OPEN ACCESS
Advances in Complex Systems
(2024) 2440003 (25 pages)
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DOI: 10.1142/S0219525924400034

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UNDERSTANDING MEMORY MECHANISMS IN SOCIO-TECHNICAL SYSTEMS: THE CASE OF AN AGENT-BASED MOBILITY MODEL

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Received 29 September 2023
Revised 9 February 2024
Accepted 24 April 2024
Published 27 June 2024

This paper explores memory mechanisms in complex socio-technical systems, using a mobility demand model as an example case. We simplify a large-scale agent-based mobility model, formulate the corresponding stochastic process, and observe that the mobility decision process is non-Markovian. This is due to its dependence on the system's history, including social structure and local infrastructure, which evolve based on prior mobility decisions. Complementing the mobility process with two history-determined components leads to an extended mobility process that is Markovian. Although our model is a very much reduced version of the original one, it remains too complex for the application of usual analytic methods. Instead, we employ simulations to examine the functionalities of the two history-determined components. We think that the structure of the analyzed stochastic process is exemplary for many socio-technical, -economic, -ecological systems. Additionally, it exhibits analogies with the framework of extended evolution, which has previously been used to study cultural evolution.

Keywords: Socio-technical system; agent-based model; Markov process; mobility demand; extended evolution.

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1. Introduction

Agent-based models (ABMs) are valuable tools for representing complex socio-economic, -ecological, and -technical systems because they allow the modeler to define the system based on its individual elements and their interactions, rather than having to specify aggregate dynamics of the system in the first place. Moreover, ABMs offer the advantage of being easily explainable to a non-technical audience: relevant actors in the system under study are represented on the computer as agents with features, governed by decision or interaction rules, and embedded in agent networks and a common environment; the system evolution is then simulated by repeatedly carrying out the specified (inter)actions. There hence is a wide usage of such models for analyzing challenges in environmental and social sustainability contexts, e.g. [1, 15, 18, 24, 19].

Owing to the detailed micro-level definitions, however, complex, empirically grounded ABMs are challenging to express mathematically [11]. In fact, there is a research gap between these models and simpler ABMs studied in physics and mathematics, which often prioritize the depiction of mechanisms over accurately representing a specific social system [4].

As collective phenomena in social contexts emerge from repeated interactions among individuals, methods from statistical physics have been applied to understand such phenomena [5, 26, 14]. The related literature offers analytical tools and mean-field approximations, particularly for binary opinion models, with the famous *voter model* being extensively analyzed, e.g. [6, 25, 29, 27]. Mathematically, such ABMs have been described as discrete-time Markov chains [13, 3] or continuous-time Markov jump processes [20, 31], also coupled with (e.g. spatial) diffusion dynamics, allowing for description by stochastic differential equations [8] or approximation by stochastic partial differential equations [12]. Analyzing well-known ABMs as Markov chains offered valuable insights, including the identification of absorbing states from which the system cannot escape once entered [13].

On the other hand, for more complex ABMs — that, e.g. include an economic, ecological, or technical dimension along with the social system, as well as feedbacks between these components — the corresponding Markov chain is hard to even write down explicitly and would in most cases be “too big for a tractable analysis of a model” [2]. One explanation for this is the significance of history in such systems: for instance, path dependencies resulting from prior (collective) decisions for a certain technology or investment have a significant impact in human decision-making processes, as do past experiences, long-term beliefs, social norms or conventions, and the social network itself. These path dependencies clearly contrast with the Markov property, which presupposes that the future will only depend on the current situation. However, in theory, it is possible to convert non-Markov processes into Markovian ones by extending the state space.

Here, this paper contributes. We focus on the topic of mobility, as an example case of a complex system in which a transition toward sustainability is required, and consider ABMs of mobility choices. The large-scale, empirically grounded Mobility Transition Model (MoTMo) [10], rather than modeling transport, abstracts from the single trips made by people and considers their choice of a means of transport from an economic demand perspective with a monthly time resolution. Based on this complex ABM, we derive a reduced model that retains the primary memory mechanisms of the original model while abstracting from much of MoTMo's detail [28]. In this paper, we formulate the reduced model as a stochastic process and transform it — through an extension of the state space — into a Markov chain. This not only enhances the explainability of the model but also points to a classification of memory mechanisms that can be helpful for understanding complex socio-technical systems more generally.

Further, the explicit formulation of our stochastic process can be viewed through two distinct lenses present in previous literature. First, the components identified in our formulation can be related to the elements of ABMs for social systems described in prior conceptual work see, e.g. [7] and references therein, that has emphasized agents, social structure, and the environment. While model components are identified based on their meaning in conceptual frameworks, in our case their distinction is one of (mathematical) form; this invites the question how far structure and function determine each other in such social systems.

Second, there have been past approaches discussing social systems from an evolutionary perspective [23]. The complexity arising from the feedback mechanisms in our model can be interpreted using more recent developments in evolutionary theory [17]. With our work, we take an initial step towards bridging the conceptual gap between sophisticated, empirically based ABMs and mathematical theory. By explicitly formulating the stochastic process, we create a foundation for better integrating empirical and theoretical aspects of modeling complex socio-technical systems.

The paper is structured as follows: In Sec. 2, we sketch the large-scale ABM MoTMo as the point of departure and introduce Reduced MoTMo (R-MoTMo), its strongly reduced version. Both models are described using natural language. In Sec. 3, we formalize (the reduced ABM) R-MoTMo by constructing the associated stochastic process. The non-Markovian mobility decision process is complemented by two history-determined components, resulting in an extended mobility process that is Markovian. Section 4 explores simulation results for different settings to analyze the effect of the history-determined components on the system's dynamics. Moving on to Sec. 5, the paper discusses how the Markov chain formulation enhances the comprehension of the model and its dynamics. It also establishes connections with previous conceptual frameworks and the theory of extended evolution, before Sec. 6 concludes.

2. An Agent-Based Mobility Model and its Reduced Version

Computational ABMs are often provided in the form of computer code together with a documentation, ideally following a standardized format like the ODD protocol [11], as their mathematical definitions would be very challenging to formulate. MoTMo is such an ABM. To have a chance of mathematically analyzing its basic structures, we created a very much reduced version of it [10]. Both the original model and its reduction are shortly presented in this section.

2.1. *The mobility transition model*

Purpose and empirical foundation. MoTMo has been developed for simulating private mobility demand in Germany until 2035. It projects the choice of (main) transportation mode of the German population in monthly time steps, i.e. it is an economic demand model rather than a transportation model. MoTMo is empirically grounded in the sense that it both uses real-world input data at the micro-level and is calibrated to real-world data at the macro-level. In terms of input, for example, a synthetic population of agents, see, e.g. [9], represents the real-world population of Germany with a spatial resolution of $5\,\mathrm{km} \times 5\,\mathrm{km}$ pixels; distributions of features such as population density, household size, and income are matched. The model is calibrated by forecasting data from the past, i.e. parameters are adjusted in a way that mobility choices from 2005 to 2017 are replicated. Its main output is the set of mobility choices of all persons over time, which can then be used to, e.g. calculate overall CO_2 emissions (by multiplying the emissions per km assumed in the model with the individual traveling distances) or to look into mobility patterns in different regions or for different social groups. MoTMo further provides different (policy) options for the simulation period of 2017–2035, so that the dynamics under different combinations of options can be compared to a business-as-usual scenario to support discussions on sustainable mobility [32].

Utility functions. Persons in MoTMo have standard economic Cobb–Douglas utility functions with four factors of which costs and convenience of a mobility choice are the dominating ones.[a] Convenience, which summarizes such aspects as velocity and safety etc., is assumed to be local and modeled as a function of population density. In order to have a relatively simple form (a few parameters) of a positive function with a maximum and an adjustable width, Gaussian functions with different parameters are used for all mobility types.[b] The exponents in the Cobb–Douglas functions, that weight the different factors and hence represent preferences, are agent-specific.[c]

Social network. Persons in MoTMo have a social network of friends, where all agents have the same number of friends on average and the connection to each friend comes with a weight. The social network is formed randomly at initialization of the

[a] The other two factors account for how innovative or ecological a choice is with respect to what other persons currently use.

[b] The parameters of the respective functions are set during calibration of MoTMo.

[c] These exponents are determined in a calibration step for the synthetic population. They correlate with a person's situation in life (in the model age, income, gender, and household type).

model; the probability to draw a person as a friend increases with spatial proximity and similarity in preferences.

Decision-making. Agents decide between five mobility types (vehicles with combustion engine, electric vehicles (EVs), public transport, car sharing, and non-motorized, i.e. walking or biking). Their decisions are based on utility optimization (with limited information) via social learning: The core idea is that agents take decisions by imitating the mobility choices of their social contacts and improve their choices by learning from trial and error. However, other factors also influence the actual mobility decision, as, e.g. a person's household has a limited budget for mobility expenses.

Memory mechanisms. Past decision-making influences present decisions via two kinds of *memory mechanisms*. The first kind comprises feedback from previous mobility decisions onto the mobility options agents can choose from. In MoTMo, this feedback takes various forms, such as growth of convenience for a mobility type with increased usage (leading to the expansion of corresponding infrastructure) and a decline of unit prices with increased usage. This feedback operates through local (conveniences) or global (prices) fields, which are influenced by decisions of the agents aggregated at one location or globally, respectively. In the general context of socio-technical systems, this structure can be associated with feedbacks related to general global or local parameters, such as unit costs, prices, technological progress, productivity, or social norms and conventions. The second memory mechanism is a network-based learning process, which in MoTMo operates as follows: individuals evaluate the usefulness of information obtained from others and revalue the links in their network accordingly. This process resembles Bayesian updating and simulates social learning dynamics. In general socio-technical systems, this learning mechanism may involve feedbacks facilitated by an evolving network structure, encompassing elements like social relations, information networks, supply chains, and local connections such as roads, railway tracks, and pipelines.

As with many ABMs, MoTMo results can be simulated but cannot be derived analytically from the rules that govern agents' behavior. Even an overseeable mathematical formulation of all the model features is far from feasible.[d] However, a mathematical analysis could be very useful to understand which parts of the model have a bigger impact on the system dynamics than others, for example on its ability to adapt to a changing environment. In the best case this understanding could also be useful to find relevant policy intervention points. As a first step toward an analytical understanding of the dynamics, the model has to be dramatically reduced in order to make it feasible to write it down in mathematical form. Such a reduced model will be presented next.

[d]Besides the elements discussed here, MoTMo has other features like households that persons interact with, sub-models for the development of unit costs for the different mobility types, and the extension of EV charging infrastructure, which are not relevant for the discussion here but would make it even more difficult to write it down in mathematical form.

2.2. *A reduction of MoTMo*

The R-MoTMo is available via the CoMSES Model Library [28], where the code and a documentation according to the ODD protocol [11] can be found. It abstracts from empirical data and serves to analyze the role of memory mechanisms. In the reduced model — as in the original one — agents are *persons* who live in one of the model's *cells* and take mobility decisions.

Utilities. While MoTMo uses a Cobb–Douglas utility function with four factors, for simplicity, the reduced model considers only one factor. To preserve the local level feedback, we chose convenience. Thus, in R-MoTMo utility equals convenience. The shape of the convenience curves is again Gaussian. Public transport is more convenient in densely populated cells (representing urban areas), while cars are more convenient in sparsely populated ones (rural areas). The utility is further influenced by endogenous factors depending on the mobility type usage history in a cell (see memory mechanism below).

Friends network. As in MoTMo, persons have a social network of *friends*. The creation of the network is analogous; only spatial location is relevant, as it is the only feature in which persons can differ. The social network is formed at initialization of the model, with all the links initially weighted equally. All persons have the same number of friends, which are drawn randomly with a higher probability of forming friendship with someone in closer proximity, as described in detail in the ODD document.

Decision-making. The available mobility choices in R-MoTMo are *cars* and *public transport*. Persons want to optimize their utilities but they do not have full information for doing so; they only know the utility of their own current mobility choice and have information about current mobility choices and utilities of their friends. Mobility decisions are taken by imitating the choice of a friend that is drawn randomly from the set of friends, where the probability increases with the friend's utility and with the weight of the connection towards this friend. Agents thus optimize their choices by trial and error, learning from the information given by their network.

Memory mechanisms. Principles of the two memory mechanisms are adopted from MoTMo. In particular, in R-MoTMo, first, the utility of a mobility type changes due to the local decision history: in the short run, more persons using the same mobility type in one cell decreases its utility there (representing traffic jams, crowded trains, etc.) while in the long run, more persons using a mobility type make it more convenient (as, e.g. infrastructure is being built up). Second, social learning is represented in the following way: The weights of connections in a person's social network co-evolve with the decisions taken, as individuals adjust the connection weights based on whether and to what extent imitating a particular friend has resulted in an increase or decrease in their own utility.

Note that although this description uses the vocabulary of a mobility demand simulation, R-MoTMo can be viewed as a prototypical model for a consumption decision between two different technologies which, in the long run, provide greater utility to the consumer the more they have been used (locally); this is known as demand-side economies of scale. The model can further be seen as prototypical for a co-evolutionary modeling approach where agents' decisions randomly depend on the network they are embedded in (which is transformed by the agents in the course of time) and on their environment that evolves as a consequence of agents' (collective) actions.

We have been able to formulate the Reduced Mobility Decision Model in strict mathematical form as a stochastic process, as will be shown in the next section. This formulation is another way of writing down the same model. The computational code contains all necessary information, as does the ODD document or the formulation as a stochastic process. The different modes of writing down the same ABM serve different purposes; the most effective approach for gaining analytical comprehension is the latter.

3. R-MoTMo as a Stochastic Process

To formalize the reduced model R-MoTMo described in words above and published in form of code and documentation in [28], we will introduce the notation and components needed (Sec. 3.1), define the steps that compose the dynamics (Sec. 3.2), and then investigate the Markov property of the dynamics (Sec. 3.3).

3.1. *Notation and process components*

Our notation is the following: There are $N \in \mathbb{N}$ agents, and referring to a specific agent, the indices $i, j \in \mathbb{I}$ are used, where $\mathbb{I} := \{1, \ldots, N\}$ is the set of agents' indices. Each agent is located in a cell on a grid. The number of cells is denoted $C \in \mathbb{N}$, and cells are indexed by $c \in \{1, \ldots, C\}$. Let $N_c \in \mathbb{N}$ be the number of agents in cell c, which means that $\sum_{c\in\{1,\ldots,C\}} N_c = N$. The location of each agent is fixed, such that also the number N_c of agents in a cell is time-independent. Each person i interacts directly with a set of friends $\mathbb{F}(i) \subset \mathbb{I}$, which is a subset of all agents of size $n \in \mathbb{N}$, i.e. all agents have the same number of friends. An agent's set of friends is also fixed throughout a simulation. Moreover, there is the set $\mathbb{M} = \{0, 1\}$ of mobility types that each agent can choose between, where $m = 0$ refers to "car" and $m = 1$ refers to "public transport".

With this notation, we can define the components of R-MoTMo as a stochastic process in discrete time, where we use $t \in \mathbb{N}_0$ as the discrete-time index throughout.

The mobility decision process $\mathcal{M}(t)$. For each person $i \in \mathbb{I}$, let $\mathcal{M}_i(t) \in \mathbb{M}$ denote the chosen mobility type of this person at time t. The state of the stochastic process of mobility choices at time t is given by

$$\mathcal{M}(t) = (\mathcal{M}_i(t))_{i=1,\dots,N} \in \mathbb{M}^N. \tag{1}$$

The starting point $\mathcal{M}(0) \in \mathbb{M}^N$ is a randomly chosen initial state. The state space of the mobility decision process thus corresponds to that in some opinion dynamics models, see, e.g. [3]. However, to describe the dynamics, and in particular to define the probabilities underlying the stochastic mobility choices, further elements will be needed.

The counting processes $\mathcal{X}(t)$. At some points in the following, we are not interested in each single agent's mobility choice, but only in how many agents have made which choice. Therefore, the information contained in $\mathcal{M}(t)$ is aggregated in the *counting process* $\mathcal{X}(t)$ in the following way: Let $\mathbb{I}_c \subset \mathbb{I}$ be the set of indices of persons living in cell $c \in \{1, \dots, C\}$. For each cell c and each mobility type m, let

$$\mathcal{X}_c^{(m)}(t) := \frac{1}{N_c} \sum_{i \in \mathbb{I}_c} 1_m(\mathcal{M}_i(t)) \in [0,1], \tag{2}$$

be the (random) share of persons in cell c using mobility type m at time t.

The infrastructure bonus process $\mathcal{B}(t)$. For each cell c and each mobility type m, the process $\mathcal{B}_c^{(m)} = (\mathcal{B}_c^{(m)}(t))_{t=0,1,\dots}$ is recursively defined by

$$\mathcal{B}_c^{(m)}(t) = \frac{1}{3} \cdot \mathcal{X}_c^{(m)}(t) + \frac{2}{3} \cdot \mathcal{B}_c^{(m)}(t-1) \in [0,1], \tag{3}$$

for $t \geq 0$, where $\mathcal{B}_c^{(m)}(-1) := 0$. Set $\mathcal{B}(t) = (\mathcal{B}_c^{(m)}(t))_{c=1,\dots,C;m=1,\dots,M}$. This process represents the assumption that — at a local level — conditions will become better for mobility types that are used more in the respective cell, e.g. as an infrastructure build-up with a positive feedback mechanism. That is, large values of $\mathcal{X}_c^{(m)}(t)$ have a positive impact onto the evolution of the bonus process $\mathcal{B}_c^{(m)}(t)$.

The utility process $\mathcal{U}(t)$. Utility of a mobility choice is defined on the level of cells, determined by the current level of the bonus process and the current number of users of this mobility type. The individual utility at time t of agent i is then given by the utility of the respective cell c_i that the agent lives in

$$\mathcal{U}_i(t) := u_{c_i}^{(\mathcal{M}_i(t))}(\mathcal{X}_{c_i}^{(\mathcal{M}_i(t))}(t), \mathcal{B}_{c_i}^{(\mathcal{M}_i(t))}(t)), \tag{4}$$

where the utility functions $u_c^{(m)} : [0,1]^2 \to (0,\infty)$ have the following form:

$$u_c^{(m)}(x,b) := a_c^{(m)}(x) \cdot \hat{u}_c^{(m)} + b, \tag{5}$$

with the *short-term malus function* $a_c^{(m)} : [0,1] \to \left[\frac{2}{3}, 1\right]$ given by

$$a_c^{(m)}(x) := 1 - \frac{1}{3}x, \tag{6}$$

and the exogenous factor

$$\hat{u}_c^{(m)} := \frac{100}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(N_c - \mu^{(m)})^2}{2\sigma^2}\right) \in \left(0, \frac{100}{\sqrt{2\pi}\sigma}\right], \tag{7}$$

where

$$\mu^{(m)} := \begin{cases} \min\limits_{c\in\{1,\ldots,C\}} N_c & \text{for } m = 0, \\ \max\limits_{c\in\{1,\ldots,C\}} N_c & \text{for } m = 1, \end{cases} \tag{8}$$

and $\sigma := \frac{1}{2}(\max_c N_c - \min_c N_c)$. The factor 100 appearing in (7) serves to bring utilities to a handy range (e.g. $u_c^{(m)}(x, 0) \in [0, 4]$ for all x), but also to weight the influence of the malus factor compared to the bonus, as only $a_c^{(m)}$ is multiplied with $\hat{u}_c^{(m)}$.

The exogenous factors $\hat{u}_c^{(m)}$ are independent of the dynamics and only depend on the mobility type m and the number of persons N_c in the cell. In Fig. 1, they are shown for both mobility types $m = 0$ (representing *car*) and $m = 1$ (representing *public transport*) for different numbers N_c of persons per cell. In contrast, the malus term $a_c^{(m)}(x)$, as given in Eq. (6), and the added value b of the bonus process, see (4) and (5), depend on the system's state and represent two kinds of feedback mechanisms associated to infrastructure: The more agents currently use a mobility type m in cell c, the lower the utility at present (e.g. due to traffic jams or crowded trains) because infrastructure starts working at capacity, which is represented by the short-term malus $a_c^{(m)}(x)$. On the other hand, the more agents use a certain mobility type, the more the related infrastructure is extended in the longer run; this is

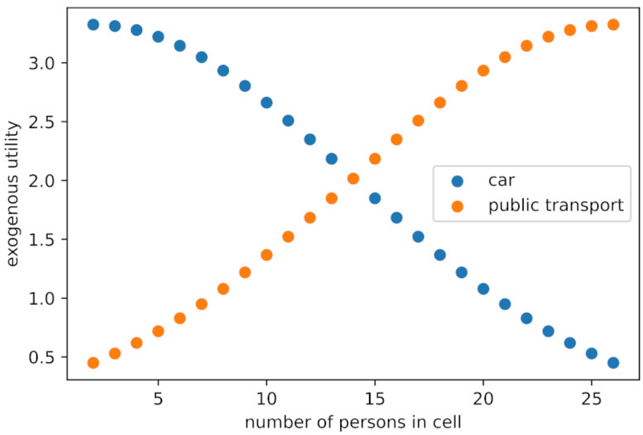


Fig. 1. Exogenous part $\hat{u}_c^{(m)}$, see (7), of the utility function $u_c^{(m)}$ over the number N_c of agents per cell for $\mu^{(0)} = 2$ ($m = 0$ represents car) and $\mu^{(1)} = 26$ ($m = 1$ stands for public transport), see (8). With these values, used in Sec. 4, the exogenous utilities are equal for both mobility types for cells with a population of 14 agents.

modeled by the infrastructure bonus process $\mathcal{B}_c^{(m)}(t)$. The two terms will be called *infrastructure feedback* in the following. To explore the role of this feedback mechanism on the dynamics of the system, R-MoTMo simulations will be compared in Sec. 4 to simulations with a model like R-MoTMo just without this feedback, i.e. a model where $u_c^{(m)}(x,b) = \hat{u}_c^{(m)}$ for all x, b. This will be called *the model without infrastructure feedback*.

Note that through the dependence on the random mobility choices (summarized in $\mathcal{X}(t)$) and on the bonus process $\mathcal{B}(t)$, the individual utilities $\mathcal{U}_i(t)$ are again stochastic processes.

The weight matrix process $\mathcal{W}(t)$. The connections between agents are weighted by time-evolving values $\mathcal{W}_{i,j}(t) \in [0,\infty)$. The larger the connection weight $\mathcal{W}_{i,j}(t)$, the more likely it is for agent i to communicate at time t with agent j, as will be seen in the recursive process rules below. The overall weight matrix process is given by $\mathcal{W}(t) = (\mathcal{W}_{i,j}(t))_{i,j=1,\dots,N}$. Its initial state is defined by

$$\mathcal{W}_{i,j}(0) = \begin{cases} 1 & \text{if } j \in \mathbb{F}(i), \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

This is a generally non-symmetric matrix with zero entries on the diagonal (because a person is never friends with itself).

3.2. *Updating scheme*

The mobility choice of an agent evolves by imitating the choice of a friend, where the probability for choosing a specific friend is the normalized product of the friend's utility times the connection weight. The friend's mobility choice is copied only if the friend's utility exceeds the agent's own utility.

More precisely, for $t \in \mathbb{N}_0$, let $\mathcal{M}(t)$, $\mathcal{W}(t)$, and $\mathcal{B}(t)$ be given, and assume that $\mathcal{X}(t)$ and $\mathcal{U}(t)$ have already been calculated using Eqs. (2) and (4), respectively.

(1) **Choice of a friend.** For each agent $i \in \mathbb{I}$ choose a friend $\mathcal{J}_i(t) \in \mathbb{F}(i)$ randomly according to the probabilities $q_{i,j(t)}$

$$\text{Prob}(\mathcal{J}_i(t) = j) = q_{i,j}(t), \tag{10}$$

where

$$q_{i,j}(t) := \frac{\mathcal{U}_j(t) \cdot \mathcal{W}_{i,j}(t)}{\sum_{k \in \mathbb{F}(i)} \mathcal{U}_k(t) \cdot \mathcal{W}_{i,k}(t)}. \tag{11}$$

(2) **Update of mobility types.** For each agent $i \in \mathbb{I}$ update the mobility type according to

$$\mathcal{M}_i(t+1) = \begin{cases} \mathcal{M}_j(t) & \text{if } j = \mathcal{J}_i(t) \quad \text{and} \quad \mathcal{U}_j(t) > \mathcal{U}_i(t), \\ \mathcal{M}_i(t) & \text{otherwise.} \end{cases} \tag{12}$$

(3) **Update of counting state and bonus.** For each cell c and each mobility type m determine $\mathcal{X}_c^{(m)}(t+1)$ according to Eq. (2) and $\mathcal{B}_c^{(m)}(t+1)$ according to Eq. (3).
(4) **Update of individual utilities.** For each agent $i \in \mathbb{I}$ determine $\mathcal{U}_i(t+1)$ according to Eq. (4).
(5) **Update of weights.** For all $i, j \in \mathbb{I}$ set

$$\mathcal{W}_{i,j}(t+1) = \begin{cases} \mathcal{W}_{i,j}(t) \cdot \dfrac{\mathcal{U}_i(t+1)}{\mathcal{U}_i(t)} & \text{if } j = \mathcal{J}_i(t) \quad \text{and} \quad \mathcal{U}_j(t) > \mathcal{U}_i(t), \\ \mathcal{W}_{i,j}(t) & \text{otherwise.} \end{cases} \tag{13}$$

(6) Set $t \mapsto t+1$ and restart with step (1).

The recursion (13) means that, when the mobility choice of a friend is copied, the respective connection to this friend is updated based on the relative utility gain or loss achieved by copying the friend's choice. Note that the weight $\mathcal{W}_{i,j}$ can change value also when agent i does not change the mobility type (namely, if the chosen friend $\mathcal{J}_i(t)$ uses the same mobility type as person i and $\mathcal{U}_j(t) > \mathcal{U}_i(t)$).

The rationale behind this updating scheme is that if a person's utility is increased by imitating somebody, it becomes more likely to imitate this friend again and vice versa. This can be seen as a way of Bayesian updating: during the simulation, person i collects information about how useful it was to copy certain friends in terms of utility optimization. Weights $\mathcal{W}_{i,j}$ are used to quantify expectations about how beneficial it is to copy the choice of friend j. In the beginning, all friends are expected to be equally useful to copy from, see the initial weights in Eq. (9). New information which is gathered during the simulation is used to update the person's expectations. To investigate the impact of this *weighting feedback* on system dynamics, subsequent R-MoTMo simulations will contrast with simulations of a model akin to R-MoTMo but lacking the weighting feedback. In this alternative model, denoted *the model without weighting feedback*, the network weights remain constant, $\mathcal{W}(t) = \mathcal{W}(0)$ for all t.

In summary, the system dynamics arise out of several interrelated process components, namely, $\mathcal{J}(t), \mathcal{M}(t), \mathcal{X}(t), \mathcal{B}(t), \mathcal{U}(t)$, and $\mathcal{W}(t)$, that cyclically influence each other in a complex manner. We note that the intrinsic stochasticity of the dynamics solely originates from the random choice of friends $\mathcal{J}(t)$, see Eq. (10), while the values of all other process components result from deterministic updating rules. Which of these components are required to understand the dynamics of the mobility choices and to make statistical predictions about their future evolution? This question will be addressed in the following section.

3.3. *Markov property*

The process of interest is given by the mobility choices $(\mathcal{M}(t))_{t=0,1,\ldots}$. This process is by itself not Markovian, as the probabilities for its future evolution are not fully determined by its current state $\mathcal{M}(t)$. Instead, the probabilities defined in (11) depend (i) on the history of previous mobility choices $(\mathcal{M}(s))_{s\leq t}$ because these determine the current value of the infrastructure bonus and hence the current utilities, and (ii) on the history of the choices $(\mathcal{J}(s))_{s\leq t}$ of friends which affects the current weights. This historical information is entirely encoded in the bonus process $\mathcal{B}(t)$ and the network weights $\mathcal{W}(t)$ which recursively evolve over time. Together, the three components $\mathcal{M}(t)$, $\mathcal{B}(t)$, and $\mathcal{W}(t)$ uniquely determine the mobility dynamics, as $\mathcal{X}(t)$ and $\mathcal{U}(t)$ are direct results of their values, and together $\mathcal{U}(t)$ and $\mathcal{W}(t)$ determine the transition probabilities.[e]

This observation motivates to extend the state space $\mathbb{M}^N$ of mobility decisions by state spaces for the processes $\mathcal{B}(t)$ and $\mathcal{W}(t)$, leading to the extended state space

$$\mathbb{Y} := \mathbb{M}^N \times [0,1]^{C\times M} \times [0,\infty)^{N\times N}. \tag{14}$$

On this state space, we introduce the *extended mobility process*

$$\mathcal{Y}(t) := (\mathcal{M}(t), \mathcal{B}(t), \mathcal{W}(t)), \tag{15}$$

with possible states

$$\boldsymbol{y} = (\boldsymbol{m}, \boldsymbol{B}, \boldsymbol{W}) \in \mathbb{Y}. \tag{16}$$

The process $(\mathcal{Y}(t))_{t=0,1,\ldots}$ is a time-homogeneous Markov process that contains all relevant historical information. Skipping any of the components of the process $\mathcal{Y}(t) = (\mathcal{M}(t), \mathcal{B}(t), \mathcal{W}(t))$ would mean to loose the Markov property.

The process $(\mathcal{Y}(t))_{t=0,1,\ldots}$ is recursively defined by the updating scheme introduced in Sec. 3.2, and its transition probabilities are determined by the probabilities $q_{i,j}(t)$ defined in Eq. (11) which in fact are functions $q_{i,j}(\boldsymbol{y})$ of the system state $\boldsymbol{y}$.

More precisely, let $\mathbb{F} := \prod_{i=1}^N \mathbb{F}(i)$ denote the Cartesian product of the sets $\mathbb{F}(i)$ of friends of agent i, which are fixed over time. For each $J \in \mathbb{F}$ there is a function f_J that maps $\mathbb{Y}$ onto itself and summarizes the steps (2)–(5) from the updating scheme. For the transition probabilities of the Markov chain $(\mathcal{Y}(t))_{t=0,1,\ldots}$, we obtain

$$\mathrm{Prob}(\mathcal{Y}(t+1) = \boldsymbol{y}' | \mathcal{Y}(t) = \boldsymbol{y}) = \sum_{\substack{J\in\mathbb{F} \\ f_J(\boldsymbol{y}) = \boldsymbol{y}'}} P(J|\boldsymbol{y}), \tag{17}$$

where

$$P(J|\boldsymbol{y}) := \prod_{i\in\mathbb{I}} q_{i,J_i}(\boldsymbol{y}), \tag{18}$$

is the probability for the choice of friends $\mathcal{J}(t) = J \in \mathbb{F}$ given $\mathcal{Y}(t) = \boldsymbol{y} \in \mathbb{Y}$.

As for the sum in Eq. (17), we note that for agents i whose weights change from t to $t+1$ on account of updating, i.e. $\mathcal{W}_{i,j}(t+1) \neq \mathcal{W}_{i,j}(t)$ for some $j \in \mathbb{F}_i$, the index $\mathcal{J}_i(t)$ of the chosen friend is uniquely determined because there is equality $\mathcal{W}_{i,j}(t+1) = \mathcal{W}_{i,j}(t)$ for all other $j \neq \mathcal{J}_i(t)$. However, for those agents i who do not

[e] Note that in case of interest in the output of $\mathcal{J}(t)$, this process would have to be stored as well.

update their mobility type and weights at time t (because the utility of the chosen friend does not exceed their own), this uniqueness is not given. Thus, there can generally be several choices of friends $\mathcal{J}(t) = J \in \mathbb{F}$ that lead to the same state $\boldsymbol{y}'$, and we need to sum their probabilities.

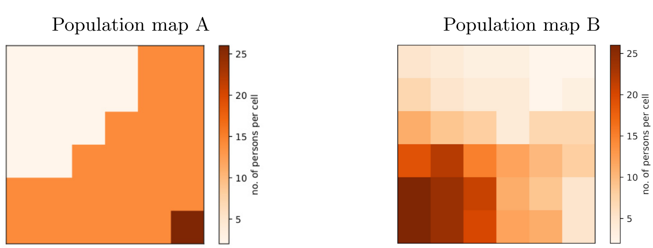
While this analysis has made the structure of the Markov process clear, the process $\mathcal{Y}(t)$ itself is still too complex for its dynamic features to be investigated in an analytic manner within this paper. In the following section, we therefore examine the contribution of the components $\mathcal{B}(t)$ and $\mathcal{W}(t)$ to the dynamics using numerical simulations, as is customary for ABMs.

4. Simulation Results

In the following, we show simulation results focusing on the role of the two history-determined process components in the dynamics of the mobility choices $\mathcal{M}(t)$: Sec. 4.2 compares the system's behavior with and without the infrastructure feedback, including $\mathcal{B}(t)$, and Sec. 4.3 analogously for $\mathcal{W}(t)$. Before, however, we provide some detail about how simulations are set up and initialized.

4.1. *Setup and initialization*

The simulations shown in this paper are performed with the R-MoTMo version that can be downloaded at the CoMSES model library [28]. All simulations take place on a map of 6×6 pixels. Two different population distributions are used, which are part of the published R-MoTMo code and shown in Fig. 2. In both of them, the maximum number of persons per cell is 26, and the minimum is 2. That means, that without any infrastructure feedback, i.e. $u_c^{(m)}(x, b) = \hat{u}_c^{(m)}$, see Sec. 3, for cells with less than 14 inhabitants car usage has a higher utility (these are called *rural cells*) and for cells with more than 14 inhabitants public transport has a higher utility (*urban cells*).



Population map A		Population map B	
$N = 360$ persons on		$N = 360$ persons on	
$C = 36$ cells of which are		$C = 36$ cells of which are	
urban cells:	1, with 26 persons	urban cells:	9, in total 197 persons
rural cells:	13, in total 26 persons	rural cells:	27, in total 173 persons
intermediate:	22, in total 308 persons	intermediate:	0 cells

Fig. 2. The different population distributions (see text for explanation).

In cells with 14 inhabitants both utility functions have the same value (*intermediate cells*), see also Fig. 1.

In the beginning of a simulation with R-MoTMo, persons are created and distributed as given by one of the input maps. They are assigned random mobility choices. For each agent, the agent-specific set of friends of size n is randomly drawn at initialization of the model, where the probabilities for being friends with persons living close by is higher than for persons further away, as described in the ODD document given in [28].[f] As mentioned, the set of friends stays fixed throughout a simulation but the intensity of a friendship can change according to (13). By construction, agent i is never friends with itself, so $i \notin \mathbb{F}(i)$.

All simulations shown here are done with $n = 15$ friends per person. As all behavior of interest is clearly seen within the first 200 time steps, all simulations were run up to this point in time.

4.2. *Coordination via infrastructure feedback*

The infrastructure feedback is two-fold: The bonus process $\mathcal{B}(t)$, given in Eq. (3), causes a mobility type that is chosen more often to provide higher utility to its users in the long run. On the other hand, there is an immediate utility malus for mobility types the more they are used, as given in Eq. (6). Both effects are local, i.e. they depend only on the amounts of users in the respective cell.

To examine the influence of infrastructure feedback on the mobility dynamics we consider population map A of Fig. 2. This map might look a bit artificial but was created to be a balanced test case in the sense that there are as many agents living in rural cells as in urban ones and the utilities for cars and public transport are maximal or minimal for urban and rural cells, respectively (see Eq. (5)). Most persons live in the intermediate area between rural and urban cells, where the exogenous part of the utility functions is equal (see Fig. 1), which means that without infrastructure feedback using either of the two mobility types does not make a difference for the utility of persons living in these cells.

With infrastructure feedback, however, agents in the intermediate area tend to coordinate on one mobility type. Simulations are found to typically end in one of two configurations: either car usage is dominant in all of the intermediate area, or almost everybody there uses public transport. Exemplary final states of cellwise car usage in both cases are shown in Fig. 3: By the time $t = 200$, the intermediate area (i.e. the cells with 14 inhabitants) has become either mainly dominated by car use (a) or by the use of public transport (b). The third graphic (c) shows a typical run of the model without infrastructure feedback, i.e. a model with no further changes to R-MoTMo but $u_c^{(m)}(x, b) = \hat{u}_c^{(m)}$. Here, no coordination emerges.

[f] We note that there is a nonzero probability for the network to split-up into disconnected components. This probability will decrease with increasing number n of friends per agent. For the value $n = 15$ chosen for the experiments of this section, the probability of such a split-up is very small and may be ignored.

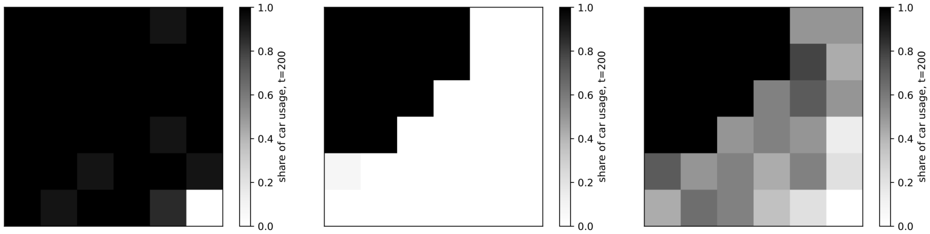


Fig. 3. Influence of the infrastructure feedback: Exemplary final states. Car usage after 200 time steps for simulations with population map A (see Fig. 2) and $n = 15$ friends per person. (a) and (b) Exemplary shares of car usages $\mathcal{X}_c^{(0)}$ for all cells c for two different runs with infrastructure feedback. (c) A typical case of car usage $\mathcal{X}_c^{(0)}$ after 200 time steps using the model without infrastructure feedback.

The statistics of the dynamics can be found in Fig. 4, where the simulation output for ensembles of 100 runs each are depicted. On the left, i.e. in Figs. 4(a) and 4(c), respectively, the mean car usage per cell at time $t = 200$ is shown. On the right, i.e. in Figs. 4(b) and 4(d), the temporal development of the overall car usages (i.e. the average over all cells) for the 100 different runs are depicted. Figures 4(a) and 4(b)

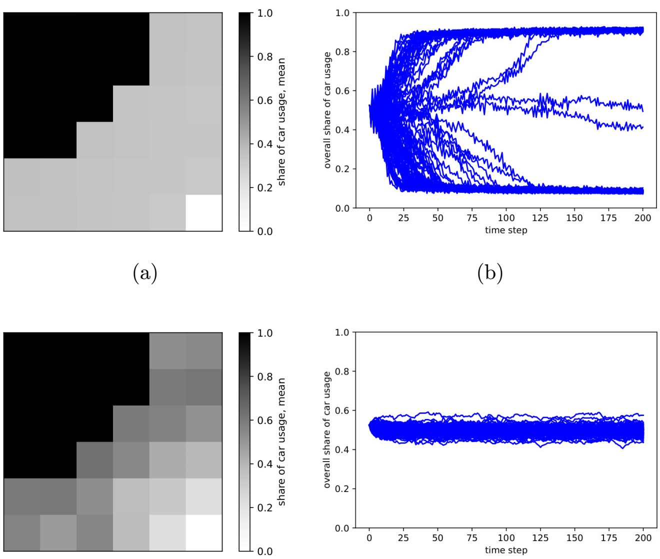


Fig. 4. Influence of the infrastructure feedback: Statistics. Simulations with infrastructure feedback (bonus and malus processes): (a) Mean $\mathcal{X}_c^{(0)}$ (car usage per cell) after 200 time steps, (b) overall car usage in the different simulation runs over time, see Sec. 4.2. Simulations using the model without infrastructure feedback: (c) Mean $\mathcal{X}_c^{(0)}$ (car usage per cell) after 200 time steps, (d) overall car usage in the different simulation runs over time. The statistics result from 100 simulations per scenario, with 200 time steps and $n = 15$ friends per person, using population map A.

show simulations with infrastructure feedback, whereas Figs. 4(c) and 4(d) refer to the model without infrastructure feedback.

For the case with infrastructure feedback, in most cases (here more than 60% of the realizations) the average car usage in the end was between 0% and 20%, which is, e.g. the case in the exemplary run shown in Fig. 3(b), and almost all the others (around 35%) had a final car usage of between 80% and 100%, like the run shown in Fig. 3(a). We find that, in fact, with infrastructure feedback there is a strong coordination on either of the two mobility types (roughly twice as much on public transport as on cars) while without infrastructure feedback the usage of both types is always around 50%, as can be seen in Fig. 4(d). The coordination also suggests that the (long-term) bonus process $\mathcal{B}(t)$, given in Eq. (3), dominates the (short-term) malus process, given in Eq. (6), because contrary to the bonus process, the malus process would rather incentivize not to coordinate. These simulation results indicate that an analysis of the asymptotic behavior of the Markov chain defined in Sec. 3.3 would be fruitful. In particular, it seems that the dynamics converge in the long run to one of two fixed points: a map where all agents except those in the urban cell use cars, and one where all agents but those in the rural cells use public transport. Such a mathematical analysis, however, goes beyond the scope of this paper.

The observed behavior can be classified as path dependency as defined by [30]. It depends on the system's history in the following way: it is triggered by contingent events (in this case, the random social networks of the individuals living in intermediate areas and their random choices of friends to copy); it possesses a self-enforcing component through the bonus process; and it results in a lock-in state.

The high abundance of intermediate cells in map A implies that coordination via the bonus process dominates the simulation dynamics. For other population distributions with less intermediate cells this mechanism plays a minor role. This is due to the fact that, already from the beginning on, one mobility type has a higher utility than the other for all cells. In Fig. A.1, the same simulations as in Fig. 4 are shown for population map B, as an example for this second scenario.

4.3. *Learning by weighting links in the social network*

In R-MoTMo, the weight matrix process $\mathcal{W}(t)$ represents social learning of agents. After an agent imitates a friend's mobility choice the usefulness of this choice for the person is determined and the link to the respective friend is re-weighted accordingly, as given in Eq. (13). This means that agents learn over time about how useful copying certain friends is in terms of utility optimization; past experiences are used to strengthen or weaken the link to the respective friend in a recursive manner.

This effect can be easily examined using map B of Fig. 2. Simulation results are illustrated in Fig. 5: Based on an ensemble of 100 runs with $n = 15$ friends per person, Figs. 5(a) and 5(c) show the mean share of car usage per cell, while in Figs. 5(b) and 5(d) the mean utilities per cell are plotted. For Figs. 5(a) and 5(b), the weighting of friends as described in Sec. 3 is used, whereas Figs. 5(c) and 5(d) refer to the model

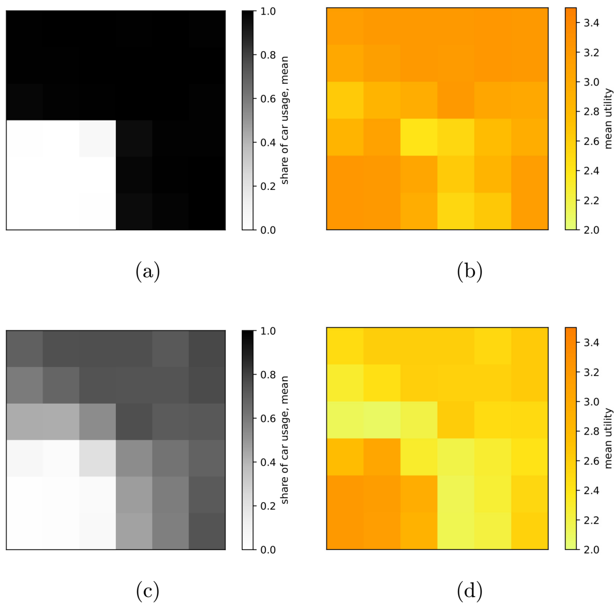


Fig. 5. Influence of the weight matrix process: Statistics. Simulations for $n = 15$ friends per person using population map B, means over 100 runs, (a) mean $\mathcal{X}_c^{(0)}$ (car usage per cell) and (b) mean utilities per cell, i.e. average over all inhabitants per cell. The second row shows the same simulations using the model without weighting feedback, i.e. all nonzero entries of $\mathcal{W}_{i,j}(t)$ are constantly set to 1: (c) Mean $\mathcal{X}_c^{(0)}$ (car usage per cell) and (d) mean utilities per cell.

without weighting feedback, i.e. the only change to R-MoTMo is that all nonzero entries of the matrix $\mathcal{W}(t)$ are kept constant at 1. Throughout large parts of the map, one can see that the scenario with social learning (by weighting of links in a recursive manner) indeed works better in terms of utility optimization.

Population map A exhibits a minor role for the weight matrix process, primarily because of the large number of individuals residing in intermediate cells and the resulting significance of coordinating individuals' choices through infrastructure feedback, as discussed in Sec. 4.2. Simulations for population map A (with the same parameter values as before) can be found in Fig. A.2.

5. Discussion

We have formulated the ABM R-MoTMo mathematically as a Markov process

$$\mathcal{Y}(t) = (\mathcal{M}(t), \mathcal{B}(t), \mathcal{W}(t))_{t=0,1,\ldots},$$

which has three components:

(1) The decision process $\mathcal{M}(t)$ recording the mobility choices of the persons, which is a random process with transition probabilities determined by the other

process components. The state space of the mobility decision process $\mathcal{M}(t)$ complies with the one found in mathematical formulations of basic opinion dynamics models.

(2) The bonus process $\mathcal{B}(t)$ that is recursively defined and thus conserves the decision history and causes a path dependency via infrastructure development.

(3) The weight matrix process $\mathcal{W}(t)$ defining the time-dependent network of agents; agents recalculate the weights recursively following a Bayesian updating rule.

How general are the components we have identified for ABMs of social systems, and can we relate them to the elements others have proposed as fundamental for agent-based simulations of social systems? Recently, Dilaver and Gilbert developed a *conceptual anatomy framework* for agent-based social simulation [7]. The elements of this framework are "agents", "social structure", "environment", "actions and interactions", as well as "temporality". In our work, the "agents" are the persons who take mobility decisions and evaluate their connections to other persons in a learning process. In the formulation of Sec. 3, agents are tracked by the index i in the processes $\mathcal{M}(t)$ and $\mathcal{W}(t)$. The matrix $\mathcal{W}(t)$ containing the connection strengths between any two agents in the system corresponds to the (evolving) "social structure". The cells with their evolving infrastructure bonus values $\mathcal{B}(t)$ (which influence the utilities of the two mobility types) correspond to the (evolving) "environment". The element "actions and interactions" can be captured as the mobility process: "actions" in the form of decision-making are here modeled as a rather simple copying of another agent's mobility choice. At the same time, they produce a two-fold "interaction" both with the environment and with the social structure: the probabilities for choosing which agent to copy depend on both the environment (given by $\mathcal{B}(t)$) and the social structure (captured by $\mathcal{W}(t)$), and the decisions taken, in turn, influence the updated values of these processes. Considering "temporality", Dilaver and Gilbert point out both time and history as relevant concepts. In our framework, time is modeled as a simple period-based process given by a discrete-time Markov chain (without an empirical interpretation of what a period represents). A building up of history occurs through the feedback mechanisms in the environment and the network processes, that through their co-evolution with the agents' actions create path dependency in the mobility decisions. In this regard, our model stands out against other mathematically formulated ABMs (e.g. for opinion dynamics), which mostly do not take memory effects into account.

While the components of our model can be related to the elements of social ABM identified by Dilaver and Gilbert as explained in the previous paragraph, our way of classifying the three components $\mathcal{M}(t), \mathcal{B}(t), \mathcal{W}(t)$ differs from their approach. They classify components looking at their meaning, e.g. whether they are part of "social structure" of the relevant society. In contrast to this *contentual* classification, our classification is rather given by the mathematical form, e.g. we identify local fields (i.e. cellwise quantities given by $\mathcal{B}(t)$) and a network structure with updating rules for the link weights (the social network $\mathcal{W}(t)$). Although we associated the infrastructure bonus process in our model with the "environment" and the social network with the "social structure", this is not necessarily a one-to-one mapping: as the buildup of infrastructure is also subject of political decisions it could also be interpreted as a different part of the "social structure".

If abstracted from the example of mobility decisions, we can classify two "memory" processes that are needed to obtain the Markovian system, depending on whether they operate through (local) fields or rather networks of agents. Many other socio-technical systems will have dynamic elements that operate through the same kind of structural features, that is, in terms of fields and networks, meaning that the dynamics will be influenced in similar ways by the respective feedback mechanisms. However, which elements are interpreted as "social structure" or "environment" may or may not correspond with our example system. If the association of mathematical structures with contentual meaning differs from our case, this would mean that the role played by these contentual components for the system's dynamics also differs. We think that this can be a starting point for interesting further research. There are probably correlations between the formal role (e.g. local fields or changing networking structures) and the meaning of elements of socio-technical ABM. Investigating these correlations may also be a contribution of agent-based modeling to the agency–structure debate in the social sciences, as it could point out different influences of agency and social structure on such socio-technical systems.

Another rather formal conceptual framework that our model's structures can be related to has been proposed by Laubichler and Renn [17]. They introduce the concept of *extended evolution* to study cultural dynamics with the help of concepts from biological evolution theory, in particular, regulatory networks and niche construction. Due to its co-evolutionary dynamics, our model can be interpreted in terms of this framework. Figure 6 gives an overview of how an agent's mobility decision

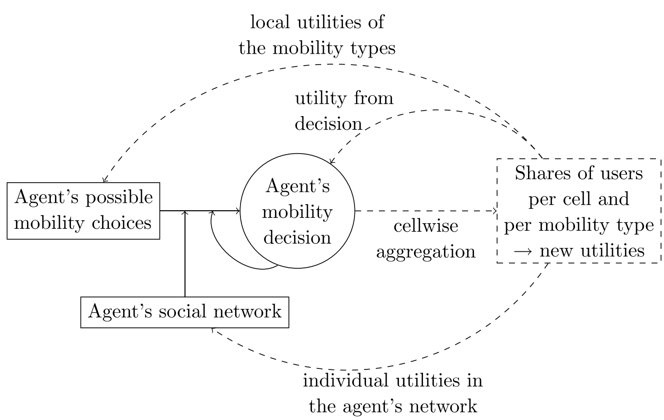


Fig. 6. Illustration of feedback mechanisms. An agent's decision-making process (solid lines) and its interactions with the agent's environment (dashed). See text for further description.

interacts with the environment and vice versa. The agent's choice relies on the options at hand, specifically, the mobility choices with their respective characteristics that are accessible at the agent's location. Which of the possible options is chosen depends on the agent's social network and on the agent's present utility as a consequence of the agent's last choice. In terms of Laubichler and Renn, the possible options available at that agent's place and time can be seen as the genomic sequence, and the agent's social networks, that determines which of them is chosen, as the *regulatory network*. All agents' choices made at one time step influence the environment in a way that resembles *niche construction*, e.g. [16, 21] in ecology. In other words, the individual decisions are externalized such that they shape the environment outside of the agent in a lasting way (due to the bonus process). These changes in the environment affect the elements that determine the agent's decision, which can be viewed as a process of internalization. In our model, this is done by the mechanisms of how the individual utilities depend on the behavior of all actors in the local environment of an agent.

Studying socio-technical systems, especially in the context of societal challenges such as sustainability transitions, may benefit from analogies to the framework of extended evolution. This is particularly relevant because the emergence of innovations, whether they are technological or behavioral in nature, carries substantial importance. To complement a perspective of statistical physics, that could focus on such transitions as regime changes, an evolutionary perspective can address questions around adaptation to changing environmental conditions (which may be caused by the populations under study). In particular, the role of regulatory networks and whether or how they can trigger faster adaptation than the process associated with evolutionary mutation of genes may deserve further investigation for a specific socio-technical system.

6. Conclusion and Outlook

In summary, we have introduced the agent-based mobility demand model R-MoTMo and have shown how formulate it as a Markov chain. As the mobility decision process $\mathcal{M}(t)$ is not Markovian, it was necessary to extend the state space by including the processes $\mathcal{B}(t)$ and $\mathcal{W}(t)$, which account for the feedback mechanisms. These make the model more complex than, e.g. a classical voter model.

Since an analytic treatment of the Markov process is more difficult than for "history-free" decision models, we have provided simulation results to investigate the impact of the process components $\mathcal{B}(t)$ and $\mathcal{W}(t)$ on the dynamics of the system, showing that both are essential in creating the model's dynamics. We found that the "infrastructure component" $\mathcal{B}(t)$ can introduce a coordination effect that results in path dependency in the system. Triggered by contingent events, the self-enforcement through the bonus process can cause the system to reach a lock-in state. The weight matrix component $\mathcal{W}(t)$ accounts for agents' learning in the system which helps them to optimize their utilities over time.

We have then compared our model's structure, made explicit in the Markov chain formulation, to two previous theoretical frameworks. As for the *conceptual anatomy framework* by Dilaver and Gilbert we found that we have the elements they describe as substantial for ABMs of social systems and can relate them to our components of the Markov process, but our classification of components is rather formal than contentual and the relation of these three formal components to the contentual elements of Dilaver and Gilbert might differ in other models that still have the same mathematical structure. For the *framework of extended evolution* proposed by Laubichler and Renn we uncovered parallels with niche construction and regulatory networks that suggest potential benefits from this perspective for understanding the emergence of innovation in socio-technical systems with similar structures. In fact, the same may hold for socio-economic and socio-ecological systems, or combinations thereof, with feedback effects that can be conceptualized as operating through (global or local) fields or network structures.

With this work, we have cast an ABM with two kinds of memory mechanism into the form of a Markov chain — a form that is well used for similar models without feedback effects, where it has been employed to better understand and analyze these systems' dynamics, e.g. by calculating transition probabilities. The analysis of the dynamics of our model in this paper was still at the level of simulation output because the mathematical structure is rather complex, but a mathematical analysis of the Markov chain defined here is part of ongoing and future work by the authors. It will include, e.g. an analysis of fixed points of the model in different settings, and an analysis of initial configurations of the friendship network between agents, as well as the influence of different network topologies on the dynamics. Such work shall further enhance the understanding and explainability of this model, R-MoTMo, and more generally, a model structure, that of agents taking decisions with environment- and network-based memory mechanisms. Those mechanisms underlie various complex socio-ecologic, socio-economic, or socio-technical systems, where feedbacks between decisions and environment are crucial, such as questions of sustainability, and more. The resulting form, a Markov chain with deterministic updates that encode the decision history into an extension of the state space, may hopefully be useful for a further analysis of these. Furthermore, working out the similarities (and probably differences) of the theory of these complex socio-technical systems with the theory of extended evolution can help to better understand change and adaptability of the former by looking for analogies for adaptation mechanisms.

Complementary to the mathematical analysis of the complex systems underlying a given societal challenge, dialogues with the respective stakeholders are a useful element to come up with possible future scenarios. In order to accompany and interweave scientific knowledge with practical and experiential knowledge as well as preferences, interests, and values [22], new methods for this are being developed.

One discussion format for such a transdisciplinary co-creation is the *Decision Theatre*,[g] and one of the main challenges in setting up a Decision Theatre is building ABMs from scratch for each new case under study [32]. The generalizable mathematical structure for a socio-technical system with memory mechanisms defined here will be helpful for conceptualizing such models in future work.

Acknowledgments

We thank Christof Schütte and Armin Haas for helpful discussions, and three anonymous reviewers for their valuable questions and comments. This research has been partially funded under Germany's Excellence Strategy, MATH+: The Berlin Mathematics Research Center (EXC-2046/1), Project No. 390685689.

Appendix A

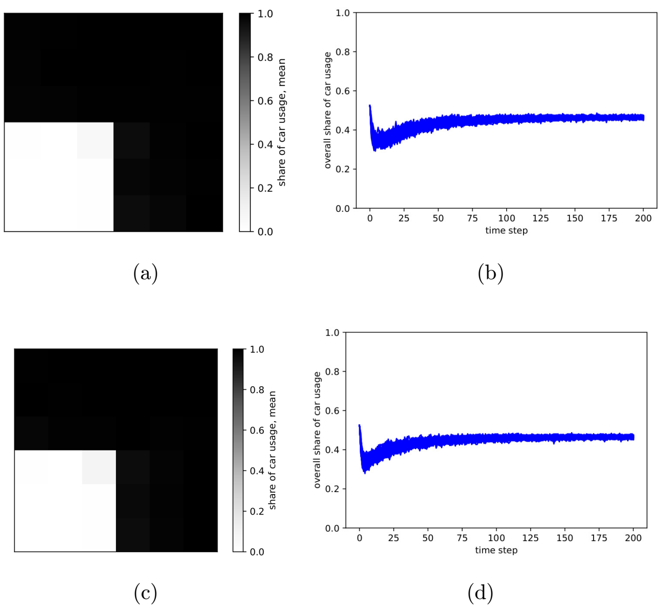


Fig. A.1. Influence of the infrastructure feedback: Statistics. Simulations with infrastructure feedback (bonus and malus processes): (a) Mean $\mathcal{X}_c^{(0)}$ (car usage per cell) after 200 time steps, (b) overall car usage in the different simulation runs over time, see Sec. 4.2. Simulations using the model without infrastructure feedback: (c) Mean $\mathcal{X}_c^{(0)}$ (car usage per cell) after 200 time steps, (d) overall car usage in the different simulation runs over time. The statistics result from 100 simulations per scenario, with 200 time steps and $n = 15$ friends per person, using population map B. Unlike as for map A (see Fig. 4), coordination via infrastructure feedback does not play a role.

[g] A Decision Theatre is a workshop in which scientists and stakeholders discuss a societal challenge supported by visualizations of data and model simulations on big screens. Participants can compose alternative model scenarios that are then explored, discussed and evaluated together, see also [32].

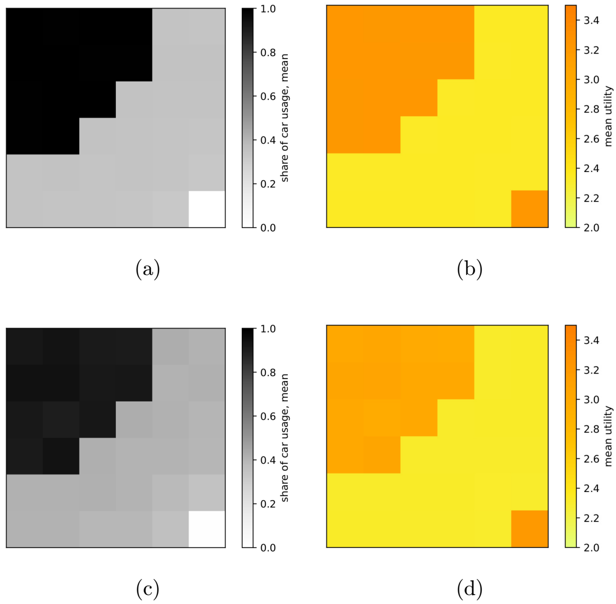


Fig. A.2. Influence of the weight matrix process: Statistics. Simulations for $n = 15$ friends per person using population map A, means over 100 runs, (a) mean $\mathcal{X}_c^{(0)}$ (car usage per cell) and (b) mean utilities per cell, i.e. average over all inhabitants per cell. The second row shows the same simulations using the model without weighting process, i.e. all nonzero entries of $\mathcal{W}_{i,j}(t)$ are constantly set to 1: (c) Mean $\mathcal{X}_c^{(0)}$ (car usage per cell) and (d) mean utilities per cell. For map A, the effect of the weight matrix process are small compared to the infrastructure feedback processes.

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