß_T$ over the stability ränge from $\frac{z}{L*}$ = - 1 to $\frac{z}{L*}$ = 0,5 we obtain:

$$ßr \quad 0.85 \qquad (51)$$

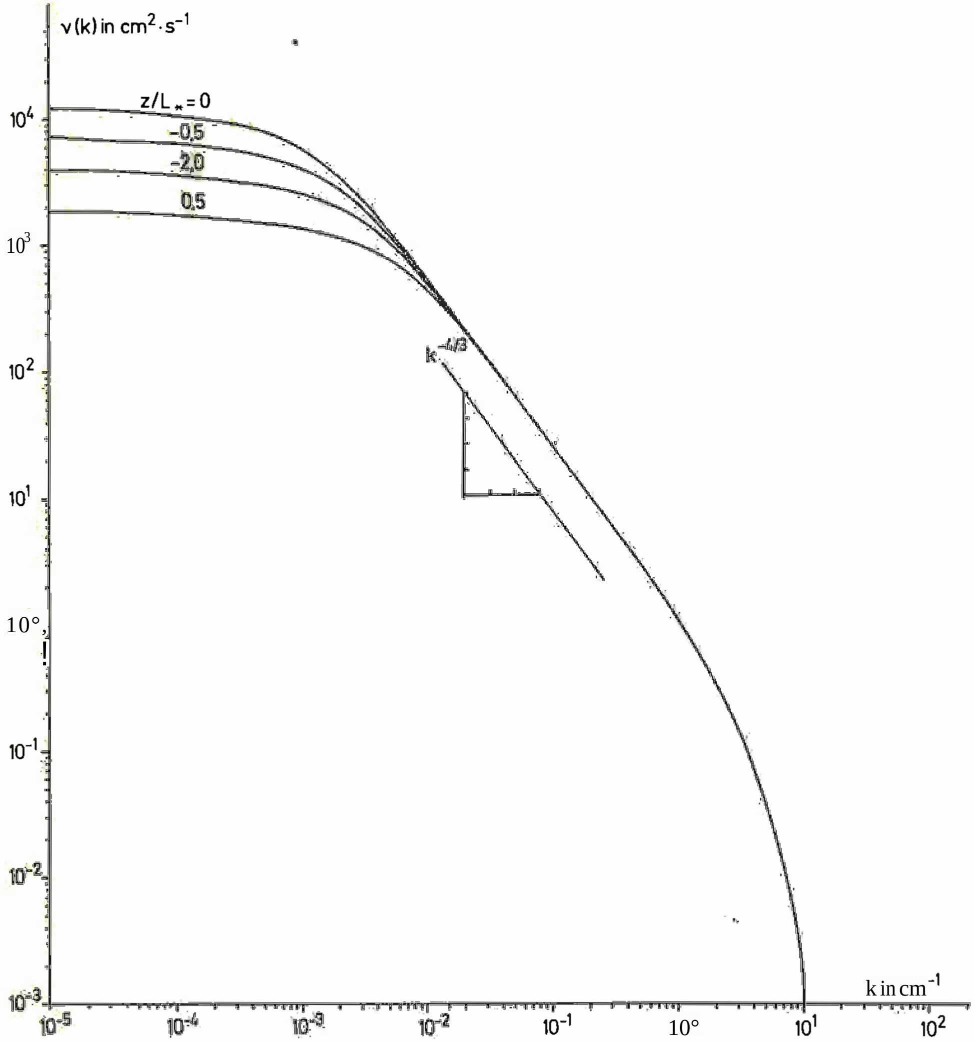


- Figure 4

  Spectral eddy diffusivity for different stabilities z/L* ranging from 0.5 to - 2.0.

- Bild 4

  Spektraler turbulenter Diffusionskoeffizient für unterschiedliche Stabilitäten z/L* im Bereich von 0.5 bis - 2.0.



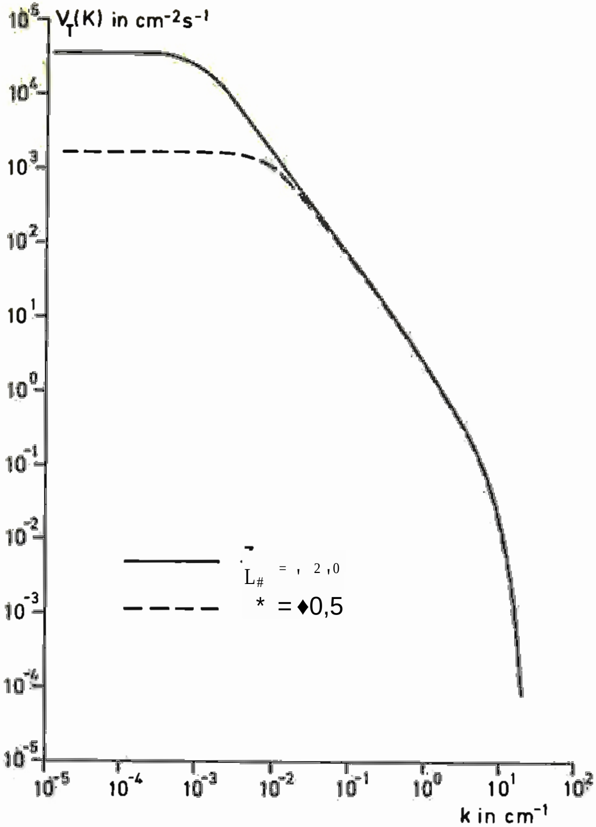
• **Figure 5.** Spectral eddy viscosity according to Heisenberg's assumption (see Equation (55)) for values of z/L* = 0, -0.5, - 2.0, and 0.5

• **Bild 5.** Spektraler turbulenter Diffusionskoeffizient entsprechend Heisenberg's Annahme (siehe Gleichung (55)) für Werte von z/L* = 0, - 0.5, - 2.0 und 0.5

■ **Table 1.** Kolmogorov constant ßp for the one-dimensional temperature spectrum computed from Equation (49) and (50) respectively

■ **Tabelle 1.** Kolmogorov Konstante ßp für das eindimensionale Temperaturspektrum berechnet aus Gleichung (49) bzw. (50)

$\frac{z}{L^*}$	-1,0	-0,6	-10^{-8}	$+10^{-8}$	+0,2	+0,4
$ß_T$	0,820	0,623	0,688	0,490	1,072	0,954



® **Figure 6**

Spectral eddy thermal conductivity for a stable (z/L# = 0.5) and unstable case (z/L* = - 2.0)

• **Bild 6**

Spektraler turbulenter Wärmeleitkoeffizient für einen stabilen (z/L* = 0.5) und einen instabilen Fall (z/L* = — 2.0)

GARRATT (1972) reported from boundary-layer measurements over water $ß_T$ = 0.8 while measurements over flat, uniform terrain supply *ßr* = 0.82 (KAIMAL, WYNGAARD, IZUMI and COTE, 1972). Both experimental values show satisfactory agreement with our estimate. In the following calculations we used *ß'P* = 0.8.

Therefore, the corresponding constant *ß* in Kolmogorov's law (39) for the three-dimensional spectrum E_t (k) will be taken to be

$$0 = 4/3 \qquad (52)$$

In consequence of (40), (41) and (42), the maxima of E_T (k) should be observed in the ränge of wave numbers from 4 X 10^{-4} to 3 X 10~^{3} cm' ^{1} (if: - 2 < z/L* < + 0.4), whereas the dissipation of temperature variance is expected to become important at kj = $(g/X^3)^{1/4}$. Using the values e = 20 $cm^2 s^{-3}$ and X = 0.1 82 $cm^2 s'^1$, we obtain k_T = 8 cm^{-1}. Hence, the point k* - - 0.09 an" ^{1} for matching the two spectral functions lies well within the inertial ränge. Therefore, E ?' (k) and Ey' (k) can be matched at the same wave number k = 0.09 cm^{-1} where E}P (k) and Eg^1 (k) bäve been connectßd.

Because of the uncertainties of the different empirical constants we define the spectrum of temperature variance E_T (k) in the ränge 0 < k < + °° by

$$E_t\,(k) = \begin{cases} c_2\, E^{TM}(k) & \text{if} \quad 0<k<k^* \\ EjP\ (k) & \text{if} \quad k^*<k<oo \end{cases} \qquad (53)$$

where

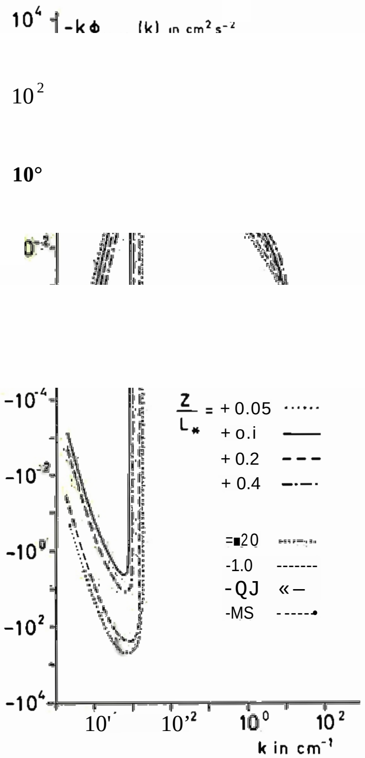
$$c_2 = \frac{E^{‘D}(k_„)}{E?’(k.)}\ '$$

As a measure of accuracy, we may again use the difference Ax between the value of x as given by (17) and

$$Xint=2X \int_0^{00} k^2 E_T(k)\, dk = 2XJ \int_0^{k*} c_2 k^2 E™(k)\, dk + 2XJ \int_{k*}^{00} k^2 E*P(k)dk. \quad (54)$$

Using the above mentioned values for X,*i*>and e, numerical calculations show that the factor c_2 remains between 0.75 and 1.63, if $k^* = 0.09\ cm'^1$ and z/L* varies between -1.0 and 0.4. x is of the Order $10“^4$ to $10^{-3}\ cm^2 s"^1$; the difference Ax = Ix$^-$ Xint I is, however, always smaller than $10"^7\ cm^2 s^{-3}$. No considerable change of this Situation was found when k* was varied between 0.06 and 0.12 cm^{-1}.
The rather large value $c_2 = 1.63$ occured for $z/L^* = +10^{-8}$. As PANOFSKY (1969) pointed out, the expressions for the temperature spectra will be erratic in the ränge of z/L* from - 0.05 to 0.05. Avoiding this ränge, then c_2 remains close to one.

These spectral densities for E_e (k) and E_T (k) have been used as input data in order to study the behavior of the spectral eddy viscosity ν(k), thermal diffusivity (k), covariance spectra $0_{U1\,U3}$(k), $0_{ÖU3}$(k), spectral transfer functions and the spectral distribution of production and dissipation.



• **Figure** 7
Cospectra of the momentuni flux (log-log-plot) for z/L* values ranging from 0.4 to —2.0

• **Bild** 7
Kospektren für den Impulsfluß (log-log-Darstellung) für z/L* Werte von 0.4 bis—2.0

3. Results and discussion

3.1. The three-dimensional spectra of turbulent energy and temperature variance

As has been pointed out in the previous section, three-dimensional spectra cannot be measured directly but have to be deduced frorn accessible one-dimensional spectra. We have found them by transforming one-dimensional spectra of the vertical velocity component valid in the energy-containing ränge and in the inertial subrange and matching it to a three-dimensional spectrum which describes spectral energy densities in the inertial and dissipation ränge. The composite spectra are shown in Figures 2 and 3. Figure 2 shows the spectral density of the turbulent kinetic energy on a log-log-plot for different stabilities. Since we held e constant, all curves are brought into coincidence in the inertial and in the dissipation ränge. The dependence on stability then appears at smaller wave numbers where the spectral curves split according to z/L*.

3.2. Spectral eddy viscosity $\nu(k)$ and thermal diffusivity ν_T (k)

The present paper differs from previous studies that ν(k) and (k) have not been introduced by hypothetical analytic functions but are deduced from empirical input spectra of turbulent kinetic energy and temperature variance.

As is evident from (15) and (16) ν(k) and p_T (k) are specified if E_e (k) and Et (k) are known.

The results for p(k) are presented in Figure 4. In neutral and stable stratification p(k) decrease monotonically. However the decrease at small wave numbers, say $k < 3 \cdot 10^{-3}$ cm^{-1}, is almost unnoticable and p(k) is approximately equal to p(0) for this whole ränge. Under neutral conditions it is easy to see from Equation (15) that

$$p(0) = K_m = \frac{\epsilon}{\left(\frac{\partial \bar{U}_1}{\partial z}\right)^2} \qquad (55)$$

In general, the level of the plateau of r(k) is determined by the thermal stratification. At larger wave numbers all curves converge into one single curve, where the slope is in a small region close to - 4/3 but then it decreases more rapidly caused by the exponential decrease of the spectral density.

For comparison, p(k) has also been determined from Heisenberg's formula

$$y(k) = a_H \int_k^\infty \sqrt{\frac{E_E(k')}{k'^3}}\, dk' \qquad (55)$$

where $a_H = 0.5$. E_e (k) was introduced according to Eqaution (27) and (29). The results for different stabilities z/L* = 0.5, 0, - 0,5, - 2.0 are summarized graphically in Figure 5. It may be noticed that both results exhibit in the inertial subrange a - 4/3 slope but deviate substantially at smaller wave numbers. The curves for p_t (k) display much the same shape as the neutral and stable curves of ν(k) as shown in Figure 6. Only the plateau at small wave numbers is somewhat narrower for p_T (k) and it Starts to decrease at smaller wave numbers than p(k). The most interesting features of p(k) are shown up under unstable conditions, when it is dominated by an increasing peak with decreasing stability at wave numbers 0.001 cm^{-1} to 0.01 cm^{-1}, This may be explained from the different locations of the maximum of spectral density for turbulent energy and temperature variance. The maximum of Ej (k) occurs at substantially smaller k than that of E_e (k). In unstable stratification ($9\bar{T}/9z < 0$) the second term in Equation (15) is negative.

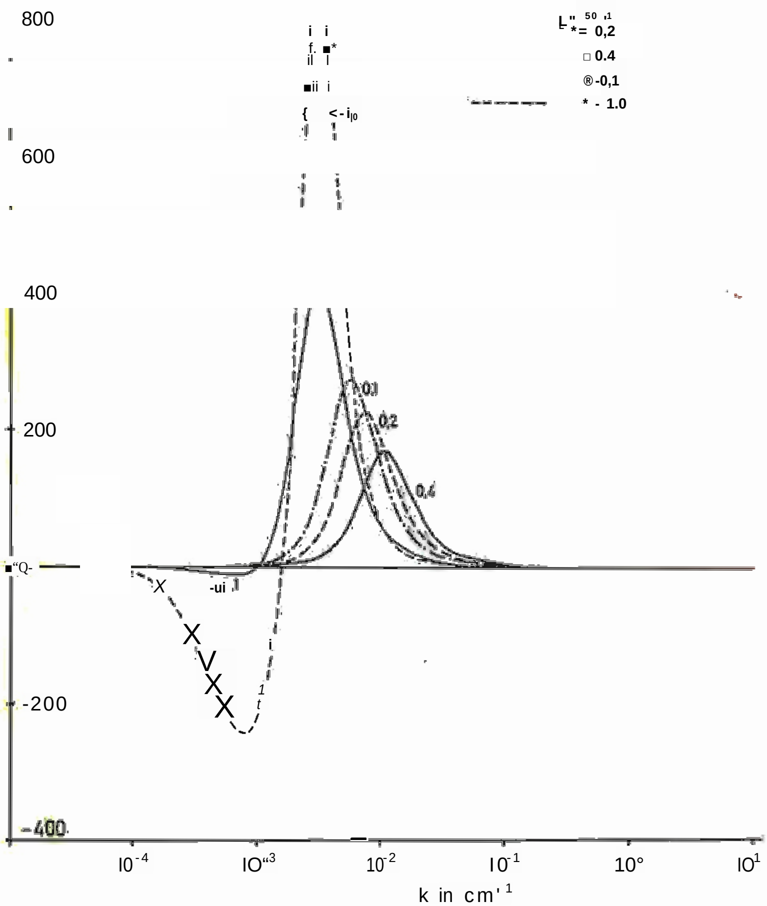
The shift of the maximum of the spectral density is determined through the given empirical data for φ_e (z/L*). These data were chosen from KAIMAL et al. (1972) and show a minimum at z/L* = 0.

A similar plot shows the spectral density of temperature variance (Figure 3). Since we did not normalize these spectra by the dissipation of temperature variance x the temperature spectra do not appear as a single curve at the high wave number ränge. The position of the maximum in these plots are again a direct consequence of the empirical functions for f_m (z/L*) and Y_m (z/L*) as given by PANOFSKY (1969).

The composite spectra show both a - 5/3 ränge which extends for the temperature spectrum to lower wave numbers compared to the turbulent kinetic energy spectrum.

However, its magnitude decreases more rapidly at small wave numbers with increasing k than the first term because E_T (k) exhibits a steeper positive slope than E_e (k).

Physically, this may be interpreted as a counteraction of thermal induced turbulence to shear produced turbulence at small wave numbers. Strong convection tends to produce organized motions and destroy favorable conditions for shear induced turbulence. Fürther consequences of this maximum of p(k) will be discussed in the following section.



• Figure 8

Cospectra of the momentum flux (linear-log-plot) for z/L* values ranging from 0.4 to -1.0

• Bild 8

Kospektren für den Impulsfluß (linear-log-Darstellung) für z/L* Werte von 0.4 bis - 1.0

3.3. Cospectra $\Phi_{u_1u_3}(k)$ and $\Phi_{\theta u_3}(k)$. Spectral densities of Transfer of energy and temperature variance

We recall again our procedure of solving the equations for the energy and temperature spectra. Instead of using one of the well-known assumptions for the spectral eddy viscosity and thermal diffusivity we constructed three-dimensional spectra for the turbulent energy and temperature variance from measured one-dimensional spectra. Therefore we find as a result eddy viscosity, thermal diffusivity and the cospectra of vertical momentum flux and vertical heat flux. These cospectra will now be discussed.

Figure 7 shows the computed cospectra of momentum flux on a log-log plot for different stabilities. The cospectra agree with a - 7/3 power law (- 4/3 for the logarithmic co-ordinate) in the inertial subrange. According to our procedure we may conclude that the measured power spectra of turbulent kinetic energy and temperature variance are in agrrement with a - 7/3 law in the momentum cospectra. Figure 8 shows the same spectra in a log-linear plot. Two interesting features may be noted. First, under stable conditions the maximum of spectral density shifts to smaller wave numbers with decreasing stability. For $z/L^* < 0$ the curves display no shift of the location where the maximum appears but an increasing maximum spectral density with decreasing z/L^*. Second, there is a very interesting behavior of the unstable cospectra at low wave numbers. For all negative z/L^* the cospectra reveal a sign reversal indicating an upward momentum flux caused by larger convective elements. Although the form of the

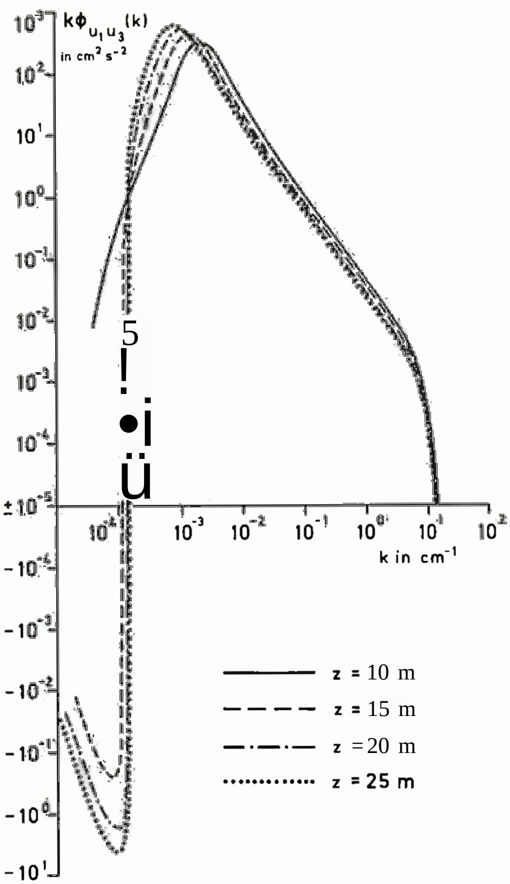


Figure 9

Cospectra of the momentum flux (log-log-plot) for $z/L^* = -0.025$ and z ranging from 10 to 25 m

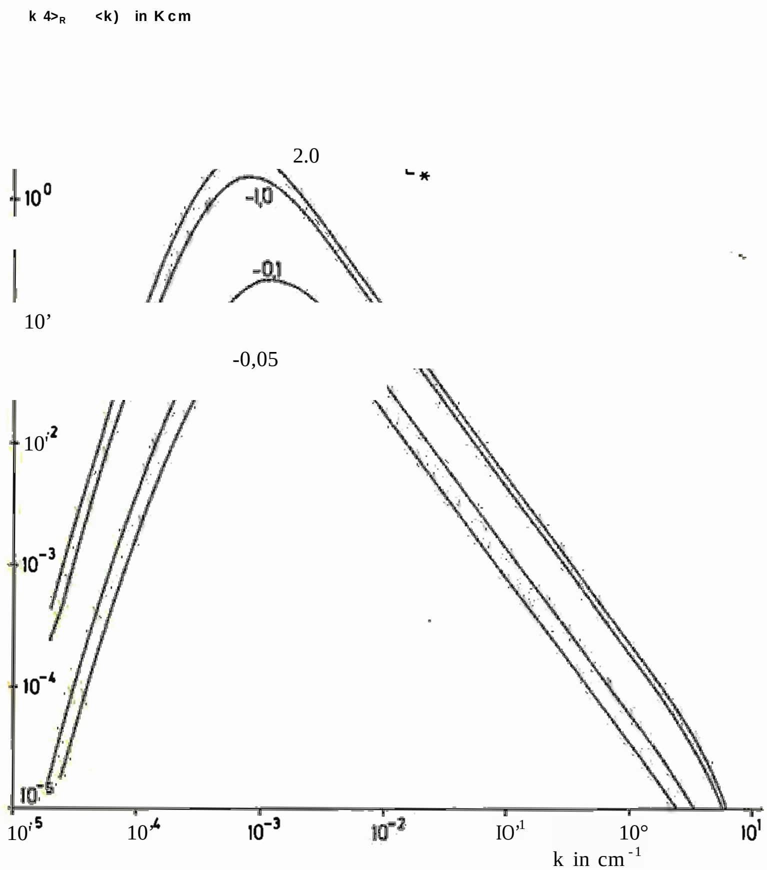
Bild 9

Kospektren für den Impulsfluß (log-log-Darstellung) für negative $z/L^* = -0.025$ und z von.10 bis 25 m

Spectrum may be uncertain in that region because of erratic estimates of the input power spectra this sign reverse seems to be a real fact. In the last years several authors have reported such a behavior of momentum flux spectra under unstable conditions. ZUBKOVSKY and KOPROV (1969) and WALK (1970) have published spectra of the vertical momentum flux. Their results showed a clear tendency to positive values at low wave numbers. The same behaviour was found by POND et al. (1971) and also by KAIMAL et al. (1972). KAIMAL et al. mentioned a tendency to positive values of the spectral momentum flux under unstable conditions for wave numbers $k < 1{,}5 \cdot 10^{-4}$ cm^{-1}. The results in Figure 8 indicate a sign reverse at wave numbers about $k = 10^{-3}$ cm^{-1}. This may be a consequence of the strong anisotropy of atmospheric turbulence at those scales. Since we used the energy spectrum of vertical velocity which is distinctly different in this ränge from the energy spectrum of the longitudimal component, we applied in an additional test one-dimensional spectra of the longitudinal and the vertical component and derived a three-dimensional spectrum from

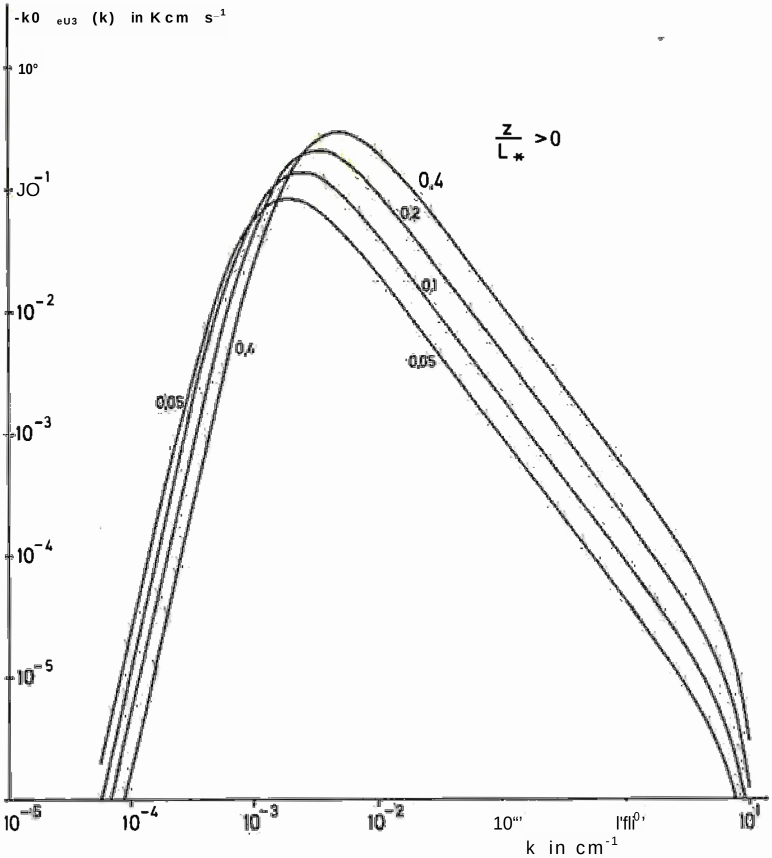
$$E_e(k) = -\frac{1}{2}\;\frac{\partial E_{1E}(k_1)}{\partial k_1}\Big|_{k_1}\;\frac{\partial E_{3E}(k)}{\partial k} \;. \qquad (56)$$

Using $E_{3E}(k)$ as given by BUSCH and PANOFSKY (1968) and the longitudinal spectrum $E_{1E}(k)$ from a paper by FICHTL and MCVEHIL (1969) a cospectrum was found with a sign reversal approximately at $k \approx 10^{-4}$ cm^{-1} (see Figure 9). This is in satisfactory agreement with observations.



- **Figure 10**
  Cospectra of the heat flux (log-log-plot) for negative z/L* values (unstable) ranging from - 0.05 to -2.0

- **Bild 10**
  Kospektren für den Wärmefluß (log-log-Darstellung) für negative z/L* Werte von - 0.05 bis - 2.0 (labile Schichtung)



- **Figure 11**
  Cospectra of the heat flux (log-log-plot) for positive z/L* values (stable) ranging from 0.05 to 0.4

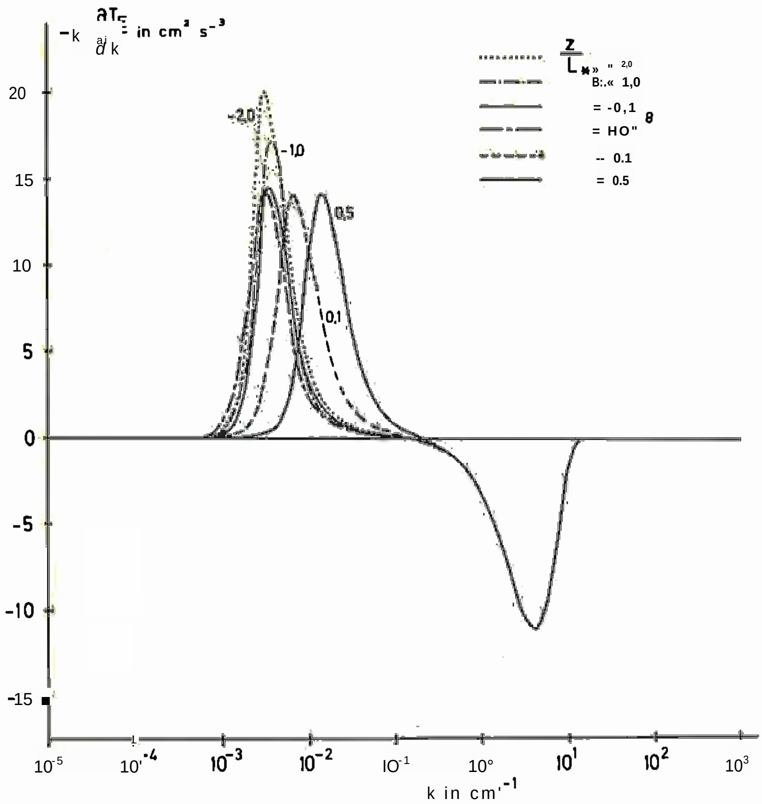
- Bild 11
  Kospektren für den Wärmefluß (log-log-Darstellung) für positive z/L* Werte von 0.05 bis 0.4 (stabile Schichtung)

Figures 10 and 11 show the computed cospectra for the vertical heat flux for unstable (Figure 10) and stable conditions (Figure 11). Again in the inertial subrange the heat flux cospectra 0_{0U3} (k) show a rather broad region with a - 7/3 shape which is supported by direct measurements by KAIMAL et al. (1972).

Under stable conditions the shift of the maximum point is more pronounced than with the unstable cospectra in much the same way as the cospectra of momentum flux. It may be noticed that heat at all scales is transported in the same direction.

Finally, we consider the spectral transfer functions for turbulent energy (Figure 12) and temperature variance (Figure 13). For large Reynolds numbers production and dissipation are separated wide enough in scale. Therefore negative transfer (at small wave numbers) may be interpreted also as a production spectrum and positive transfer as a dissipation Spectrum.

As a consequence of our input parameters with *e* being constant the transfer functions for different stabilitites converge to a single curve where it is identical to the dissipation spectrum. The area of the dissipation spectrum (positive values of transfer) is equal to the area at the negative side, the total production having the same value for all stabilities. The production region shows a systematic progression to smaller wave numbers with decreasing z/L* which breaks down as z/L* becomes negative. There is a tendency at the low wave number end of the transfer spectrum to decrease again with decreasing z/L* under unstable conditions so that production becomes confined to a narrow band in the spectrum. This



• **Figure 12**
Spectral transfer of turbulent kinetic energy for z/L* from 0.5 to - 2.0

• **Bild 12**
Spektraler Transport turbulenter kinetischer Energie für z/L* von 0.5 bis - 2.0

is a consequence of the behaviour of the cospectra of momentum and heat flux. Since the production of kinetic energy is given by

$$- k \frac{dP_E(k)}{3k} = \text{äÜ!} \{- k\, 0ui\, u_3(k)\} + g\, k\, 0_{\ddot{O}U3}(k),$$

the sign reversal of $0_{Uj\,U3}(k)$ at low wave numbers under unstable conditions causes a decrease of production of turbulent energy and narrows the production region.

From both Figures 12 and 13 the inertial subrange may be indentified as a region with small values of production and dissipation. The ideal assumption, often applied with similarity arguments of vanishing production and dissipation is only satisfied in a small region. The transfer function of temperature variance is qualitatively similar to the energy transfer except for the lack of a counteracting mechanism to production at low wave numbers. Naturally, the transfer increase with increasing z/L*.

4. Conclusion

The dynamical equations for the spectrum of turbulent kinetic energy and for temperature variance has been used to simulate the spectral transfer of kinetic energy and temperature variance. Since the production terms were parameterized by a relation to the spectrum of kinetic energy and temperature variance it was also possibele to describe the cospectra of vertical fluxes of momentum and heat.

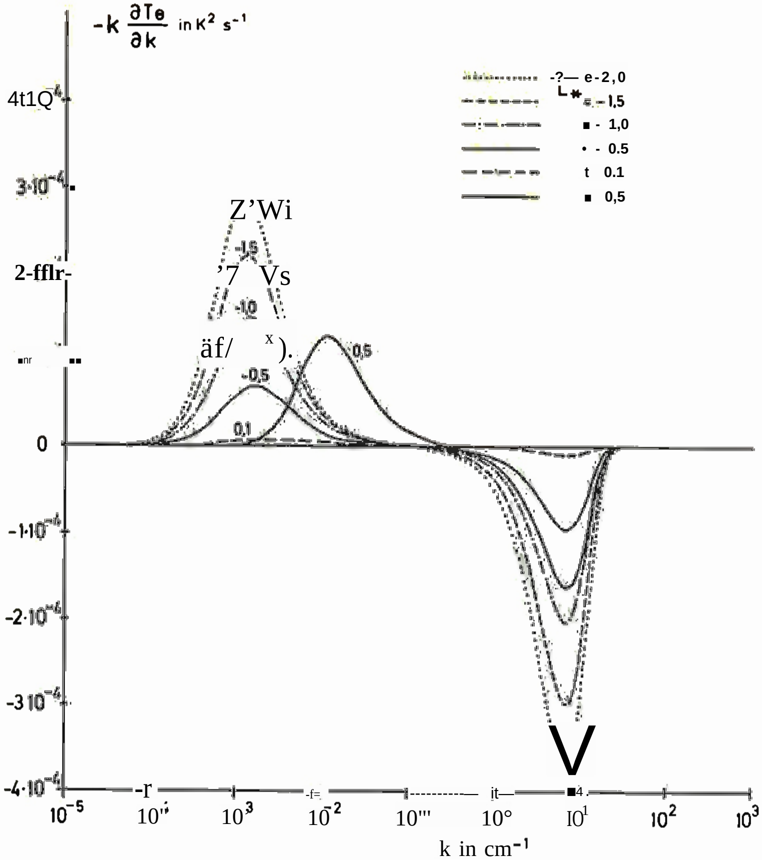


Figure 13
Spectral transfer of temperature variance for z/L* from 0.5 to -2.0

Bild 13
Spektraler Transport von Temperaturvarianz für z/L* von 0.5 bis -2.0

Several features of the spectral components are of primary interest. The cospectra derived via the elementary parameterization from empirical one-dimensional power spectra of turbulent energy and temperature variance exhibit displacements of the maximum spectral density as is also given in observed cospectra. This displacement is most pronounced in stable thermal conditions and disappears under unstable conditions. The momentum cospectra show a sign reversal at low wave numbers for negative z/L* which compares well with observations. The physical mechanism leading to an upward momentum transport at low wave numbers is, however, still not very clear. In the heat flux cospectra no sign reversal was found.

The spectral transfer is negative at low wave umbers and positive at high wave numbers, corresponding to a continual transformation of low wave number energy to high wave numbers by eddy Stretching. *

In the framework of the present model several assumptions had to be introduced, some of which are connected with isotropy. The results indicate however, that useful Information on the turbulence in the atmosphere may be derived.

In this way it is possible to derive a consistent picture of the different spectral components which are known from measurements. Therefore, it seems promissing to proceed with similar studies although it might be necessary to modify some of the introduced assumptions.

Appendix

We consider a stationary and homogeneous turbulent flow with constant gradients of the mean velocity $\overline{U}i$ (in the Xj direction; $\overline{U}_2 = \overline{U}_3 = 0$) and of mean temperature $\overline{T}$ in the vertical, i.e. $d\overline{U}i/dx_3$ = const and $d\overline{T}/dx_3$ = const. A Cartesian coordinate System (xj, x_2, x_3) is used with x_3 increasing vertically upward.

Applying Boussinesq's approximation (LUMLEY and PANOFSKY, 1964) to velocity fluctuations Ui = Uj - $\overline{U}i$ (i = 1, 2, 3) and to temperature fluctuations $6 = T - \overline{T}$ (an overbar denotes an average value) the following equations are obtained:

$$\frac{öui}{3t} + u_j \frac{3Ui}{3xj} + \overline{U}j \frac{3uj}{3xj} + u_j \frac{öui}{3xj} - m \frac{3ui}{3xj} = - \frac{i}{P}\frac{3p}{3x,} + v \frac{3^2 Ui}{3xj 3xj} + \frac{g}{T_o} 6 S_{3i} \tag{A1}$$

(i = 1,2,3)

$$\frac{30}{3t} + Uj \frac{3T}{3xj} + j \frac{3Ö}{3xj} \; Uj \frac{30}{3xj} \; U1 \frac{30}{3xj} \; \frac{3^2 0}{3xj 3xj} \tag{A2}$$

$$\frac{\partial u_i}{dXj} = 0 \qquad \text{(continuity equation)} \tag{A3}$$

where p, p, g, T_o, v and X are respectively pressure fluctuations, density, gravitational acceleration, mean temperature of the flow field, kinematic viscosity and molecular thermal conductivity.

The covariances Cn(ir) = u_{iA} Uj_B and C_Ty(ir) and $\overline{0_A \, 0_B}$ between velocities and temperatures at two points A and B a distance ir = $(r_{1s}\, r_2, r_3) = \overrightarrow{AB}$ apart are then given by (HINZE, 1959)

$$\frac{3}{\partial t} Cü(ir) = 0 = - \frac{d\overline{U}j}{} \left[2C_{i3}(ir) + r_3 \frac{313}{\partial r_1} C_{ii}(ir) \right] - \{S_{i>ki}(ir) - S_{ik>i}(ir)\} \tag{A4}$$

$$+ 2 \, \frac{3^2}{äÄ} \, c_{«}(r) + 2 \frac{g}{f;} T_{«.} 3(,r)$$

$$\frac{3}{ät} Ctt\,(ir) = 0 = \frac{r}{l?} \frac{3T}{äT,} T_{''} 3(lr) + 3Ü. \; 3 \frac{a}{äT,} Ctt\,(,r) 1$$

$$\frac{a}{3r_k} [Sfl,kö(,^r) \; S0_k,0\,(ir)] + 2X \frac{a^2}{dr_k 3r_k} C_{TT}\,(ir) \tag{A5}$$

where

Si,ki(^{lr}) - ^{u} iA ^{u} kB ^{u} iB , Sj_{kj} i(ir) - U_{iA} U_{kA} U_{iB} , (ir) - $\overline{0_A \, U_{3B}}$

and

S«,ko ($,^r$) ~ $\overline{0_A \, u_{kB} \, 0_B}$, $S_{0k>ö}$ (ir) - $\overline{0_A \, u_{kA} \, 0_B}$

Introducing the Fourier transform of the covariance fuctions and averaging in the wave-number space over spherical surfaces of the radius k = lIkI = $\sqrt{k^2_{} + k_{} + k_{}}$ we find the dynamic equation for the spectrum of kinetic energy Ee (k) and of one half of the temperature variance Ep (k):

$$\frac{3}{} E_e\,(k,t) = 0 = - 0_{U]u},(k) \frac{3\overline{U},}{} + F_e\,(k) - 2pk^2 E_e\,(k) + \frac{g}{} 0_{8u},(k) \tag{A6}$$

$$\frac{}{\partial t} Et\,(k,t) = 0 = - 0_{8U3}(k) \; jl- + F_t\,(k) - 2Xk^2 Et\,(k) \tag{A7}$$

0m u_3 (k), 0öu_3 (k), Fg (k) and F_T (k) are respectively the spectrum of the vertical momentum flux UiU_3, vertical temperature flux 0u_3, spectral transfer of one half of temperature variance. In addition, $2 v k^2 E_e$ (k) and $2 X k^2 E_T$ (k) give the spectral distribution of viscous energy and temperature dissipation.

By integrating over all wave-numbers one obtains

$$E = J_0^{00} Eß\,(k)\,dk \qquad | \; Ä^2 = J_0^{00} E_T\,(k)\,dk$$

$$uTu; = J_0^{00} \langle\!\rangle_{U1\,U3}\,(k)\,dk \qquad 0u_3 = J_0^{00} 0_{0U3}\,(k)\,dk$$

$$e = 2 v J_0^{00} k^2 E_e\,(k)\,dk \qquad X = 2 X J_0^{00} k^2 E_T\,(k)\,dk \tag{A8}$$

$$0 = \int_0^{\infty} F_E\,(k)\,dk \qquad 0 = \int_0^{\infty} F_t\,(k)\,dk$$

E is the mean turbulent kinetic energy, $\overline{0^2}$ temperature variance, Ü4Ü3the vertical momentum flux, $\overline{0u_3}$ vertical temperature flux, e the viscous dissipation of turbulent kinetic energy and x temperature dissipation.

Integrating Equation (A6) and (A7) both first over all k(0, °o) and subtracting the same equations integrated over the Intervall (0, k) leads to the budget equations of turbulent kinetic energy and temperature variance for wave-numbers from k to infinity (for the “smaller eddies”)

$$\frac{3}{} \int_k^{00} E_E\,(k)\,dk = 0 = - \int_0^{k} F_E\,(k)\,dk - \frac{\overline{3Ui}}{} \int_k^{00} 0_{U1\,U3}\,(k)\,dk + \frac{g}{} \int_k^{00} 0_{ÖU3}\,(k)\,dk - $$

$$- 2 v J_k^{00} k^2 E_e\,(k)\,dk \tag{A9}$$

$$\frac{\partial}{\partial t} J_k^{00} E_T\,(k)\,dk = 0 = j_0^{k} F_T\,(k)\,dk - |J_k^{00} [\; 0_{0U3}\,(k)\,dk - 2X j_k^{00} k^2 E_T\,(k)\,dk \tag{A10}$$

In accordance with the transfer model proposed by **HEISENBERG** (1948) we can write

$$T_E\,(k) = -J_0^{k} F_E\,(k)\,dk = 2\,p(k)\,J_0^{k} k^2 E_e\,(k)\,dk \tag{A11}$$

and

$$T_t\ (k) = - \int_0^k Ft\ (k)\,dk = 2 p_t\ (k) \int_0^k k^2 E_T\ (k)\,dk \qquad (A12)$$

where the spectral eddy viscosity p(k) and the spectral thermal diffusivity p_t (k) have been introduced.

For the terms $\int_k^\infty 0_{U1\,U3}(k)\,dk$ and $\int_k^\infty 0_{\ddot{O}U3}(k)\,dk$ we follow the assumptions introduced by TCHEN (1953, 1954)

$$- \int_k^\infty 0_{U1}\, u_3\,(k)\,dk = p^*(k)\, \frac{\partial \overline{U}_1}{\partial x_3} \qquad (A13)$$

and by MONIN(1962)

$$- \int_k^\infty 0\ddot{o}u_3\,(k)\,dk = Py\,(k)\, \frac{\partial \overline{T}}{\partial x_3} \qquad (A14)$$

with new spectral coefficients p(k) and Py(k). TCHEN (1954) concluded from a physical point of view that the diffusivities appearing in the production term in the region (k, oo), i.e.

$- \int_k 0_{U1\,U3}(k)\,dk$ and the ransfer term should be equal. Since passive contaminents of the atmosphere such as temperature fluctuations are carried by velocity fluctuations, it seems reasonable to choose the corresponding assumptions for the temperature variance budget. Therefore we use

$$p(k) = p^*(k) \qquad (A15)$$

$$pt\ (k) = p|\ (k) = a\,(k)\, p(k) \qquad (A16)$$

where a(k) may be considered as a spectral reciprocal Prandtl number. This leads to our final model equations

$$e - 2p \int_0^k k^2 E_e\ (k)\,dk = 2v(k) \int_0^k k^2 E_e\ (k)\,dk + v(k) \left[\frac{\partial \overline{U}_1}{\partial x_3} - «\,(k)\, \frac{g}{T_o}\, \frac{\partial \overline{T}}{9x_3} \right] \qquad (A17)$$

$$X - 2X \int_0^k k^2 E_t\ (k)\,dk = 2a\,(k)\, v(k) \int_0^k k^2 E_t\ (k)\,dk + a(k)\ v(k) \left(\frac{\partial \overline{T}}{\partial x_3} \right)^2 \qquad (A18)$$

with the definition of dissipation of kinetic energy $e = 2p \int_0^\infty k^2 E_e\ (k)\,dk$ and dissipation of temperature variance

$$X = 2X \int_0^\infty k^2 E_T\ (k)\,dk.$$

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