Appendix

A Choice Tasks

Table A.1: The 49 decision tasks faced by the participants

task	unchanging	changing	average							
	$(m_1,...,m_M;m)$	$(n_1,...,n_N;n)$	$\#balls$	$\#priors$	$\|\Delta(\#priors)\|$	$\|\Delta(\#win\text{-}balls)\|$	$\|\hat{\Delta}_{EU}\|$	$\|\hat{\Delta}_{SM}\|$	$\|\hat{\Delta}_{RD}\|$	$\|\hat{\Delta}_{AM}\|$
1	1,2;3	0,1,2,3;3	13.5	4.5	1	1.70	.153	.183	.158	.158
2	0,1,2,3;3	0,1,2,3;3	19.5	6.5	1.5	2.13	.152	.155	.157	.149
3	0,1,2,3,4;4	0,1,2,3;3	27.5	7.5	2.5	6.31	.146	.153	.151	.138
4	0,3;3	0,1,2,3,4;4	18	5	1.4	3.68	.132	.170	.138	.129
5	1,2;3	0,1,2,3,4;4	18	5	1.4	3.47	.134	.161	.139	.138
6	0,1,2,3;3	0,1,2,3,4;4	24	7	1.4	2.81	.133	.143	.138	.126
7	0,1,2,3,4;4	0,1,2,3,4;4	32	8	2	3.85	.143	.149	.148	.140
8	0,1,2,3,4,5;5	0,1,2,3,4;4	42	9	3	9.04	.132	.140	.137	.126
9	0,3;3	0,1,2,3,4,5;5	23.5	5.5	1.8	6.06	.118	.150	.124	.108
10	1,2;3	0,1,2,3,4,5;5	23.5	5.5	1.8	6.18	.126	.145	.131	.127
11	0,1,2,3,4;4	0,1,2,3,4,5;5	37.5	8.5	1.8	4.26	.123	.124	.129	.113
12	0,3;3	0,1,2,3,4,5,6;6	30	6	2.3	9.33	.114	.155	.121	.105
13	0,1,2,3,4,5;5	0,1,2,3,4,5,6;6	54	10	2.3	5.85	.117	.120	.123	.107
14	0,1,3,4;4	0,1,2,3;3	23.5	6.5	1.5	4.26	.149	.157	.153	.145
15	0,4;4	0,1,2,3,4;4	20	5	1.4	3.10	.131	.166	.136	.126
16	0,4;4	0,1,2,3,4,5;5	25.5	5.5	1.8	5.42	.126	.157	.132	.116
17	0,4;4	0,1,2,3,4,5,6;6	32	6	2.3	8.50	.110	.152	.116	.103
18	2,3;5	0,1,2,3;3	17.5	4.5	1	1.88	.153	.185	.159	.160
19	0,2,3,5;5	0,1,2,3;3	27.5	6.5	1.5	6.25	.148	.153	.153	.142
20	0,5;5	0,1,2,3,4;4	22	5	1.4	2.88	.138	.167	.144	.131
21	2,3;5	0,1,2,3,4;4	22	5	1.4	2.88	.135	.162	.141	.141
22	0,2,3,5;5	0,1,2,3,4;4	32	7	1.4	4.05	.134	.140	.139	.127
23	0,5;5	0,1,2,3,4,5;5	27.5	5.5	1.8	4.94	.121	.162	.127	.116
24	2,3;5	0,1,2,3,4,5;5	27.5	5.5	1.8	4.91	.123	.143	.129	.127
25	0,2,3,5;5	0,1,2,3,4,5;5	37.5	7.5	1.5	4.14	.121	.124	.127	.110
26	0,5;5	0,1,2,3,4,5,6;6	34	6	2.3	7.62	.114	.146	.121	.105
27	2,3;5	0,1,2,3,4,5,6;6	34	6	2.8	7.69	.116	.142	.122	.119
28	0,1,5;6	0,1,2,3;3	25.5	5.5	1	6.93	.226	.228	.236	.186
29	2,3,4;6	0,1,2,3;3	25.5	5.5	1	5.27	.158	.191	.163	.163
30	0,2,4,6;6	0,1,2,3;3	31.5	6.5	1.5	8.22	.159	.163	.164	.154
31	0,1,3,5,6;6	0,1,2,3;3	37.5	7.5	2.5	11.28	.148	.156	.153	.142
32	0,2,3,4,6;6	0,1,2,3;3	37.5	7.5	2.5	11.33	.158	.168	.163	.151
33	2,3,4,5,6;6	0,1,2,3;3	37.5	7.5	2.5	13.79	.235	.252	.243	.238
34	0,1,2,4,5,6;6	0,1,2,3;3	43.5	8.5	3.5	14.24	.157	.161	.161	.152
35	0,1,2,3,4,5,6;6	0,1,2,3;3	49.5	9.5	4.5	17.23	.149	.158	.153	.143
36	0,6;6	0,1,2,3,4;4	24	5	1.4	2.68	.136	.167	.142	.131
37	2,4;6	0,1,2,3,4;4	24	5	1.4	2.72	.128	.148	.133	.131
38	0,1,5;6	0,1,2,3,4;4	30	6	1.2	5.55	.227	.217	.240	.169
39	0,3,6;6	0,1,2,3,4;4	30	6	1.2	3.66	.132	.145	.138	.125
40	0,1,2,6;6	0,1,2,3,4;4	36	7	1.4	7.46	.179	.170	.199	.122
41	0,2,4,6;6	0,1,2,3,4;4	36	7	1.4	6.04	.132	.140	.137	.125
42	0,1,2,4,5,6;6	0,1,2,3,4;4	48	9	3	12.17	.132	.139	.137	.122
43	0,6;6	0,1,2,3,4,5;5	29.5	5.5	1.8	4.61	.126	.156	.132	.113
44	2,4;6	0,1,2,3,4,5;5	29.5	5.5	1.8	4.61	.123	.141	.129	.125
45	2,3,4;6	0,1,2,3,4,5;5	35.5	6.5	1.5	3.94	.118	.136	.123	.119
46	0,1,2,5;6	0,1,2,3,4,5;5	41.5	7.5	1.5	6.71	.206	.198	.220	.150
47	0,1,2,3,5;6	0,1,2,3,4,5;5	47.5	8.5	1.8	8.37	.182	.182	.192	.153
48	0,1,2,4,5,6;6	0,1,2,3,4,5;5	53.5	9.5	2.5	9.27	.126	.130	.133	.115
49	0,6;6	0,1,2,3,4,5,6;6	36	6	2.3	7.14	.118	.151	.126	.108

Note: Each task is made of two two-stage lotteries, an unchanging lottery and a changing lottery. The unchanging lottery is composed of M one-stage lotteries, each made of m balls, with the k'th containing m_k blue balls. The changing lottery is composed of N one-stage lotteries, each made of n balls, with the j'th containing n_j blue balls. In the tasks above, it is assumed that the blue is the winning colour. The average value assumed by some task-specific variables used in Table 6 is reported. Note that, while the variables $\#balls$, $\#priors$ and $|\Delta(\#priors)|$ change within task (depend only on the round) but not between subjects, the variables $|\Delta(\#win\text{-}balls)|$ and $|\hat{\Delta}_\tau|$, $\tau \in \{EU, SM, RD, AM\}$, depend on the elimination sequence and, consequently, change between subjects. The calculation of $|\hat{\Delta}_\tau|$, $\tau \in \{EU, SM, RD, AM\}$, is based on the expectation of the parameter of interest characterising that type's functional, calculated individually for each subject conditional on being of that particular type (see Conte and Hey, 2013).

B How the Different Preference Functionals were Fitted

Here we describe how C&H used the data – using only the data on the decisions for all 149 subjects for all 49 tasks, consisting of a total of 256 decision problems for each subject,[1] giving a grand total of 19,668 observations; the mixture estimation was based in these 19,668 observations. The crucial point is that we are interested in knowing whether there are different types of decision-makers in the population for the type of problem considered here. This was assessed in C&H by fitting a *Finite Mixture Model* (henceforth, mixture model or, more simply, mixture) from choice data.

A mixture model is a representation of the heterogeneity of the population. There are different types of decision-makers and types differ in the decision rules, or more technically, in the functional that describes their choice process. In the case of our experiment, "type" refers to a subject's choice process that is recognised as being compatible with one of the four theories of decision-making under ambiguity. We will describe these theories later. The crucial point here is that a mixture model enables us to estimate *jointly* the parameters of the preference functionals which characterise our types and the proportions of types in the population, namely the "mixing proportions". Put differently, the parameters of each type's functional are automatically estimated only from those subjects who appear to be "statistically" compatible with that type.[2] Note that, in the experiment, subjects can either win or not win, therefore the utility function, $u(.)$, is normalised such that $u(\text{win}) = 1$ and $u(\text{not win}) = 0$ and there is no attitude to risk parameter to estimate.

In a finite mixture model, the mixing proportion of a type represents the probability of a randomly chosen subject being of that particular type. The larger the mixing proportion of a type, the higher the probability of the subject being of that particular type. C&H assign subjects to types following a majority rule: a subject is assigned to a specific type if her posterior probability of that type is the highest.[3]

[1]The number of questions in each task depended upon the initial number of one-stage lotteries in the changing task: to be precise if there were N such one-stage lotteries then there would be N decision-problems in that task. N varied across task.

[2]This is what differentiates a mixture from a pooled estimation procedure assuming a single type at a time.

[3]The power of a mixture model is higher, the neater its capacity to assign subjects to types with high probability. As shown in Conte and Hey (2013, Figure 4), in 88.58% of subjects are assigned to type with posterior probability higher than 90%.

Using the parameter estimates and the assignment of subjects to types of the mixture model, we can isolate and investigate the effect of decision type on decision time.

B.1 Expected Utility Theory (EU)

According to the Expected Utility Theory, the valuation of a two-stage lottery used in this experiment is

$$V_{EU}\left[\mathcal{R}(r_1,\ldots,r_k,\ldots,r_R;r)\right]=\left[\frac{r_1}{r}+\ldots+\frac{r_k}{r}+\ldots+\frac{r_R}{r}\right]\frac{1}{R} \tag{B.1}$$

This valuation follows by the property of EU of being linear in the probabilities and by the fact that each one-stage lottery is equally likely. It is worth noting that the EU preference functional in this formulation does not require any parameter estimation.

As an example, let us consider the choice problem depicted in Figure 1, where the two-stage lotteries A (left lottery) and B (right lottery) include five and two one-stage lotteries, respectively. Assuming that red is the winning colour, the valuations of these lotteries according to Expected Utility Theory are

$$\begin{aligned} V_{EU}\left[A(0,1,2,3,4;4)\right] &= \left[\frac{0}{4}+\frac{1}{4}+\frac{2}{4}+\frac{3}{4}+\frac{4}{4}\right]\cdot\frac{1}{5}=0.5 \\ V_{EU}\left[B(0,3;3)\right] &= \left[\frac{0}{3}+\frac{3}{3}\right]\cdot\frac{1}{2}=0.5 \end{aligned}$$

Under EU, the two lotteries have the same valuation.[4]

B.2 Smooth Model (SM)

Smooth Model evaluates a two-stage lottery of our experiment as follows:

$$V_{SM}\left[\mathcal{R}(r_1,\ldots,r_k,\ldots,r_R;r)\right]=\left[\phi\left(\frac{r_1}{r}\right)+\ldots+\phi\left(\frac{r_k}{r}\right)+\ldots+\phi\left(\frac{r_R}{r}\right)\right]\frac{1}{R} \tag{B.2}$$

Here, $\phi\left(x\right)=\left(1-e^{-sx}\right)/\left(1-e^{-s}\right)$, where $-\infty<s<+\infty$ is the only characterising parameter to be estimated. SM reduces to EU when $s\rightarrow 0$.

[4] This valuation is deterministic and does not include any error terms.

Now, let us consider how the Smooth Model evaluates the lotteries of the decision problem depicted in Figure 1. As a value for s we use the mean of the normal distribution assumed for it whose moments are estimated in C&H. This is $s = 1.377$.

$$\begin{aligned} V_{SM}\left[A(0,1,2,3,4;4)\right] &= \left[\phi\left(\frac{0}{4}\right)+\phi\left(\frac{1}{4}\right)+\phi\left(\frac{2}{4}\right)+\phi\left(\frac{3}{4}\right)+\phi\left(\frac{4}{4}\right)\right]\cdot\frac{1}{5} \\ &= \left[0+0.39+0.67+0.86+1\right]/5 = 0.583 \\ V_{SM}\left[B(0,3;3)\right] &= \left[\phi\left(\frac{0}{3}\right)+\phi\left(\frac{3}{3}\right)\right]\cdot\frac{1}{2} = [0+1]\cdot\frac{1}{2} = 0.5 \end{aligned}$$

We conclude that the average SM subject gives a higher valuation to lottery A than to lottery B.

B.3 Rank Dependent Expected Utility (RD)

Given that $r_1 < \ldots < r_k < \ldots < r_R$, a two-stage lottery of our experiment is evaluated as follows:

$$V_{RD}\left[\mathcal{R}(r_1,\ldots,r_k,\ldots,r_R;r)\right] = \sum_{k=1}^{R} h\left(\frac{r_k}{r}\right)\left[h\left(\frac{R-k+1}{R}\right)-h\left(\frac{R-k}{R}\right)\right] \tag{B.3}$$

Here, $h(p) = p^g/\left[(p^g+(1-p)^g)\right]^{1/g}$, with $h(0)=0$ and $h(1)=1$. The only parameter to be estimated is $g>0$. When $g=1$, RD reduces to EU.

The Rank Dependent Expected Utility evaluates the lotteries of the decision problem depicted in Figure 1, using as a value of g the mean of the lognormal distribution as estimated in C&H, which is $g = 1.139$, as follows.

$$\begin{aligned} V_{RD}\left[A(0,1,2,3,4;4)\right] &= 0+h\left(\frac{1}{4}\right)\left[h\left(\frac{4}{5}\right)-h\left(\frac{3}{5}\right)\right]+h\left(\frac{2}{4}\right)\left[h\left(\frac{3}{5}\right)-h\left(\frac{2}{5}\right)\right] \\ &\quad +h\left(\frac{3}{4}\right)\left[h\left(\frac{2}{5}\right)-h\left(\frac{1}{5}\right)\right]+h(1)\cdot\left[h\left(\frac{1}{5}\right)-0\right] = 0.492 \\ V_{RD}\left[B(0,3;3)\right] &= 0+1\cdot\left[h\left(\frac{1}{2}\right)-0\right] = 0.494 \end{aligned}$$

Here, lottery B receives a higher valuation than lottery A by the average RD subject.

B.4 Alpha Expected Utility Model (AM)

According to the Alpha Expected Utility Model, the valuation of a generic two-stage lottery is a weighted average of the value of the worst and the best one-stage lotteries within the two-stage lottery:

$$V_{AM}\left[\mathcal{R}(r_1, \ldots, r_k, \ldots, r_R; r)\right] = a\frac{r_1}{r} + (1-a)\,\frac{r_R}{r} \tag{B.4}$$

The only parameter to be estimated is the weight $0 \leq a \leq 1$.

The Alpha Expected Utility Model evaluates the lotteries of the decision problem depicted in Figure 1 as follows. As a value for the parameter a we use the mean of the beta distribution estimated in C&H, which is $a = 0.542$.

$$\begin{aligned} V_{AM}\left[A(0,1,2,3,4;4)\right] &= a\frac{0}{4} + (1-a)\,\frac{4}{4} = 0.458 \\ V_{AM}\left[B(0,3;3)\right] &= a\frac{0}{3} + (1-a)\,\frac{3}{3} = 0.458 \end{aligned}$$

Again, we see that the two lotteries are equally evaluated by the average AM subject, as for EU, but here only the extreme priors of each lottery are used in the calculation.

B.5 Stochastic and Distributional assumptions

C&H assume that subjects make mistakes in their choices, more specifically, that they make errors when calculating the difference in the valuations of the two lotteries: $V_\tau(A) - V_\tau(B) + u_\tau$, with $\tau \in \{EU, SM, RD, AM\}$; $u_\tau \sim N(0, \sigma_\tau^2)$ is an additive error term, independent across questions. They further consider a probabilistic tremble of magnitude w and estimate the parameters of the models in consideration (s in the SM, g in the RD, and a in the AM) together with σ_τ^2 and w. Finally, they estimate the mixing proportions of types, that is the proportions of the population who are of each of the considered types, where a type is described by one of the four above-mentioned models.

For all the details concerning the estimation of the mixture model and the computation of posterior probabilities not directly relevant to the present paper, we refer the reader to C&H.

C Application of the Analysis to Data from Choice under Risk

This section describes a replication of our analysis using a different data set. The purpose of this exercise is not only to demonstrate that the main results of this work are not coincidental but also to show that they extend to a context different from the one involving choice under ambiguity. Here, we report only the most important results; the complete set of results is available from the authors upon request.

We use the same data set that Moffatt (2005) used in his analysis of decision times in choice under risk. The data set contains decision times from 53 subjects who selected their preferred lottery in a pair of lotteries 500 times. The experiment and the data set are described in Hey (2001).

Moffatt (2005), assumes the existence of only one type of decision-maker, characterised by a Rank-Dependent utility function with Constant Absolute Risk Aversion (CARA) utility function and a Prelec Probability Weighting Function (PWF). In contrast, we introduce a mixture hypothesis by adding a second type: an Expected Utility maximiser with a CARA utility function.[5] We select 300 decision problems per subject for estimating the mixture model. The decision times associated with these problems are also used for estimating models of decision time, while the remaining 200 are reserved for validation.

The estimate of the mixture model shows that the proportion of RD decision-makers is 0.5441 (s.e. = 0.0719), with the remaining portion consisting of EU decision-makers.

Table C.1: Summary statistics of decision time per type

		Estimation sample				Validation sample			
type	subjects	observations	mean	std. dev.	median	observations	mean	std. dev.	median
EU	24	7,200	4.09	4.98	2.80	4,800	4.17	5.52	2.80
RD	29	8,700	5.16	5.84	3.62	5,800	5.38	6.47	3.68

Table C.1 clearly demonstrates a significant difference in decision times between the two types, with the EU decision-maker taking less time on average than the RD decision-maker. Such a difference is confirmed by both the variance-comparison and mean-comparison tests of decision times between the two types resulting in the rejection of the null hypothesis of

[5] This EU type is nested within the RD type since it can be obtained from the latter by setting the two parameters of the Prelec PWF to 1.

equality against a two-tailed alternative hypothesis. This rejection is strong (p-value< 0.0000) for both the estimation and validation samples.

Table C.2: Results from model random effects model of log(decision time) by type

	EU	RD
Dependent: log(decision time)	Coeff. (s.e.)	Coeff. (s.e.)
Constant	2.1347 (0.0953)	2.0649 (0.0838)
Complexity level 1 (base)	–	–
Complexity level 2	0.2634 (0.0212)	0.2298 (0.0199)
Complexity level 3	0.3523 (0.0307)	0.4042 (0.0289)
$\tau^d - 1$	-0.0144 (0.0017)	-0.0073 (0.0005)
$\left(\tau^d - 1\right)^2/100$	0.0110 (0.0021)	0.0024 (0.0002)
$\left(\tau^d - 1\right)^3/1000$	-0.0004 (0.0001)	-0.0000 (2.96e-06)
$\left(\tau^d - 1\right)^4/1000000$	0.0008 (0.0002)	
$\left(\tau^d - 1\right)^5/100000000$	-0.0000 (0.0000)	
τ		0.0005 (0.0002)
$\lvert\hat{\Delta}\rvert$	-10.4914 (0.3429)	-3.2185 (0.1960)
$\lvert\hat{\Delta}\rvert^2$	39.3324 (2.0292)	4.2292 (0.6324)
$\lvert\hat{\Delta}\rvert^3$	-55.3443 (3.7301)	-1.9788 (0.5149)
$\lvert\hat{\Delta}\rvert^4$	24.5716 (1.9936)	
Δ^O	0.0601 (0.0192)	0.0353 (0.0182)
σ_u (between)	0.3971	0.4059
σ_s (within)	0.5885	0.6098
n	24	29
T	7,200	8,700

Note: Regressors are defined as in Moffatt (2005): $\tau^d - 1$ and τ represent the position in the sequence of decision problems within the whole experiment and its parts, respectively, accounting for experience; $\lvert\hat{\Delta}\rvert$ captures closeness to indifference; while Δ^O measures objective similarity. The powers are selected using the likelihood-ratio test for nested specifications and the Akaike Information Criterion for non-nested ones.

Table C.2 reports the estimation results of the logarithm of decision time against various regressors that account for the same task characteristics we consider (E, C, S and I). It is noteworthy that the results reveal significant differences, particularly in the learning process and in terms of the impact of closeness to indifference.

Table C.3 presents the results of a validation exercise similar to the one described in Section 6. Just as with decision times in choice under ambiguity, the best predictors for a specific type are consistently those based on that same type also in the case of decision times in choice under risk.

Table C.3: VALIDATION OF DECISION TIMES

EU **validation sample**			*RD* **validation sample**		
	τ			τ	
	EU	SM		EU	SM
decision time predicted by τ	0.9945	1.0581	*decision time predicted by* τ	0.9374**	0.9691
	(0.0347)	(0.0363)		(0.0223)	(0.0206)
intercept	0.0063	-0.0583	*intercept*	0.1052***	0.0067
	(0.0372)	(0.0374)		(0.0262)	(0.0248)
observations	4,800		observations	5,800	
subjects	24		subjects	29	
$F(2,23)$	0.01	1.30	$F(2,28)$	13.27	0.57
p-value	**0.9855**	**0.2931**	p-value	0.0001	**0.5719**
R^2	**0.5148**	0.4820	R^2	0.4178	**0.4420**
RMSE	**0.3593**	0.3847	RMSE	0.4019	**0.3845**

Note: See Note to Table **??**.