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THIRD-DEGREE PRICE DISCRIMINATION IN OLIGOPOLY WHEN MARKETS ARE COVERED*

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We analyze oligopolistic third-degree price discrimination relative to uniform pricing when markets are covered. Pricing equilibria are critically determined by supply-side features such as the number of firms and their marginal cost differences. It follows that each firm's Lerner index under uniform pricing is equal to the weighted harmonic mean of the firm's relative margins under discriminatory pricing. Uniform pricing then lowers average prices and raises consumer surplus. We can calculate the gain in consumer surplus and loss in firms' profits from uniform pricing based only on the market data of the discriminatory equilibrium (i.e., prices and quantities).

I. INTRODUCTION

I(i). *Motivation and Contribution*

Third-degree price discrimination is a key management practice, for instance, in the form of geographic market segmentation, whereby different retail prices are charged in different countries of the European Union.[1] Due to its explicit policy objective to create a *Single Market*, the European Union, however, recently passed a geo-blocking directive (EU Regulation 2018/302) that prevents such discriminatory pricing practices at least in parts, that is,

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[1] See ECB [2011] for a documentation of the substantial price differences of international brands (in the food industry) across Euro-area countries.

for online stores that discriminated against final consumers on the grounds of their geographic (i.e., country) location. This directive has spurred an ongoing policy debate over whether geo-blocking and corresponding market segmentation practices[2] which support discriminatory practices should be banned or not.[3] Also outside of the European Union, initiatives seeking to enforce uniform pricing regimes and geo-blocking bans have been launched (see, e.g., Picht [2021], for a recent initiative in Switzerland).

While much is now known about the welfare effects and the profitability of third-degree price discrimination in a monopoly, our knowledge about oligopolistic third-degree price discrimination is less clear-cut. Based on the logic of the widely used Hotelling model of product differentiation, we show that under price discrimination the increase in prices in markets with low competitive intensities is higher than the decrease of prices in markets with high competitive intensities. As a consequence of which, preventing discriminatory practices (for instance, banning geo-blocking) undoubtedly benefits consumers as a whole.

In detail, our contribution is to analyze the consumer welfare effects of third-degree price discrimination in an oligopoly, where firms have different marginal production costs. Usually when the price regime moves from discriminatory to uniform pricing, there is an output effect, a reallocation effect and an average price effect (see, for instance, equation (5) in Chen *et al.* [2021]). The Hotelling-demand model—that we generalize to the case of m brands and n markets—has the special feature that a rival's price change affects a firm's demand in the same magnitude (but in opposite direction) as the firm's own price change. We first show that this special feature leads to the result that the output in each market by each individual firm remains unchanged when the price regime changes, and thus eliminates the output effect and the reallocation effect. Thus, price discrimination does not affect social welfare in our model. But there is still an average price effect. Relative to price discrimination, uniform pricing lowers average prices due to a reduction in market power, and increases consumer surplus and reduces firm profit.

We show that each firm's aggregate price elasticity under uniform pricing is the weighted arithmetic mean of the firm's market-specific price elasticities under discriminatory pricing, where the weights are given by the firm's output in market j relative to its total output. We also show that the relative margin (or Lerner index) under uniform pricing is given by the weighted harmonic mean of the firm's relative margins (or Lerner indices) under discriminatory

[2] The European Commission recently fined *AB InBev*, the world's largest beer brewer, for implementing "territorial supply constraints" which facilitated price discrimination across countries by preventing cross-border sales at the wholesale level. The Commission declared such discrimination by geographical markets as incompatible with the rules of the Single Market (see the Commission Decision C (2019) 3465, case AT.40134 – AB InBev beer trade restrictions).

[3] See, for example, *https://www.europarl.europa.eu/thinktank/en/search.html?word=geo-blocking&page=1* (accessed on May 6, 2021).

pricing, where the weights are given by the firm's output in market j relative to its total output. The harmonic mean logic implies that the relative margin under uniform pricing is always strictly lower than the weighted arithmetic mean of the relative margins under discriminatory pricing; in other words, market power is reduced. This translates into the aggregate Lerner index being smaller under uniform than under discriminatory pricing.

A practically important finding of our analysis is that the consumer surplus loss from price discrimination can be calculated based only on observables under discriminatory pricing. Using market data including sales and posted prices under discriminatory pricing, the counterfactual gain (loss) in consumer surplus and firm profits when the price regime moves to uniform pricing can be easily calculated.

Due to best-response symmetry—whereby firms "agree" on the ordering of their discriminatory prices across markets—firms have clear incentives to collectively achieve the discriminatory outcome. For instance, firms may want to segment markets and prevent arbitrage to make price discrimination possible. From the firms' perspective, the discriminatory equilibrium represents a Pareto-improvement vis-à-vis the equilibrium outcome under uniform pricing.

Our demand system is closely related to the one proposed by Somaini and Einav [2013], who generalized the Hotelling duopoly model to the case of $m \geq 2$ firms. Demand is covered, all firms are directly linked and compete symmetrically with each other. In an analogy to the monopoly benchmark, which exclusively highlights the demand-side determinants of the welfare effects of price discrimination, we analyze an oligopoly with inelastic market demands to focus the analysis on the supply-side determinants of price discrimination and its welfare effects. We achieve this modeling approach with a "covered demand" model, where a firm's own price change has the same impact on its demand as a rival's price change, and where firms' products are symmetrically differentiated. Even though the demand system is restrictive, demand characteristics affect the size of the different markets, the price levels, and firms' market shares that can vary across markets. While price discrimination has no effect on social welfare, it affects consumer surplus, which represents the objective of most antitrust authorities (see, e.g., Davies and Lyons [2007] or Whinston [2007]). But of course, in markets that are not covered, such as new markets (for instance, the music streaming market), price discrimination can increase quantities substantially and therefore also raise consumer welfare (see Waldfogel [2020] for evidence from the music streaming market).[4]

[4] For the music streaming market, Waldfogel [2020] estimates the gains in revenues and changes in consumer surplus associated with the switch from country-specific pricing to uniform world pricing, and he does so both for the assumption that Spotify is a monopolist and for the assumption that Spotify and Apple music are duopolists. Due to positive quantity effects arising from country-specific pricing, discriminatory pricing raises both firm revenues and consumer

In a first extension, we show that our insights also hold if price discrimination is constrained by arbitrage costs. Practically, unconstrained price discrimination can only become effective if arbitrageurs cannot resell goods sourced in the low-price region to the high-price region (see Armstrong [2008]). Thus, when policymakers wish to discourage price discrimination, they will often take the indirect route of ensuring that consumer arbitrage is as easy as possible, for instance, by integrating markets (see Armstrong [2008]). In the EU, the creation of a *Single Market* is an explicit policy objective. Accordingly, the European Union passed the geo-blocking directive (EU Regulation 2018/302), which has banned price discrimination of online stores vis-à-vis final consumers on the grounds of their geographic (i.e., country) location since 2018. This recent geo-blocking directive fits this strategy, as it tries to enhance cross-border arbitrage by consumers. If markets are perfectly integrated in the sense that consumers can buy a certain good in any other country under the terms posted in that country, then any international price discrimination is doomed to fail, so that the products of any firm i must be traded at the same price in the integrated market area. According to our analysis, such market integration—which makes arbitrage as easy as possible and effectively yields uniform pricing—is desirable from a consumer point of view.

In a second and a third extension, we show that our analysis also applies to more flexible demand conditions, as long as each firm's total output is independent of the pricing regime. Here, we investigate demand structures where a firm's own price change has a larger impact on its demand relative to a rival's price change and where firms' products are asymmetrically differentiated in each market. In those instances, consumers are harmed not only by the price-level effect (which is the focus of our analysis) but also by the misallocation effect that arises from price discrimination.

I(ii). *Related Literature*

The related literature can be divided into the literature on monopolistic and oligopolistic third-degree price discrimination. The literature on monopolistic third-degree price discrimination has focused on the demand conditions which determine the welfare effects of price discrimination. This welfare effect results from a trade-off between the misallocation effect and the output effect relative to the uniform pricing rule. While Pigou [1920] considered the linear (downward sloping) demand case, Robinson [1933], Schmalensee [1981], and more recently Aguirre *et al.* [2010] derived complementary results for convex

surplus. Hence, our model is clearly not applicable if quantity effects can be expected to be large. Moreover, the advantage of our model—that it allows for differences in marginal costs across firms—should not play an important role for the music streaming market. So altogether, our model is not well-suited to represent this market.

and concave demands. The main takeaway of those works is, that a large (low) quantity response to a price decrease (increase) in the weak (strong) market favors price discrimination over uniform pricing; which holds if demand in the weak (strong) market is convex (concave). Varian [1985] extends Schmalensee [1981] by allowing for imperfect arbitrage when marginal costs are constant or increasing, and Schwartz [1990] extends Varian [1985] for the case where marginal costs are decreasing. Cowan [2012, 2016] focuses on how social welfare and consumer surplus effects of monopolistic third-degree price discrimination depend on market demands. Beside other things, he identifies "reasonable" demand conditions such that price discrimination increases consumer surplus. Similar results for a multi-market monopolist are obtained by Vickers [2020], who applies a new methodology of assuming that firms directly choose levels of consumer surplus in order to analyze the regulation of *relative* prices; Vickers [2020] also investigates monopolistic profit maximization under constrained price differences, which we analyze (for the oligopolistic case) in an extension when discussing imperfect arbitrage. The main insight from this literature is that when switching from uniform pricing to third-degree price discrimination the price rises in one (the "strong") and falls in the other (the "weak") market, while the curvatures of demand functions are critical for the resulting output and welfare effect. It remains an open question, however, in how far these insights apply to oligopolistic markets.

The literature on oligopolistic third-degree price discrimination is relatively sparse. It has to be divided into approaches that build on best-response symmetry—where firms agree on where to set higher prices—and those that build on best-response asymmetry—where firms disagree on where to set higher prices (see Corts [1998]). Under best-response asymmetry, firms disagree on where to set higher and where to set lower prices; in this case, firms find themselves in a prisoner's dilemma as price discrimination intensifies competition (see, e.g., Thisse and Vives [1988]; Armstrong [2008]). Firms then have a collective incentive to prevent price discrimination (see, e.g., Stole [2007]). The literature on best-response symmetry started out with Holmes [1989], who mainly showed that the output effect of third-degree price discrimination is the sum of Schmalensee's [1981] adjusted concavity condition (which mirrors the market demand effect) and the elasticity-ratio condition (which picks up the oligopolistic competition effect).[5] Subsequent work on oligopolistic third-degree price discrimination with symmetric firms has been further studied in Armstrong and Vickers [2001], Weyl and Fabinger [2013], Adachi and Matsushima [2014],

[5] In a spatial model of monopolistic competition that is not analytically tractable, Borenstein [1985] compares different sorting criteria for firms, and finds, using computer simulations, that price discrimination based on consumers' reservation prices tends to be better for consumers than discrimination based on the strength of consumer brand preferences.

Adachi [2022], and Miklos-Thal and Shaffer [2021]. For instance, in a symmetric oligopoly model with linear demand, Adachi and Matsushima [2014] show that price discrimination never raises consumer surplus but can raise social welfare. Armstrong and Vickers [2001] show that for sufficiently competitive markets, price discrimination increases profits and reduces welfare.

In the context of two-sided markets, Armstrong [2006] has shown that in a symmetric platform duopoly where the covered-market assumption holds, consumers benefit from uniform pricing if network effects are not too strong (as is the case in our model where they are zero). Tan and Zhou [2021] extend Armstrong [2006] by analyzing oligopolistic price competition with multi-sided markets. As a side finding for the case without externalities, they derive the same key inequality of comparing the arithmetic with the harmonic mean under the same assumption of full-market coverage, a general demand system, and symmetric costs (see the working paper version Tan and Zhou [2019]; Corollary 3 in the Appendix). Tan and Zhou [2019] also show that if cross-market externalities are strong enough, the comparison of pricing regimes can reverse.

Building on earlier work for the monopolistic case (Chen and Schwartz [2015]), Chen *et al.* [2021] analyze differential pricing in oligopolies where market-delivery costs differ across markets. With such market-specific delivery costs, uniform pricing necessarily induces an allocative inefficiency as cost differences cannot be reflected in prices. As all our results also apply when firms have uniform market-specific delivery costs, we also contribute to this strand of the literature. Finally, Adachi [2022] extends Aguirre *et al.* [2010] to the symmetric oligopoly case.[6] Altogether, the existing literature thoroughly analyzes monopolistic price discrimination and oligopolistic price discrimination when oligopolists are symmetric, but it does not pin down the effects of third-degree price discrimination in asymmetric oligopolies.[7] Moreover, our approach allows for an easy calculation of the uniform pricing counterfactual only based on observed prices and outputs under discriminatory pricing.

We proceed as follows. In Sections II and III we present the covered demand model and analyze the relation between discriminatory and uniform pricing. Section IV discusses several extensions to our basic model. Finally, Section V concludes. In the Appendix, we relate our model to a generalized multi-market Hotelling model.

[6] In the last part of his paper, Adachi [2022] also discusses heterogeneous firms (see his Proposition 6), but do not derive results based on exogenous model inputs.

[7] One notable exception to this is Galera and Zaratiegui [2006], who show that in asymmetric Cournot oligopolies price discrimination can increase welfare even if output shrinks. This is because the low-cost firm cuts prices by so much that its cost savings outweigh the negative welfare effect from output shrinkage.

II. THE MODEL

We build on the (linear-) covered demand model (in short, LCD-model), which is closely related to the generalized Hotelling model proposed by Somaini and Einav [2013]. Assume m firms sell their products in n markets, where $m, n > 1$. Throughout the paper, subscripts refer to firms and superscripts to markets. Each firm produces a single product and firm i's marginal production cost is $c_i \geq 0$.

Market demands are independent and completely inelastic. The demand of firm i in market j is a linear function of its own price p_i^j and all other firms' prices in that market. We assume symmetry in all substitutability relations. In addition, all products are directly linked, so that consumers as a whole can substitute among all other products. Taken together, we obtain a (linear-) covered demand model $LCD := \left\{D_i^j\right\}_{i=1,\dots,m}^{j=1,\dots,n}$, where the demand of firm i in market j is given by

$$(1) \qquad D_i^j(p_1^j, \dots, p_m^j) = a^j + b^j \sum_{i' \neq i} (p_{i'}^j - p_i^j), \text{ with } a^j > 0 \text{ and } b^j > 0.$$

The LCD-model nests the Hotelling duopoly model and the Salop circular model for two and three firms.[8] It does not nest the Salop model for four and more firms. To understand the difference, take $m = 4$. In the Salop model each firm only competes directly with its two neighbors and not with the remaining competitor. This kind of asymmetry of the Salop model is eliminated in our model, where all firms compete directly. In the LCD-model, the four-firm case can be illustrated by a tetrahedron, namely by six equally long lines such that all four firms are bilaterally connected with each other. The interpretations of the parameters a^j and b^j follow directly from the Hotelling model (see the Appendix). While a^j measures the size of market j in terms of the consumer mass, b^j is inversely related to the transportation parameter which stands for the competitive intensity. The larger (lower) the transportation cost parameter in market j, the lower (higher) the value of b^j. We formally derive the LCD-model from a horizontal-differentiation model (a generalized Hotelling model) in the Appendix.

Notably, there are other generalizations of the Hotelling duopoly toward m goods such as the *spokes model* by Chen and Riordan [2007]. This model can be visualized by points on a circle that give firms' locations, and all firms' locations are connected via a hub-and-spoke structure. Here, the brands are physically identical but are differentiated by their different locations. Consumers are uniformly distributed on the network of spokes, and each consumer has one most preferred brand (i.e., the firm that sits at the end of that spoke on

[8] In fact, the demand function (1) captures the Hotelling model if we have an inner solution, that is, if the price the marginal consumer pays plus her transportation costs do not exceed her valuation for the product.

which the consumer sits), and unlike in our model there is not only a single second preferred brand, but each of the other $m-1$ brands is equally likely to be her second preferred brand. While the model we build on is more tractable, in fact, the models are quite similar and nest special cases of each other (for a comparison, see the discussion in Somaini and Einav [2013]).

Our demand model applies to markets where a certain number of (independently supplied) substitutable goods compete in n separate markets and where all brands are sold in every market. Each consumer in any market has a most preferred brand she would choose if all prices were equal. Moreover, if all prices are equal (and not prohibitively large), then demands for all goods would be the same (i.e., symmetric demand assumption) and all consumers would buy one unit of one of the goods (covered market assumption). Similar to Salop's extension of the two-goods Hotelling model—but in contrast to the spokes model—each consumer considers only one other good as an effective alternative to her most preferred brand. Importantly, the considered m goods fully specify the choice sets of the consumers, so that "not buying at least one of them" (e.g., by reverting to an outside good) is not relevant for any consumer.

The demand model, therefore, best fits to product categories consisting of m international brands (e.g., automobiles, breakfast cereals, detergence products, or sports shoes), which are offered in all n (geographically distinct) markets, and where consumers buy one unit of the considered brands. In the context of the European Single market, we think of international brands offered in several EU countries. Demand in each market is differentiated and a single consumer effectively only chooses between two of the m brands: a most preferred brand and an alternative one being part of the considered m brands. A consumer, therefore, compares the net utilities from the most preferred brand and the second best choice. If this difference is larger than the price difference of the brands, then the consumer buys the most preferred brand and otherwise the second best one. It follows that individual demand in any market only depends on the price difference of the considered brands and is independent of the other brands' prices. Aggregation of all individual demands in any market preserves this property when all consumers decide accordingly. Finally, the market is a mature market and exhibits a stable and inelastic market demand.

III. ANALYSIS

This LCD-model has several convenient properties that we list in the following:

(A1) $\frac{\partial D_i^j}{\partial p_i^j} = -(m-1)b^j$ for all i and j,

(A2) $\sum_i D_i^j(p_1, \ldots, p_m) = ma^j$ for all j,

(A3) $D_i^j - D_{i'}^j = b^j m(p_{i'}^j - p_i^j)$ for all i, i', and j.

As a consequence of these properties, firm i's demand is linear in its price (A1), aggregate demand is inelastic (A2), and the demand differences between two firms are pinned down by the difference in the prices these two firms set and therefore independent from other prices charged (A3).

Throughout the paper we maintain the assumption that the discriminatory pricing equilibrium, $\{\overline{p}_i^j\}_{i=1,\dots,m}^{j=1,\dots,n}$, and the uniform pricing equilibrium, $\{\widehat{p}_i\}_{i=1,\dots,m}$, are unique and interior. Obviously, there exists a unique interior equilibrium under both discriminatory and uniform pricing if neither the demand characteristics nor costs are too heterogeneous (see also Somaini and Einav [2013]).[9]

In the following proposition, we compare the Nash equilibrium when firms simultaneously charge uniform prices across markets and when firms engage in third-degree price discrimination, thereby charging different prices in the markets.

Proposition 1. Assume an LCD-model and constant marginal production costs $c_i \geq 0$ for all $i = 1, \dots, m$. Then, the discriminatory equilibrium and the uniform pricing equilibrium fulfill the following properties:

(i) All bilateral price differences are the same under discriminatory and uniform pricing, such that $\overline{p}_{i'}^j - \overline{p}_i^j = \widehat{p}_{i'} - \widehat{p}_i = \frac{m-1}{2m-1}(c_{i'} - c_i)$ holds for all i, i' and j.
(ii) All firms' output levels in all markets are the same in the discriminatory and the uniform pricing equilibrium; that is, $D_i^j(\overline{p}_1^j, \dots, \overline{p}_m^j) = D_i^j(\widehat{p}_1, \dots, \widehat{p}_m) = a^j + b^j\left(\frac{m-1}{2m-1}\right)\sum_{i' \neq i}(c_{i'} - c_i)$ for all i and j.

Proof. Under discriminatory pricing each firm i solves

$$\max_{p_i^1,\dots,p_i^n \geq 0} \pi_i = \sum_{j=1}^{n} D_i^j(p_1^j, \dots, p_m^j)(p_i^j - c_i).$$

The unique and interior Nash equilibrium prices $\{\overline{p}_i^j\}_{i=1,\dots,m}^{j=1,\dots,n}$ fulfill

$$\frac{\partial D_i^j}{\partial p_i^j}(\overline{p}_i^j - c_i) + D_i^j = 0 \text{ for all } i \text{ and all } j. \tag{2}$$

[9] If firms and markets are perfectly symmetric (i.e., $c_i = c_{i'}$, $a^j = a^{j'}$, $b^j = b^{j'}$), there exists a unique interior equilibrium under both pricing regimes. By continuity, this also holds if $|a^j - a^{j'}|$, $|b^j - b^{j'}|$, and $|c_i - c_{i'}|$ are sufficiently small for all admissible i, j, i', j'. Under price discrimination, existence and uniqueness are obviously fulfilled when the least efficient firm produces a strictly positive quantity in the market j with the lowest a^j/b^j. Under uniform pricing, the exact conditions for the existence of the equilibrium are complex as already the corresponding analysis in Adachi and Matsushima [2014] for a symmetric duopoly shows.

Fix some j and take two firms $i \neq i'$. The equilibrium price difference $\overline{p}^j_{i'} - \overline{p}^j_i$ follows from subtracting the first-order conditions $\frac{\partial \pi_{i'}}{\partial p^j_{i'}} = 0$ and $\frac{\partial \pi_i}{\partial p^j_i} = 0$, which gives

$$\frac{\partial D^j_{i'}}{\partial p^j_{i'}}(\overline{p}^j_{i'} - c_{i'}) - \frac{\partial D^j_i}{\partial p^j_i}(\overline{p}^j_i - c_i) + D^j_{i'} - D^j_i = 0.$$

Using (A1) and (A3) we get

$$(3) \qquad -(m-1)b^j(\overline{p}^j_{i'} - \overline{p}^j_i) - b^j m(\overline{p}^j_{i'} - \overline{p}^j_i) = -(m-1)b^j(c_{i'} - c_i) \text{ or}$$

$$\overline{p}^j_{i'} - \overline{p}^j_i = \frac{m-1}{2m-1}(c_{i'} - c_i).$$

Under uniform pricing each firm i solves

$$\max_{p_i \geq 0} \pi_i = \sum_{j=1}^{n} D^j_i(p_1, \dots, p_m)(p_i - c_i).$$

The unique and interior Nash equilibrium prices $\{\widehat{p}_i\}_{i=1,\dots,m}$ fulfill

$$(4) \qquad \sum_{j=1}^{n}\left[\frac{\partial D^j_i}{\partial p_i}(\widehat{p}_i - c_i) + D^j_i\right] = 0 \text{ for all } i.$$

Take two firms $i \neq i'$. The equilibrium price difference $\widehat{p}_{i'} - \widehat{p}_i$ follows from subtracting the first-order conditions $\frac{\partial \pi_{i'}}{\partial p_{i'}} = 0$ and $\frac{\partial \pi_i}{\partial p_i} = 0$, which gives

$$\sum_{j=1}^{n}\frac{\partial D^j_{i'}}{\partial p_{i'}}(\widehat{p}_{i'} - c_{i'}) - \sum_{j=1}^{n}\frac{\partial D^j_i}{\partial p_i}(\widehat{p}_i - c_i) + \sum_{j=1}^{n} D^j_{i'} - \sum_{j=1}^{n} D^j_i = 0.$$

Using (A1) and (A3) we get

$$(5) \quad -(m-1)\sum_{j=1}^{n} b^j(\widehat{p}_{i'} - \widehat{p}_i) - m(\widehat{p}_{i'} - \widehat{p}_i)\sum_{j=1}^{n} b^j = -(m-1)\sum_{j=1}^{n} b^j(c_{i'} - c_i) \text{ or}$$

$$\widehat{p}_{i'} - \widehat{p}_i = \frac{m-1}{2m-1}(c_{i'} - c_i).$$

From (3) it follows that the price difference between two firms i and i' is the same in all markets j under discrimination. Comparison with (5) shows that the price difference under uniform pricing yields exactly the same difference. Finally, part (ii) of the proposition follows from substituting (3) for all $i' \neq i$ into (1). ■

Price competition yields the same price differences under discriminatory and uniform pricing (part (i) of Proposition 1). Consequently, firms' output levels in any market j are independent of the pricing regime (part (ii) of Proposition 1).[10] In addition, when the number of firms increases, price differences increase.[11] The underlying demand system ensures that price differences are fully driven by supply-side features; namely marginal cost asymmetries and the number of firms m.

Interestingly, even though price differences between the firms are, under discriminatory pricing, the same in every market, a firm's market share may differ across markets. The market share of firm i in market j is given by

$$s_i^j := \frac{D_i^j}{\sum_{i=1}^m D_i^j} = \frac{1}{m}\left[1 + \frac{b^j}{a^j}\left(\frac{m-1}{2m-1}\right)\sum_{i'\neq i}(c_{i'} - c_i)\right],$$

where the last equality follows from (A2) and from part (ii) of Proposition 1. Note also that $\sum_{i'\neq i}(c_{i'} - c_i) = m(c^e - c_i)$, where $c^e := \sum_{i=1}^m c_i/m$ stands for average marginal costs. Suppose $b^j/a^j > b^{j'}/a^{j'}$ holds. Then, $s_i^j > s_i^{j'}$ ($s_i^j < s_i^{j'}$) follows if and only if $c_i < c^e$ ($c_i > c^e$). A firm with below-average marginal costs, therefore, gets a larger market share in market j than in j', when the competitive intensity (as measured by b^j/a^j) increases.[12] This result is also mirrored in (A3), whereby the demand difference between two firms gets larger when the parameter b^j increases.

Proposition 1 implies that the consumer surplus difference under uniform and discriminatory pricing, $\widehat{CS} - \overline{CS}$, which must be equal to the reversed difference of total profits, $\sum_i \overline{\pi}_i - \sum_i \widehat{\pi}_i$, can be derived directly from comparing the uniform and the discriminatory prices.

Corollary 1. The consumer surplus difference and the total profit difference under uniform and discriminatory pricing are given by

$$\widehat{CS} - \overline{CS} = \sum_{i=1}^m \overline{\pi}_i - \sum_{i=1}^m \widehat{\pi}_i = \sum_{i=1}^m \sum_{j=1}^n (\overline{p}_i^j - \widehat{p}_i) D_i^j. \tag{6}$$

[10] In the following, we drop the arguments of D_i^j, which from now on stands for the equilibrium values $D_i^j(\overline{p}_1^j, \ldots, \overline{p}_m^j)$ or $D_i^j(\widehat{p}_1, \ldots, \widehat{p}_m)$.

[11] Under both pricing regimes, the price difference is equal to the marginal cost difference times the term $(m-1)/(2m-1)$, which increases monotonically in m over the interval $[1/3, 1/2)$. In the limiting case of $m \to \infty$ it approaches $1/2$.

[12] In the Appendix, we shown how a^j and b^j can be derived from a generalized Hotelling model. In particular, b^j/a^j increases when the transportation costs parameter (t^j) decreases or the length of the Hotelling line (L^j) shortens. Here, we also provide a scenario with additional loyal consumers, in which case b^j/a^j decreases when the share of loyal consumers increases.

Based on Proposition 1, we can easily calculate the Nash equilibrium prices under both pricing regimes. In the discriminatory regime, firm i's first-order condition in market j is given by (2). Solving for $\overline{p}_i^j$ we get

$$(7)\qquad \overline{p}_i^j = c_i - \frac{D_i^j}{\frac{\partial D_i^j}{\partial p_i^j}} = c_i + \frac{a^j}{(m-1)b^j} + \left(\frac{1}{2m-1}\right)\sum_{i'\neq i}(c_{i'} - c_i).$$

This formula implies that the ordering of a firm's discriminatory equilibrium prices mirrors the ordering of markets according to $\frac{a^j}{b^j}$, that is $\overline{p}_i^j > \overline{p}_i^{j'} \Leftrightarrow \frac{a^j}{b^j} > \frac{a^{j'}}{b^{j'}}$ for all i and j with $j \neq j'$. Hence, the LCD demand system ensures that the discriminatory equilibrium is characterized by best-response symmetry.

For the uniform pricing regime, the Nash equilibrium price of firm i can be obtained from firm i's first-order condition (4). Solving for $\widehat{p}_i$ we get

$$(8)\qquad \widehat{p}_i = c_i - \frac{\sum_{j=1}^n D_i^j}{\sum_{j=1}^n \frac{\partial D_i^j}{\partial p_i}} = c_i + \frac{\sum_{j=1}^n \left[a^j + b^j\left(\frac{m-1}{2m-1}\right)\sum_{i'\neq i}(c_{i'} - c_i)\right]}{(m-1)\sum_{j=1}^n b^j}.$$

We next examine how the discriminatory and uniform pricing equilibrium are related. Define firm i's equilibrium price elasticity in market j under discriminatory pricing by

$$(9)\qquad \overline{E}_i^j := E_i^j(\overline{p}_1^j, \dots, \overline{p}_m^j) := -\frac{\partial D_i^j}{\partial p_i^j}\frac{\overline{p}_i^j}{D_i^j}$$

and firm i's aggregate equilibrium price elasticity under uniform pricing by

$$(10)\qquad \widehat{E}_i := E_i(\widehat{p}_1, \dots, \widehat{p}_m) := -\frac{\sum_{j=1}^n \frac{\partial D_i^j}{\partial p_i}\widehat{p}_i}{\sum_{j=1}^n D_i^j}.$$

Firm i's Lerner index under discriminatory pricing is equal to the weighted arithmetic mean of its market-specific Lerner indices, $\overline{L}_i^j := \frac{\overline{p}_i^j - c_i}{\overline{p}_i^j}$, where the weights are given by firm i's output in market j, D_i^j, relative to its total output, $\sum_{j=1}^n D_i^j$; that is,

$$\overline{L}_i := \sum_{j=1}^n \left[\frac{D_i^j}{\sum_{j=1}^n D_i^j}\overline{L}_i^j\right].$$

Define the aggregate Lerner index under discriminatory pricing by $\overline{L} := \sum_i s_i \overline{L}_i$, where $s_i := \frac{\sum_j D_i^j}{\sum_j \sum_i D_i^j}$ stands for firm i's overall market share.

In the case of uniform pricing, $\hat{L}_i := \frac{\hat{p}_i - c_i}{\hat{p}_i}$ and $\hat{L} := \sum_i s_i \hat{L}_i$ stand for firm i's Lerner index and for the aggregate Lerner index, respectively. The following lemma and the next proposition pin down the relation between all these values under uniform and discriminatory pricing.

Lemma 1. Assume an LCD-model. The comparison of the discriminatory and the uniform pricing equilibrium gives the following relations:

(i) Firm i's aggregate equilibrium price elasticity under uniform pricing is given by the weighted Arithmetic Mean Formula:

$$\hat{E}_i = \sum_{j=1}^{n} \left[\frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \overline{E}_i^j \right] \text{ holds for all } i.$$

(ii) If $c_i > 0$ for all i, firm i's Lerner index under uniform pricing is given by the weighted Harmonic Mean Formula:

$$\hat{L}_i = \frac{1}{\sum_{j=1}^{n} \left[\frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \frac{1}{\overline{L}_i^j} \right]} \text{ holds for all } i. \quad (11)$$

Proof. Assume discriminatory pricing. Summing up firm i's first-order conditions over all markets j gives

$$\sum_{j=1}^{n} \left[\frac{\partial D_i^j}{\partial p_i^j} (\overline{p}_i^j - c_i) \right] + \sum_{j=1}^{n} D_i^j = 0.$$

Under uniform pricing, firm i's first-order condition is given by (4). From Proposition 1 it follows that firm i's equilibrium demand in every market is the same under both pricing regimes, which implies that, in particular, the total output of each firm is independent of the pricing regime; that is

$$\sum_{j=1}^{n} D_i^j(\overline{p}_1^j, \ldots, \overline{p}_m^j) = \sum_{j=1}^{n} D_i^j(\hat{p}_1, \ldots, \hat{p}_m). \quad (12)$$

It thus follows that

$$\sum_{j=1}^{n} \frac{\partial D_i^j}{\partial p_i} (\hat{p}_i - c_i) = \sum_{j=1}^{n} \left[\frac{\partial D_i^j}{\partial p_i^j} (\overline{p}_i^j - c_i) \right]. \quad (13)$$

Simplifying and expanding both sides we get

$$\frac{\sum_{j=1}^{n} \frac{\partial D_i^j}{\partial p_i} \hat{p}_i}{\sum_{j=1}^{n} D_i^j} \left(\sum_{j=1}^{n} D_i^j \right) = \sum_{j=1}^{n} \left[\frac{\partial D_i^j}{\partial p_i^j} \frac{\overline{p}_i^j}{D_i^j} D_i^j \right].$$

Using (9) and (10) we get

$$(14)\qquad \widehat{E}_i \sum_{j=1}^{n} D_i^j = \sum_{j=1}^{n} \left[\overline{E}_i^j D_i^j \right] \quad \text{or}$$

$$\widehat{E}_i = \sum_{j=1}^{n} \left[\frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \overline{E}_i^j \right].$$

The equilibrium aggregate demand elasticity of firm i under uniform pricing is equal to the weighted arithmetic mean of firm i's demand elasticities under discriminatory pricing. The weight of firm i's demand elasticity in market j is given by the share of firm i's total output sold in market j. This gives part (i).

Next, we can re-write firm i's first-order condition under uniform pricing (see (4)) as

$$\frac{\widehat{p}_i - c_i}{\widehat{p}_i} = \frac{1}{\widehat{E}_i}.$$

Likewise, under discriminatory pricing we can re-write each of firm i's first-order conditions (see (2)) as

$$\frac{\overline{p}_i^j - c_i}{\overline{p}_i^j} = \frac{1}{\overline{E}_i^j}.$$

Taken together and using (14) we get

$$(15)\qquad \frac{\widehat{p}_i - c_i}{\widehat{p}_i} = \frac{1}{\sum_{j=1}^{n} \left[\frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \overline{E}_i^j \right]} = \frac{1}{\sum_{j=1}^{n} \left[\frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \left(\frac{\overline{p}_i^j - c_i}{\overline{p}_i^j} \right)^{-1} \right]}.$$

Using the definitions of $\widehat{L}_i$ and $\overline{L}_i^j$, we get the formula stated in part (ii). ■

According to part (i) of Lemma 1, each firm's aggregate equilibrium elasticity under uniform pricing is the weighted arithmetic mean of a firm's equilibrium elasticities under discriminatory pricing, which follows from the fact that a firm's total equilibrium output (see equation 13) does not change with the pricing regime (Proposition 1). Part (ii) shows that the Lerner index of any firm i under uniform pricing is the weighted harmonic mean of its market-specific Lerner indices under discriminatory pricing, where the weights are given by firm i's output in market j, relative to its total output.

Lemma 1 gives rise to the following proposition.

Proposition 2. Assume an LCD-model. Comparing the discriminatory and the uniform pricing equilibrium gives the following relations:

(i) If $c_i > 0$ for all i and there are j and j' so that $b^j/a^j \neq b^{j'}/a^{j'}$, then all firm-level Lerner indices and the aggregate Lerner index are strictly smaller under uniform pricing than under discriminatory pricing; that is,

$$\widehat{L}_i < \overline{L}_i \text{ holds for all } i \text{ and } \widehat{L} < \overline{L}.$$

(ii) If there are *j and j′* so that $b^j/a^j \neq b^{j'}/a^{j'}$, then firm i's uniform price is strictly smaller than the weighted arithmetic mean of its discriminatory prices; that is,

$$\widehat{p}_i < \sum_{j=1}^{n} \frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \overline{p}_i^j \text{ holds for all } i.$$

Proof. According to Jensen's inequality,[13] it must be that

$$\sum_{j=1}^{n} \left[\frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \frac{1}{\overline{L}_i^j} \right] > \frac{1}{\sum_{j=1}^{n} \left[\frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \overline{L}_i^j \right]},$$

which implies $\widehat{L}_i < \overline{L}_i$ and also $\widehat{L} < \overline{L}$, because s_i is independent of the pricing regime. This proves part (i) of the proposition. Thus, part (i) follows from Lemma 1, part (ii).

Next, for $c_i > 0$, we show that part (ii) follows from part (i) (namely $\widehat{L}_i < \overline{L}_i$) and is, therefore, also a consequence of the harmonic mean formula. It is straightforward to see that all derivations are independent of c_i and therefore also hold for $c_i = 0$. First note that we can re-write $\overline{L}_i$ as

$$\overline{L}_i = 1 - c_i \sum_{j=1}^{n} \frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \frac{1}{\overline{p}_i^j}.$$

Thus, $\widehat{L}_i < \overline{L}_i$ is equivalent to

(16) $$\frac{\widehat{p}_i - c_i}{\widehat{p}_i} < 1 - c_i \sum_{j=1}^{n} \frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \frac{1}{\overline{p}_i^j} \quad \text{or}$$

$$\frac{1}{\widehat{p}_i} > \sum_{j=1}^{n} \frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \frac{1}{\overline{p}_i^j}.$$

[13] Jensen's inequality implies that for any positive random variable X with a strictly positive expected value $\mathbb{E}(X)$, the inequality $\mathbb{E}\left[\frac{1}{X}\right] > \frac{1}{\mathbb{E}(X)}$ holds.

According to Jensen's Inequality, the right-hand side of (16) is strictly larger than the inverse of the weighted arithmetic mean of the discriminatory prices, so that

$$\frac{1}{\widehat{p}_i} > \frac{1}{\sum_{j=1}^{n} \frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \overline{p}_i^j}$$

follows, from which we directly get the inequality stated in part (ii) of the proposition. ■

Part (i) of Proposition 2 states that all firms' Lerner indices and the aggregate Lerner index are lower under uniform pricing than under discriminatory pricing, which directly follows from the preceding Lemma. This relation gives a clear-cut assessment of the overall effect of uniform pricing on market power. Uniform pricing unambiguously constrains firms' market power, so that firms' ability to raise prices above marginal costs is smaller than under discriminatory pricing.

The harmonic mean formula implies that all firms' uniform prices are strictly smaller than the weighted arithmetic mean of their discriminatory prices (part ii) of Proposition 2. Using Corollary 1, we then know that consumer surplus must be strictly larger under uniform than under discriminatory pricing. This follows from noticing that

$$\widehat{CS} - \overline{CS} = \sum_{i=1}^{m} \sum_{j=1}^{n} (\overline{p}_i^j - \widehat{p}_i) D_i^j = \sum_{i=1}^{m} \left[\sum_{j=1}^{n} D_i^j \left(\sum_{j=1}^{n} \left(\frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \overline{p}_i^j \right) - \widehat{p}_i \right) \right] > 0,$$

where the inequality follows from part (ii) of Proposition 2. As all firms realize lower relative margins under uniform pricing according to the harmonic mean formula, prices decrease on average, consumer surplus increases, and total producer surplus is reduced. This is intuitive, as all output levels do not change under both pricing regimes.

Proposition 2 generalizes Holmes' [1989, p. 248] conjecture that average prices increase under discriminatory pricing to an oligopoly with asymmetric firms. Holmes assumed symmetric firms and a constant elasticity demand at the firm level with inelastic market demand to show his conjecture. Relatedly, Armstrong [2007] has shown that this conjecture holds true for symmetric firms in a model closely related to ours, namely in a multi-market Hotelling model. Proposition 2 shows that Holmes' conjecture is also valid when firms are asymmetric as long as markets are fully covered, demand is linear, and products are symmetrically differentiated.

Finally, we state our central result, that the counterfactual uniform price as well as the consumer surplus gain from non-discriminatory prices can be calculated based only on market data under discriminatory pricing (i.e., observed prices and quantities).

Corollary 2. Each firm's price under uniform pricing as well as the consumer surplus gain from uniform pricing can be calculated based only on market data under discriminatory pricing:

$$\widehat{p}_i = \sum_{j=1}^{n} \left[\frac{b^j}{\sum_{j=1}^{n} b^j} \overline{p}_i^j \right].$$

The respective gain in consumer surplus is given by

$$\widehat{CS} - \overline{CS} = \frac{1}{m-1} \sum_{i=1}^{m} \left(\sum_{j=1}^{n} \frac{\left(D_i^j\right)^2}{b^j} - \frac{\left(\sum_{j=1}^{n} D_i^j\right)^2}{\sum_j b^j} \right),$$

where b^j can be determined from observables by (A3).

Proof. From (13) and (A1) we get

$$\sum_{j=1}^{n} \frac{\partial D_i^j}{\partial p_i} \widehat{p}_i = \sum_{j=1}^{n} \left[\frac{\partial D_i^j}{\partial p_i^j} \overline{p}_i^j \right] \quad \text{or} \tag{17}$$

$$\widehat{p}_i = \sum_{j=1}^{n} \left[\frac{b^j}{\sum_{j=1}^{n} b^j} \overline{p}_i^j \right].$$

Using (A3), which gives $b^j = \frac{D_i^j - D_{i'}^j}{m(p_{i'}^j - p_i^j)}$, we get $\widehat{p}_i$ directly from observed prices $\overline{p}_i^j$. Substituting (17) into (6), we get the consumer surplus gain from uniform pricing as stated above. ■

Usually, when discriminatory instead of uniform prices are set, there is an output, a reallocation, and an average price effect. The LCD-demand adopted in this paper has the feature that a rival's price change affects a firm's demand in the same magnitude (but in the opposite direction) as the firm's own price change. This special feature leads to the result that output in each market by each individual firm remains unchanged when the price regime changes, and thus eliminates the output effect and the reallocation effect. But there is still an average price effect. Relative to price discrimination, uniform pricing lowers the average price due to a reduction in market power, reduces firm profit, but increases consumer surplus.

Thus, consumers as a whole are better off when firms must charge a uniform price across markets. Correspondingly, every firm realizes a higher profit when all firms engage in price discrimination. From the firms' perspective, the discriminatory equilibrium Pareto-dominates the uniform pricing equilibrium.

It follows that firms have a joint incentive to coordinate market segmentation (e.g., by geo-blocking or, more generally, by restricting buyer arbitrage between markets). Thus, our results appear to be relevant for price discrimination along national markets (as in the EU).

Let us finally relate our results to the effect of price discrimination in a monopoly and in perfectly competitive markets. In a monopoly with linear demand, our demand structure would yield, as shown by Pigou [1920], a (negative) allocation effect and no quantity effect. Notably, the harmonic mean formula applies here also, as straightforward calculations show that it holds whenever each firm's total output is independent of the pricing regime (see condition (12)). Perfect competition, in contrast, would yield marginal-cost pricing and therefore uniform and discriminatory would be indistinguishable.[14]

Comparative Statics. We are interested in the comparative statics of our "pricing-regime effect," that is, in learning how $\widehat{CS} - \overline{CS}$ changes with demand and supply characteristics. For this, it helps to consider the weak/strong market distinction. Suppose there are two markets, that is, $n = 2$, and suppose $a^1/b^1 \neq a^2/b^2$. We take a^j/b^j as a measure of competitiveness, and without loss of generality, let market 1 be the strong and market 2 the weak market, that is, $a^1/b^1 > a^2/b^2$. The pricing-regime effect then equals

$$\widehat{CS} - \overline{CS} = \frac{2}{b^1 b^2} \frac{\left(a^1 b^2 - a^2 b^1\right)^2}{b^1 + b^2}. \tag{18}$$

Under price discrimination, the price difference of firm i across markets equals $p_i^1 - p_i^2 = \frac{a^1}{b^1} - \frac{a^2}{b^2}$. Intuitively, the larger these price differences across markets are, the larger the pricing-regime effect. Formally,

$$\frac{\partial(\widehat{CS} - \overline{CS})}{\partial b^1} = -\frac{2}{\left(b^1\right)^2 \left(b^1 + b^2\right)^2} \left(2a^1 b^1 + a^1 b^2 + a^2 b^1\right) \left(a^1 b^2 - a^2 b^1\right) \tag{19}$$

is strictly negative by our assumption that market 1 is the strong market. Increasing b^1 decreases the asymmetry between markets and therefore lowers the pricing-regime effect. Conversely, $\frac{\partial(\widehat{CS} - \overline{CS})}{\partial b^2} > 0$, as increasing b^2 increases the asymmetry between markets and increases the pricing-regime effect. The same conclusion can be drawn from comparative statics regarding a^1 and a^2:

$$\frac{\partial(\widehat{CS} - \overline{CS})}{\partial a^1} = \frac{4}{b^1} \frac{a^1 b^2 - a^2 b^1}{b^1 + b^2}$$

[14] With market-delivery costs as in Chen and Schwartz [2015], however, uniform and discriminatory pricing would yield different outcomes.

is positive (as market 1 is the strong market). Altogether, the pricing-regime effect increases in the asymmetry of markets.

Equation (18) also shows that the pricing-regime effect is independent of differences in marginal costs, which is due to the fact that price differences between firms are the same under discrimination and uniform pricing.[15]

Market-specific Delivery Costs. It is also possible to include market-specific delivery costs $c^j \geq 0$ per unit of the good for all j which affect all firms equally. In this case, firm i's marginal cost of selling products in market j is given by $c_i + c^j$. Clearly, this does not affect the price differences in any market, so that all the results of Proposition 1 remain valid. For all of our other results, anologous versions can be derived.

IV. DISCUSSION: EXTENSIONS AND LIMITATIONS

In this section we discuss three assumptions we have made in our analysis. First, we assumed that the uniform pricing rule works perfectly and bans price differences of a firm's product altogether. Second, we invoked the full market coverage assumption. Third, we assumed symmetric product differentiation among the firms' products in each market. In the following we show that these three assumptions can be relaxed to some extent without affecting our main results. While constraints on the relative prices (e.g., in the form of arbitrage costs or in the form of relative price regulation as in Vickers [2020]) can be accounted for in a simplified version of our model with two firms and two markets, a discussion of the latter two assumptions reveals limitations of our analysis when demand systems are such that firms' total outputs change across pricing regimes. Throughout this section, we maintain the assumption that all firms want to serve all markets under both pricing regimes and that this equilibrium is unique.

IV(i). *Arbitrage Costs*

We show that the harmonic mean formula can be extended to take care of arbitrage costs (or binding relative price regulations, as in Vickers [2020]). Assume that buyers can arbitrage among markets with arbitrage costs of $r \geq 0$ per unit. We focus on the case with $n, m = 2$. Thus, discriminatory prices, $\left\{\overline{p}_i^j(r)\right\}_{i=1,2}^{j=1,2}$, must fulfill the requirement $\overline{p}_i^1(r) - \overline{p}_i^2(r) \leq r$ for $i = 1, 2$. Suppose that the constraints bind. The following proposition states the main features of the arbitrage-constrained third-degree price discrimination equilibrium.

Proposition 3. Assume an LCD-model with $n, m = 2$. Assume the unconstrained discriminatory prices fulfill $\overline{p}_i^1 > \hat{p}_i > \overline{p}_i^2$. Suppose the arbitrage

[15] The model is not well-suited to conducting comparative statics regarding the number of firms m because an increase in the number of firms changes the entire demand architecture (e.g., for three firms a market can be illustrated by a triangle and for four firms by a tetrahedron).

constraint is binding for both firms; that is, $\overline{p}_i^1 - \overline{p}_i^2 \geq r$ for $i = 1, 2$. Then, the arbitrage-constrained discriminatory Nash equilibrium prices $\{\overline{p}_i^j(r)\}_{i=1,2}^{j=1,2}$ are given by

$$\overline{p}_i^1(r) = \widehat{p}_i + r\overline{\alpha} \text{ and } \overline{p}_i^2(r) = \widehat{p}_i - r(1 - \overline{\alpha}),$$

where $\overline{\alpha} := \frac{b^2}{b^1+b^2}$, with $\overline{\alpha} \in (0, 1)$. All price differences $p_{i'}^j - p_i^j$ and each firm's output in any market remains the same as under unconstrained discrimination or uniform pricing.

Proof. Each firm $i = 1, 2$ maximizes its profit $\pi_i = \sum_{j=1}^2 \left[D_i^j(p_i^j - c_i)\right]$ subject to $p_i^1 - p_i^2 \leq r$ for $i = 1, 2$. We obtain two first-order conditions of the constrained maximization problems:

$$(20) \quad \sum_{j=1}^{2} \left[\frac{\partial D_i^j}{\partial p_i^j}(p_i^j(r) - c_i) + D_i^j\right] = 0 \text{ with } \overline{p}_i^1(r) - \overline{p}_i^2(r) \leq r \text{ for } i = 1, 2.$$

Of course, the constraints must bind, so that $\overline{p}_i^1(r) - \overline{p}_i^2(r) = r$. Substitute $p_i^1(r) = \widehat{p}_i + \alpha r$ and $p_i^2(r) = \widehat{p}_i - (1 - \alpha)r$, with $\alpha \in [0, 1]$, so that $p_i^1 - p_i^2 = r$ holds for $i = 1, 2$. This gives

$$(21) \quad \frac{\partial D_i^1}{\partial p_i^1}(\widehat{p}_i + \alpha r - c_i) + D_i^1 + \frac{\partial D_i^2}{\partial p_i^2}(\widehat{p}_i - (1 - \alpha)r - c_i) + D_i^2 = 0 \text{ for } i = 1, 2.$$

or

$$(22) \quad \underbrace{\left[\left(\frac{\partial D_i^1}{\partial p_i^1} + \frac{\partial D_i^2}{\partial p_i^2}\right)(\widehat{p}_i - c_i) + D_i^1 + D_i^2\right]}_{\textit{first term}} + r\underbrace{\left(\frac{\partial D_i^1}{\partial p_i^1}\alpha - \frac{\partial D_i^2}{\partial p_i^2}(1 - \alpha)\right)}_{\textit{second term}} = 0$$

for $i = 1, 2$.

Note that each firm's equilibrium output levels do not change under the proposed solution, because $p_{i'}^j(r) - p_i^j(r) = \widehat{p}_{i'} - \widehat{p}_i$ for all i, i', and j. For each firm i, the *first term* of (22) is equal to its first-order condition under uniform pricing (4). Thus, the *first term* in the first-order conditions of firms 1 and 2 is zero at $\{p_i^j(r)\}_{i=1,2}^{j=1,2}$. The *second term* is zero at

$$\overline{\alpha} = \frac{\frac{\partial D_i^2}{\partial p_i^2}}{\frac{\partial D_i^1}{\partial p_i^1} + \frac{\partial D_i^2}{\partial p_i^2}} = \frac{b^2}{b^1 + b^2}.$$

Thus, $\widehat{p}_i^1(r) = \widehat{p}_i + \overline{\alpha}r$ and $\widehat{p}_i^2(r) = \widehat{p}_i - (1 - \overline{\alpha})r$ solves the system of first-order conditions (20). ■

From Proposition 3 it follows that

$$\overline{\alpha} = \frac{\overline{p}_i^1 - \widehat{p}_i}{\overline{p}_i^1 - \overline{p}_i^2} \text{ and } 1 - \overline{\alpha} = \frac{\widehat{p}_i - \overline{p}_i^2}{\overline{p}_i^1 - \overline{p}_i^2},$$

so that a lower value of the arbitrage parameter r must decrease the average price $\frac{D_i^1}{D_i^1+D_i^2}\overline{p}_i^1(r) + \frac{D_i^2}{D_i^1+D_i^2}\overline{p}_i^2(r)$ and thus increases consumer surplus. In other words, any policy that makes cross-market arbitrage more effective is to the benefit of consumers as a whole.

IV(ii). *More Flexible Demand*

For a general demand function, switching from price discrimination to uniform pricing gives rise to an output effect, a reallocation effect, and an average price effect. Our LCD-model focuses on the price effect, as there is no misallocation and no output effect. In a monopoly model with linear demand, the total output effect is also zero, but there is a misallocation effect, which lowers consumer surplus. At the same time there is a price effect, which—if demand is linear—is also given by the harmonic mean formula. In an oligopoly model with a more general demand structure, a quantity effect can occur and the social welfare effect can go in both directions (as shown for the symmetric duopoly in Holmes [1989] and the linear demand model in Adachi and Matsushima [2014]).

In an oligopoly model that combines our price effect and the misallocation effect occurring in a monopoly, we also obtain the unambiguous result that uniform pricing benefits consumers as a whole. In other words, in a model that abstracts from quantity effects, both the misallocation effect and the price effect (and with that the logic of the harmonic mean formula as stated in Lemma 1) associated with price discrimination negatively affect consumer welfare. For this to happen, we need to assume that there is no total quantity effect, because both Lemma 1 and Proposition 2 depend on the absence of a total quantity effect for each firm.

We can, therefore, extend the harmonic mean formula toward a more general demand system, where a firm's own price change has a larger impact upon its demand relative to a rival's price change while the total output of each oligopolist is the same under both pricing regimes. As in the previous extension we consider the case $n = m = 2$. Moreover, let the demand system be given by

$$D_i^j = a^j - (1 - b^j)p_i^j + b^j(p_{i'}^j - p_i^j), \text{ for } i, j = 1, 2, \; i \neq i', \; j \neq j',$$

with parameters $a^j > 0$ and $b^j \in [0, 1]$ for $j = 1, 2$.

Notably, this demand specification—which is similar to the one proposed in Holmes [1989], formula (12), and Chen *et al.* [2021], Example 1—nests both our LCD-model (for $b^1 = b^2 = 1$) and a specific monopoly model (for $b^1 = b^2 = 0$).

It is straightforward to check that, under the assumption that price effects are symmetric in both markets (i.e., $b^1 = b^2$), the overall quantity produced is the same under uniform and discriminatory pricing, and the harmonic mean formula (11) still holds. The welfare effect of uniform pricing stays unambiguously positive: a uniform pricing regime lowers the average price, which positively affects welfare, and it also prevents the misallocation effect that emerges from price discrimination (as delineated for the monopoly case by Pigou [1920] and Robinson [1933]). Clearly, a more general demand system introduces quantity effects which can counter the misallocation and the price effect.

IV(iii). *Asymmetric Product Differentiation*

So far, we assumed that firms' products are symmetric for each market. What happens when products are asymmetrically differentiated, meaning that demand parameters a^j and b^j also depend on the firms' products? We first think of parameter a^j. If this parameter is firm-specific, meaning that we have $a_i^j \neq a_{i'}^j$, then—given the Hotelling-model interpretation of our demand system—this relates to differences in the gross utilities the products provide for all consumers; that is, products become vertically differentiated.[16] To analyze this type of asymmetry, we assume $m = n = 2$ (as in the previous extensions), and the demand system

$$D_i^j = a_i^j + b^j(p_{i'}^j - p_i^j), \text{ for } i,j = 1,2, \ i \neq i'.$$

This differs from our original demand system (1) by allowing for vertically differentiated products with $a_i^j \neq a_{i'}^j$ for $i,j = 1,2$, $i \neq i'$. A comparison of the equilibrium outcomes under discriminatory pricing and uniform pricing shows that each firms' total output stays the same whenever $b^j = b^{j'}$ for $j = 1,2, j \neq j'$ (in fact, this result remains true for $m > 2$ as long as b remains the same across all markets). Given $b = b^j = b^{j'}$ for $j = 1,2, j \neq j'$, we get the equilibrium price levels

$$\overline{p}_i^j = \frac{2a_i^j + a_{i'}^j + b(2c_i + c_{i'})}{3b} \quad \text{for } i,j = 1,2, \ i \neq i' \text{ and}$$

$$\widehat{p}_i = \frac{2(a_i^j + \alpha_i^{j'}) + a_{i'}^j + a_{i'}^{j'} + 2b(2c_i + c_{i'})}{6b} \quad \text{for } i,j = 1,2, \ i \neq i', \ j \neq j',$$

[16] In the Appendix, we assume in each market j a utility function of the form $U_i^j(x) = v^j - p_i^j - t^j|x_i - x|$, where x is the consumer's location on the Hotelling line, v^j is the gross utility of the good, and $t^j|x_i - x|$ stands for the transportation costs the consumer located at x has to incur to buy product i. If we assume different values of v for different products i, with $v_i^j \neq v_{i'}^j$, we obtain demand functions $D_i^j = a_i^j + b^j \sum_{i \neq i'}(p_{i'} - p_i)$, with $a_i^j \gtreqless a_{i'}^j \Leftrightarrow v_i^j \gtreqless v_{i'}^j$.

and the equilibrium output levels

$$\overline{D}_i^j = \frac{2a_i + a_{i'} - b(c_i - c_{i'})}{3} \text{ for } i,j = 1,2,\ i \neq i' \text{ and}$$

$$\widehat{D}_i^j = \frac{5a_i^j + a_{i'}^j - a_i^{j'} + a_{i'}^{j'} - 2b(c_i - c_{i'})}{6} \text{ for } i,j = 1,2,\ i \neq i',\ j \neq j'.$$

It is easily checked that the sum of each firms' outputs is independent of the pricing regime:

$$\overline{D}_i^1 + \overline{D}_i^2 = \widehat{D}_i^1 + \widehat{D}_i^2 \text{ for } i = 1,2,$$

as a consequence of which analogous versions of Lemma 1 and Proposition 2 can be derived. We can conclude that our analysis also applies for asymmetries with respect to parameter a^j—at least when $b^j = b^{j'}$ for $j = 1,2,\ j \neq j'$. In particular, the harmonic mean formula applies, so that

$$\frac{\widehat{p}_i - c_i}{\widehat{p}_i} = \frac{1}{\frac{\overline{D}_i^j}{\overline{D}_i^j + \overline{D}_i^{j'}}\left(\frac{\overline{p}_i^j - c_i}{\overline{p}_i^j}\right)^{-1} + \frac{\overline{D}_i^{j'}}{\overline{D}_i^j + \overline{D}_i^{j'}}\left(\frac{\overline{p}_i^{j'} - c_i}{\overline{p}_i^{j'}}\right)^{-1}} \text{ for } i,j = 1,2,\ i \neq i',\ j \neq j'.$$

In this case, price discrimination leads to the price effect (that was central in our main model specification and that disadvantages consumers as a whole) and the misallocation effect (that is also disadvantageous for consumers as a whole). As in the extension presented in Section IV(ii), our central results remain valid as long as the price regime does not affect a firm's total output: in those instances, the negative effect of discriminatory prices on consumer surplus is reinforced as the price effect is accompanied by the misallocation effect. Of course, under general demand conditions, the output effect has to be taken into account as well, and this can overturn our results whenever it leads to substantial total output expansions of the firms.

We finally turn to asymmetries regarding parameter b^j. This parameter refers to consumers' transportation costs on a Hotelling line that connects two firms. In our main model specification, we assumed symmetry, so that the Hotelling lines connecting all firms' products in a market j are characterized by the same transportation cost parameter. As we show in the Appendix, this assumption gives rise to the parameter b^j in our demand system (1), which is the same for all firms in market j. This assumption can be challenged as consumers with different choice sets (i.e., consumers on different Hotelling lines) may have different transportation costs. Thus, the problem becomes relevant when there are more than two products: then, a price change of one product may lead to different demand responses vis-á-vis rival products, even when these have the same price. Consequently, the demand difference between two brands in any market j not only depends on the prices of the two brands (see property (A3) of our demand system), but also on all other brand prices. In such a situation total demand effects associated with a change in the pricing regime are, by and large, inevitable.

V. CONCLUSION

In this paper, we analyzed the effects of oligopolistic third-degree price discrimination on consumer surplus. Under the assumption of full market coverage, consumer surplus is always lower, but firms' profits are always higher if price discrimination is feasible. Specifically, firms' equilibrium outputs are the same under discriminatory and uniform pricing, which also extends to the case of arbitrage-constrained prices. It then follows that the Lerner index of each product under uniform pricing is given by the weighted harmonic mean of the market-specific Lerner indices under discriminatory pricing. Consequently, average prices are higher and aggregate consumer welfare is lower under discriminatory pricing. We present a simple formula that allows us to calculate the consumer surplus loss and firm profit gain of third-degree price discrimination based solely on observable market data under discriminatory pricing (prices and quantities).

But for which markets could our model provide insights? First, our model is rather applicable to saturated markets than to markets for new products, as for the latter quantity effects might play an important role. Second, it is better applicable for non-digital goods for which firm heterogeneity in production costs could be important. For digital goods (such as software or music streaming services), on the contrary, marginal production costs are rather negligible, and therefore asymmetric costs structures across firms are not of crucial importance; instead, for such markets network effects (as included in Armstrong [2006] and Tan and Zhou [2021]) are decisive. Thus, when drawing policy conclusions from our study it is important to reflect upon the plausibility of our core assumptions for the market under consideration.

In particular, our clear-cut results no longer hold when price discrimination leads to quantity effects by either changing firms' overall output across markets and/or changing market demand when it is not inelastic. For instance, productive efficiency may increase under discrimination when more efficient firms tend to expand their outputs. Likewise, when firms' demands are non-linear, firms' market-specific demand elasticities may change in a way which is more favorable for discrimination, so that prices do not increase much in less competitive markets and decrease overproportionally in more competitive markets. But our results can be empirically tested against such alternative hypotheses: the harmonic mean formula predicts that price discrimination affects market prices (in absolute terms) the more the lower the competitive intensity of the market is.

APPENDIX

DERIVATION OF THE LCD-MODEL

The LCD-model can be derived from a horizontal product differentiation model in the spirit of the Hotelling duopoly model, as suggested by Somaini and Einav [2013]. There are $i = 1, \dots, m$ firms each producing a horizontally differentiated product. The firms sell their goods in $j = 1, \dots, n$ independent markets. In each market j, there are $l_m := \frac{m(m-1)}{2}$ Hotelling lines, such that all firms are directly linked with each other. The length of each line in market j is L^j. As in the Hotelling duopoly model, two firms i and i', with $i \neq i'$ are always located at the end points of a line. Let there be a total mass of consumers of M^j in market j, which is uniformly distributed over all lines. Thus, the consumer distribution has a constant density $f^j := M^j/(l_m L^j)$ over each line of length L^j. Every consumer is distributed along one of the l_m lines and is identified by its address $x \in [0, L^j]$ on this line. All consumers have unit demands. A consumer x, located on a line connecting firms i and i', obtains a utility of $U_i^j(x) = v^j - p_i^j - t^j|x_i - x|$ from consuming product i at price p_i and incurring "transportation" costs $t^j > 0$ per unit of distance, where x_i stands for firm i's location on the line. The parameter v^j stands for the gross utility of consuming one unit of the good a consumer obtains in market j.

Take the line between the firms i and i', with $i \neq i'$. Firm i's demand on the respective line is determined by the location of the indifferent consumer x' which follows from

$$U_i^j(x') = v^j - p_i^j - t^j x' = v^j - p_{i'}^j - t^j \left(L^j - x'\right) = U_{i'}^j(x'),$$

where we assumed that firm i is located at $x = 0$ and firm i' is located at $x = L^j$. Solving for x' we get the indifferent consumer and thus firm i's demand on the line connecting firms i and i':

$$D_{ii'}^j(p_i^j, p_{i'}^j) := x' f^j = \frac{1}{2}\left[L^j + \frac{1}{t^j}\left(p_{i'}^j - p_i^j\right)\right] \frac{2M^j}{m(m-1)L^j}.$$

The total demand of firm i in market j is then given by summing the "line-demands," $D_{ii'}^j(p_i^j, p_{i'}^j)$, over all $i' \neq i$, which gives

$$D_i^j(p_1^j, \dots, p_m^j) = \frac{M^j}{m} + \frac{M^j}{m(m-1)L^j t^j} \sum_{i' \neq i} \left(p_{i'}^j - p_i^j\right).$$

Thus, the overall demand of firm i in market j follows from (1), with $a^j = \frac{M^j}{m}$ and $b^j = \frac{M^j}{m(m-1)L^j t^j}$.[17]

We finally note that the LCD-model can take care of loyal consumers, who always buy from one of the firms as long as the price does not exceed their reservation prices. Suppose the mass of loyal consumers is K^j in market j, so that the total mass of consumers in market j becomes $M^j + K^j$. The mass of loyal consumers is

[17] We assumed that the prices are such that consumers are willing to buy at the posted prices; that is, their gross utilities v^j are sufficiently large. In addition, we suppose that the utilities from buying are larger than their reservation utilities.

equally distributed among the firms, so that every firm serves a mass of K^j/m of loyal consumers. Assume that a firm never wants to serve only its loyal consumers and that the loyals' reservation price is large enough, so that they are willing to buy at the price the indifferent consumers pay (for instance, it is v^j). In this scenario, firm i's demand is given by

$$D_i^j(p_1^j, \dots, p_n^j) = \frac{M^j + K^j}{m} + \frac{M^j}{m(m-1)L^j t^j} \sum_{i' \neq i} \left(p_{i'}^j - p_i^j\right),$$

so that the demand of firm i in market j follows from (1), with $a^j = \frac{M^j + K^j}{m}$ and $b^j = \frac{M^j}{m(m-1)L^j t^j}$.

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