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The dynamics of within-group and between-group interaction

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Abstract

The article strengthens and provides a dynamic extension of the theory on collective rent seeking and private provision of a public good. Each individual agent within each group chooses in continuous or discrete time a continuous or discrete effort level. The combined effort within each group provides within-group public goods which are used as an input in the between-group n-group competition for an external prize. Intergroup mobility and intergroup warfare are allowed for. Each group and each individual agent within each group get a fraction of the prize based on a linear combination of equity and relative effort. A model/algorithm is developed generating analytical results and simulations illustrating how the interaction within and between groups proceeds through time.

JEL classification: C72; C73; D72

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1. Introduction

The article strengthens and provides a dynamic extension of the theory on rent seeking, pioneered by Tullock (1967, 1980), supplemented by Katz et al. (1990) and Nitzan (1991). A model/algorithm is developed generating analytical results and simulations illustrating the interaction within and between groups of generally different and arbitrary size, allowing for intergroup mobility and intergroup warfare. The means employed in the between-group n-group competition are public goods generated by within-group interaction in each group.

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Each individual agent A_{ij} within each group A_i, $i = 1, \dots, n$, chooses at time t a cost S_{ij} of effort, $0 \leq S_{ij} \leq c_i$. The combined cost S_i of effort within group A_i provides within-group public goods which are used in the between-group n-group competition for an external prize E. Each group A_i and each individual agent A_{ij} within each group get a fraction of the prize E based on a linear combination of equity and relative effort.

Section 2 presents the static model involving within-group and between-group interaction. Section 3 provides static equilibrium analysis. Section 4 develops an eight-step continuous and discrete time algorithm for an arbitrary number n of groups A_i each consisting of an arbitrary number $n_i(t)$ of individual agents A_{ij}. Section 5 presents the dynamic model assuming fixed group sizes $n_i(t)$, and employs it to a continuous and discrete strategy within-group $n_i(t)$-person prisoner's dilemma in each group A_i, accounting for between-group n-group competition for an external prize E. A simulation allowing the two strategies 'always cooperate' and 'always defect' for two groups is performed. Section 6 presents the dynamic model assuming intergroup mobility, and performs simulations. Section 7 presents the dynamic model assuming intergroup warfare, illustrating with simulations. Section 8 considers the combined operation of intergroup mobility and intergroup warfare, and provides simulations. Section 9 introduces the $n_i(t)$-person reactive strategies 'tit-for-tat' and 'bully' and gives simulations for one group in isolation.

2. Static model

Assume n groups A_i, $i = 1, \dots, n$, with $n_i(t) = n_i$ individual agents A_{ij}, $j = 1, \dots, n_i$, in each group (Fig. 1). Each individual agent A_{ij} chooses a strategy S_{ij}, by which we mean that he has a cost S_{ij} of effort, $0 \leq S_{ij} \leq c_i$, so that each group A_i has a cost of effort

$$S_i \overset{\text{def}}{=} \sum_{j=1}^{n_i} S_{ij}, \tag{2.1}$$

where $0 \leq S_i \leq n_i c_i$. All the n groups A_i have a cost of effort

$$S \overset{\text{def}}{=} \sum_{i=1}^{n} S_i = \sum_{i=1}^{n} \sum_{j=1}^{n_i} S_{ij}, \tag{2.2}$$

where $0 \leq S \leq \sum_{i=1}^{n} n_i c_i$.

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Level 2   ____A1____    ____A2____...............____An____
Level 1   A11...A1n1    A21...A2n2               An1...Ann_n
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Fig. 1. Hierarchy represented by matrix A.

Each group A_i is involved in a within-group game which determines the provision of a public good that is used as an input in the competition for a between-group prize, E, against the other groups. Let an effort of c_i in group A_i produce within-group public goods $B_i(t) = B_i$. An effort of S_{ij} by an individual agent A_{ij} thus generates within-group public goods $B_i S_{ij}/c_i$, and an effort S_i by group A_i generates within-group public goods $B_i S_i/c_i$. Interesting cases of B_i are when B_i is constant and when B_i is proportional to n_i. B_i being constant means that the public goods produced by a cooperative act c is divided between the group members A_{ij}, giving a smaller share to each as the group size n_i increases. B_i being proportional to n_i means that the benefits reaped by one individual group member do not reduce the benefits received by another group member.

The between-group n-group competition for the prize E gives a fraction f_i, i.e. $f_i E$ to group A_i, and a fraction f_{ij} of this, i.e. $f_{ij} f_i E$ to individual agent A_{ij}. The one-period (static) payoff P_i to each group A_i is thus

$$P_i \overset{\text{mod}}{=} \frac{S_i}{c_i} B_i + f_i E - S_i, \tag{2.3}$$

where mod means that this is a model of nature. (2.3) says that group A_i has a total cost S_i of effort, which generates a fraction $S_i/n_i c_i$ of the within-group public goods $n_i B_i$, and a fraction f_i of the between-group prize E. Similarly, the one-period payoff P_{ij} to each individual agent A_{ij} is

$$P_{ij} \overset{\text{mod}}{=} f_{ij} \left[\frac{S_i}{c_i} B_i + f_i E \right] - S_{ij}. \tag{2.4}$$

A variety of different distribution rules f_{ij} and f_i can be considered. Nitzan (1991, p. 1524) uses the rule

$$f_{ij} \overset{\text{mod}}{=} \frac{(1 - a_i) S_{ij}}{S_i} + \frac{a_i}{n_i}, \tag{2.5}$$

which means that a proportion a_i is distributed within group A_i on egalitarian grounds (i.e. equal portion to each) and the remaining proportion $1 - a_i$ is distributed according to the relative amount of public goods produced by each individual agent A_{ij}, which corresponds to the relative effort of each individual agent A_{ij}. Analogously for the between-group distribution, set

$$f_i \overset{\text{mod}}{=} \frac{(1 - a) \dfrac{B_i}{c_i} S_i}{\sum_{i=1}^{n} \dfrac{B_i}{c_i} S_i} + \frac{a}{n}, \tag{2.6}$$

where a proportion a is distributed between the n groups A_i on egalitarian grounds and the remaining proportion $1-a$ is distributed according to the relative amount of public goods produced by each group A_i. An alternative to (2.6) is

$$f_1 = \ldots = f_m \overset{\text{mod}}{=} 1/m,\ f_{m+1} = \ldots = f_n \overset{\text{mod}}{=} 0 \tag{2.7}$$

where $S_1 = \ldots = S_m > S_{m+1} \geq \ldots \geq S_n$ and $1 \leq m \leq n$, which means that the group (assume A_1 without loss of generality) producing the highest effort S_1 gets the entire between-group prize E, or, if m groups equally produce the highest effort, that these equally share the prize E among themselves. In the remainder of the article I assume (2.6). Inserting (2.5) and (2.6) into (2.4) gives

$$\begin{aligned} P_{ij} &= \left[\frac{(1-a_i)S_{ij}}{S_i} + \frac{a_i}{n_i}\right]\left[\frac{S_i}{c_i}B_i + \left(\frac{(1-a)\frac{B_i}{c_i}S_i}{\sum_{i=1}^{n}\frac{B_i}{c_i}S_i} + \frac{a}{n}\right)E\right] - S_{ij} \\ &= \frac{(1-a_i)B_i}{c_i}S_{ij} + \frac{a_iB_i}{n_ic_i}S_i + (1-a_i)E\frac{(1-a)\frac{B_i}{c_i}S_{ij}}{\sum_{i=1}^{n}\frac{B_i}{c_i}S_i} \\ &\quad + (1-a_i)E\frac{a}{n}\frac{S_{ij}}{S_i} + \frac{a_iE}{n_i}\frac{(1-a)\frac{B_i}{c_i}S_i}{\sum_{i=1}^{n}\frac{B_i}{c_i}S_i} + \frac{a_iE}{n_i}\frac{a}{n} - S_{ij}. \end{aligned} \tag{2.8}$$

3. Static equilibrium analysis

The first order condition for a global interior maximum of P_{ij} for individual agent A_{ij} is

$$\begin{aligned} \frac{\partial P_{ij}}{\partial S_{ij}} &= \frac{(1-a_i)B_i}{c_i} + \frac{a_iB_i}{n_ic_i} + (1-a_i)E(1-a)\frac{B_i}{c_i}\left(\frac{\left(\sum_{i=1}^{n}\frac{B_i}{c_i}S_i\right) - \frac{B_i}{c_i}S_{ij}}{\left(\sum_{i=1}^{n}\frac{B_i}{c_i}S_i\right)^2}\right) \\ &\quad + (1-a_i)E\frac{a}{n}\left(\frac{S_i - S_{ij}}{S_i^2}\right) + \frac{a_iE}{n_i}(1-a)\frac{B_i}{c_i}\left(\frac{\left(\sum_{i=1}^{n}\frac{B_i}{c_i}S_i\right) - \frac{B_i}{c_i}S_i}{\left(\sum_{i=1}^{n}\frac{B_i}{c_i}S_i\right)^2}\right) \\ &\quad - 1 = 0. \end{aligned} \tag{3.1}$$

In the remainder of the article I set $a_i = 1$ and $a = 0$, which means that the payoff available to each group A_i is divided equally among its members A_{ij}, and that each group A_i gets a fraction of the between-group prize E proportional to the relative amount of public goods it produces compared with the other groups. (3.1) then becomes

$$E\frac{B_i}{c_i}\left(\frac{\left(\sum_{i=1}^{n}\frac{B_i}{c_i}S_i\right)-\frac{B_i}{c_i}S_i}{\left(\sum_{i=1}^{n}\frac{B_i}{c_i}S_i\right)^2}\right)=n_i-\frac{B_i}{c_i}. \tag{3.2}$$

Setting $B_i = B$, $c_i = c$, and summing both sides in (3.2) from $i = 1$ to $i = n$, i.e. $\sum_{i=1}^{n}(\cdot)$, gives

$$E\frac{S(n-1)}{S^2}=\left(\sum_{i=1}^{n}n_i\right)-\frac{nB}{c}, \tag{3.3}$$

i.e.

$$S=\frac{(n-1)E}{\left(\sum_{i=1}^{n}n_i\right)-\frac{nB}{c}}. \tag{3.4}$$

If we assume that all the groups A_i are equally large, i.e. $n_1 = n_2 = \ldots = n_n$, and that identical individual agents A_{ij} in a symmetric Nash equilibrium choose the same effort level S_{ij}, which implies $S = nS_i = nn_iS_{ij}$, (3.4) gives that

$$S_{ij}=\frac{(n-1)E}{nn_i\left(nn_i-\frac{nB}{c}\right)}=\frac{(n-1)Ec}{n^2n_i^2\left(c-\frac{B}{n}\right)}. \tag{3.5}$$

Checking the border conditions $0 \le S_{ij} \le c$, (3.5) shows that the former implies $c > B/n_i$ and the latter implies

$$E\le\frac{n^2n_i^2}{(n-1)}\left(c-\frac{B}{n_i}\right). \tag{3.6}$$

Inserting $S_{ij} = c$ and (3.5) into (2.8) with $a_i = 1$ and $a = 0$ shows that the former maximizes P_{ij} when (3.6) is not satisfied. That is, when E is above that level given by (3.6), it is beneficial for an individual agent A_{ij} to limit his effort to $S_{ij} = c$ rather than choose the higher S_{ij} according to (3.5).

If we assume $0 < B_i/n_i < c_i < B_i$, the within-group game is a 'continuous' version of an n_i-person prisoner's dilemma, in the sense that each individual agent A_{ij} can choose any effort level S_{ij}, i.e. degree of cooperation, on a continuous scale from 0 to c_i.

If we assume the two discrete strategies cooperation c (effort level $S_{ij}=c_i$) and defection d (effort level $S_{ij}=0$), we get the standard (discrete) n_i-person prisoner's dilemma. Define $h_{ic}-1$ as the number of cooperating individual agents aside from individual agent A_{ij} in group A_i. If individual agent A_{ij} chooses $S_{ij}=c_i$, giving h_{ic} cooperators in group A_i, he gets according to (2.8) with $a_i=1$ and $a=0$, a payoff

$$P_{ijc}=\frac{1}{n_i}\left[h_{ic}B_i+\frac{h_{ic}B_i}{\sum_{i=1}^{n}h_{ic}B_i}E\right]-c_i \text{ if } S_{ij}=c_i. \tag{3.7}$$

If individual agent A_{ij} chooses $S_{ij}=0$, giving $h_{ic}-1$ cooperators in group A_i, he gets according to (2.8) with $a_i=1$ and $a=0$, a payoff

$$P_{ijd}=\frac{1}{n_i}\left[(h_{ic}-1)B_i+\frac{(h_{ic}-1)B_i}{\left(\sum_{i=1}^{n}h_{ic}B_i\right)-B_i}E\right] \text{ if } S_{ij}=0. \tag{3.8}$$

When the between-group prize $E=0$, the unique Nash equilibrium for each individual agent A_{ij} in group A_i is the well-known mutual defection $S_{ij}=0$. As E increases, a minimum value of E can be determined which guarantees cooperation $S_{ij}=c_i$ by all the n_i individual agents A_{ij} in the n groups A_i. The most restrictive requirement for E, to ensure cooperation by all the individual agents A_{ij}, is derived by assuming there are exclusively cooperators in the n groups, and that one individual agent A_{ij} in group A_i contemplates whether to defect. Hence set $h_{ic}=n_i$ in (3.7) and (3.8). Cooperation by the individual agent A_i follows if (3.7) is larger than (3.8), i.e. if

$$\frac{1}{n_i}\left[n_iB_i+\frac{n_iB_i}{\sum_{i=1}^{n}n_iB_i}E\right]-c_i>\frac{1}{n_i}\left[(n_i-1)B_i+\frac{(n_i-1)B_i}{\left[\left(\sum_{i=1}^{n}n_iB_i\right)-B_i\right]}E\right], \tag{3.9}$$

i.e. if

$$E>\frac{\left(\sum_{i=1}^{n}n_iB_i\right)\left[\left(\sum_{i=1}^{n}n_iB_i\right)-B_i\right]}{\left[\left(\frac{1}{n_i}\sum_{i=1}^{n}n_iB_i\right)-B_i\right]B_i}\left[c_i-\frac{B_i}{n_i}\right], \tag{3.10}$$

where $n_i>0\ \forall i=1,\dots,n$, to avoid possible division with 0. That is, groups A_i without members A_{ij} are not included in (3.10), which is consistent with these not getting any fraction of the between-group prize E. Note from (3.10) that the requirement for E when $B_i = B$ is most strict for the largest group and least strict for the smallest group. Calculating the right-hand side of (3.10) $\forall i = 1, \ldots, n$, choose that i that generates the strictest requirement for E.

4. The algorithm

One common algorithm to determine the relative fitnesses of several strategies, when there is only one group, is employed by Martinez-Coll and Hirshleifer (1991), Maynard Smith (1982), Schuster and Sigmund (1985), Taylor and Jonker (1978), and Zeeman (1981). That algorithm makes five major simplifications:

1. It assumes only one group, in isolation/vacuum, not several groups, nor a hierarchy.
2. It assumes for each individual agent A_{ij} a finite number of discrete strategies S_{ij} rather than a continuous effort level S_{ij}, say $0 \le S_{ij} \le c_i$.
3. It does not specify group size, nor specific numbers of individual agents playing the various strategies. It specifies instead what fractions of the group play the various strategies. These fractions change through time dependent upon what strategies are most fit.
4. It assumes that the fractions play two-person games, not $n_i(t)$-person games.
5. It assumes that the reactive strategies, tit-for-tat and bully, have instantaneous response capability. For a two-person game this implies that a tit-for-tat player will be able to distinguish between pure cooperators and pure defectors after having met one pure defector once. This is because the tit-for-tat player will cooperate and be exploited the first time it meets a defector, the defector enjoying a free ride, while, in subsequent time periods, whenever a tit-for-tat player meets a defector, the tit-for-tat player will also defect.

Generalizing that algorithm, this section rectifies these five simplifications:

1. It assumes a two-level hierarchy A and an arbitrary number n of groups A_i.
2. It assumes a continuous and discrete time model. It posits for each individual agent A_{ij} a continuous strategy model with a continuous effort level S_{ij}, where $0 \le S_{ij} \le c_i$, and a discrete strategy model allowing for a finite number of discrete strategies S_{ij}.
3. It specifies the exact number $n_i(t)$ of individual agents A_{ij} in each group A_i at time t, and the exact fraction $p_i(s, t)$ or $p_{is}(t)$, and exact number $h_{is}(t)$ playing each strategy $s = S_{ij}$, in group A_i at time t.
4. It allows $n_i(t)$ to change through intergroup mobility, intergroup warfare, or analogs of reproduction and death of individual agents A_{ij} in each group A_i. The

changes $\mathrm{d}n_i(t)/\mathrm{d}t$ and $\mathrm{d}p_i(s,t)/\mathrm{d}t$ in $n_i(t)$ and $p_i(s,t)$ (or $p_{is}(t)$) depend upon the within-group $n_i(t)$-person games, and between-group n-group games.
5. It acknowledges in a discrete time multi-period game that the $n_i(t)$-person reactive strategies, tit-for-tat and bully, account for the conditions and strategies S_{ij} played both in the current time period t and the previous time period $t-1$.

The eight-step algorithm is as follows:

1. Set initial conditions: specify the number n of groups A_i, and the initial number $n_i(0)$ of individual agents A_{ij} in each group A_i at time $t=0$. Specify the strategy set S_{ij} of available strategies S_{ij} for each individual agent A_{ij}. For the continuous strategy model, define the function $p_i(s,t)$ as the fraction of individual agents A_{ij} that play the strategy $s=S_{ij}$, $0 \le s = S_{ij} \le c_i$ at time t, where t is continuous or discrete, $t=0, 1, 2, \ldots$. For the discrete strategy model, define the initial number $h_{is}(0)$ of individual agents A_{ij} playing each strategy S_{ij} at time $t=0$. Define

$$p_{is}(t) \overset{\text{def}}{=} \frac{h_{is}(t)}{n_i(t)} \tag{4.1}$$

as the fraction of individual agents A_{ij} in group A_i that chooses strategy $s=S_{ij}$ at time t. Observe that

$$\int_0^{c_i} p_i(s,t)\,\mathrm{d}s = 1, \quad \sum_{S_{ij}} p_{is}(t) = 1, \text{ and } \sum_{S_{ij}} h_{is}(t) = n_i(t). \tag{4.2}$$

2. Determine the mean yield $Y_i(s,t)$ (for continuous strategies $s=S_{ij}$, $0 \le s = S_{ij} \le c_i$) or $Y_{is}(t)$ (for discrete strategies S_{ij}) that individual agent A_{ij} in group A_i can expect to receive by playing strategy S_{ij} at time t, using (2.8) (continuous strategy version), (3.7) and (3.8) (discrete strategy version), or the model to be analyzed.
3. The overall mean yield $Y_i(t)$ for an individual agent A_{ij} in group A_i is

$$Y_i(t) \overset{\text{mod}}{=} \int_0^{c_i} p_i(s,t) Y_i(s,t)\,\mathrm{d}s \tag{4.3}$$

for the continuous strategy model and

$$Y_i(t) \overset{\text{mod}}{=} \sum_{S_{ij}} p_{is}(t) Y_{is}(t) \tag{4.4}$$

for the discrete strategy model.

The overall mean yield $Y(t)$ for an individual agent A_{ij} in any group A_i, $i = 1, \dots, n$, is

$$Y(t) \overset{\text{mod}}{=} \frac{1}{N(t)} \sum_{i=1}^{n} n_i(t) Y_i(t), \tag{4.5}$$

where $Y_i(t)$ is given by (4.3) or (4.4) and

$$N(t) \overset{\text{def}}{=} \sum_{i=1}^{n} n_i(t) \tag{4.6}$$

is the total number of individual agents A_{ij} in all the groups A_i, $i = 1, \dots, n$.
4. The relative fitness $F_i(s, t)$ for strategy S_{ij} in group A_i is

$$F_i(s, t) \overset{\text{def}}{=} Y_i(s, t) - Y_i(t) \tag{4.7}$$

for the continuous strategy model, substituting $F_i(s, t) \to F_{is}(t)$ and $Y_i(s, t) \to Y_{is}(t)$ for the discrete strategy model.
5. The change $\Delta p_i(s, t) = p_i(s, t+1) - p_i(s, t)$ in the fraction of individual agents A_{ij} in group A_i that play strategy S_{ij} is

$$\begin{aligned} p_i(s, t+1) - p_i(s, t) &\overset{\text{mod}}{=} k_i p_i(s, t) F_i(s, t) \\ &= k_i p_i(s, t) [Y_i(s, t) - Y_i(t)] \end{aligned} \tag{4.8}$$

for the continuous strategy model, substituting $p_i(s, t) \to p_{is}(t)$, $F_i(s, t) \to F_{is}(t)$, and $Y_i(s, t) \to Y_{is}(t)$ for the discrete strategy model. The change $\Delta p_i(s, t)$ is positive if the mean yield $Y_i(s, t)$ that individual agent A_{ij} can expect to receive by playing strategy $s = S_{ij}$ is larger than the overall mean yield $Y_i(t)$ that an individual agent A_{ij} in group A_i can expect to receive, and negative if the reverse is the case. k_i is the rapidity of change, or 'sensitivity' of the process. If k_i is too large we may get irregular movement of the process for group A_i. To ensure stability, k_i must be sufficiently small. If k_i is too small, there will be no change in the fraction. (4.8) is a difference equation. Assuming $\Delta t = 1$, it can be written as a differential equation, viz.

$$\frac{\mathrm{d} p_i(s, t)}{\mathrm{d} t} \overset{\text{mod}}{=} k_i p_i(s, t) [Y_i(s, t) - Y_i(t)] \tag{4.9}$$

for the continuous strategy model, with the above substitutions for the discrete strategy model.

The change $\mathrm{d} n_i(t)/\mathrm{d} t$ in the number $n_i(t)$ of individual agents A_{ij} in group A_i is given by

$$\frac{\mathrm{d} n_i(t)}{\mathrm{d} t} \overset{\text{mod}}{=} g(r_i, n_i(t), p_i(s, t), Y_i(s, t), Y_i(t), Y(t)), \tag{4.10}$$

for the continuous strategy model, with the above substitutions for the discrete strategy model, where r_i is the 'sensitivity' of the process for group A_i. Examples of functions $g(\cdot)$ are considered in Sections 6 (intergroup mobility) and 7 (intergroup warfare).
6. For the discrete time model, let $t+1 \to t$. For the discrete strategy model, calculate $h_{is}(t) = p_{is}(t)/n_i(t)$, and round off the specific number $h_{is}(t)$ of individual agents A_{ij} playing strategy S_{ij} to the nearest integer value:

$$h_{is}(t) \leftarrow \text{round}(h_{is}(t)). \tag{4.11}$$

7. Print out the relevant curves; $Y_i(s, t)$, $p_i(s, t)$, $h_i(s, t)$, $Y_{is}(t)$, $p_{is}(t)$, $h_{is}(t)$, etc.
8. Go to 2.

5. Dynamic model with fixed group sizes $n_i(t)$

Using the dynamic version of (2.8) to express (4.9) involves writing the effort level S_i for group A_i in terms of $p_i(s, t)$, accounting on the one hand for the effort level of the other $n_i(t) - 1$ individual agents in group A_i, and on the other hand for the specific effort level $S_{ij} = s$ chosen by the particular individual agent A_{ij}. Hence, referring to (2.1), S_i can be written as

$$S_i \overset{\text{def}}{=} (n_i(t) - 1)\int_0^{c_i} p_i(s, t)\, s\, \mathrm{d}s + s. \tag{5.1}$$

Inserting (5.1) into (2.8) with $a_i = 1$ and $a = 0$ gives

$$\begin{aligned} Y_i(s, t) = {} & \left[\left(1 - \frac{1}{n_i(t)}\right)\frac{1}{c_i}\int_0^{c_i} p_i(s, t)\, s\, \mathrm{d}s + \frac{s}{n_i(t) c_i}\right] B_i(t) \\ & + \left\{\left[\left(1 - \frac{1}{n_i(t)}\right)\frac{1}{c_i}\int_0^{c_i} p_i(s, t)\, s\, \mathrm{d}s + \frac{s}{n_i(t) c_i}\right] B_i(t)\, E\right\} \\ & \times \left\{\left[\sum_{i=1}^{n} \frac{B_i(t)}{c_i} n_i(t) \int_0^{c_i} p_i(s, t)\, s\, \mathrm{d}s\right]\right. \\ & \left. + \left(\frac{s}{c_i} - \frac{1}{c_i}\int_0^{c_i} p_i(s, t)\, s\, \mathrm{d}s\right) B_i(t)\right\}^{-1} - s. \end{aligned} \tag{5.2}$$

Inserting (4.3) and (5.2) into (4.9) gives

$$\begin{aligned}
&\frac{\mathrm{d}p_i(s,t)}{\mathrm{d}t}\\
&= k_i p_i(s,t)\Bigg[\Bigg[\Bigg(1-\frac{1}{n_i(t)}\Bigg)\frac{1}{c_i}\int_0^{c_i} p_i(s,t)s\,\mathrm{d}s+\frac{s}{n_i(t)c_i}\Bigg]B_i(t)\\
&\quad+\Bigg\{\Bigg[\Bigg(1-\frac{1}{n_i(t)}\Bigg)\frac{1}{c_i}\int_0^{c_i} p_i(s,t)s\,\mathrm{d}s+\frac{s}{n_i(t)c_i}\Bigg]B_i(t)E\Bigg\}\\
&\quad\times\Bigg\{\Bigg[\sum_{i=1}^{n}\frac{B_i(t)}{c_i}n_i(t)\int_0^{c_i} p_i(s,t)s\,\mathrm{d}s\Bigg]\\
&\quad+\Bigg(\frac{s}{c_i}-\frac{1}{c_i}\int_0^{c_i} p_i(s,t)s\,\mathrm{d}s\Bigg)B_i(t)\Bigg\}^{-1}-s\\
&\quad-\int_0^{c_i} p_i(s,t)\Bigg(\Bigg[\Bigg(1-\frac{1}{n_i(t)}\Bigg)\frac{1}{c_i}\int_0^{c_i} p_i(s,t)s\,\mathrm{d}s+\frac{s}{n_i(t)c_i}\Bigg]B_i(t)\\
&\quad+\Bigg\{\Bigg[\Bigg(1-\frac{1}{n_i(t)}\Bigg)\frac{1}{c_i}\int_0^{c_i} p_i(s,t)s\,\mathrm{d}s+\frac{s}{n_i(t)c_i}\Bigg]B_i(t)E\Bigg\}\\
&\quad\times\Bigg\{\Bigg[\sum_{i=1}^{n}\frac{B_i(t)}{c_i}n_i(t)\int_0^{c_i} p_i(s,t)s\,\mathrm{d}s\Bigg]\\
&\quad+\Bigg(\frac{s}{c_i}-\frac{1}{c_i}\int_0^{c_i} p_i(s,t)s\,\mathrm{d}s\Bigg)B_i(t)\Bigg\}^{-1}-s\Bigg)\,\mathrm{d}s\Bigg],
\end{aligned}\tag{5.3}$$

where $i = 1, 2, \ldots, n$ and $0 \leq s \leq c_i$. (5.3) is an integro-differential equation which is most easily solved using the explicit method (i.e. rewriting with (4.8)) for the time integration, and a standard numerical method for the strategy integrals.

To derive the analog of (5.3) for discrete strategies S_{ij}, set

$$p_i(s,t) \overset{\text{mod}}{=} \sum_{S_{ij}} p_{is}(t)\,\delta(s-S_{ij}) \underset{\substack{S_{ij}=c_i\\ S_{ij}=0}}{=} p_{ic}(t)\,\delta(s-c_i)+p_{id}(t)\,\delta(s), \tag{5.4}$$

where the rightmost two terms give the special case where only the two strategies $S_{ij} = c_i$ and $S_{ij} = 0$ are allowed, and where δ is Dirac's delta 'function' defined by

$$\int_{-\infty}^{\infty} p_i(s,t)\,\delta(s-S_{ij})\,\mathrm{d}s \overset{\text{def}}{=} p_i(S_{ij},t), \text{ and } \delta(s-S_{ij}) \overset{\text{def}}{=} 0 \text{ for } s \neq S_{ij}. \tag{5.5}$$

With the two discrete strategies cooperation c (effort level $S_{ij} = c_i$) and defection d (effort level $S_{ij} = 0$), where $0 < B_i(t)/n_i(t) < c_i < B_i(t)$, the within-group game is the standard (discrete) $n_i(t)$-person prisoner's dilemma. Inserting (5.4) into (5.3) and simplifying gives the dynamic model for discrete strategies S_{ij}, i.e.

$$\begin{aligned}
&\frac{\mathrm{d}p_{ic}(t)}{\mathrm{d}t}\\
&= k_i p_{ic}(t)(1-p_{ic}(t))\Bigg(\frac{B_i(t)}{n_i(t)} - c_i\\
&\quad + \left\{\left(\left(1-\frac{1}{n_i(t)}\right)p_{ic}(t) + \frac{1}{n_i(t)}\right)B_i(t)E\right\}\\
&\quad \times \left\{\left[\left(\sum_{i=1}^{n} p_{ic}(t)n_i(t)B_i(t)\right) + (1-p_{ic}(t))B_i(t)\right]\right\}^{-1}\\
&\quad - \left\{\left(1-\frac{1}{n_i(t)}\right)p_{ic}(t)B_i(t)E\right\}\\
&\quad \times \left\{\left[\left(\sum_{i=1}^{n} p_{ic}(t)n_i(t)B_i(t)\right) - p_{ic}(t)B_i(t)\right]\right\}^{-1}\Bigg)\\
&= k_i p_{ic}(t)(1-p_{ic}(t))\Bigg(\frac{B_i(t)}{n_i(t)} - c_i\\
&\quad + \left\{\left[\left(\frac{1}{n_i(t)}\sum_{i=1}^{n} p_{ic}(t)n_i(t)B_i(t)\right) - p_{ic}(t)B_i(t)\right]B_i(t)E\right\}\\
&\quad \times \Bigg\{\left[\left(\sum_{i=1}^{n} p_{ic}(t)n_i(t)B_i(t)\right) + (1-p_{ic}(t))B_i(t)\right]\\
&\quad \times \left[\left(\sum_{i=1}^{n} p_{ic}(t)n_i(t)B_i(t)\right) - p_{ic}(t)B_i(t)\right]\Bigg\}^{-1}\Bigg),
\end{aligned} \tag{5.6}$$

where $i = 1, 2, \ldots, n$, and $p_{id}(t) = p_{ic}(t) - 1$. (5.6) has a very nice interpretation. Within the rightmost parentheses, the first two terms $B_i(t)/n_i(t) - c_i$ describe the influence of the within-group $n_i(t)$-person prisoner's dilemma, and have a negative effect upon $\mathrm{d}p_{ic}(t)/\mathrm{d}t$ to a degree corresponding to the 'severity' of the within-group $n_i(t)$-person prisoner's dilemma. Note that if $c_i < B_i(t)/n_i(t)$, the within-group game is a pure cooperation game. If $c_i > B_i(t)$, the within-group game is a pure defection game. The range of c_i from $B_i(t)/n_i(t)$ to $B_i(t)$ thus specifies how 'severe' the within-group $n_i(t)$-person prisoner's dilemma is. The last two terms within these parentheses, simplified to one term in the lower part of the equation, describe the influence of the between-group n-group competition for the external prize E, and have a positive effect upon $\mathrm{d}p_{ic}(t)/\mathrm{d}t$ to a degree corresponding to the size of E. (5.6) is a non-linear coupled ordinary differential equation of the first order. Although it has no straightforward analytical solution, it is richly endowed with information.

Given $0 \le p_{ic}(t) \le 1$, (5.6) specifies three qualitatively different stationary values for $p_{ic}(t)$, i.e. the values that $p_{ic}(t)$ eventually tends towards as t approaches infinity. The stationary values of $p_{ic}(t)$ are found by setting the left-hand side $\mathrm{d}p_{ic}(t)/\mathrm{d}t$ equal to zero. The first stationary value, $p_{ic}(t) = 1$, follows from setting the second term on the right-hand side of (5.6) equal to zero, i.e. $1 - p_{ic}(t) = 0$, and requiring that the term within the rightmost parentheses is strictly positive, which ensures that $p_{ic}(t)$ is strictly monotonically increasing towards its highest possible value $p_{ic}(t) = 1$. Expressing this requirement in terms of E gives

$$E > \left[\left(\sum_{i=1}^{n} p_{ic}(t) n_i(t) B_i(t)\right) + (1 - p_{ic}(t)) B_i(t)\right] \times \left[\left(\sum_{i=1}^{n} p_{ic}(t) n_i(t) B_i(t)\right) - p_{ic}(t) B_i(t)\right] \times \left\{\left[\left(\frac{1}{n_i(t)} \sum_{i=1}^{n} p_{ic}(t) n_i(t) B_i(t)\right) - p_{ic}(t) B_i(t)\right] B_i(t)\right\}^{-1} \times \left[c_i - \frac{B_i(t)}{n_i(t)}\right], \tag{5.7}$$

which is equivalent to the static requirement (3.10) when $p_{ic}(t) = 1\ \forall i = 1, \ldots, n$. In other words, if (5.7) is satisfied due to a sufficiently large between-group prize E, $p_{ic}(t)$ will eventually and inevitably be driven towards one as t approaches infinity, ensuring that all the groups A_i will consist exclusively of cooperating individual agents A_{ij}.

If the rightmost parenthesis in (5.6) is negative, $p_{ic}(t)$ is monotonically decreasing. The most extreme form for monotonic decrease involves setting $E = 0$, which means exclusively within-group and not between-group interaction. In this case (5.6) has an analytical solution given by

$$p_{ic}^{E=0}(t) = \frac{1}{1 - \left(1 - \dfrac{1}{p_{ic}(0)}\right) \mathrm{e}^{(c_i - B_i(t)/n_i(t)) k_i t}}, \tag{5.8}$$

where $p_{ic}(0)$ is the initial value of $p_{ic}(t)$ when $t = 0$. We see that $p_{ic}^{E=0}(t) \to 0$ when $t \to \infty$ given that $c_i > B_i(t)/n_i(t)$, which gives the second stationary value

$p_{ic}(t)=0$. This means that the number of cooperating individual agents A_{ij} in each group A_i will eventually vanish, as is implied by an $n_i(t)$-person prisoner's dilemma specified by $0<B_i(t)/n_i(t)<c_i<B_i(t)$. (We also see from (5.8) that $p_{ic}^{E=0}(t)\to 1$ when $t\to\infty$ given that $c_i<B_i(t)/n_i(t)$, since the within-group game then is a pure cooperation game.)

The third stationary value of $p_{ic}(t)$, for those i, $i=1,\dots,n$, where $E>0$ and (5.6) is not satisfied, is found by setting the term within the rightmost parentheses in (5.6) equal to zero for those i, and then solving the equations to find $p_{ic}(t)=p_{ic}(\infty)$. These values $p_{ic}(\infty)$ may be larger, equivalent, or smaller than their corresponding initial values $p_{ic}(0)$, meaning that $p_{ic}(t)$ may be monotonically increasing, constant, or monotonically decreasing for different i, $i=1,\dots,n$, as this follows from the rightmost term being positive, zero, or negative for different i, given the parameter values and how the various $p_{ic}(t)$ proceed over time.

(5.7) contains ample information of how the parameters n, $n_i(t)$, $B_i(t)$, c_i, and $p_{ic}(t)$ affect the requirement for E to ensure cooperation by the individual agents A_{ij} in group A_i. The requirement increases (becomes more strict) with c_i and decreases with $B_i(t)$, as is consistent with the within-group $n_i(t)$-person prisoner's dilemma becoming more 'severe' as $c_i-B_i(t)/n_i(t)$ increases. The requirement also increases with $n_i(t)$, as this gives more individual agents A_{ij} among which to divide the fraction of the prize E that group A_i receives.

Based on (5.6) and (5.7) we can specify how $p_{ic}(t)$ depends upon various characterizations of E. Assume $B_i(t)$ is proportional to $n_i(t)$ (i.e. $B_i(t)\sim n_i(t)$), and set for simplicity $B_i(t)=B_i$. If E is constant or $E\sim n_i(t)$ then $p_{ic}(t)\to 0$ when any $n_u(t)\to\infty$, i, $u=1,\dots,n$. If $E\sim n_i^2(t)$ then $p_{ic}(t)\to 0$ when $n_i(t)\to\infty$. If $E\sim n_u(t)$ then $0\le p_{ic}(t)\le 1$ when $n_u(t)\to\infty$, $u\ne i$, i, $u=1,\dots,n$. If $E\sim n_i^3(t)$ then $0\le p_{ic}(t)\le 1$ when $n_i(t)\to\infty$. If $E\sim n_u^2(t)$ then $p_{ic}(t)\to 1$ when $n_u(t)\to\infty$, $u\ne i$. If $E\sim n_i^4(t)$ then $p_{ic}(t)\to 1$ when $n_i(t)\to\infty$.

Simulating (5.3) and/or (5.6) enables us to predict a solution for generally all parameter values n, $n_i(t)$, $B_i(t)$, c_i, and E, accounting for the combined within-group $n_i(t)$-person prisoner's dilemma and between-group n-group competition for a prize E over time. Simulate (5.6) allowing for the two strategies cooperation c (effort level $S_{ij}=c_i$) and defection d (effort level $S_{ij}=0$). Set $n=2$ and posit fixed group sizes $n_i(t)=n_i=1000$, $i=1,2$. Considering the case with $B_i(t)$ proportional to $n_i(t)$, set $B_i(t)=n_i(t)$. c_i has to be chosen so that $0<B_i(t)/n_i(t)<c_i<B_i(t)$. Assuming a low degree of 'severity' for the within-group $n_i(t)$-person prisoner's dilemma, set $c_i=2$. With these parameter values, the requirement (5.7) for E to guarantee cooperation by all the individual agents A_{ij} in the static case is $E>3{,}998{,}000$. If $E=0$, the fraction $p_{ic}(t)$ of cooperating individual agents A_{ij} will go exponentially towards 0 as described by (5.8). If $E>3{,}998{,}000$, the fraction $p_{ic}(t)$ of cooperating individual agents A_{ij} will go exponentially towards one. Interesting cases of E are thus $0<E<3{,}998{,}000$, which gives an intermediate degree of cooperation in each group A_i.

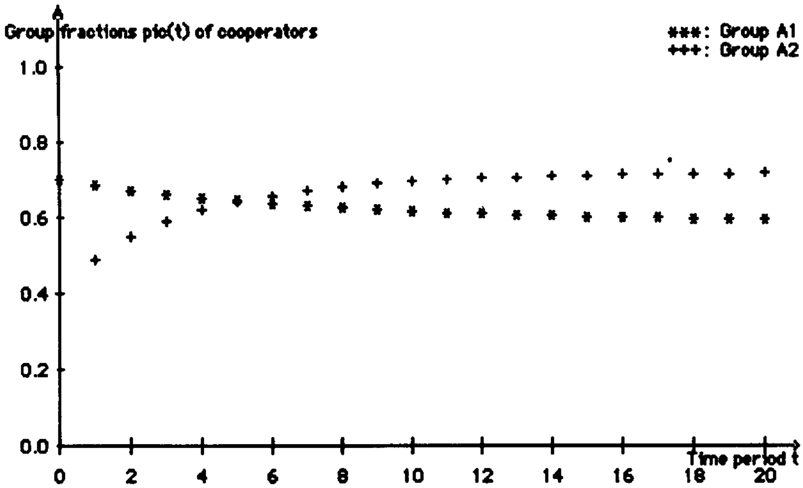

Fig. 2. Evolution of group fractions $p_{ic}(t)$ of cooperators, $E = 2{,}700{,}000$, $k_i = 0.5$.

Choose $E = 2{,}700{,}000$. With the above parameter values, the fraction $p_{ic}(t)$ of cooperating individual agents A_{ij} in the two groups A_i, $i = 1, 2$, will eventually be equal, i.e. $p_{1c}(\infty) = p_{2c}(\infty)$, regardless of initial conditions $p_{ic}(0)$. I will change this by setting $B_l(t) = 0.9n_1(t)$ and $B_2(t) = 1.1n_2(t)$, which means that a given degree of cooperation, ceteris paribus, gives fewer public goods in group A_1 than in group A_2, which implies that eventually $p_{1c}(\infty) < p_{2c}(\infty)$. Note that by setting $B_1(t) = 0.9n_1(t)$ and $B_2(t) = 1.1n_2(t)$, the requirement (5.7) for group A_1 increases to $E > 4{,}442{,}444.4$, while for group A_2 it decreases to $E > 3{,}634{,}363.6$. Fig. 2 sets the initial fractions of cooperators c to $p_{1c}(0) = 0.7$ and $p_{2c}(0) = 0.4$ (i.e. $h_{1c}(0) = 700$ and $h_{2c}(0) = 400$) and simulates over 20 time periods with 'sensitivity' factor $k_i = 0.5$.

We see that the defectors in the least cooperative group A_2 switch to cooperation while the cooperators in the most cooperative group A_1 switch to defection. As the number of cooperating individual agents A_{ij} in the two groups becomes more equal, an equilibrium arises with a mixture of cooperators and defectors in each group A_i. This equilibrium is such that no single individual agent A_{ij} has any incentive to deviate unilaterally. Consistent with (5.7), $p_{ic}(\infty)$, $i = 1, 2$, increases with E and $B_i(t)$, and decreases with c_i and $n_i(t)$.

6. Dynamic model with intergroup mobility

Allowing for intergroup mobility means positing a model (4.10) specifying a change $\mathrm{d}n_i(t)/\mathrm{d}t$ in the number $n_i(t)$ of individual agents A_{ij} in each group A_i

over time. This change $\mathrm{d}n_i(t)/\mathrm{d}t$ is such that individual agents A_{ij} flow between the groups A_i, $i = 1, \ldots, n$, preserving the total number $N(t)$, as given by (4.6), of individual agents A_{ij} in all the n groups A_i, $i = 1, \ldots, n$, i.e. $N(t) = N =$ constant. I consider the model

$$\frac{\mathrm{d}n_i(t)}{\mathrm{d}t} \stackrel{\text{mod}}{=} (\gamma_i - \mu_i) n_i(t) [Y_i(t) - Y(t)]$$

$$= r_i n_i(t) \left[Y_i(t) - \frac{1}{N} \sum_{i=1}^{n} n_i(t) Y_i(t) \right], \tag{6.1}$$

using (4.5), where $Y_i(t)$ is given by (4.3) inserting (5.2) for the continuous strategy model, and (4.4) inserting (5.2) and (5.4) for the discrete strategy model. $r_i = \gamma_i - \mu_i > 0$ is the 'compound' sensitivity of the process, in the sense that γ_i is the sensitivity without switching costs, and μ_i is the cost of switching group A_i. Switching costs μ_i account for both entry costs (if $Y_i(t) - \dot{Y}(t)$ is positive) and exit costs (if $Y_i(t) - Y(t)$ is negative) for individual agents A_{ij} switching to and from, respectively, group A_i. Switching costs μ_i slow down the rapidity of change $\mathrm{d}n_i(t)/\mathrm{d}t$, or sensitivity, of the process, and thus have a negative sign in (6.1). The change $\mathrm{d}n_i(t)/\mathrm{d}t$ is positive if the overall mean yield $Y_i(t)$ for individual agent A_{ij} in group A_i is larger than the overall mean yield $Y(t)$ for an individual agent A_{ij} in any group A_i, $i = 1, \ldots, n$, and negative if the reverse is the case. Aside from the economic interpretation, (6.1) can be interpreted as the Malthusian law of population growth, where the sign of the proportionality factor $r_i[Y_i(t) - Y(t)]$ specifies whether the size $n_i(t)$ of group A_i (the population) increases or decreases.

To derive the requirement for preserving the total number $N(t) = N$ of individual agents A_{ij} in all the n groups A_i, $i = 1, \ldots, n$, sum both sides in (6.1) from $i = 1$ to $i = n$, i.e. $\Sigma_{i=1}^{n}(\cdot)$. Inserting (4.6) gives

$$\sum_{i=1}^{n} \frac{\mathrm{d}n_i(t)}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t} \sum_{i=1}^{n} n_i(t)$$

$$= \frac{\mathrm{d}}{\mathrm{d}t} N(t) = \frac{\mathrm{d}}{\mathrm{d}t} N = \sum_{i=1}^{n} r_i n_i(t) [Y_i(t) - Y(t)] = 0. \tag{6.2}$$

(6.2) can be simplified by setting the "sensitivity" factor r_i constant across the groups A_i, $i = 1, \ldots, n$, i.e. $r_i = r$. Using (4.5) gives

$$\sum_{i=1}^{n} r_i n_i(t) [Y_i(t) - Y(t)] = r \left(\sum_{i=1}^{n} n_i(t) Y_i(t) \right) - rY(t) \sum_{i=1}^{n} n_i(t)$$

$$= rNY(t) - rY(t)N = 0. \tag{6.3}$$

which means that the requirement for preserving the total number $N(t)=N$ of individual agents A_{ij} in all the n groups A_i, $i=1,\dots,n$, is automatically satisfied if the 'sensitivity' factor $r_i = r =$ constant.

The final requirement for (6.1) is that $0 \le n_i(t) \le N \ \forall i=1,\dots,n$. I set n constant so that $n_i(t)=0$ means that group A_i exists but has no members A_{ij}. If $n_i(t)=N$ then all the groups A_u, $u \neq i$, i, $u=1,\dots,n$, except group A_i has no members A_{uj}.

Inserting (5.2) and (5.4) into (4.4) allowing for the two discrete strategies cooperation c (effort level $S_{ij}=c_i$) and defection d (effort level $S_{ij}=0$), where $0 < B_i(t)/n_i(t) < c_i < B_i(t)$, gives

$$\begin{aligned} Y_i(t) &= p_{ic}(t)(B_i(t)-c_i) \\ &+ \Bigg\{\Bigg[\left(\sum_{i=1}^{n} p_{ic}(t)n_i(t)B_i(t)\right) + \left(1-\frac{1}{n_i(t)}+\frac{p_{ic}(t)}{n_i(t)}-2p_{ic}(t)\right) \\ &\times B_i(t)\Bigg] p_{ic}(t)n_i(t)B_i(t)E\Bigg\} \\ &\times \Bigg\{\Bigg[\left(\sum_{i=1}^{n} p_{ic}(t)n_i(t)B_i(t)\right) + (1-p_{ic}(t))B_i(t)\Bigg] \\ &\times \Bigg[\left(\sum_{i=1}^{n} p_{ic}(t)n_i(t)B_i(t)\right) - p_{ic}(t)B_i(t)\Bigg]\Bigg\}^{-1}, \end{aligned} \tag{6.4}$$

defined for $n_i(t)>0$, $i=1,\dots,n$. (6.1) inserting (6.4) has no straightforward analytical solution. With one available strategy cooperation c (effort level $S_{ij}=c_i$), giving $p_{ic}(t)=1$, (6.1) inserting (6.4) becomes

$$\begin{aligned} \frac{\mathrm{d}n_i(t)}{\mathrm{d}t} &= r_i\frac{n_i(t)}{N(t)} \\ &\times \sum_{\substack{u=1\\u\neq i}}^{n} n_u(t)\left[(B_i(t)-B_u(t)-c_i+c_u)+\frac{(B_i(t)-B_u(t))E}{\sum_{i=1}^{n} n_i(t)B_i(t)}\right]. \end{aligned} \tag{6.5}$$

(6.5) illustrates that with one available strategy cooperation c there is no inter-group mobility if the parameters in the n groups A_i, $i=1,\dots,n$ are equivalent, i.e. $B_i(t)=B(t)$ and $c_i=c \ \forall i=1,\dots,n$. If $B_i(t)=B(t)$ then all the individual agents A_{ij} will, compatible with the principle of competitive exclusion in population biology, eventually seek that group A_i having the lowest cost c_i of effort for a cooperative act c, i.e. $c_i < c_u \ \forall i, u=1,\dots,n, i \neq u$, the other groups A_u becoming extinct in the sense of having no members A_{uj}. If $c_i=c$ then all

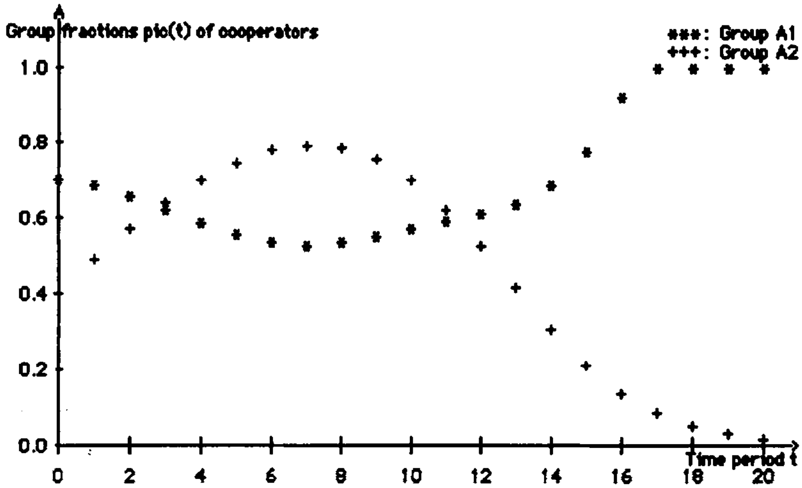

Fig. 3. Evolution of fractions $p_{ic}(t)$, $E = 2{,}700{,}000$, $k_i = 0.5$, $r_i = 0.00021$.

individual agents A_{ij} will eventually seek that group A_i that can generate the highest amount of within-group public goods $B_i(t)$ from a cooperative act c, the other groups A_u becoming extinct. Note that this latter process is quicker if E is large. If $B_i(t) \neq B(t)$ and $c_i \neq c$ then all the individual agents A_{ij} will eventually seek that group A_i where the bracketed term in (6.5) is positive when two groups A_i and A_u remain, i, $u = 1, \dots n$, $i \neq u$. Observe that in this case $B_i(t) < B_u(t)$ that is not beneficial for individual agent A_{ij} can be offset by a beneficial $c_i \ll c_u$, and vice versa.

Simulate (6.1) inserting (6.4) allowing for the two discrete strategies cooperation c (effort level $S_{ij} = c_i$) and defection d (effort level $S_{ij} = 0$). Figs. 3 and 4 assume the same parameters as in Fig. 2, i.e. $n = 2$, $n_i(0) = 1000$, $B_1(t) = 0.9n_1(t)$, $B_2(t) = 1.1n_2(t)$, $E = 2{,}700{,}000$, initial fractions $p_{1c}(0) = 0.7$ and $p_{2c}(0) = 0.4$, and 'sensitivity' factors $k_i = 0.5$ and $r_i = 0.00021$.

Fig. 3 should be compared with Fig. 2. During the first time periods the defectors in the least cooperative group A_2 switch to cooperation at a high rate, $B_2(t) = 1.1n_2(t)$, while the cooperators in the most cooperative group A_1 switch to defection at a slightly slower rate, $B_1(t) = 0.9n_1(t)$. Because of the higher group fraction $p_{1c}(t)$ of cooperators in group A_1 initially, group A_1 is considered more attractive, some individual agents A_{2j} switching from group A_2 to group A_1, giving $n_1(t) > n_2(t)$ (cf. Fig. 4). From time period four, the fraction $p_{2c}(t)$ of cooperators in group A_2 is larger than the fraction $p_{1c}(t)$ of cooperators in group A_1, which makes group A_2 more attractive. Hence from time periods four to eight, $n_1(t)$ decreases and $n_2(t)$ increases. From time period eight, $n_2(t) > n_1(t)$, and the between-group prize E is no longer large enough to sustain cooperation within the

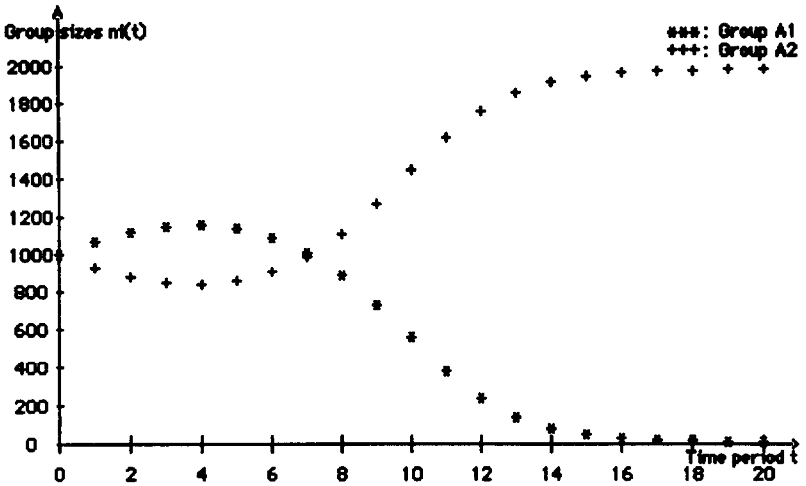

Fig. 4. Evolution of group sizes $n_i(t)$, $E = 2{,}700{,}000$, $k_i = 0.5$, $r_i = 0.00021$.

larger group A_2. Observe from (5.7) and (3.10) that the requirement for the between-group prize E gets more restrictive for the larger group A_2 and less restrictive for the smaller group A_1 as the group size $n_2(t)$ gets increasingly larger than $n_1(t)$. The individual agents A_{1j} in group A_1 thus have an incentive to cooperate, but the small size $n_1(t)$ makes group A_1 command a relatively small proportion of the between-group prize E. Hence group A_2 can function more as if in isolation. When group A_1 eventually becomes extinct, no between-group prize E is large enough to counteract the logic of the within-group $n_2(t)$-person prisoner's dilemma, which dictates defection as the strategy that will flourish.

7. Dynamic model with intergroup warfare

Describing intergroup warfare means positing a model (4.10) specifying a change $\mathrm{d}n_i(t)/\mathrm{d}t$ based on how the groups A_i, $i = 1, \ldots, n$, wage war against each other over time. There is no preservation of the total number $N(t)$ of individual agents A_{ij} in the n groups A_i. A generalized n-group version of Lanchester's square law for conventional warfare is

$$\frac{\mathrm{d}n_i(t)}{\mathrm{d}t} \overset{\text{mod}}{=} -\lambda_i \left[\sum_{\substack{u=1 \\ u \neq i}}^{n} n_u(t) Y_u(t) \right] + h_i(t), \tag{7.1}$$

where λ_i is the 'sensitivity' factor for the process, $h_i(t)$ is the reinforcement rate, and $0 \leq n_i(t) \leq N(t)\ \forall i = 1, \ldots, n$. Operational loss due to non-combat factors is

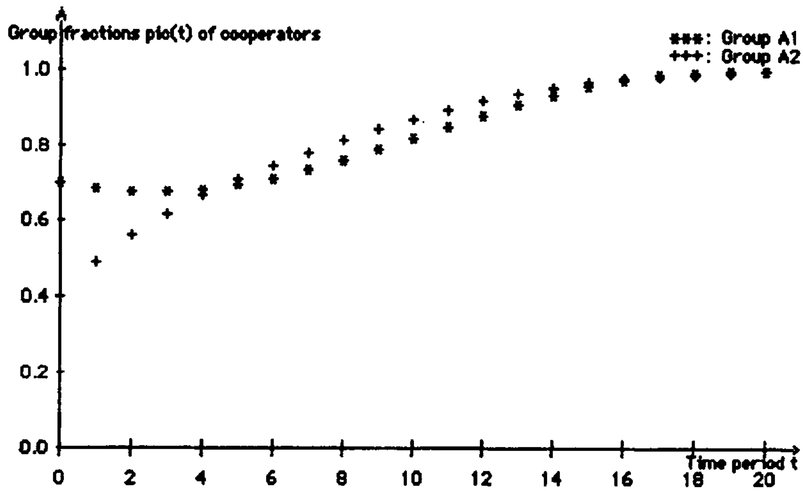

Fig. 5. Evolution of fractions $p_{ic}(t)$, $E = 2{,}700{,}000$, $k_i = 0.5$, $\lambda_i = 0.000014$.

assumed to be negligible. $Y_u(t)$, $u \neq i$, i, $u = 1, \ldots, n$, is given by (4.3) inserting (5.2) for the continuous strategy model and (4.4) inserting (5.2) and (5.4) for the discrete strategy model. Consider n groups (forces) A_i, $i = 1, \ldots, n$, out in the open, each within kill range of its $n-1$ enemies A_u, $u \neq i$, i, $u = 1, \ldots, n$. If one group A_i suffers a loss, fire is concentrated on the remaining individual agents A_{ij} in group A_i. This gives the combat loss rate for group A_i as described by the first term on the right-hand side of (7.1). That is, each group A_i suffers a loss proportional to the sum of the 'strengths' of the opposing $n-1$ groups A_u, $u \neq i$, i, $u = 1, \ldots, n$. Note that 'strength' in (7.1) is defined as $n_u(t)Y_u(t)$ and not $n_u(t)$, where $Y_u(t)$ is given by (4.3). This means that $\mathrm{d}n_i(t)/\mathrm{d}t$ is affected not only by the numbers $n_u(t)$, $u \neq i$, i, $u = 1, \ldots, n$, of individual agents A_{uj} in the other $n-1$ groups A_u, as is commonly done in the simulation of Lanchester's square law, but also by the mean yields $Y_u(t)$ for these individual agents, as these depend upon the parameters for the groups A_u and the various strategies S_{uj} that the individual agents A_{uj} choose. Hence the different capacities of the various individual agents A_{uj} are accounted for, as these capacities depend upon the parameters and strategy sets S_{uj} for the groups A_u.

(7.1) inserting (6.4) has no straightforward analytical solution. Simulate (7.1) inserting (6.4) allowing for the two discrete strategies cooperation c (effort level $S_{ij} = c_i$) and defection d (effort level $S_{ij} = 0$). Figs. 5 and 6 assume the same parameters as in Figs. 2, 3, and 4, i.e. $n = 2$, $n_i(0) = 1000$, $B_1(t) = 0.9n_1(t)$, $B_2(t) = 1.1n_2(t)$, $E = 2{,}700{,}000$, initial fractions $p_{1c}(0) = 0.7$ and $p_{2c}(0) = 0.4$, reinforcement rate $h_i(t) = 0$, and 'sensitivity' factors $k_i = 0.5$ and $\lambda_i = 0.000014$.

Fig. 5 should be compared with Fig. 2. The between-group competition is now more fierce, in the sense that the two groups A_i and A_u compete not only for the

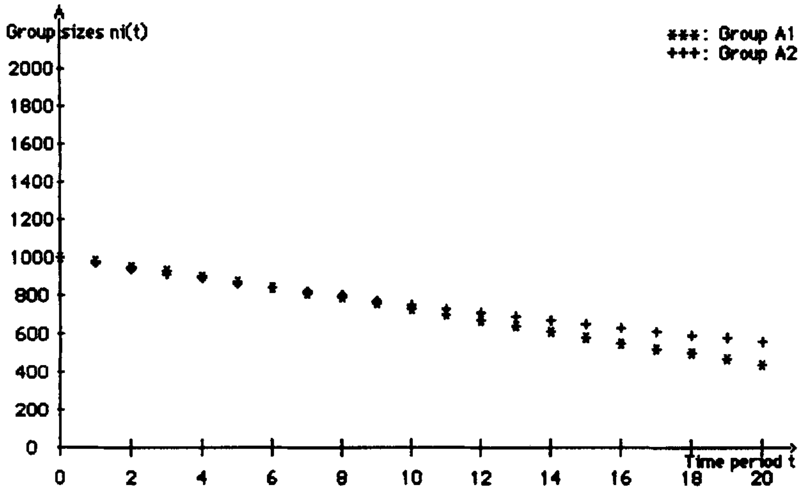

Fig. 6. Evolution of group sizes $n_i(t)$, $E = 2{,}700{,}000$, $k_i = 0.5$, $\lambda_i = 0.000014$.

between-group prize E; they also compete to exterminate each other's individual agents A_{ij} and A_{uj}. This provides an additional incentive for each individual agent A_{ij} and A_{uj}, $i \neq u$, $i,u = 1,\ldots,n$, within each group A_i and A_u to choose cooperation c, which increases the fractions $P_{ic}(t)$ and $p_{uc}(t)$, respectively, of cooperators. Conversely, by comparison with Fig. 2, a given fraction $p_{ic}(t)$ of cooperation within each group can be obtained with a lower between-group prize E. These results are compatible with the common claim by Hart (1983) and others that more fierce competition in the product-market (i.e. between organizations) reduces managerial slack, i.e. increases managerial cooperation. I will return to this issue in a future article. Fig. 6 illustrates the evolution of the group sizes $n_i(t)$, where eventually $n_1(t) < n_2(t)$ because the parameters $B_1(t) = 0.9n_1(t)$ and $B_2(t) = 1.1n_2(t)$ make group A_2 more favorable, more cooperative, and stronger than group A_1.

Generalizing (7.1) to guerilla warfare, set

$$\frac{\mathrm{d}n_i(t)}{\mathrm{d}t} \overset{\text{mod}}{=} -\lambda_i\left[\sum_{\substack{u=1\\u\neq i}}^{n} n_i(t)n_u(t)Y_u(t)\right] + h_i(t). \tag{7.2}$$

Assume that each group A_i, $i = 1,\ldots,n$, occupies a certain geographical region and is invisible to all its opposing $n-1$ groups A_u, $u \neq i$, i, $u = 1,\ldots,n$. The groups A_u fire into the region but cannot know when a kill of an individual agent A_{ij} has been made. As for conventional warfare, each group A_i suffers a loss proportional to the sum of the 'strengths' of the opposing $n-1$ groups A_u, $u \neq i$, i, $u = 1,\ldots,n$. Guerilla warfare assumes in addition that each group A_i suffers a loss proportional to the number $n_i(t)$ of individual agents A_{ij} in its own group A_i, since the larger $n_i(t)$ is, the greater is the probability that a shot from the other $n-1$ groups A_u will kill a member from group A_i.

8. Dynamic model with intergroup mobility and intergroup warfare

The combined operation of intergroup mobility and intergroup warfare can be modeled combining (6.1) and e.g. a generalized version of (7.2), giving

$$\frac{dn_i(t)}{dt} \overset{\text{mod}}{=} r_i n_i(t)\left[Y_i(t) - \frac{1}{N(t)}\sum_{i=1}^{n} n_i(t)Y_i(t)\right] - \lambda_i\left[\sum_{\substack{u=1\\u\neq i}}^{n}\sum_{x=0}^{x_{\max}}\sum_{y=1}^{y_{\max}} n_i^x(t)n_u^y(t)Y_u^y(t)\right] + h_i(t), \tag{8.1}$$

where $n_i(t)n_u(t)Y_u(t)$ in (7.2) is substituted with a Taylor series, and where $N(t)$ is no longer a constant. The 'sensitivity' factors r_i and λ_i determine the relative weight of intergroup mobility and intergroup warfare.

(8.1) inserting (6.4) has no straightforward analytical solution. Simulate (8.1) inserting (6.4) allowing for the two discrete strategies cooperation c (effort level $S_{ij}=c_i$) and defection d (effort level $S_{ij}=0$). Figs. 7 and 8 assume the same parameters as in Figs. 2, 3, and 4, i.e. $n=2$, $n_i(0)=1000$, $B_1(t)=0.9n_1(t)$, $B_2(t)=1.1n_2(t)$, $E=2{,}700{,}000$, $x_{\max}=0$, $y_{\max}=1$, initial fractions $p_{1c}(0)=0.7$

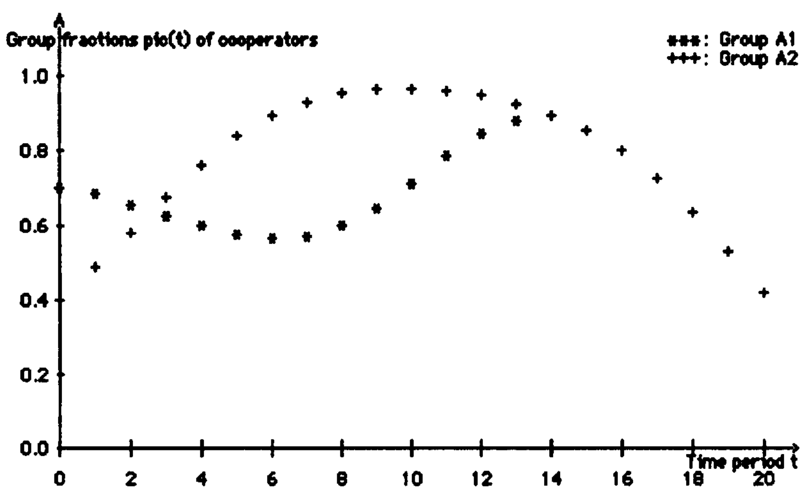

Fig. 7. Evolution of $p_{ic}(t)$, $E=2{,}700{,}000$, $k_i=0.5$, $r_i=0.00021$, $\lambda_i=0.000014$.

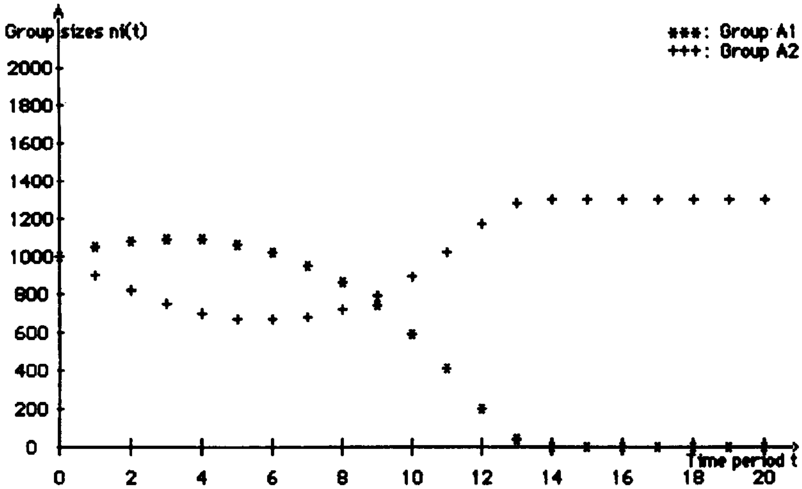

Fig. 8. Evolution of $n_i(t)$, $E = 2{,}700{,}000$, $k_i = 0.5$, $r_i = 0.00021$, $\lambda_i = 0.000014$.

and $p_{2c}(0) = 0.4$, reinforcement rate $h_i(t) = 0$, and 'sensitivity' factors $k_i = 0.5$, $r_i = 0.00021$, and $\lambda_i = 0.000014$.

Figs. 7 and 8 illustrate the combined operation of intergroup mobility from Section 6 and intergroup warfare. The intergroup warfare makes $n_1(t)$ approach zero faster than when exclusively assuming intergroup mobility. As $n_1(t)$ approaches zero, group A_2 can function more in isolation, implying a decline in $p_{2c}(t)$ eventually giving $p_{2c}(\infty) = 0$. $n_1(14) = 0$ makes $p_{1c}(t)$ undefined for $n_1 \geq 14$, while $n_2(t) = 1304$ for $t \geq 14$.

9. Simulation of the algorithm with four strategies in one group

This section shows that the introduction of the $n_i(t)$-person reactive strategies, tit-for-tat and bully, into the within-group one-group $n_i(t)$-person prisoner's dilemma does not facilitate within-group cooperation in the way the between-group competition for an external prize E does. Define the two reactive strategies as follows:

1. T_e; cooperate in the initial time period $t = 0$, and then cooperate in each subsequent time period t, $t = 1, 2, \ldots$ if e or more of the other $n_i(t) - 1$ individual agents in group A_i chose cooperation c during the previous time period $t - 1$ (Taylor, 1976).
2. G_e; defect in the initial time period $t = 0$, and then defect in each subsequent time period t, $t = 1, 2, \ldots$ if e or more of the other $n_i(t) - 1$ individual agents in group A_i chose cooperation c during the previous time period $t - 1$.

Inserting (5.4) into (5.2) allowing for the two discrete strategies cooperation c (effort level $S_{ij} = c_i$) and defection d (effort level $S_{ij} = 0$), where $0 < B_i(t)/n_i(t) < c_i < B_i(t)$, gives

$$Y_{ic}(t) = \left(\left(1 - \frac{1}{n_i(t)}\right)p_{ic}(t) + \frac{1}{n_i(t)}\right)B_i(t) + \frac{\left(\left(1 - \frac{1}{n_i(t)}\right)p_{ic}(t) + \frac{1}{n_i(t)}\right)B_i(t)E}{\left[\left(\sum_{i=1}^{n} p_{ic}(t)n_i(t)B_i(t)\right) + (1 - p_{ic}(t))B_i(t)\right]} - c_i \tag{9.1}$$

and

$$Y_{id}(t) = \left(1 - \frac{1}{n_i(t)}\right)p_{ic}(t)B_i(t) + \frac{\left(1 - \frac{1}{n_i(t)}\right)p_{ic}(t)B_i(t)E}{\left[\left(\sum_{i=1}^{n} p_{ic}(t)n_i(t)B_i(t)\right) - p_{ic}(t)B_i(t)\right]}. \tag{9.2}$$

The payoffs $Y_{iT}(t)$ and $Y_{iG}(t)$ for playing the reactive strategies T_e and G_e are determined from (9.1) and (9.2) based on the definitions of T_e and G_e given above, i.e. depending upon the parameter e related to how many of the other $n_i(t) - 1$ individual agents in group A_i chose cooperation c during the previous time period. Set $n_i(t) = 1000$, $B_i(t) = n_i(t)$, $c_i = 2$, 'sensitivity' factor $k_i = 0.5$, and simulate over 20 time periods with initial conditions $p_{ic}(0) = 0.7$, $p_{id}(0) = p_{iT}(0) = p_{iG}(0) = 0.1$. Fig. 9 sets $e = n_i(t) - 1$. In the initial time period $t = 0$, d and G_e go to prey upon c and T_e. In time period $t = 1$, G_e switches to cooperation and T_e switches to defection. Hence thereafter d and T_e prey upon c and G_e, the latter two becoming extinct. If T_e defects in the initial time period $t = 0$, T_e and d coincide throughout giving $p_{id}(\infty) = p_{iT}(\infty) = 0.5$.

Fig. 10 decreases e to $e = 0.7n_i(t)$. G_e defects and T_e cooperates through time period $t = 3$, after which they switch roles, giving a similar evolution as Fig. 9. Fig. 11 decreases e further to $e = 0.1n_i(t)$. G_e defects and T_e cooperates through time period $t = 8$, after which they switch roles. Since T_e is almost extinct at $t = 8$, d thereafter preys upon G_e, the latter becoming extinct, d approaching $p_{id}(\infty) = 1$. With $e = 0$, d and G_e coincide throughout giving $p_{id}(\infty) = p_{iG}(\infty) = 0.5$.

Figs. 9–11 show the superiority of defection d. The $n_i(t)$-person tit-for-tat strategy T_e is not as successful as the two-person tit-for-tat strategy (Axelrod, 1984), mainly because of the constellation of one against many in $n_i(t)$-person

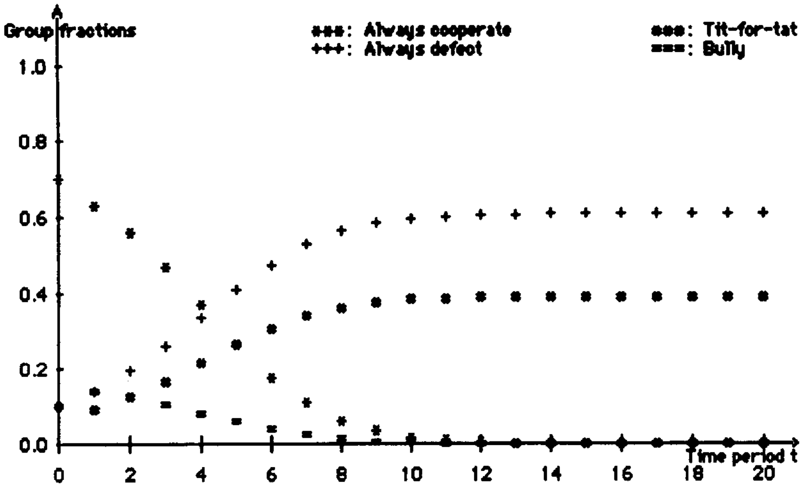

Fig. 9. Evolution of fractions $p_{is}(t)$ for $e = n_i(t) - 1$ for one group ($E = 0$), $k_i = 0.5$.

games, not present in two-person games, and the rigidity of the parameter e. This can be partly remedied by assuming a 'continuous' version of an $n_i(t)$-person prisoner's dilemma which, in addition to complicating the model, allows individual agent A_{ij} to choose a degree S_{ij}, $0 \le S_{ij} \le c_j$, of cooperation based upon the overall degree S_i of cooperation in group A_i in the previous time period. This remedy is insufficient because defectors are not punished with sufficient degree of specificity, the punishment being spread out over all the other $n_i(t) - 1$

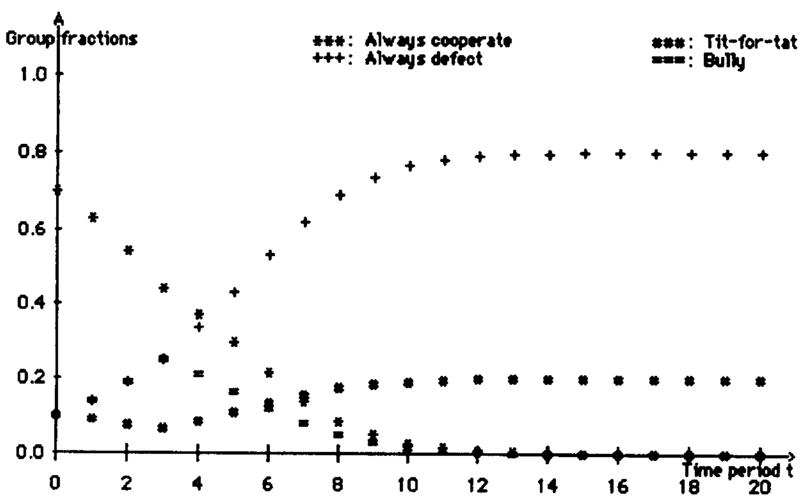

Fig. 10. Evolution of fractions $p_{is}(t)$ for $e = 0.7n_i(t)$ for one group ($E = 0$), $k_i = 0.5$.

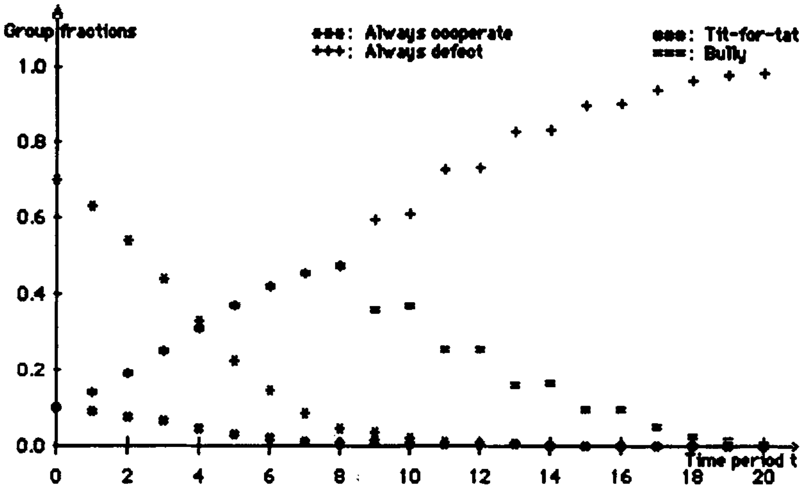

Fig. 11. Evolution of fractions $p_{is}(t)$ for $e = 0.1\, n_i(t)$ for one group ($E = 0$), $k_i = 0.5$.

individual agents in group A_i. A more appropriate remedy considered in this article involves introducing an external structure in the form of between-group n-group competition for a specific prize E.

Appendix: Between-group competition where the more cooperative group A_i gets the entire between-group prize E

This article has assumed between-group competition where each group A_i gets a fraction of the between-group prize E proportional to the relative amount of public goods it produces compared with the other groups. An alternative is to assume between-group competition where the group A_i producing the highest amount of public goods gets the entire between-group prize E, i.e. 'winner takes all', and that if m of the n groups produce an equally high amount of public goods, that these share the prize E equally. Assume for expositional convenience $B_i = B_1$, $c_i = c_1$, and $a_i = 1$. Assume without loss of generality that $n_1 = \ldots = n_m > \ldots \geq n_n$, where $1 \leq m \leq n \geq 2$.

Assume the within-group n_i-person prisoner's dilemma from Section 3 where $0 < B_1/n_i < c_1 < B_1$. Define $h_{1c} - 1$ as the number of cooperating individual agents aside from individual agent A_{1j} in group A_1. If individual agent A_{1j} chooses $S_{1j} = c_1$, giving h_{1c} cooperators in group A_1, he gets, analogous to (3.7), a payoff

$$P_{1jc} = \frac{1}{n_1}[h_{1c}B_1 + E] - c_1 \text{ if } h_{1c} > h_{2c} \geq \ldots \geq h_{nc}$$

$$= \frac{1}{n_1}\left[h_{1c}B_1 + \frac{E}{m}\right] - c_1 \text{ if } h_{1c} = \dots = h_{mc} > \dots \geq h_{nc}$$

$$= \frac{1}{n_1}\left[h_{1c}B_1 + \frac{E}{n}\right] - c_1 \text{ if } h_{1c} = \dots = h_{nc}$$

$$= \frac{1}{n_1}h_{1c}B_1 - c_1 \text{ if } h_{1c} < h_{kc} \text{ for at least one } k,\ k = 2,\dots,n. \tag{A.1}$$

If individual agent A_{ij} chooses $S_{ij} = 0$, there are $h_{ic} - 1$ cooperators in group A_i. Assume that h_{uc}, $u \neq i$, $u = 1,\dots,n$, is the number of cooperators in the other $n-1$ groups A_u. Individual agent A_{ij} then gets, analogous to (3.8), a payoff

$$P_{1jd} = \frac{1}{n_1}\left[(h_{1c} - 1)B_1 + E\right] \text{ if } h_{1c} - 1 > h_{2c} \geq \dots \geq h_{nc}$$

$$= \frac{1}{n_1}\left[(h_{1c} - 1)B_1 + \frac{E}{m}\right] \text{ if } h_{1c} - 1 = \dots = h_{mc} > \dots \geq h_{nc}$$

$$= \frac{1}{n_1}\left[(h_{1c} - 1)B_1 + \frac{E}{n}\right] \text{ if } h_{1c} - 1 = \dots = h_{nc}$$

$$= \frac{1}{n_1}(h_{1c} - 1)B_1 \text{ if } h_{1c} - 1 < h_{kc} \text{ for at least one } k,\ k = 2,\dots,n. \tag{A.2}$$

When m of the n groups are equally large, where $2 \leq m \leq n$, then the most restrictive requirement for E to ensure cooperation by all the individual agents in the m groups arises when everyone in the m groups cooperates, and one individual agent A_{1j} in group A_1 contemplates whether to defect. To derive this requirement, set $h_{1c} = n_1$ in (A.1) and (A.2). Cooperation by the individual agent A_{1j} is guaranteed if (A.1) is larger than (A.2), i.e. if

$$\frac{1}{n_1}\left[n_1 B_1 + \frac{E}{m}\right] - c_1 > \frac{1}{n_1}(n_1 - 1)B_1, \tag{A.3}$$

i.e. if

$$E > mn_1\left(c_1 - \frac{B_1}{n_1}\right) \text{ if } 2 \leq m \leq n. \tag{A.4}$$

If $m = 1$, which means that $n_1 > n_2$, then the individual agents in group A_1 will coordinate such that $h_{1c} = n_2 + 1$, given that the between-group prize E is sufficiently large, which precludes any other group getting access to E. To derive the most restrictive requirement for E, assume $n_2 > n_3$, set $h_{1c} = n_2 + 1$ in (A.1) and (A.2). Cooperation by the individual agent A_{1j} is guaranteed if (A.1) is larger than (A.2), i.e. if

$$\frac{1}{n_1}[(n_2+1)B_1+E]-c_1>\frac{1}{n_1}\left[n_2B_1+\frac{E}{2}\right], \tag{A.5}$$

i.e. if

$$E>2n_1\left(c_1-\frac{B_1}{n_1}\right) \text{ if } m=1. \tag{A.6}$$

(A.6) has a very nice interpretation. The requirement for having a within-group prisoner's dilemma is $0<B_1/n_i<c_1<B_1$. For the largest group A_1 the expression c_1-B_1/n_1 thus gives the extent to which the cost c_1 of cooperation for individual agent A_{1j} in the largest group A_1 exceeds the lower limit B_1/n_1 where the within-group game A_1 goes from being a within-group prisoner's dilemma to becoming a within-group pure cooperation game. This expression c_1-B_1/n_1 is then multiplied with twice the number of individual agents A_{1j} in the largest group A_1. This gives the minimum value of E required to ensure cooperation by $h_{1c}=n_2+1$ individual agents A_{1j} in the largest group A_1.

Observe that inserting $n_1=2$ into (A.6) gives

$$E>4(c_1-B_1/2), \tag{A.7}$$

which is three times less restrictive than

$$E>12(c_1-B_1/2), \tag{A.8}$$

which follows from inserting $n=n_i=2$, $B_i=B_1$, and $c_i=c_1$ into (3.10), for the proportional allocation rule.

(A.4) gets more restrictive as the number m of those largest groups having equal size increases, because the fraction of E available to each group then gets smaller, and may get too small to facilitate within-group cooperation. If E represents a fixed amount of resources in the environment, (A.4) may be realistic. However, if the nature of the between-group prize E is such that it increases with the number m, (A.4) may be too restrictive. More specifically, if the m groups compete for a between-group prize E that is proportional to m, and if we, in order to illustrate compatibility with the two-group case, choose a proportionality factor such that the m groups compete for a between-group prize $mE/2$, then (A.4) becomes (A.6).

Using (A.4) and (A.6) we can specify six different intervals for the between-group prize E, and what degree of cooperation within the n groups corresponds:

1. $E>mn_1(c_1-B_1/n_1)$ and $m\geq 2$: n_1 cooperators in groups A_1 to A_m (all remaining are defectors).

Table A1
Game with two defectors in the other group

		Individual agent 2	
		c	d
Individual agent 1	c	$B_1 - c_1 + E/2,\ B_1 - c_1 + E/2$	$B_1/2 - c_1 + E/2,\ B_1/2 + E/2$
	d	$B_1/2 + E/2,\ B_1/2 - c_1 + E/2$	$E/4,\ E/4$

2. $(m-q)n_1(c_1 - B_1/n_1) < E \leq (m-q+1)(c_1 - B_1/n_1)$ and $m \geq 3$: n_1 cooperators in groups A_1 to A_{m-q}, $q = 1, \ldots, m-2$.
3. $E > 2n_1(c_1 - B_1/n_1)$ and $m = 1$: $n_2 + 1$ cooperators in group A_1.
4. $2n_{m+1}(c_1 - B_1/n_{m+1}) < E \leq 2n_1(c_1 - B_1/n_1)$: $n_{m+2} + 1$ cooperators in group A_{m+1}.
5. $2n_n(c_1 - B_1/n_n) < E \leq 2n_{n-1}(c_1 - B_1/n_{n-1})$: 1 cooperator in group A_1.
6. $E \leq 2n_n(c_1 - B_1/n_n)$: no cooperators.

In the remainder of the appendix I analyze in detail the case of two groups A_i with two individual agents A_{ij} in each group. An individual agent A_{ij} gets from between-group competition a payoff $E/2$ if he is part of the more cooperative group, and a payoff $E/4$ if the two groups are equally cooperative. The payoff matrix for the individual agents in each group, given that the two individual agents in the other group defect, is as in Table A1.

Table A1 is derived from the basic two-person prisoner's dilemma by adding the payoff $E/2$ to each entry except the two entries in the lower right corner, where the payoff $E/4$ is added. Table A1 shows a chicken game and not a prisoner's dilemma when

$$B_1/2 - c_1 + E/2 > E/4. \tag{A.9}$$

Given this, and given that there are four defectors in the other group, any defector has an incentive to switch to cooperation to increase his payoff.

Given that there are one cooperator and one defector in the other group, the payoff matrix for a group is as in Table A2.

Table A2 is derived from the basic two-person prisoner's dilemma by adding the payoff $E/2$ to the upper left entries, adding the payoff $E/4$ to the entries on

Table A2
Game with one cooperator and one defector in the other group

		Individual agent 2	
		c	d
Individual agent 1	c	$B_1 - c_1 + E/2,\ B_1 - c_1 + E/2$	$B_1/2 - c_1 + E/4,\ B_1/2 + E/4$
	d	$B_1/2 + E/4,\ B_1/2 - c_1 + E/4$	0, 0

Table A3
Game with two cooperators in the other group

		Individual agent 2	
		c	d
Individual agent 1	c	$B_1 - c_1 + E/4,\ B_1 - c_1 + E/4$	$B_1/2 - c_1,\ B_1/2$
	d	$B_1/2,\ B_1/2 - c_1$	0, 0

the diagonal from lower left to upper right, and keeping the lower right entries unchanged. Table A2 shows a mutual cooperation game when

$$B_1 - c_1 + E/2 > B_1/2 + E/4. \tag{A.10}$$

Given this, if cooperation gets started in one group, it will be perpetuated to mutual cooperation in the other group.

Given that there are two cooperators in the other group, the payoff matrix for a group is as in Table A3. Table A3 is derived from the basic two-person prisoner's dilemma by adding the payoff $E/4$ to the upper left entries, and keeping the other entries unchanged. Table A3 shows a cooperation game when (A.10) is satisfied. Given this, the individual agents can be expected to coordinate their actions so that the unique mutual cooperation equilibrium $(B_1 - c_1 + E/4,\ B_1 - c_1 + E/4)$ is attained.

Both (A.9) and (A.10) imply that

$$E > 4(c_1 - B_1/2), \tag{A.11}$$

which is thus the requirement for the between-group prize E to ensure the unique mutual cooperation equilibrium developed in this appendix.

(A.11) has a very nice interpretation. The requirement for having a within-group prisoner's dilemma is $0 < B_1/2 < c_1 < B_1$. The expression $c_1 - B_1/2$ gives the extent to which the cost c_1 of cooperation for an individual agent exceeds the lower limit $B_1/2$ where the within-group game goes from being a within-group prisoner's dilemma to becoming a within-group pure cooperation game. This expression $c_1 - B_1/2$ is then multiplied with the total number, four, of individual agents in the two groups. This gives the minimum value of E required to ensure cooperation by all four individual agents in the two groups.

Tables A1–A3 can be illustrated with the four-dimensional hypercube in Fig. A1, where $c_1 = 3$ and $B_1 = 4$ dictate that we have a within-group prisoner's dilemma, while $E = 8$ nevertheless guarantees that the unique mutual cooperation equilibrium is attained.

Fig. A1 illustrates how the between-group assumption of a value E to be competed for between the two groups turns the basic within-group prisoner's

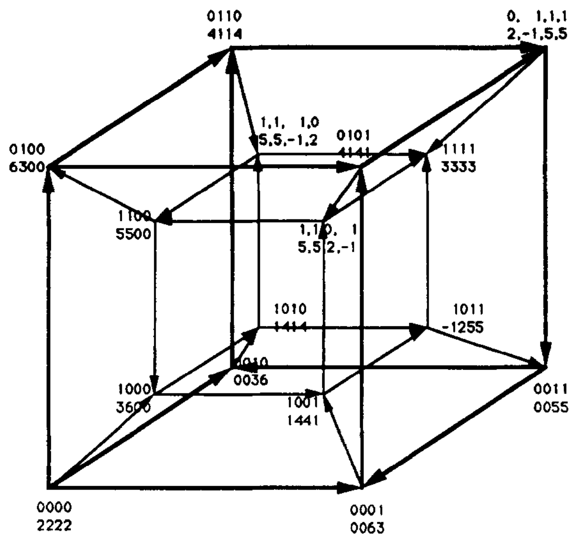

Fig. A1. Four-dimensional hypercube for payoffs to four individual agents divided into two groups.

dilemma into a game with mutual cooperation as the unique equilibrium. The hypercube in Fig. A1 has 16 nodes, corresponding to the 16 possible combinations of cooperation and defection by the four individual agents in the two groups. Each node has eight numbers attached to it, which are to be read as follows. The four numbers in the upper row are zero or one, corresponding to defection and cooperation, respectively, by the four individual agents. The two leftmost numbers in the upper row are for the two individual agents in what we may call group A_1, and the two rightmost numbers in the upper row are for the two individual agents in group A_2. The four numbers in the lower row are payoffs to the four individual agents, when the combination of cooperation and defection by the four individual agents is as specified in the upper row.

All nodes adjacent to each other in Fig. A1 are such that only one of the four individual agents switches strategy, either from cooperation to defection, or from defection to cooperation. A switch of strategy will occur if a higher payoff is attained by switching strategy, which is illustrated in Fig. A1 with an arrow in the direction of the higher payoff. There are thus 32 arrows in Fig. A1.

We observe in Fig. A1 that there is only one node that has four arrows coming into it, corresponding to the unique mutual cooperation equilibrium (1, 1, 1, 1) with payoff (3, 3, 3, 3). There is also only one node with four arrows going out from it, corresponding to mutual defection (0, 0, 0, 0) with payoff (2, 2, 2, 2), which is thus as far from the equilibrium as one can get. The quickest route from mutual defection (0, 0, 0, 0) to mutual cooperation (1, 1, 1, 1) goes through four steps, and involves a zigzagging movement, i.e. a switch of strategy alternately by an individual agent from each of the two groups.

We also observe that regardless of the node in Fig. A1 in which one is located initially, one will eventually end up in the unique mutual cooperation equilibrium (1, 1, 1, 1), with one exception. There are five interconnected circles in Fig. A1, all going through four nodes, and all going through the two nodes (1, 1, 0, 0) and (0, 0, 1, 1) and thereafter one of the four chicken equilibria in Table A1. Two of the circles are on the inner cube, one is on the outer cube, and two are between the two cubes. These circles open up the possibility of infinite circular movement. No node along any of the five circles corresponds to a final resting point. The circular movements along all five circles can be broken at any point in time when at least three of the four individual agents cooperate, which happens at one of the four nodes in each of the five circles. Once any of the circles is broken, the unique mutual cooperation equilibrium is attained, from which no single individual agent has any incentive to deviate unilaterally.

One may impose a simple restriction to avoid the possibility of circular movement altogether. Fig. A1 shows that when three of the four individual agents cooperate, the single defector improves his payoff from two to three by switching to cooperation, while his companion cooperator, if he has a shorter reaction time to switch, improves his payoff from minus one to zero by switching to defection. These two gains are of equal magnitude one. However, the first switch is preferable from a utilitarian point of view, because it involves an increase in the overall payoff to the two groups from 11 to 12, whereas the second switch involves a decrease in the overall payoff to the two groups from 11 to 10. Hence if indifference is involved, and the cultural characteristics are such that switches from cooperation to defection occur faster and more often than switches from defection to cooperation, a utilitarian argument resolves the indifference. This restriction is not consistent with methodological individualism, and may well be abandoned.

The conclusion of this example is that simple competition between two groups for a between-group prize of value E turns a static within-group prisoner's dilemma into a game, where mutual cooperation by all the individual agents in the two groups is a unique equilibrium, from which no single individual agent thus has any incentive to deviate unilaterally. That is, a within-group mixed-motive game is turned into a total pure coordination game for the four individual agents. This unique mutual cooperation equilibrium is reachable from any combination of strategies initially played by the individual agents in the two groups.

Let us make some considerations of the size of the between-group prize E compared with the parameters B_1 and c_1 of the within-group prisoner's dilemmas. If the prisoner's dilemmas are 'intermediately severe', say $c_1 = 3B_1/4$, then (A.11) becomes $E > B_1$, which means that pure cooperation by the four individual agents A_{ij} is a unique equilibrium if the between-group prize E is larger than the public goods, B_1, produced by one cooperative act c by one individual agent A_{ij}. If the prisoner's dilemmas are 'maximally severe' in the sense of bordering towards pure defection games, i.e. $c_1 = B_1$, then (A.11) becomes $E > 2B_1$, which means that pure cooperation by the four individual agents A_{ij} is a unique equilibrium if the between-group prize E is larger than the public goods, $2B_1$, produced by two individual agents A_{ij} choosing cooperative acts c. If the prisoner's dilemmas are 'minimally severe' in the sense of bordering towards pure cooperation games, i.e. $c_1 = B_1/2$, then (A.11), of course, becomes $E > 0$, which means a minuscule requirement for the between-group prize E to ensure a unique cooperation equilibrium for the four individual agents A_{ij}.

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