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UNDERSTANDING MEMORY MECHANISMS IN SOCIO-TECHNICAL SYSTEMS: THE CASE OF AN AGENT BASED MOBILITY MODEL

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Understanding Memory Mechanisms in Socio-Technical Systems: the Case of an Agent-based Mobility Model

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ABSTRACT

This paper explores memory mechanisms in complex socio-technical systems, using a mobility de-mand model as an example case. We simplified a large-scale agent-based mobility model into a Markov process and discover that the mobility decision process is non-Markovian. This is due to its dependence on the system's history, including social structure and local infrastructure, which evolve based on prior mo-bility decisions. To make the process Markovian, we extend the state space by incorporating two history-dependent components. Although our model is a very much reduced version of the original one, it remains too complex for the application of usual analytic methods. Instead, we employ simulations to examine the functionalities of the two history-dependent components. We think that the structure of the analyzed stochastic process is exemplary for many socio-technical, -economic, -ecological systems. Additionally, it exhibits analogies with the framework of extended evolution, which has previously been used to study cultural evolution.

1 Introduction

Agent-based models (ABMs) are valuable tools for representing complex socio-economic, -ecological, and -technical systems because they allow the modeller to define the system based on its individual elements and their interactions, rather than having to specify aggregate dynamics of the system in the first place. Moreover, ABMs offer the advantage of being easily explainable to a non-technical audience: relevant actors in the system under study are represented on the computer as agents with features, governed by decision or interaction rules, and embedded in agent networks and a common environment; the system evolution is then simulated by repeatedly carrying out the specified (inter)actions. There hence is a wide usage of such models for analysing challenges in environmental and social sustainability contexts (e.g. 1; 14; 17; 23; 18).

Owing to detailed micro-level definitions, however, complex, empirically grounded ABMs are challenging to express mathematically (10). In fact, there is a research gap between these models and simpler ABMs studied in physics and mathematics, which often prioritize the depiction of mechanisms over accurately representing a specific social system (4).

Generally, in social contexts, where collective phenomena emerge from repeated interactions among individuals, there is growing interest in applying statistical physics to understand such phenomena (5; 25; 13). The related literature offers analytical tools and mean-field approximations, particularly for binary opinion models, with the famous *voter model* being extensively analyzed (e.g., 6; 24; 28; 26).

Mathematically, such agent-based models have been described as discrete-time Markov chains (12; 3) or continuous-time Markov jump processes (19; 30), also coupled with (e.g. spatial) diffusion dynamics, allowing for description by stochastic differential equations (8) or approximation by stochastic partial differential equations (11). In a seminal work, Izquierdo et al. (12) demonstrated how analyzing well-known ABMs as Markov chains offered valuable insights, including the identification of absorbing states from which the system cannot escape once entered.

On the other hand, for more complex ABMs – that e.g. include an economic, ecological, or technical dimension along with the social system, as well as feedbacks between these components – the corresponding Markov chain is hard to even write down explicitly and would in most cases be "too big for a tractable analysis of a model" (2). One explanation for this is the significance of history in such systems: for instance, path dependencies resulting from prior (collective) decisions for a certain technology or investment have a significant impact in human decision-making processes, as do past experiences, long-term beliefs, social norms or conventions, and the social network itself. These path dependencies clearly contrast with the Markov property, which presupposes that the future will only depend on the current situation. However, in theory, it is possible to convert non-Markov processes into Markovian ones by extending the state space.

Here this paper contributes. We focus on an agent-based model of mobility decisions, as an example case of a complex system in which a transition towards sustainability is required. Based on a large-scale, empirically-grounded ABM – the Mobility Transition Model (MoTMo) (9) – we derived a reduced model that retains the primary memory mechanisms of the original model (27). In this paper, we formulate the reduced model as a stochastic process and transform it – through an extension of the state space – into a Markov chain. This not only enhances the explainability of the model but also points to a classification of memory mechanisms that can be helpful for understanding complex socio-technical systems more generally.

MoTMo has been developed for simulating private mobility demand in Germany until 2035. While MoTMo itself is highly detailed and complex, making it challenging to express it entirely as a stochastic process, it contains two general kinds of memory mechanisms. The first kind comprises feedback from previous mo-

bility decisions that influences the mobility options agents can choose from. In MoTMo, this feedback takes various forms, such as the growth of convenience for a mobility type with increased usage (leading to the expansion of corresponding infrastructure) and the decline of unit prices with increased usage. This feedback operates through local (conveniences) or global (prices) fields, which are influenced by aggregate decisions of the agents. That means that it is not of importance who among the agents takes which decisions but rather how many of them choose what. In the general context of socio-technical systems, this structure can be associated with feedbacks related to general global or local parameters, such as unit costs, prices, technological progress, productivity, or social norms and conventions. The second memory mechanism is a network-based learning process, which in MoTMo operates as follows: individuals base their mobility decisions on the choices made by others within their social network. Afterward, they evaluate the usefulness of the information obtained and revalue the links in their network accordingly. This process resembles Bayesian updating and simulates social learning dynamics. In general socio-technical systems, this learning mechanism involves feedbacks facilitated by an evolving network structure, encompassing elements like social relations, information networks, supply chains, and local connections such as roads, railway tracks, and pipelines.

In this work, we present a highly condensed version of MoTMo which serves as an abstract model to analyze these two memory mechanisms. The Reduced Mobility Transition Model (R-MoTMo) is available via the CoMSES Model Library (27), where the code and a documentation according to the ODD protocol (10) can be found. We will show how formulating the (mobility) decision process as a Markov chain by extending the state space helps understand the system's structure and characteristic properties. The definition of this stochastic process explicitly reveals the required information for defining probabilities to predict future behaviour. The extended (Markovian) process not only contains the agents' mobility choices $\mathcal{M}(t)$, but also an infrastructure bonus process $\mathcal{B}(t)$ aggregating the convenience of the mobility types induced by previous mobility decisions (related to the first memory mechanism) and time-evolving weights $\mathcal{W}(t)$ for the social network (capturing the second memory mechanism of social learning).

The explicit formulation of our stochastic process can be viewed through two distinct lenses present in previous literature. Firstly, the components identified in our formulation can be related to the elements of ABMs for social systems described in prior conceptual work (see, e.g., 7, and references therein), that has emphasized agents, social structure, and the environment. However contrary to Dilaver and Gilbert, we identify components rather formally than contentual, so this relation is not an equation but maybe a step towards understanding in how far function might determine structure (or vice versa) in such systems.

Secondly, there have been past approaches discussing social systems from an evolutionary perspective (22). The complexity arising from the feedback mechanisms in our model can be interpreted using more recent developments in evolutionary theory (16). With our work, we take an initial step towards bridging the conceptual gap between sophisticated, empirically based ABMs and mathematical theory. By explicitly formulating the stochastic process, we create a foundation for better integrating empirical and theoretical aspects of modeling complex socio-technical systems.

The paper is structured as follows: In Section 2 we introduce R-MoTMo and define it as a Markov chain. Section 3 explores simulation results for different settings to analyse the effect of the memory mechanisms $\mathcal{B}(t)$ and $\mathcal{W}(t)$ on the system's dynamics. Moving on to Section 4, the paper discusses how the Markov chain formulation enhances the comprehension of the model and its dynamics. It also establishes connections with previous conceptual frameworks and the theory of extended evolution, before Section 5 concludes.

2 R-MoTMo as a Markov chain

In the Reduced Mobility Transition Model (R-MoTMo), agents are *persons* who live in one of the model's *cells* and take mobility decisions. They can choose between *cars* and *public transport*, and using one of these mobility types provides them with *utility*. The level of utility depends on the population density of their cell: Roughly, public transport is more convenient in densely populated cells (representing urban areas), while cars are more convenient in sparsely populated ones (rural areas). This convenience translates into utility. It is further influenced by endogenous factors depending on the mobility type usage: in the short run, more persons using the same mobility type decreases its utility (representing traffic jams, crowded trains, etc.) while in the long run, more persons using a mobility type make it more convenient (as, e.g., infrastructure is being built up). Persons want to optimise their utilities as in standard economic models, but – unlike in standard economics – they do not have full information for doing so; they only know their current utility and have information from other persons, who are their *friends* in a social network, about their utility stemming from their mobility choice. Taking mobility decisions by copying a random friend (where the probability of choosing a specific friend depends on the friend's utility and on the connection strength in the social network) they rather optimise their choices by trial and error, learning from the information given by this network. The intensity of connections in the social network also co-evolves with the decisions taken, as a person adapts the connection weight according to whether copying a specific friend has led to an increase or decrease of the utility experienced.

Note that, while this description uses the vocabulary of a mobility demand simulation, R-MoTMo can be seen as a prototypical model for a consumption decision taken between two different technologies, which in the long run deliver a higher utility to the consumers the more they have been used (locally); this is known as the network effect or as demand-side economies of scale, and in our model it does not operate through a network but through a local field. The model can further be seen as prototypical for co-evolutionary modelling approach where agents' decisions randomly depend on the network the agents are embedded in, which is transformed by the agents in the course of time, and on their environment that evolves as a consequence of agents' (collective) actions.

To formalize the model described in words above, we will introduce the notation and components needed (Section 2.1), define the steps that compose the dynamics (Section 2.2), and then assert the Markov property of the defined stochastic process (Section 2.3).

2.1 Notation and Process Components

Our notation here is the following: There are $N \in \mathbb{N}$ agents, and referring to a specific agent, the indices $i, j \in \mathbb{I}$ are used, where $\mathbb{I} := \{1, ..., N\}$ is the set of agents' indices. Each agent is located in a cell on a grid. The number of cells is denoted $C \in \mathbb{N}$, and cells are indexed by $c \in \{1, ..., C\}$. Let $N_c \in \mathbb{N}$ be the number of agents in cell c, which means that $\sum_{c \in \{1,...,C\}} N_c = N$. The location of each agent is fixed, such that also the number N_c of agents in a cell is time-independent. Each person i interacts directly with a set of friends $\mathbb{F}(i) \subset \mathbb{I}$, which is a subset of all agents of size $n \in \mathbb{N}$, i.e. all agents have the same number of friends. An agent's set of friends is also fixed throughout a simulation. Moreover, there is a set $\mathbb{M} = \{0, ..., M-1\}$ of M different mobility types that each agent can choose between. In the reduced model, this set is given by $\mathbb{M} = \{0, 1\}$ where $m = 0$ refers to "car" and $m = 1$ refers to "public transport".

With this notation, we can define the components of R-MoTMo as a stochastic process in discrete time, where we use $t \in \mathbb{N}_0$ as the discrete time index throughout.

The mobility decision process $\mathcal{M}(t)$**.** For each person $i \in \mathbb{I}$, let $\mathcal{M}_i(t) \in \mathbb{M}$ denote the chosen mobility type of this person at time t. The state of the stochastic process of mobility choices at time t is then given by

$$\mathcal{M}(t) = (\mathcal{M}_i(t))_{i=1,\ldots,N} \in \mathbb{M}^N. \tag{1}$$

The starting point $\mathcal{M}(0) \in \mathbb{M}^N$ is a randomly chosen initial state. The state space of the mobility decision process thus corresponds to that in some opinion dynamics models (see, e.g., 3). However, to describe the dynamics, and in particular to define the probabilities underlying the stochastic mobility choices, further elements will be needed.

The counting processes $\mathcal{X}(t)$**.** At some points in the following, we are not interested in each single agent's mobility choice, but only in how many agents have made which choice. Therefore the information contained in $\mathcal{M}(t)$ is aggregated in the *counting process* $\mathcal{X}(t)$ in the following way: Let $\mathbb{I}_c \subset \mathbb{I}$ be the set of indices of persons living in cell $c \in \{1, \ldots, C\}$. For each cell c and each mobility type m, let

$$\mathcal{X}_c^{(m)}(t) := \frac{1}{N_c} \sum_{i \in \mathbb{I}_c} 1_m(\mathcal{M}_i(t)) \in [0,1] \tag{2}$$

be the (random) share of persons in cell c using mobility type m at time t.

The infrastructure bonus process $\mathcal{B}(t)$**.** For each cell c and each mobility type m, the process $\mathcal{B}_c^{(m)} = \left(\mathcal{B}_c^{(m)}(t)\right)_{t=0,1,\ldots}$ is recursively defined by

$$\mathcal{B}_c^{(m)}(t) = \frac{1}{3} \cdot \mathcal{X}_c^{(m)}(t) + \frac{2}{3} \cdot \mathcal{B}_c^{(m)}(t-1) \in [0,1] \tag{3}$$

for $t \geq 0$, where $\mathcal{B}_c^{(m)}(-1) := 0$. Set $\mathcal{B}(t) = \left(\mathcal{B}_c^{(m)}(t)\right)_{c=1,\ldots,C;m=1,\ldots,M}$. This process represents the assumption that – at a local level – conditions will become better for mobility types that are used more in the respective cell, e.g. as an infrastructure build-up with a positive feedback mechanism. That is, large values of $\mathcal{X}_c^{(m)}(t)$ have a positive impact onto the component $\mathcal{B}_c^{(m)}(t)$ of the bonus process.

The utility process $\mathcal{U}(t)$**.** Utility of a mobility choice is defined on the level of cells, determined by the current level of the bonus process and the current number of users of this mobility type. The individual utility at time t of agent i is then given by the utility of the respective cell c_i that the agent lives in:

$$\mathcal{U}_i(t) := u_{c_i}^{(\mathcal{M}_i(t))}\left(\mathcal{X}_{c_i}^{(\mathcal{M}_i(t))}(t), \mathcal{B}_{c_i}^{(\mathcal{M}_i(t))}(t)\right), \tag{4}$$

where the utility functions $u_c^{(m)} : [0,1]^2 \to (0,\infty)$ have the form

$$u_c^{(m)}(x,b) := a_c^{(m)}(x) \cdot \hat{u}_c^{(m)} + b, \tag{5}$$

with the *short-term malus function* $a_c^{(m)} : [0,1] \to \left[\frac{2}{3}, 1\right]$ given by

$$a_c^{(m)}(x) := 1 - \frac{1}{3}x \tag{6}$$

and the exogenous factor

$$\hat{u}_c^{(m)} := \frac{100}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(N_c - \mu^{(m)})^2}{2\sigma^2}\right) \in \left(0, \frac{A}{\sqrt{2\pi}\sigma}\right], \tag{7}$$

where

$$\mu^{(m)} := \begin{cases} \min_{c \in \{1,\dots,C\}} N_c & \text{for } m = 0, \\ \max_{c \in \{1,\dots,C\}} N_c & \text{for } m = 1, \end{cases} \tag{8}$$

and $\sigma := \frac{1}{2}(\max_c N_c - \min_c N_c)$. The factor 100 appearing in (7) just serves to bring utilities to a handy range (e.g., $u_c^{(m)}(x,0) \in [0,4]$ for all x).

The exogenous factors $\hat{u}_c^{(m)}$ are independent of the system and only depend on the mobility type m and the number of persons N_c in the cell. In Figure 1, they are shown for both mobility types $m = 0$ (representing *car*) and $m = 1$ (representing *public transport*) for different numbers N_c of persons per cell. In contrast, the malus term $a_c^{(m)}(x)$, as given in Eq. (6), and the added b, see (5), depend on the system's state and represent two kinds of feedback mechanisms associated to infrastructure: The more agents currently use a mobility type m in cell c, the lower the utility at present (e.g. due to traffic jams or crowded trains) because infrastructure starts working at capacity, which is represented by the short-term malus $a_c^{(m)}(x)$. On the other hand, the more agents use a certain mobility type, the more the related infrastructure is extended in the longer run; this is modelled by the infrastructure bonus process $\mathcal{B}_c^{(m)}(t)$. The two terms will be called *infrastructure feedback* in the following. The case where $u_c^{(m)}(x,b) = \hat{u}_c^{(m)}$ for all x, b will be called a *scenario without infrastructure feedback*.
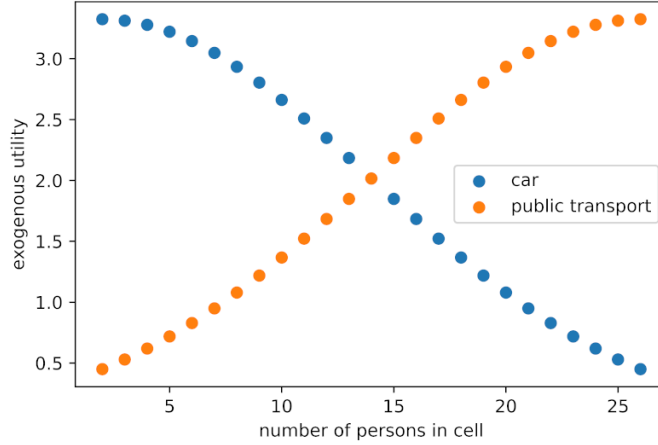

Figure 1: Exogenous part $\hat{u}_c^{(m)}$, see (7), of the utility function $u_c^{(m)}$ over the number N_c of agents per cell for $\mu^{(0)} = 2$ ($m = 0$ represents car) and $\mu^{(1)} = 26$ ($m = 1$ stands for public transport), see (8). With these values, used in Section 3, the exogenous utilities are equal for both mobility types for cells with a population of 14 agents.

Note that through the dependence on the random mobility choices (summarized in $\mathcal{X}(t)$) and on the bonus process $\mathcal{B}(t)$, the individual utilities $\mathcal{U}_i(t)$ are again stochastic processes.

The weight matrix process $\mathcal{W}(t)$. The connections between agents are weighted by time-evolving values $\mathcal{W}_{i,j}(t) \in [0, \infty)$. The larger the connection weight $\mathcal{W}_{i,j}(t)$, the more likely it is for agent i to communicate at time t with agent j, as will be seen in the recursive process rules below. The overall weight matrix process is given by $\mathcal{W}(t) = (\mathcal{W}_{i,j}(t))_{i,j=1,\dots,N}$. Its initial state is defined by

$$\mathcal{W}_{i,j}(0) = \begin{cases} 1, & \text{if } j \in \mathbb{F}(i), \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

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This is a generally non-symmetric matrix with zero entries on the diagonal (because a person is never friends with itself).

2.2 Updating Scheme

The mobility choice of an agent evolves by copying the choice of a friend, where the probability for choosing a specific friend is the normalized product of the friend's utility times the connection weight. The copying only takes place if the friend's utility exceeds the agent's own utility.

More precisely, for $t \in \mathbb{N}_0$, let $(\mathcal{M}(t), \mathcal{W}(t), \mathcal{B}(t))$ be given, and assume that $\mathcal{X}(t)$ and $\mathcal{U}(t)$ have already been calculated using Eq. (2) and Eq. (4), respectively.

1. **Choice of a friend.** For each agent $i \in \mathbb{I}$ choose a friend $\mathcal{J}_i(t) \in \mathbb{F}(i)$ randomly according to the probabilities $q_{i,j(t)}$,

$$\text{Prob}(\mathcal{J}_i(t) = j) = q_{i,j}(t), \tag{10}$$

   where

$$q_{i,j}(t) := \frac{\mathcal{U}_j(t) \cdot \mathcal{W}_{i,j}(t)}{\sum_{k \in \mathbb{F}(i)} \mathcal{U}_k(t) \cdot \mathcal{W}_{i,k}(t)}. \tag{11}$$

2. **Update of mobility types.** For each agent $i \in \mathbb{I}$ update the mobility type according to

$$\mathcal{M}_i(t+1) = \begin{cases} \mathcal{M}_j(t), & \text{if } j = \mathcal{J}_i(t) \text{ and } \mathcal{U}_j(t) > \mathcal{U}_i(t), \\ \mathcal{M}_i(t), & \text{otherwise.} \end{cases} \tag{12}$$

3. **Update of counting state and bonus.** For each cell c and each mobility type m determine $\mathcal{X}_c^{(m)}(t+1)$ according to Eq. (2) and $\mathcal{B}_c^{(m)}(t+1)$ according to Eq. (3).

4. **Update of individual utilities.** For each agent $i \in \mathbb{I}$ determine $\mathcal{U}_i(t+1)$ according to Eq. (4).

5. **Update of weights.** For all $i, j \in \mathbb{I}$ set

$$\mathcal{W}_{i,j}(t+1) = \begin{cases} \mathcal{W}_{i,j}(t) \cdot \frac{\mathcal{U}_i(t+1)}{\mathcal{U}_i(t)}, & \text{if } j = \mathcal{J}_i(t) \text{ and } \mathcal{U}_j(t) > \mathcal{U}_i(t), \\ \mathcal{W}_{i,j}(t), & \text{otherwise.} \end{cases} \tag{13}$$

6. Set $t \mapsto t+1$ and restart with step 1.

The recursion (13) means that, when the mobility choice of a friend is copied, the respective connection to this friend is updated based on the relative utility gain or loss achieved by copying the friend's choice. Note that the weight $\mathcal{W}_{i,j}$ can change value also when agent i does not change the mobility type (namely if the chosen friend $\mathcal{J}_i(t)$ uses the same mobility type as person i and $\mathcal{U}_j(t) > \mathcal{U}_i(t)$).

The rationale behind this updating scheme is that if a person's utility is increased by copying somebody, it becomes more likely to copy this friend again and vice versa. This can be seen as a way of Baysian updating: during the simulation, person i collects information about how useful it was to copy certain friends in terms of utility optimisation. Weights $\mathcal{W}_{i,j}$ are used to quantify expectations about how beneficial it is to copy the choice of friend j. In the beginning, all friends are expected to be equally useful to copy from, see the initial weights Eq. (9). New information which is gathered during the simulation is used to update the person's expectations.

2.3 Markov Property

Let

$$\mathcal{Y}(t) := (\mathcal{M}(t), \mathcal{B}(t), \mathcal{W}(t)) \tag{14}$$

denote the combined state of the process at time t. The process $(\mathcal{Y}(t))_{t=0,1,\ldots}$ recursively defined by the rules above is a time-homogeneous Markov process with possible states

$$\boldsymbol{y} = (\boldsymbol{m}, \boldsymbol{W}, \boldsymbol{B}) \in \mathbb{M}^N \times [0,1]^{C \times M} \times [0,\infty)^{N \times N} =: \mathbb{Y}. \tag{15}$$

Its transition probabilities are determined by the probabilities $q_{i,j}(t)$ defined in Eq. (11) which in fact are functions $q_{i,j}(\boldsymbol{y})$ of the whole system state $\boldsymbol{y}$, as they depend on the weights $\mathcal{W}(t)$ and on the utilities $\mathcal{U}(t)$ which themselves are determined by $\mathcal{M}(t)$ and $\mathcal{B}(t)$, see Eq. (4).

More precisely, let $\mathbb{F} := \prod_{i=1}^N \mathbb{F}(i)$ denote the Cartesian product of the sets of friends. For each $J \in \mathbb{F}$ there is a function f_J that maps $\mathbb{Y}$ onto itself and summarizes the steps 2-5 from the updating scheme. For the transition probabilities of the Markov chain $(\mathcal{Y}(t))_{t=0,1,\ldots}$, we obtain

$$\text{Prob}(\mathcal{Y}(t+1) = \boldsymbol{y}'|\mathcal{Y}(t) = \boldsymbol{y}) = \sum_{\substack{J \in \mathbb{F} \\ f_J(\boldsymbol{y}) = \boldsymbol{y}'}} P(J|\boldsymbol{y}), \tag{16}$$

where

$$P(J|\boldsymbol{y}) := \prod_{i \in \mathbb{I}} q_{i,J_i}(\boldsymbol{y}) \tag{17}$$

is the probability for the choice of friends $\mathcal{J}(t) = J \in \mathbb{F}$ given $\mathcal{Y}(t) = \boldsymbol{y} \in \mathbb{Y}$.

As for the sum in Eq. (16), we note that for agents i whose weights change from t to $t+1$ on account of updating, i.e. $\mathcal{W}_{i,j}(t+1) \neq \mathcal{W}_{i,j}(t)$ for some $j \in \mathbb{F}_i$, the index $\mathcal{J}_i(t)$ of the chosen friend is uniquely determined because there is equality $\mathcal{W}_{i,j}(t+1) = \mathcal{W}_{i,j}(t)$ for all other $j \neq \mathcal{J}_i(t)$. However, for those agents i who do not update their mobility type and weights at time t (because the utility of the chosen friend does not exceed their own), this uniqueness is not given. Thus, there can generally be several choices of friends $\mathcal{J}(t) = J \in \mathbb{F}$ that lead to the same state $\boldsymbol{y}'$, and we need to sum their probabilities.

Skipping any of the components of the process $\mathcal{Y}(t) = (\mathcal{M}(t), \mathcal{B}(t), \mathcal{W}(t))$ would mean to loose the Markov property: Because of the recursive equation (3) the utility functions depend on the past of the process, just as the weights do, owing to (13). With these two, the transition probabilities depend on the past of the process as well. Hence the processes $\mathcal{M}(t)$ and $\mathcal{X}_c^{(m)}(t)$, which are the components of main interest, are not Markovian by themselves. They become Markovian when artificially setting $\mathcal{B}(t) = 0$ and $\mathcal{W}(t) = \mathcal{W}(0)$ for all times t, so by ignoring the bonus process and by skipping the update of weights. We note that the intrinsic stochasticity of the dynamics solely originates from the random choice of a friend $\mathcal{J}_i(t)$, see Eq. (10). The values of all other process components result from deterministic rules.

While this analysis has made the structure of the Markov process clear, the process itself is still too complex for its dynamic features to be investigated in an analytic manner within this paper. In the following section, we therefore examine the contribution of the components $\mathcal{B}(t)$ and $\mathcal{W}(t)$ to the dynamics using numerical simulations, as is customary for agent-based models.
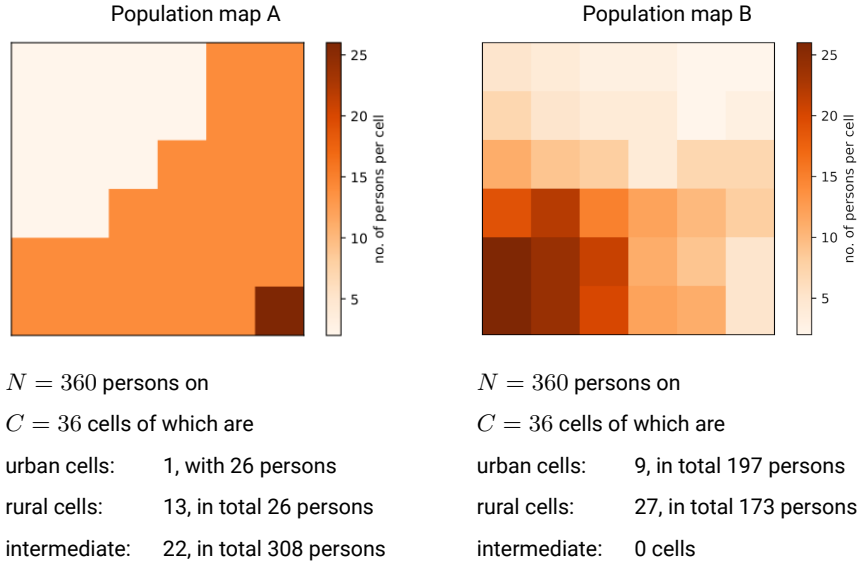
	Population map A	Population map B
	$N = 360$ persons on	$N = 360$ persons on
	$C = 36$ cells of which are	$C = 36$ cells of which are
urban cells:	1, with 26 persons	9, in total 197 persons
rural cells:	13, in total 26 persons	27, in total 173 persons
intermediate:	22, in total 308 persons	0 cells

Figure 2: **The different population distributions.** For both maps the maximum population density per cell is 26 and the minimum is 2. Cells with more than 14 inhabitants are called "urban", and cells with less than 14 "rural"; cells with 14 persons are neither urban nor rural but "intermediate" (see text for explanation).

3 Simulation results

In addition to the mobility choices $\mathcal{M}(t)$, the stochastic process $\mathcal{Y}(t)$ defined in Eq. (14) contains the two components $\mathcal{B}(t)$, $\mathcal{W}(t)$ that convey the feedbacks of the agents' decisions on the system and conserve the past dynamics of the system which thereby influence the present behaviour. In the following, we show simulation results focusing on the role of these two feedback processes:

1. *The influence of the bonus process*: As mentioned above, the bonus process $\mathcal{B}(t)$ can be interpreted as infrastructure development in the mobility context. It is steadily updated according to Eq. (3) and describes how a certain behaviour – in this case the usage of a certain mobility type – shapes the system in a way that enforces this behaviour. In the mobility case this could be an extension of the infrastructure for this type. We study the influence of this process by comparing the system's behaviour with and without this effect in Section 3.2.

2. *The influence of the weight matrix process*: Agents' connections to their friends are re-weighted according to Eq. (13) in every time step. This can be interpreted as social learning in the sense that the person learns whether it was useful or adverse to copy the behaviour of a certain friend and thus makes it more likely or unlikely to copy them again. The influence of this weight matrix process $\mathcal{W}(t)$ on the dynamics of the system is studied by comparing the system's behaviour with and without this effect, see Section 3.3.

Before that, we give some more details about how simulations are set up and initialised.

3.1 Setup and Initialisation

The simulations shown in this paper are performed with the R-MoTMo version that can be downloaded at the CoMSES model library (27). All simulations take place on a map of 6×6 pixels. Two different population distributions are used, which are part of the published R-MoTMo code and shown in Figure 2. In both of them, the maximum number of persons per cell is 26, and the minimum is 2. That means, that without any infrastructure feedback, i.e. $u_c^{(m)}(x, b) = \hat{u}_c^{(m)}$, see Section 2, for cells with less than 14 inhabitants car usage has a higher utility (rural cells) and for cells with more than 14 inhabitants public transport has a higher utility (urban cells). In cells with 14 inhabitants both utility functions have the same value (intermediate cells), see also Figure 1.

In the beginning of a simulation with R-MoTMo, persons are created and distributed as given by one of the input maps. They are assigned random mobility choices. For each agent, the agent-specific set of friends of size n is randomly drawn at initialization of the model, where the probabilities for being friends with persons living close by is higher than for persons further away, as described in the ODD document given in (27).[1] As mentioned, the set of friends stays fixed throughout a simulation but the intensity of a friendship can change according to (13). By construction, agent i is never friends with itself, so $i \notin \mathbb{F}(i)$.

All simulations shown here are done with $n = 15$ friends per person. As all behaviour of interest is clearly seen within the first 200 time steps, all simulations were run up to this point in time.

3.2 Coordination via Infrastructure Feedback

The infrastructure feedback is twofold: there is the bonus process $\mathcal{B}(t)$, given in Eq. (3). This process causes a mobility type that is chosen more often to provide a higher utility to its users in the long run. On the other hand, there is a immediate utility malus for mobility types the more they are used, as given in Eq. (6). Both effects are local, i.e. they depend only on the amounts of users in the respective cell.

Population map A of Figure 2 looks a bit artificial but was created to be a balanced test case in the sense that there are as many agents living in rural cells as in urban ones and the utilities for cars and public transport are maximal or minimal for urban and rural cells, respectively (see Eq. (5)). Most persons live in the intermediate area between rural and urban cells, where the exogenous part of the utility functions is equal (see Figure 1), which means that without bonus and malus processes (infrastructure feedback) using either of the two mobility types does not make a difference for the utility of persons living in these cells.

With infrastructure feedback, however, agents in the intermediate area tend to coordinate on one mobility type. Simulations are found to typically end in one of two configurations: either car usage is dominant in all of the intermediate area, or almost everybody there uses public transport. Typical simulation results for cellwise car usage in both cases are shown in Figure 3: By the time $t = 200$, the intermediate area (i.e. the cells with 14 inhabitants) has become either mainly dominated by car use (a) or by the use of public transport (b). The third graphic (c) shows a typical run without the infrastructure feedback, i.e. with $u_c^{(m)}(x, b) = \hat{u}_c^{(m)}$. Here, no coordination emerges.

The statistics of the dynamics can be found in Figure 4, where the simulation output for ensembles of 100 runs each are depicted for scenarios with and without infrastructure feedback. On the left, i.e. in Figure 4(a) and (c), respectively, the mean car usage per cell at time $t = 200$ is shown. On the right, i.e. in Figure 4(b) and

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[1] We note that there is a non-zero probability for the network to split up into disconnected components. This probability will decrease with increasing number n of friends per agent. For the value $n = 15$ chosen for the experiments of this Section, the probability of such a split-up is very small and may be ignored.
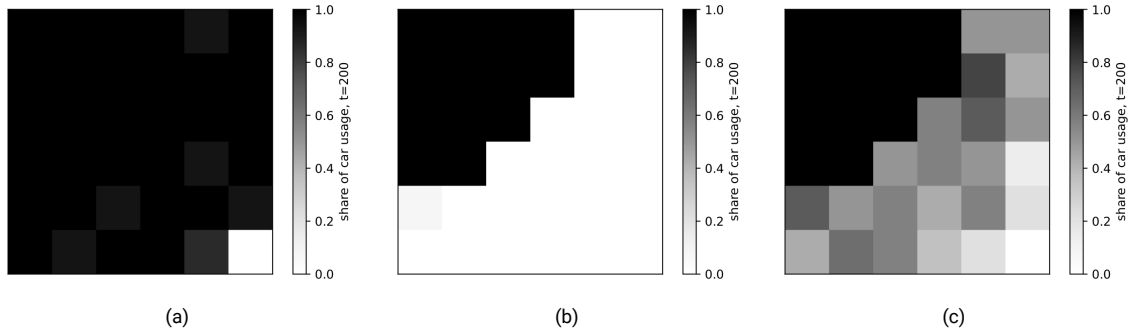
Figure 3: **Influence of the infrastructure feedback: Exemplary final states.** Car usage after 200 time steps for simulations with population map A (see Figure 2) and $n = 15$ friends per person. (a) and (b) show exemplary shares of car usages $\mathcal{X}_c^{(0)}$ for all cells c for two different runs with infrastructure feedback. (c) shows a typical case of car usage $\mathcal{X}_c^{(0)}$ after 200 time steps without infrastructure feedback.

(d), the temporal development of the overall (i.e. the average over all cells) car usages for the 100 different runs are depicted. (a) and (b) show simulations with infrastructure feedback (bonus process $\mathcal{B}$, and malus function as described above), whereas the simulations of (c) and (d) are performed without infrastructure feedback, i.e. the utility functions (5) are set constant as $u_c^{(m)} = \hat{u}_c^{(m)}$.

For the case with infrastructure feedback, in most cases (here more than 60% of the realizations) the average car usage in the end was between 0 and 20%, which is e.g. the case in the exemplary run shown in Figure 3(b), and almost all the others (around 35%) had a final car usage of between 80% and 100%, like the run shown in Figure 3(a). We find that, in fact, with infrastructure feedback there is a strong coordination on either of the two mobility types (roughly twice as much on public transport as on cars) while without infrastructure feedback the usage of both types is always around 50%, as can bee seen in Figure 4(d). The coordination also suggests that the (long-term) bonus process $\mathcal{B}(t)$, given in Eq. (3), dominates the (short-term) malus process, given in Eq. (6), because contrary to the bonus process, the malus process would rather incentivise not to coordinate. These simulation results indicate that an analysis of the asymptotic behaviour of the Markov chain defined in Section 2 would be fruitful. In particular, it seems that the dynamics converge in the long run to one of two fixed points: a map where all agents except those in the urban cell use cars, and one where all agents but those in the rural cells use public transport. Such a mathematical analysis, however, goes beyond the scope of this paper.

The observed behaviour can be classified as path dependency as defined by Vergne and Durand (29). It depends on the system's history in the following way: it is triggered by contingent events (in this case, the random social networks of the individuals living in intermediate areas and their random choices of friends to copy); it possesses a self-enforcing component through the bonus process; and it results in a lock-in state.

For population map A, which has many intermediate cells, the coordination via the bonus process dominates the simulation dynamic. For other population distributions with less intermediate cells this mechanism plays a minor role. This is due to the fact that, already from the beginning on, one mobility type has a higher utility than the other for all cells. In Figure 7 in the Appendix, the same simulations as in Figure 4 are shown for population map B, as an example for this second scenario.
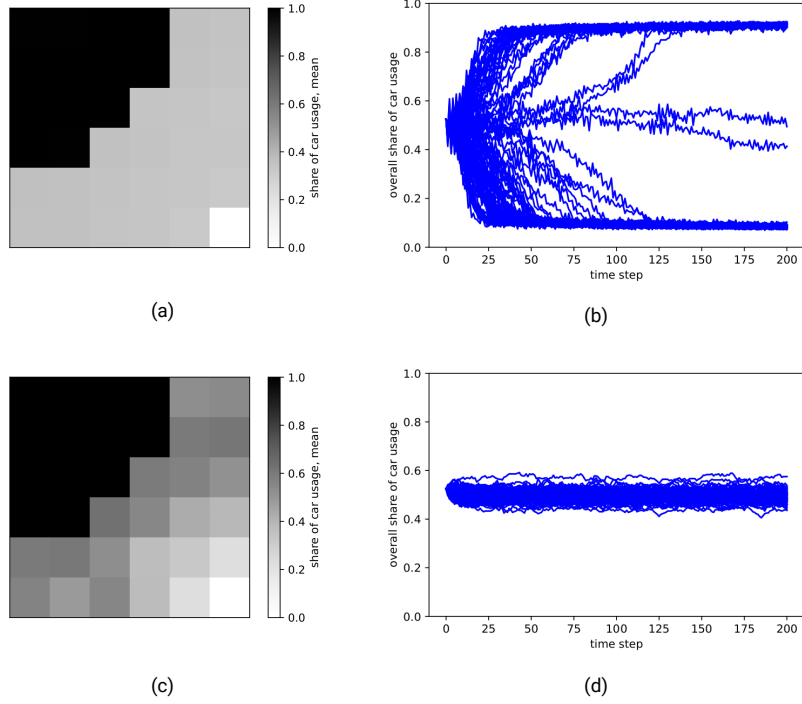
Figure 4: **Influence of the infrastructure feedback: Statistics.** Simulations with infrastructure feedback (bo-nus and malus processes): (a) Mean $\mathcal{X}_c^{(0)}$ (car usage per cell) after 200 time steps, (b) overall car us-age in the different simulation runs over time, see Section 3.2. Simulations without infrastructure feedback (i.e. $u_c^{(m)}(x, b) = \hat{u}_c^{(m)}$): (c) Mean $\mathcal{X}_c^{(0)}$ (car usage per cell) after 200 time steps, (d) overall car usage in the different simulation runs over time. The statistics result from 100 simulations per scenario, with 200 time steps and $n = 15$ friends per person, using population map A.

3.3 Learning by Weighting Links in the Social Network

In the model, the weight matrix process $\mathcal{W}(t)$ represents social learning of agents. After an agent copies a friend's mobility choice the usefulness of this choice for the person is determined and the link to the respec-tive friend is re-weighted accordingly, as given in Eq. (13). This means that agents learn over time about how useful copying certain friends is in terms of utility optimisation; past experiences are used to strengthen or weaken the link to the respective friend in a recursive manner.

Figure 5 shows the effect of the weighting process for population map B. Based on an ensemble of 100 runs with $n = 15$ friends per person, (a) and (c) show the mean share of car usage per cell, while in (b) and (d) the mean utilities per cell are plotted. For (a) and (b) the weighting of friends as described in Section 2 is used, whereas (c) and (d) are done without the weighting process, i.e. all non-zero entries of the matrix $\mathcal{W}(t)$ are kept constant at 1 (as they are initialised). Throughout large parts of the map, one can see that the scenario with social learning (by weighting of links in a recursive manner) indeed works better in terms of utility optimisation.

Population map A exhibits a minor role for the weight matrix process, primarily because of the large number of individuals residing in intermediate cells and the resulting significance of coordinating individuals' choices through infrastructure feedback, as discussed in Section 3.2. Simulations for population map A (with the same parameter values as before) can be found in Figure 8 in the Appendix.
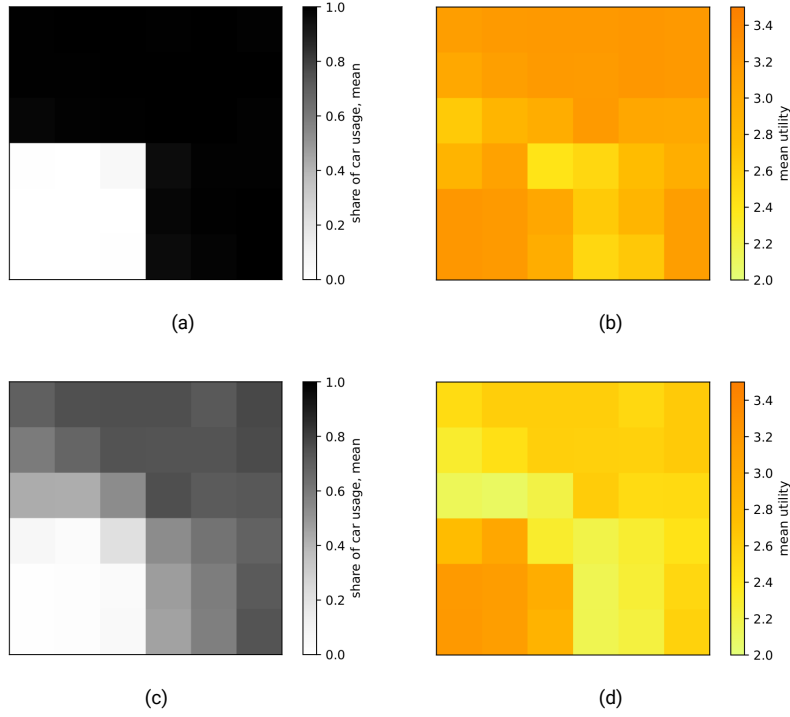
Figure 5: **Influence of the weight matrix process: Statistics.** Simulations for $n = 15$ friends per person using population map B, means over 100 runs, (a) mean $\mathcal{X}_c^{(0)}$ (car usage per cell) and (b) mean utilities per cell, i.e. average over all inhabitants per cell. The second row shows the same simulations without weighting process, i.e. all non-zero entries of $\mathcal{W}_{i,j}(t)$ are constantly set to 1: (c) mean $\mathcal{X}_c^{(0)}$ (car usage per cell) and (d) mean utilities per cell.

4   Discussion

We have formulated the agent-based model R-MoTMo mathematically as a Markov process

$$(\mathcal{M}(t), \mathcal{B}(t), \mathcal{W}(t))_{t=0,1,\ldots}$$

which has three components:

1. The decision process $\mathcal{M}(t)$ recording the mobility choices of the persons, which is a random process with transition probabilities determined by the other process components.

2. The bonus process $\mathcal{B}(t)$ that is recursively defined and thus conserves the decision history and causes a path dependency via infrastructure development.

3. The weight matrix process $\mathcal{W}(t)$ defining the time-dependent network of agents; agents recalculate the weights recursively following a Bayesian updating rule.

The state space of the mobility decision process $\mathcal{M}(t)$ complies with the one found in mathematical formulations of basic opinion dynamics models. However, to describe the model's dynamics by defining the

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probabilities underlying the mobility choices, two further components are necessary. These are $\mathcal{B}(t)$ and $\mathcal{W}(t)$ which are related to the agents' environment and the network between them, both co-evolving with the agents' decisions. By including the processes $\mathcal{B}(t)$ and $\mathcal{W}(t)$ which store the decision history, the path-dependency of the dynamics is turned into memoryless state dependency in the sense of the Markov property: The statistics of the process' future dynamics solely depends on the present system state $(\mathcal{M}(t), \mathcal{B}(t), \mathcal{W}(t))$, while being independent of the dynamics' history.

How general are the components we have identified for ABMs of social systems, and can we relate them to the elements others have proposed as fundamental for agent-based simulations of social systems? Recently, Dilaver and Gilbert (7) developed a *conceptual anatomy framework* for agent-based social simulation. The elements of this framework are 'agents', 'social structure', 'environment', 'actions and interactions', as well as 'temporality'. In our work, the 'agents' are the persons who take mobility decisions and evaluate their connections to other persons in a learning process. In the formulation of Section 2, agents are tracked by the index i in the processes $\mathcal{M}(t)$ and $\mathcal{W}(t)$. The matrix $\mathcal{W}(t)$ containing the connection strengths between any two agents in the system corresponds to the (evolving) 'social structure'. The cells with their evolving utility values for the two mobility types correspond to the (evolving) 'environment'. The element 'actions and interactions' can be captured as the mobility process: 'actions' in the form of decision making are here modelled as a rather simple copying of another agent's mobility choice. At the same time, they produce a two-fold 'interaction' both with the environment and with the social structure: the probabilities for choosing which agent to copy depend on both the environment (given by $\mathcal{B}(t)$) and the social structure (captured by $\mathcal{W}(t)$), and the decisions taken, in turn, influence the updated values of these processes. Considering 'temporality', Dilaver and Gilbert point out both time and history as relevant concepts. In our framework, time is modelled as a simple period-based process given by a discrete-time Markov chain (without an empirical interpretation of what a period represents). A building up of history occurs through the feedback mechanisms in the environment and the network processes, that through their co-evolution with the agents' actions create path-dependency in the mobility decisions. In this regard, our model stands out against other mathematically formulated ABMs (e.g., for opinion dynamics), which mostly do not take memory effects into account.

Although the components of our model can be related to the elements of social ABM identified by Dilaver and Gilbert, our way of classifying the three components – decision process, infrastructure, and network – differs fundamentally from their classification as ours is rather formal while they classify regarding content. If abstracted from the example of mobility decisions, we can classify two "memory" processes that are needed to make the system Markovian, depending on whether they operate through (local) fields or rather networks of agents. Other socio-technical systems may have the same structural features in terms of fields and networks, but what is interpreted as 'social structure' might not only be represented in a network process[2]. We think that comparing the different approaches of component identification can be fruitful. The formal and the contentual role of elements are not completely arbitrary but there are probably strong correlations, e.g. between network component and social structure. Investigating these correlations will be the topic of further research, and this may also be a contribution of agent-based modelling to the agency-structure debate in the social sciences.

Another rather formal conceptual framework that our model's structures can be related to has been proposed by Laubichler and Renn (16). They introduce the concept of *extended evolution* to study cultural dynamics with the help of concepts from biological evolution theory, in particular, regulatory networks and niche

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[2]Also in our case, the infrastructure development could be seen as influenced by a different part of 'social structure' since it depends upon political decisions.

construction. Due to its co-evolutionary dynamics, our model can be interpreted in terms of this framework. Figure 6 gives an overview of how an agent's mobility decision interacts with the environment and vice versa. The agent's choice relies on the options at hand, specifically, the mobility choices with their respective characteristics that are accessible at the agent's location. Which of the possible options is chosen depends on the agent's social network and on the agent's present utility as a consequence of the agent's last choice. In terms of Laubichler and Renn, the possible options available at that agent's place and time can be seen as the genomic sequence, and the agent's social networks, that determines which of them is chosen, as the *regulatory network*. All agents' choices made at one time step influence the environment in a way that resembles *niche construction* (e.g. 15; 20) in ecology. In other words, the individual decisions are externalised such that they shape the environment outside of the agent in a lasting way (due to the bonus process). These changes in the environment affect the elements that determine the agent's decision, which can be viewed as a process of internalisation. In our model this is done by the mechanisms of how the individual utilities depend on the behaviour of all actors in the local environment of an agent.
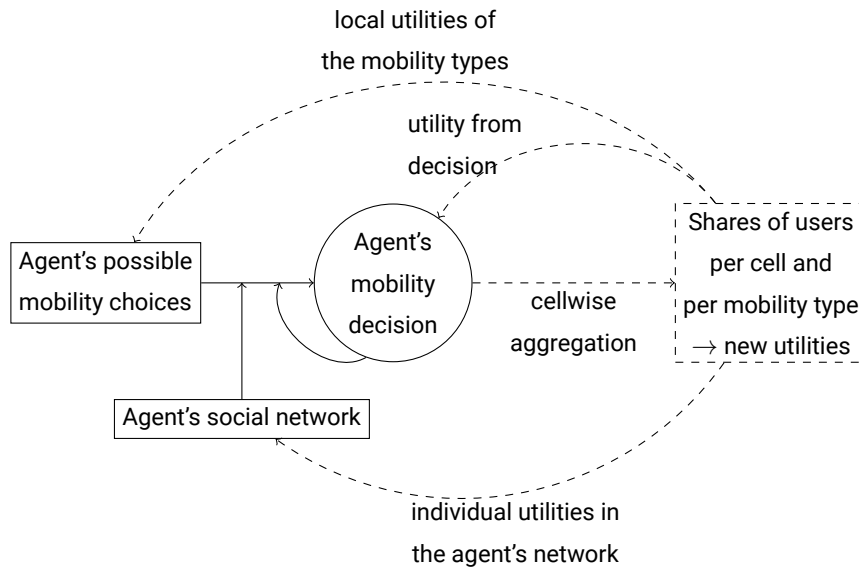

Figure 6: **Illustration of feedback mechanisms.** An agent's decision making process (solid lines) and its interactions with the agent's environment (dashed). See text for further description.

Studying socio-technical systems, especially in the context of societal challenges such as sustainability transitions, may benefit from analogies to the framework of extended evolution. This is particularly relevant because the emergence of innovations, whether they are technological or behavioral in nature, carries substantial importance. To complement a perspective of statistical physics, that could focus on such transitions as regime changes, an evolutionary perspective can address questions around adaptation to changing environmental conditions (which may be caused by the populations under study). In particular, the role of regulatory networks and whether or how they can trigger faster adaptation than the process associated with evolutionary mutation of genes may deserve further investigation for a specific socio-technical system.

5   Conclusion and Outlook

In summary, we have introduced the agent-based mobility demand model R-MoTMo and have shown how formulate it as a Markov chain. As the mobility decision process $\mathcal{M}(t)$ is not Markovian, it was necessary to extend the state space by including the processes $\mathcal{B}(t)$ and $\mathcal{W}(t)$, which account for the feedback mechanisms. These make the model more complex than, e.g., a classical voter model.

Since an analytic treatment of the Markov process is more difficult than for "history-free" decision models, we have provided simulation results to investigate the impact of the process components $\mathcal{B}(t)$ and $\mathcal{W}(t)$ on the dynamics of the system, showing that both are essential in creating the model's dynamics. We found that the "infrastructure component" $\mathcal{B}(t)$ can introduce a coordination effect that results in path dependency in the system. Triggered by contingent events, the self-enforcement through the bonus process can cause the system to reach a lock-in state. The weight matrix component $\mathcal{W}(t)$ accounts for agents' learning in the system which helps them to optimise their utilities over time.

We have then compared our model's structure, made explicit in the Markov chain formulation, to two previous theoretical frameworks. As for the *conceptual anatomy framework* by Dilaver and Gilbert we found that we have the elements they describe as substantial for ABMs of social systems and can relate them to our components of the Markov process, but our classification of components is rather formal than contentual and the relation of these three formal components to the contentual elements of Dilaver and Gilbert might differ in other models that still have the same mathematical structure. For the *framework of extended evolution* proposed by Laubichler and Renn we uncovered parallels with niche construction and regulatory networks that suggest potential benefits from this perspective for understanding the emergence of innovation in socio-technical systems with similar structures. In fact, the same may hold for socio-economic and socio-ecological systems, or combinations thereof, with feedback effects that can be conceptualized as operating through (global or local) fields or network structures.

With this work, we have cast an agent-based model with two kinds of memory mechanism into the form of a Markov chain – a form that is well used for similar models without feedback effects, where it has been employed to better understand and analyse these systems' dynamics, e.g. by calculating transition probabilities. The analysis of the dynamics of our model in the present paper was still at the level of simulation output because the mathematical structure is rather complex, but a mathematical analysis of the Markov chain defined here is part of ongoing and future work by the authors. It will include, e.g., an analysis of fixed points of the model in different settings, and an analysis of initial configurations of the friendship network between agents. Such work shall further enhance the understanding and explainability of this model, R-MoTMo, and more generally, a model structure, that of agents taking decisions with environment- and network-based memory mechanisms. Those mechanisms underlay various complex socio-ecologic, socio-economic, or socio-technical systems, where feedbacks between decisions and environment are crucial, such as questions of sustainability, and more. The resulting form, a Markov chain with deterministic updates that encode the decision history into an extension of the state space, may hopefully be useful for a further analysis of these. Furthermore, working out the similarities (and probably differences) of the theory of these complex socio-technical systems with the theory of extended evolution can help to better understand change and adaptability of the former by looking for analogies for adaption mechanisms.

Complementary to the mathematical analysis of the complex systems underlying a given societal challenge, dialogues with the respective stakeholders are a useful element to come up with possible future scenarios. In order to accompany and interweave scientific knowledge with practical and experiential knowledge

as well as preferences, interests, and values (21), new methods for this are being developed. One discussion format for such an transdisciplinary co-creation is the *Decision Theatre*[3], and one of the main challenges in setting up a Decision Theatre is building agent-based models from scratch for each new case under study (31). The generalisable mathematical structure for a socio-technical system with memory mechanisms defined here will be helpful for conceptualising such models in future work.

Acknowledgements

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[3]A Decision Theatre is a workshop in which scientists and stakeholders discuss a societal challenge supported by visualisations of data and model simulations on big screens. Participants can compose alternative model scenarios that are then explored, discussed and evaluated together, (see also 31).

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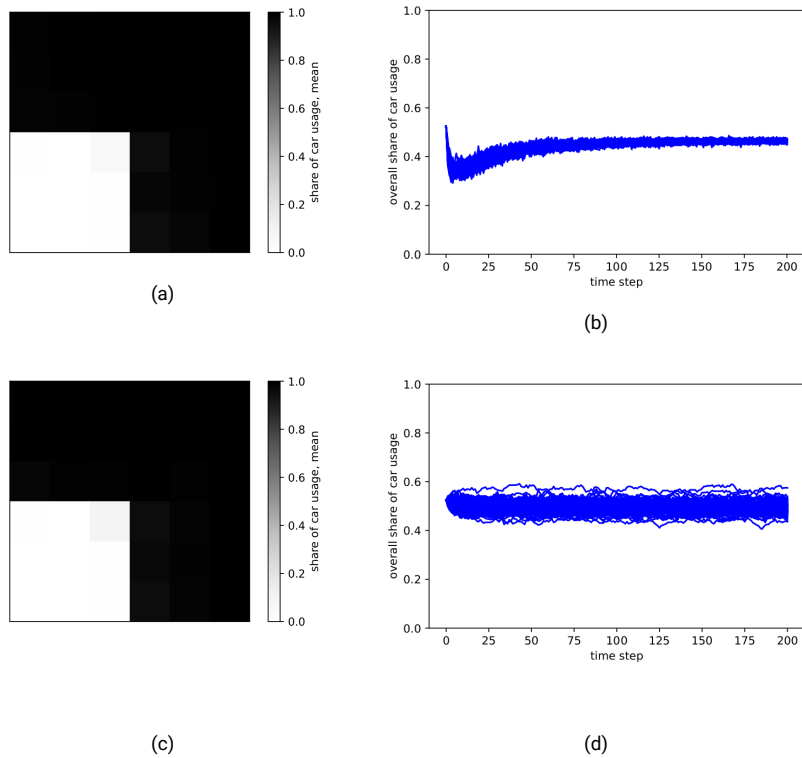
Appendix

(a)

(b)

(c)

(d)

Figure 7: **Influence of the infrastructure feedback: Statistics.** Simulations with infrastructure feedback (bonus and malus processes): (a) Mean $\mathcal{X}_c^{(0)}$ (car usage per cell) after 200 time steps, (b) overall car usage in the different simulation runs over time, see Section 3.2. Simulations without infrastructure feedback (i.e. $u_c^{(m)}(x, b) = \hat{u}_c^{(m)}$): (c) Mean $\mathcal{X}_c^{(0)}$ (car usage per cell) after 200 time steps, (d) overall car usage in the different simulation runs over time. The statistics result from 100 simulations per scenario, with 200 time steps and $n = 15$ friends per person, using population map B. Unlike as for map A (see Figure 4), coordination via infrastructure feedback does not play a role.
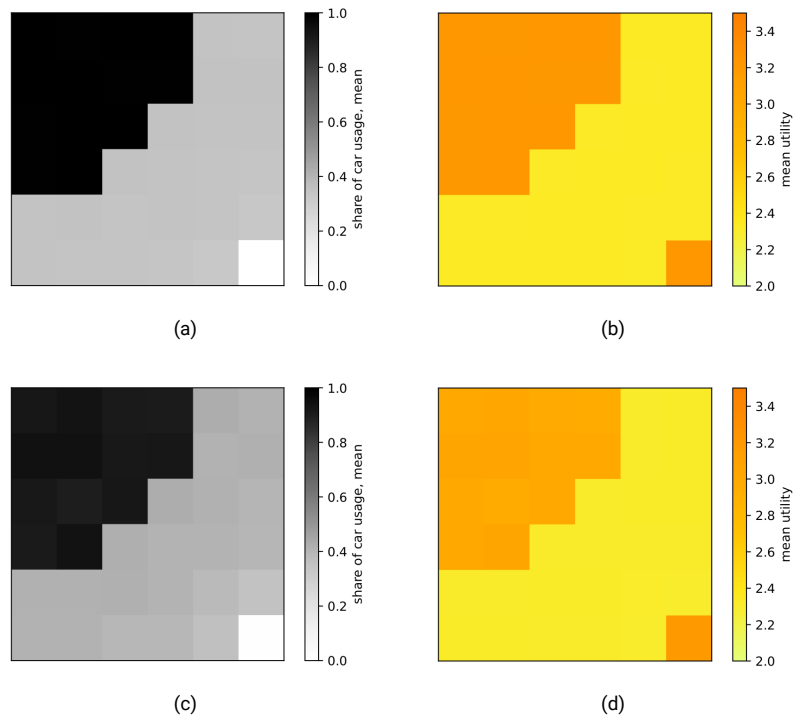
Figure 8: **Influence of the weight matrix process: Statistics.** Simulations for $n = 15$ friends per person using population map A, means over 100 runs, (a) mean $\mathcal{X}_c^{(0)}$ (car usage per cell) and (b) mean utilities per cell, i.e. average over all inhabitants per cell. The second row shows the same simulations without weighting process, i.e. all non-zero entries of $\mathcal{W}_{i,j}(t)$ are constantly set to 1: (c) mean $\mathcal{X}_c^{(0)}$ (car usage per cell) and (d) mean utilities per cell. For map A, the effect of the weight matrix process are small compared to the infrastructure feedback processes.