Infomorphic networks: Locally learning neural networks derived from partial information decomposition

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Abstract

Understanding the intricate cooperation among individual neurons in performing complex tasks remains a challenge to this date. In this paper, we propose a novel type of model neuron that emulates the functional characteristics of biological neurons by optimizing an abstract local information processing goal. We have previously formulated such a goal function based on principles from partial information decomposition (PID). Here, we present a corresponding parametric local learning rule which serves as the foundation of "infomorphic networks" as a novel concrete model of neural networks. We demonstrate the versatility of these networks to perform tasks from supervised, unsupervised and memory learning. By leveraging the explanatory power and interpretable nature of the PID framework, these infomorphic networks represent a valuable tool to advance our understanding of cortical function.

Keywords: information theory, partial information decomposition, neural networks, local learning

1 Introduction

The human neocortex is an impressive information processing system involved in performing a wide variety of tasks from visual, auditory, tactile and gustatory perception via various forms of memory to complex planning and motor actions (Lodato and Arlotta, 2015). Despite this diverse range of responsibilities, the neocortex is widely believed to consist of structurally similar, yet functionally flexible circuits (Creutzfeldt, 1977; Rockel et al., 1980). On the smallest scale, these circuits consist of individual neurons whose firing is dependent on only local

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factors, such as the firing of other, connected neurons and the local biochemical environment – without taking into account any global or semantic knowledge about the task (Douglas and Martin, 2004, 2007).

However, the how intricate cooperation among individual neurons helps performing complex tasks is still not well understood. To enhance our understanding of how neurons cooperate it is crucial to develop a model system in which the relevant dynamics of collaboration, specialization and self-organization can be readily observed and promoted. To grasp the fundamental and universal factors underlying these dynamics, these model systems should eliminate unnecessary idiosyncrasies associated with the specific task or the biological neurons. Such semantics-free information processing can be quantified using the framework of information theory (Wibral et al., 2017). From an information-theoretic perspective, neurons can be interpreted as information channels that convert the incoming signals into spiking activity (Wibral et al., 2015). Previous research by Kay (1994) has demonstrated the feasibility of this information-theoretic approach by creating model neurons which directly optimize for certain information-theoretic objectives (Kay and Phillips, 2011).

Despite these successes, the framework of classical information theory is limited in its ability to adequately account for all relevant aspects of neural information processing: Information theory focuses primarily on quantities such as mutual information, which capture only information channels with a single source – albeit thuis source possibly being multivariate and high-dimensional. Biological neurons, on the other hand, often receive signals from several *classes* of inputs, such as bottom-up, top-down or lateral connections, each playing a distinct role in the information processing (Rolls and Treves, 1997; Shu et al., 2003; Manita et al., 2015). A comprehensive description of the complex interactions of multiple such sources with respect to a single target variable has only recently been made possible by an extension to classical information theory known as Partial Information Decomposition (PID) (Williams and Beer, 2010; Lizier et al., 2018; Gutknecht et al., 2021). PID allows the information between multiple sources and a target variable to be dissected into unique, redundant and synergistic contributions called *atoms*, through which it paints a richer picture of the information processing underlying the transformation from sources to target (Williams and Beer, 2010). Recently, PID has been used to describe the function of cortical neurons (Schulz et al., 2021) and the representation of information in artificial and biological neural networks (Luppi et al., 2022; Ehrlich et al., 2023; Varley et al., 2023) and has been proposed as a unifying framework to describe cortical function (Wibral et al., 2017).

A litmus test for assessing the adequacy of the PID framework in capturing all relevant information processing would be the construction of model neurons which learn by directly optimizing certain PID objectives. While this idea has been present for some time (Wibral et al., 2017), a viable demonstration of feasibility has been lacking so far, primarily due to the absence of a *differentiable* PID measure that lends itself to gradient descent learning. However, drawing upon our recently developed differentiable PID measure I^{sx} (Makkeh et al., 2021; Schick-Poland et al., 2021), we here demonstrate for the first time that it is indeed possible to create artificial neurons that learn based on directly interpretable information-processing goal functions derived from PID. Networks build from these novel artificial neurons can learn in a supervised or unsupervised manner, and perform classification, representation learning and memory tasks. As the topology or connection structure is shaped by the input information and the information theoretic goal of learning itself we term these neurons and networks *infomorphic* – a portmanteau of "information" and "morphous" to indicate that they are directly shaped by the information they process. By studying these infomorphic neurons and

their information-processing capabilities, valuable insights can be gained into the collaborative dynamics and self-organization principles that govern the function of complex neural networks, such as the neocortex.

The main contributions of this paper are (1) the derivation and implementation of the PID-based learning rule set out by Wibral et al. (2017) by deriving analytical gradients of the relevant PID atoms and (2) demonstrations of the usefulness and flexibility of the infomorphic neurons in different learning paradigms.

The remaining sections of the paper are structured in the following way: First, we explain how neurons can be interpreted as information channels (Section 2.1) and how this view can be extended to account for the information processing occurring between two distinct input classes to a channel (or, alternatively, two channels with a common output) using PID (Section 2.2). Based on these insights, we introduce the model of the infomorphic neuron (Section 3) and demonstrate its usefulness on a collection of learning scenarios involving supervised (Section 4.1), unsupervised (Section 4.2) and memory learning (Section 4.3). We conclude with a discussion of strengths, limitations and next steps (Section 5).

2 Using information theory to describe the information processing of a neuron

In order to devise local learning rules that optimize the information processing of the individual neurons, it is necessary to first comprehend how to effectively describe the information processing of a neuron using the unifying framework of information theory. To this end, this section introduces an abstract model of the information processing of neurons.

2.1 The neuron as an information channel

As a first step towards understanding how neural networks as a whole process information, it is important to quantify the amount of information that can be extracted from a single neuron. The most relevant type of information for computation is the *active* information of a neuron, which is the short-lived information expressed in the neuron's firing activity. At any given moment, the neuron's activity Y can be in different states y – in the simplest model either "firing" or "not firing" – with corresponding probabilities p(y). These probabilities define the information content of each observation based on the intuition that observing a more uncommon event is more surprising and thus conveys a larger amount of information. This notion is captured mathematically by the Shannon information content defined as h(y) = $-\log_2 p(y)$ (Shannon, 1948; Cover and Thomas, 2006). The expectation value of this quantity is the Shannon entropy $H(Y) = \mathbb{E}_y h(y)$, which thus quantifies the total amount of information that can be extracted from a neuron on average and which could therefore in principle be used by other neurons down the line.

However, not all of a neuron's entropy constitutes information which is actually important for its computation, as some parts merely reflect random spontaneous activity due to variations in the local biochemical environment (Alving, 1968; Llinás, 1988; Getting, 1989). To understand the processing of information in a neuron, it thus becomes crucial to quantify how much of the variability of a neuron is due to stochasticity and how much is induced in the neuron by the activity of presynaptic neurons. Mathematically, these driving neurons can be incorporated into the model by making the probabilities of different activity states



Figure 1: Output information of an infomorphic neuron consists of 5 distinct information contributions. A The total output information H(Y) of a neuron that consists of the mutual information with the inputs $I(Y : \mathbf{X}_R, \mathbf{X}_C)$ and the stochasticity $H(Y \mid \mathbf{X}_R, \mathbf{X}_C)$ that does not originate from the inputs, but is intrinsic to the neuron. In its turn, $I(Y : \mathbf{X}_R, \mathbf{X}_C)$, the information about the output that originates from the inputs, consists of four information contributions, namely, (i) $I_{\rm red}$ the redundant information that is part of \mathbf{X}_R and of \mathbf{X}_C , (ii) $I_{\text{ung},R}$ the unique information of \mathbf{X}_R that is only part of \mathbf{X}_R but not of \mathbf{X}_C , (iii) $I_{unq,C}$ the unique information of \mathbf{X}_C that is only part of \mathbf{X}_C but not of \mathbf{X}_R , and (iv) \mathbf{I}_{syn} the synergistic information that is part of \mathbf{X}_R and \mathbf{X}_C only when taken jointly but not of \mathbf{X}_R or \mathbf{X}_C taken individually. These four contribution of $I(Y : \mathbf{X}_R, \mathbf{X}_C)$ and the term $H(Y \mid \mathbf{X}_R, \mathbf{X}_C)$ constitute the basis of the local goal function G which each infomorphic neuron possess. This goal function G is a refinement of the local goal function F introduced by Kay (1994). **B** The different information contributions of F that are coarsegrained contributions of those in G: The co-information $I(Y : \mathbf{X}_R : \mathbf{X}_C)$ is the difference of redundant information R and synergistic information S, the conditional mutual information $I(Y: \mathbf{X}_R \mid \mathbf{X}_C)$ is the unique information U_R and the synergistic information S, the conditional mutual information $I(Y : \mathbf{X}_C | \mathbf{X}_R)$ is the unique information U_C and the synergistic information S, and the term $H(Y \mid \mathbf{X}_R, \mathbf{X}_C)$.

y depend on the current activity \mathbf{x} of the presynaptic neurons \mathbf{X} , which can be expressed as a conditional probability distribution $p(y \mid \mathbf{x})$. The amount of information conveyed by a single invocation of this channel is captured by the local Shannon Mutual Information $i(y:\mathbf{x}) = \log_2 p(y|\mathbf{x})/p(y)$, which is equal to the difference between the information content h(y) of y being observed without prior knowledge and the conditional information content $h(y \mid \mathbf{x}) = -\log_2 p(y \mid \mathbf{x})$ of observing y when \mathbf{x} is known. The expectation value of this quantity, the mutual information $I(Y:\mathbf{X}) = \mathbb{E}_{y,\mathbf{x}}i(y:\mathbf{x})$ then quantifies how much of the entropy in Y is determined by \mathbf{X} (Shannon, 1948).

With these definitions, the entropy of a neuron can be dissected into two parts: The mutual information $I(Y : \mathbf{X})$, which captures the part of the amount of information conveyed from \mathbf{X} to Y and the conditional entropy $H(Y \mid \mathbf{X})$ which captures the remaining stochasticity in Y's responses, such that

$$H(Y) = I(Y : \mathbf{X}) + H(Y \mid \mathbf{X}).$$
(1)

2.2 Beyond simple channels: Differentiating different input classes

The picture of neurons as simple information channels has to be refined in light of the insight that biological neurons often have multiple *classes* of inputs with distinct information processing characteristics (Larkum and Nevian, 2008). An example of a biological neuron with two distinct input classes can be found in layer-5 pyramidal neurons (Cajal et al., 1893). Layer-5 pyramidal neurons are ubiquitous in cortex, involved in sensory, cognitive, and motor tasks, and have been hypothesized to play a role in conscious awareness (Lodato and Arlotta, 2015: Aru et al., 2020). They are typically embedded in a relatively stereotyped cortical microcircuit, at the junction of feed-forward and feed-back information streams in the cortical hierarchy (Bastos et al., 2012). To achieve this, pyramidal neurons possess two distinct types of dendrites, the basal and apical dendrites (Cajal et al., 1893). Basal dendrites receive input from hierarchically lower cortical areas and play a role in encoding the external features of the environment that are processed along the cortical hierarchy. Apical dendrites, in contrast, receive contextual input from higher cortical areas and have been shown to play an important role in modulating perception (Takahashi et al., 2016, 2020). This connectivity is similar across a range of different brain areas and cognitive domains, giving rise to the assumption that their function is independent of the semantics of their input (Rockel et al., 1980).

To account for the two classes of inputs in the information-theoretic analysis, the source variable \mathbf{X} needs to be reinterpreted as being a composite variable $\mathbf{X} = (\mathbf{X}_R, \mathbf{X}_C)$ of the *receptive* input \mathbf{X}_R , which is inspired by the basal dendrites, and the *contextual* input \mathbf{X}_C , which is inspired by the apical dendrites. Analogous to Eq. (1), the total entropy of the neuron Y can now be dissected into parts like

$$H(Y) = I(Y : \mathbf{X}_R, \mathbf{X}_C) + H(Y \mid \mathbf{X}_R, \mathbf{X}_C).$$
⁽²⁾

However, in addition to this joint channel from both input classes to the output, the dissection of \mathbf{X} also allows to consider the individual channels of the receptive or contextual inputs to the target, which are characterized by the mutual information terms $I(Y : \mathbf{X}_R)$ or $I(Y : \mathbf{X}_C)$, respectively. Note, however, that these two channels do not simply add up to the total mutual information $I(Y : \mathbf{X}_R, \mathbf{X}_C)$, because in the sum $I(Y : \mathbf{X}_R) + I(Y : \mathbf{X}_C)$ information which is redundantly present in both input classes will be double-counted, while synergistic information which only becomes apparent if one considers \mathbf{X}_C and \mathbf{X}_R

simultaneously will be overlooked (McGill, 1954; Cover and Thomas, 2006). By introducing the co-information

$$I(Y: \mathbf{X}_R : \mathbf{X}_C) = I(Y: \mathbf{X}_R, \mathbf{X}_C) - I(Y: \mathbf{X}_R \mid \mathbf{X}_C) - I(Y: \mathbf{X}_C \mid \mathbf{X}_R),$$
(3)

the decomposition in Eq. (2) can be refined to

$$H(Y) = I(Y : \mathbf{X}_R : \mathbf{X}_C) + I(Y : \mathbf{X}_R \mid \mathbf{X}_C) + I(Y : \mathbf{X}_C \mid \mathbf{X}_R) + H(Y \mid \mathbf{X}_R, \mathbf{X}_C), \quad (4)$$

where the conditional mutual information $I(Y : \mathbf{X}_R | \mathbf{X}_C)$ captures the remaining dependence of Y on \mathbf{X}_R when \mathbf{X}_C is the expected mutual information between Y and \mathbf{X}_R given that the value of \mathbf{X}_C is known and $I(Y : \mathbf{X}_C | \mathbf{X}_R)$ is defined analogously (Cover and Thomas, 2006).

This decomposition is the starting point that Kay (1994) used to construct their models of learning neurons with information theoretic objective functions (Section 3.1). In our work, we build on this concept by exploiting the superior expressiveness provided by the framework of partial information decomposition to build infomorphic neurons.

2.3 Uncovering the information processing between different input classes using Partial Information Decomposition

The perspective of viewing a neuron as a collection of information channels still paints an incomplete picture of the information processing within a neuron because it cannot account for all the different ways in which the different information sources combine and determine the output information: While some of the information in the neuron's output activity Y might be provided *uniquely* by either the receptive input \mathbf{X}_R or the contextual input \mathbf{X}_R , other parts might be *redundantly* supplied by both of them while yet others only become available synergistically when both sources are considered jointly (Wibral et al., 2017). Classical information: While one can compute the total amount of information coming from each source or from both sources together using mutual information, there is no way of quantifying how much of it is the same, i.e., redundant between the variables (Williams and Beer, 2010).

Dissecting the mutual information between multiple source variables and a single target variable into non-overlapping additive information *atoms* is the subject of partial information decomposition (Williams and Beer, 2010; Gutknecht et al., 2021). Using PID, we can subdivide the entropy H(Y) into five parts (Fig. 1.A)

$$H(Y) = I_{unq}(Y : \mathbf{X}_R) + I_{unq}(Y : \mathbf{X}_C) + I_{red}(Y : \mathbf{X}_R, \mathbf{X}_C) + I_{syn}(Y : \mathbf{X}_R, \mathbf{X}_C) + H(Y | \mathbf{X}_R, \mathbf{X}_C),$$
(5)

where $I_{\text{unq}}(Y : \mathbf{X}_R)$ and $I_{\text{unq}}(Y : \mathbf{X}_C)$ are the unique information atoms of the receptive and contextual inputs, respectively, $I_{\text{red}}(Y : \mathbf{X}_R, \mathbf{X}_C)$ refers to the redundant (shared) information, and $I_{\text{syn}}(Y : \mathbf{X}_R, \mathbf{X}_C)$ refers to the synergistic (complementary) information. These four atoms can describe the information processing in Y of \mathbf{X}_R and \mathbf{X}_C in versatile ways. These atoms have meaningful interpretations: For example, if a neuron encodes the coherent parts of its inputs, this would be reflected in a high redundant information. Alternatively, a neuron might encode the information in its receptive input \mathbf{X}_R that is specifically *not* present in the contextual input \mathbf{X}_C , which would translate to a high unique information contribution from \mathbf{X}_R . Finally, if the neuron's output contains information which cannot be obtained from any single source alone, for instance if the output Y reflected the logical exclusive "or" of its inputs, the synergy between the sources would be high. Overall, PID provides a decomposition framework with well-defined and intuitive interpretations for understanding a neuron's information processing.

Note that while the co-information $I(Y : \mathbf{X}_R : \mathbf{X}_C)$ (Eq. (3)) can be computed as the difference between redundant and synergistic information

$$I(Y: \mathbf{X}_R : \mathbf{X}_C) = I_{\text{red}}(Y: \mathbf{X}_R, \mathbf{X}_C) - I_{\text{syn}}(Y: \mathbf{X}_R, \mathbf{X}_C),$$
(6)

classical information theory provides no tool to disentangle the two components.

From these information atoms, the two conditional mutual information terms can be constructed from the synergy atom and the corresponding unique atoms as

$$I(Y : \mathbf{X}_R \mid \mathbf{X}_C) = I_{\text{unq}}(Y : \mathbf{X}_R) + I_{\text{syn}}(Y : \mathbf{X}_R, \mathbf{X}_C)$$

$$I(Y : \mathbf{X}_C \mid \mathbf{X}_R) = I_{\text{unq}}(Y : \mathbf{X}_C) + I_{\text{syn}}(Y : \mathbf{X}_R, \mathbf{X}_C).$$
(7)

To analyze the information processing of a neuron, the aforementioned PID atoms need to be quantified. Note that despite their strong relation to classical information-theoretic quantities through Eq. (5) and Eq. (7), the size of the PID atoms cannot be determined from classical information-theoretic quantities alone as there are four atoms with only three equations providing constraints (Williams and Beer, 2010). This underdeterminedness is the reason that an additional quantity has to be defined for PID, which is typically the redundant information (Williams and Beer, 2010; Lizier et al., 2018, and references therein) – but see (Gutknecht et al., 2021). By now, a multitude of different measures for redundant information exist in literature, each fulfilling a number of partly mutually exclusive desiderata and drawing on concepts from different fields such as decision or game theory, e.g. (Harder et al., 2013; Bertschinger et al., 2014; Ince, 2017; Finn and Lizier, 2018; Lizier et al., 2018, and references therein). In this work, we use the PID measure i_{Ω}^{sx} defined by Makkeh et al. (2021) due to the fact that it is based only on ideas of probability theory and matches the operational interpretation of inferring a target variable from multiple inputs. Furthermore, the i_{Ω}^{sx} measure is differentiable with respect to the underlying probability distribution $\mathbb{P}(Y, \mathbf{X}_R, \mathbf{X}_C)$ allowing for learning of the PID quantities and deriving their learning rules using gradient descent techniques (Makkeh et al., 2021).

3 Infomorphic Neurons

As outlined before, the goal for building infomorphic neurons is to create a succinct, yet functional model of biological neurons in order to study their cooperative information-processing abilities. In this paper, we aim to build infomorphic neurons that embody core operating principles of layer-5 pyramidal neurons and to demonstrate their applicability in supervised, unsupervised and memory learning.

The idea of using information theory to model layer-5 pyramidal neurons dates back to Kay (1994) and has been extended in subsequent research (Kay and Phillips, 1997; Kay, 1999; Kay and Phillips, 2011). In the following, we first outline the most important information

processing properties of the chosen biological neuron and then explain the PID-based goal function first envisioned by Wibral et al. (2017), before outlining how analytic gradients can be derived for this goal function to allow for a gradient-ascent optimization procedure.

3.1 Information-theoretic neurons inspired by pyramidal neurons

As a first step towards building infomorphic neurons inspired by cortical layer-5 pyramidal neurons, we introduce two of their most central computational paradigms as laid out by Kay (1994), namely their *multi-compartment computation* to produce the output during inference and *local learning* for training.

Multi-compartment computation. In analogy to the basal and apical dendrites of layer-5 pyramidal neurons, an infomorphic neuron distinguishes between two classes of input synapses, namely "receptive" inputs \mathbf{X}_R and "contextual" inputs \mathbf{X}_C . Inspired by how the inputs of different input classes are individually aggregated in separate compartments in the biological neurons, the inputs of the two classes of the infomorphic neuron are separately combined in a weighted sum to produce the aggregate inputs $R = \mathbf{w}_R \cdot \mathbf{X}_R - w_{0,R}$ and $C = \mathbf{w}_C \cdot \mathbf{X}_C - w_{0,C}$ (Kay, 1994). Here, \mathbf{w}_R and \mathbf{w}_C reflect the weights associated with the receptive and contextual inputs, respectively, while $w_{0,R}$ and $w_{0,C}$ denote constant bias values.

These aggregate inputs are subsequently passed to the activation function A. While the activation function can in principle be chosen arbitrarily, a biology-inspired choice of Amay draw inspiration from layer-5 pyramidal neurons: By making the activation function be primarily dependent on the receptive inputs, one can imitate the privileged role that basal dendrites play in driving pyramidal neurons (Kay and Phillips, 1997).

In turn, the output of the activation function A is mapped to a number between zero and one to produce the probability of firing θ . Finally, depending on this probability, the neuron's output is stochastically set to either "HIGH" (Y = 1) or "LOW" (Y = -1).

Local learning. Biological neurons learn *locally*, in the sense that the learning signal must be constructed from information that is available at the location of the neuron. This locality entails that the infomorphic neuron should likewise only optimize a similarly local goal function. This goal function is chosen to model the information processing of the receptive \mathbf{X}_R and contextual \mathbf{X}_C input classes by the neuron. While Kay (1994) first envisioned this goal function to be formulated on the level of two channels using mutual information (see Section 2.2), the local goal function in an infomorphic neuron is formulated on the level of the interaction between the two channels using PID as established by Wibral et al. (2017).

The process of maximizing this local goal function is an abstraction of biological learning. While learning in biological neurons involves synaptic plasticity, learning in infomorphic neurons amounts to changing the weights \mathbf{w}_R and \mathbf{w}_C (including the bias terms).

3.2 Modelling the activity of infomorphic neurons

Infomorphic neurons operate in discrete time and output values $Y \in \{-1, +1\}$ (referred to as "LOW" and "HIGH"), following the McCulloch-Pitts model and in analogy to timebinned spike trains of biological neurons (McCulloch and Pitts, 1943). At any time step, the probability θ of a neuron to be in the HIGH state, given that it receives a receptive input \mathbf{x}_R



Figure 2: Illustration of the basic structure of an infomorphic neuron, inspired by the morphology of cortical pyramidal neurons. A Overlay of coherent infomax neural processor developed originally by Kay and Phillips (2011) on layer 5 pyramidal cells, highlighting potential parallels to existing physiological mechanisms. Layer 5 cell created with the TREES toolbox (Cuntz et al., 2010), courtesy of Hermann Cuntz. Figure (A) from Wibral et al. (2017). B The infomorphic neuron has two functionally distinct groups of inputs that get multiplied in a scalar product with their respective set of weights. Based on the resulting R and C values, the probabilities for the LOW and HIGH output values are calculated. The neuron learns to adapt its output Y to optimize a prescribed informationtheoretic goal function G.

and a contextual input \mathbf{x}_C , depends on these inputs only through their respective aggregate values $r = \mathbf{w}_R \cdot \mathbf{x}_R - w_{0,R}$ and $c = \mathbf{w}_C \cdot \mathbf{x}_C - w_{0,C}$. These aggregate values are passed into the activation function A and finally the sigmoid function $\sigma(\xi) = 1/(1 + e^{-\xi})$, so that the expression for computing the firing probability becomes

$$\theta(r,c) := \mathbb{P}(Y=1 \mid R=r, C=c) := \sigma(A(r,c)).$$

The activation function $A : R \times C \to \mathbb{R}$ is chosen to match the requirements posed by the different tasks. Receptive inputs \mathbf{X}_R will typically stem from the previous layer in the hierarchy, whereas the contextual inputs \mathbf{X}_C can be, for instance, intra-layer recurrent connections or feedback connections from higher hierarchical layers. Generally, the receptive inputs are to be considered as the primary driving inputs, whereas the contextual inputs serve a primarily modulatory role. However, the degree to which the contextual input influences the output can differ for different learning paradigms. For this reason, the activation function Aneeds to be tailored to the learning paradigm and will be individually motivated and discussed in the corresponding parts of Section 4.

3.3 Locally learning optimal information processing

To imitate the local learning of pyramidal neurons, an infomorphic neuron needs to be able to learn how to locally process the information in its inputs \mathbf{X}_R and \mathbf{X}_C by tuning the two sets of weights \mathbf{w}_R and \mathbf{w}_C . This information processing can take on very different shapes: For some tasks, optimal information processing could mean encoding the coherent part of the receptive and contextual inputs, while for other tasks, optimal processing might entail extracting any piece of information (e.g. a feature) exclusively provided by the receptive inputs that is not present in the contextual input.

We argue that a wide variety of such desired information processing objectives can be formalized as a goal function involving a linear combination of the four PID atoms that make up the mutual information $I(Y : \tilde{R}, \tilde{C})$ between the output and binned versions of the aggregated inputs (Wibral et al., 2017). The use of the binned variables \tilde{R} and \tilde{C} as proxys for the continuous-valued R and C inputs is a technical necessity caused by the lack of a usable differentiable PID measure for continuous variables and other conceptual difficulties of information theory in continuous networks (Saxe et al., 2019; Goldfeld et al., 2019). Therefore, to define the information processing of the infomorphic neuron, we equip it with a local goal function that characterizes such information processing with the four PID atoms – namely the unique information by \tilde{R} and \tilde{C} , the redundant information and the synergy – and the intrinsic stochasticity of the output as follows:

$$G(Y:\tilde{R},\tilde{C}) = \Gamma_0 I_{\text{unq}}(Y:\tilde{R}) + \Gamma_1 I_{\text{unq}}(Y:\tilde{C}) + \Gamma_2 I_{\text{red}}(Y:\tilde{R},\tilde{C}) + \Gamma_3 I_{\text{syn}}(Y:\tilde{R},\tilde{C}) + \Gamma_4 H(Y \mid \tilde{R},\tilde{C}),$$
(8)

where the choice of Γ_i determines the neuron's local goal G, and thereby the information processing function it implements. Throughout learning, the neuron adapts its weights \mathbf{w}_R and \mathbf{w}_C in order to maximize G.

Optimizing the goal function. Now that a general PID-based goal function has been established, the next step is to determine a *learning rule* for each neuron to maximize this goal function. Mathematically, the objective of learning is to infer the optimal weight vectors \mathbf{w}_R and \mathbf{w}_C from the local information available at the neuron when a set of data samples is presented to the network. A general toolbox for optimizing a parameterized function using data samples can be found in gradient-ascent techniques.

By virtue of the differentiability of the I_{\cap}^{sx} measure, the gradients of G with respect to the weight vectors \mathbf{w}_R and \mathbf{w}_C can be derived analytically as

$$\frac{\partial G}{\partial \mathbf{w}_R} = \frac{1}{N} \sum_{\mathbf{x}_R, \mathbf{x}_C} f_{p_{R,C}}(\tilde{r}, \tilde{c}) \frac{\partial A}{\partial r} \mathbf{x}_R$$
(9)

and

$$\frac{\partial G}{\partial \mathbf{w}_C} = \frac{1}{N} \sum_{\mathbf{x}_R, \mathbf{x}_C} f_{p_{R,C}}(\tilde{r}, \tilde{c}) \ \frac{\partial A}{\partial c} \ \mathbf{x}_C,\tag{10}$$

where N is the number of samples in the dataset and f is a function that depends functionally on the full probability distribution $p_{R,C}$ of r and c over the dataset and the explicit current values of those variables. The complete derivation of the full gradients is deferred to Section 5.3.

These gradients can now be used to optimize the goal function by computing the gradient for the current weight vectors and making a step of predefined size in the direction that the gradients suggest. However, to accelerate the rate of convergence, a weight update can be made more often. This can be achieved by not taking the averages in Eqs. (9) and (10) over the whole dataset but only over small subsets thereof (often referred to as "minibatches"), which are also used to create the probability distribution $p_{R,C}$. This procedure is known in machine learning as *stochastic* gradient ascent.

4 Infomorphic networks encompass various learning paradigms

The neuron model and the learning rule derived above have the potential to serve as a very general approach to learning. In the following we demonstrate the broad applicability of this learning scheme by providing several example applications of infomorphic neurons. These examples will also highlight the decisions that need to be made when establishing a local learning scheme for a concrete example and how to approach these choices. We will show how groups of infomorphic neurons can collaborate and successfully perform tasks together. The tasks were purposefully chosen from a diverse set of learning paradigms to illustrate the flexibility and universality of infomorphic networks while also suggesting avenues for future research. Our examples are intentionally kept minimalistic to simplify the understanding of the learning approach and setups.

For each of the tasks, we need to define the architecture of the network and the local goal functions for the individual neurons. We approach these decisions by relating the (global) goal on the task-level to the (local) goals of the individual neurons and the information necessary for learning this local goal.

When establishing this local-global network relationship to determine the local goal functions and the network architecture, one can utilize two approaches: either a *top-down* or a *bottom-up* approach. With a top-down approach, the idea is to state the desired global goal of the task in terms of a global information-theoretic goal and then translate this to local goals for individual neurons while deciding for an appropriate connectivity scheme between them. This top-down approach can be beneficial for learning paradigms where the task defines a clear information-theoretic goal that can be split into local goals for individual neurons. We will demonstrate this in our first example, which is a supervised learning task. However, in many scenarios, there is no clear global function or there is no obvious way to dissect this global goal function into local elements. For such cases a bottom-up approach that involves more self-organization of the neurons is more effective. In this approach, we first examine the local processes and determine how the architecture could lead to a well-defined global effect or approximate the intended global goal function. This method was used in our examples involving unsupervised and associative memory learning.

4.1 Supervised learning using infomorphic neurons

To investigate the capabilities of infomorphic neurons we first apply infomorphic neurons to a supervised learning task and implement a simple one-layer network. In our setup, a group of infomorphic neurons each learn to classify the presence of handwritten digits in an image. Thereby they act together to solve the MNIST handwritten digit classification task with its 10 classes to a level that is comparable to established simple machine learning methods like logistic regression.

Supervised classification tasks have the goal of learning a function that maps samples from an input space \mathcal{X} , such as the pixels of an image, to a discrete set of classes \mathcal{L} . In more formal terms, the goal is to find a mapping $f : \mathcal{X} \to \mathcal{L}$. The task is defined by a dataset \mathcal{D} consisting individual samples $d \in \mathcal{X}$ and the corresponding desired class label $\ell \in \mathcal{L} = \{1, \ldots, k\}$ that the network should predict. The correct label ℓ is used as a teacher signal during training to gradually learn this mapping and adjust the parameters of the model. The ultimate aim in supervised learning is to achieve generalization, where the learned function demonstrates high accuracy in predicting the correct class labels even for samples that were not encountered during training and for which the correct label is not provided.

Structure To solve a task using a network of infomorphic neurons, we first need to decide on the structure of the network and the local goal functions. For a supervised classification problem, we approach these decisions in a *top-down* manner. The output layer of the network must completely represent the information of the label to solve the task. This means it must have at least as many neurons as there are bits of label information. We chose to use a simple and intuitive way to spread the desired label information over multiple neurons, where each neuron is expected to represent one class only: we use a layer with k neurons along with a one-hot encoding of the label information $\ell \in \mathcal{L} = \{1, \ldots, k\}$, i.e. a vector in which each boolean element corresponds to one class in the task. Given this one-to-one correspondence it is enough to provide each neuron with the respective element of the one-hot label vector as teaching signal. Our minimal network consists just of this single layer with k neurons, with no additional layers, thus acting both as input and output layer.

As the teaching signal conveying the label information is not available to the neuron in the evaluation phase after training has concluded, the individual neurons must not just directly pass on the label information of the teaching signal, but must learn to extract the relevant information from the network input even in the absence of the label. To this end, we chose (i) that each neuron receives its respective element of the teaching signal via the contextual (non-driving) input x_C , (ii) that the input data d is provided to each neuron via the receptive input \mathbf{x}_R that forms r (i.e. full connectivity), and (iii) the activation function of each neuron to be $A_{\sigma}(r,c) := r(0.5 + \sigma(2rc))$ such that neurons are also active in absence of a teacher signal. During the training phase, the neurons receive an input $x_C = -1$ for absence of the class and $x_C = 1$ for its presence, while in the testing phase the neuron simply receive $x_C = 0$. See also Fig. 3A for an illustration of the architecture.

Goal As outlined before, the individual neuron needs to learn how to extract the task-relevant information from the input data. During training, this task-relevant information (i.e. the correct label) is provided directly via the contextual neuron input c. Since, however, the training signal will not be available to the network at test time, the goal becomes to extract the same information from the inputs r instead. In terms of the PID terms introduced in Section 2.3, this amounts to promoting the redundant information between r and c. The goal function for the individual neuron therefore takes the shape $G_{\text{theory}} = I_{\text{red}}$.

Results In experiments with the MNIST handwritten digit classification task, we find that indeed the neurons can learn to solve the classification task to a test accuracy of approximately 90 % (Fig. 3B, see Section 5.3 for the chosen training parameters). Given the small number of 10 neurons overall, this test accuracy is to be expected, especially as it is almost matching approximately 92 % reached by a comparable standard machine learning algorithm, i.e. logistic regression. In practice we found that a slightly adjusted goal function $G_{\text{experiment}} = 0.1I_{\text{unq},R} + 0.1I_{\text{unq},C} + 1.0I_{\text{red}} + 0.1I_{\text{syn}}$ leads to solving the task more reliably for some of the runs, which we believe has to do more with the learning dynamics than the



Figure 3: Infomorphic networks implements MNIST supervised learning achieving an accuracy similar to that of logistic regression. A Illustration of the architecture of the infomorphic network that implements the MNIST task using a single layer one-hot encoding. Each neuron has an activation function $A(r,c) = r(0.5 + \sigma(2rc))$ and receives on \mathbf{X}_R the 32-by-32 MNIST image pixels while on \mathbf{X}_C the labels. During training, each neuron maximizes the redundant information $I_{red}(Y : \mathbf{X}_R, \mathbf{X}_C)$. **B**, Top The average training and test accuracy of 100 network are plotted over the course of training. The average accuracy converges to that of the logistic regression at about 92%. **B**, Bottom The average of the five goal function quantities over all the neurons in the 100 networks. The average entropy approaches the analytical entropy of each neuron $H(Y) = 0.1 \log_2(10) + 0.9 \log_2(1.1)$, which is all the information in any designated MNIST digit. The divergence of $I_{\rm red}$ from this analytical entropy reflects an imperfect classification. C presents two observables from a randomly chosen network run out of the 100 networks. C, Top The receptive fields for the neurons encoding digits "0" and "6", resp. C, Bottom The corresponding five goal function quantities for these two neurons. We observe that the goal function is $I_{\rm red}$ and the two neurons converge to the analytical entropy values labeled as the "correct H(Y)".

ultimate optimization goal, as it helps speeding up the beginning of the learning process. As expected, the redundant information $I_{\rm red}$ increases with training. The total output entropy of the neurons H(Y) decreases and approaches the entropy of the label vector elements of on average 0.47 bits. However, even at the end of training with $G_{\rm experiment}$, the neurons still encode some unique information from the data input $I_{\rm unq,R}$, as they do not perfectly match the label and are driven by the data input. The residual entropy H(Y|R,C) decreases fast, which indicates that the neuron's firing becomes less and less stochastic. For individual neurons (Fig. 3C), we find that the the general trends in terms of information are very consistent, however some digits are more difficult to classify, which leads to differences in the redundant information $I_{\rm red}$ that they can achieve. By the end of training, the weights \mathbf{w}_R of individual neurons (i.e., their receptive fields) reveal the digit that they encode and qualitatively match the optimal parameters that can be found using logistic regression.

4.2 Unsupervised learning of independent features by locally maximizing unique information with respect to activity of other neurons

In this section we will explore the application of infomorphic networks to an unsupervised learning task. Unsupervised learning aims to model the underlying structure or distribution of a data set and represent the data in a new, often simpler and more usable way. One typical goal of unsupervised learning is compression, meaning the preservation of as much information about the data as possible using minimal resources. In this experiment, we explore a minimal infomorphic network that strives to learn maximally informative independent features of the inputs.

Structure As this global goal can not be trivially split into local goals we will determine the structure and goal function in a bottom-up approach that involves self-organization between the neurons. To determine the structure of the infomorphic network in the case of unsupervised learning of independent features, several requirements can be identified. In order to learn a coherent global representation, the neurons need to self-organize and coordinate their learning, i.e. exchange information about their own output with each another and adjust their own learning accordingly. Therefore, (i) neurons need to be connected in a recurrent fashion to communicate to each other what they are encoding. The goal for the individual neuron is to transmit information about the data, but encode independent information with respect to other neurons, leading to the following additional requirement: (ii) the data input should be driving the activation of the neuron, while the recurrent input should only guide learning but not contribute much information about other neurons' activation to the current neuron's output. In our experiment we meet requirement (ii) by feeding the data d as input as X_R and the recurrent input from other neurons as X_C to each neuron, while choosing an asymmetric activation function of the form $A(r,c) = r(0.5 + \sigma(2rc))$. Overall, the structure used is a single layer of recurrently connected neurons that each receive full data sample d(see Fig. 4A).

Goal For setting the local goal functions we reconsider the goal of the task, which is that the neurons learn to maximize the *independence* of their own outputs from the other neurons' outputs. The objective of each neuron is thus to encode information that is *unique* with respect to all other neurons, which directly leads to the goal function $G = I_{\text{unq},R}$ that was used for our experiments.



Figure 4: Infomorphic networks implement an unsupervised learning task such that each neuron encodes for a unique feature. A The architecture of the infomorphic network implementing a simple task representing an input with 8 independently (co)occurring features, the network consists of 8 neurons. Each neuron has an activation function A(r, c) = $r(0.5 + \sigma(2rc))$, receives on \mathbf{X}_R a 8-by-8 binary pixels, and on \mathbf{X}_C the outputs of all other neurons. During training, each neuron maximizes $I_{unq}(Y:\mathbf{R})$ while \mathbf{w}_C are fixed to random values. While the neuron is informed about the encoding of other neurons via fixing \mathbf{w}_{C} , $I_{\rm ung}$ fosters the extraction of unique features in the input that are not yet encoded by other neurons. B The average five information quantities of the 8 neurons over 300 networks. Each experiment consists of two distinct training phases to foster self-organization: (i) in the first 50 training steps the weights are subjected to weight decay and neurons compete to each encode a single bar; (ii) after 50 epochs the weight decay is shut down and neurons affirm their encoding. The corresponding receptive fields visualize the encoded bars. C The average mutual information $I(X_R : Y_1, \ldots, Y_8)$ over the 300 networks. This mutual information approaches $H(\mathbf{X}_R) = 8$, the number of bits to encode all 8 bars. **D** Relevant observables from a randomly chosen run out of the 300 tested networks. D, Left The weights of the neurons encoding bars "1" and "2", resp. In each plot, only 8 different weights are visible since the weights corresponding to a single bar match almost perfectly. D, Middle The five information quantities of the two neurons. The goal function tracks $I_{unq}(Y:R)$. D, Right The receptive fields of all the neurons in this network, where each encodes a single unique bar.

Results In the experiments we trained the single layer, recurrently connected network on an image dataset consisting of 8x8 binary pixel images, each containing 8 horizontal bars each appearing independently with probability $p_{\text{bar}} = 0.5$ (see Fig. 4A). We set the number of neurons to 8 to construct a simple optimal solution in which each neuron learns to encode a different single bar and reaching at best 1.0 bits of independent information about the 8 bits of total information in the data. Note that the recurrent structure of our network intrinsically creates a time delay where the output of each neuron given the current input can only be conveyed to other neurons in the consecutive time step. Therefore, we resorted to repeating each sample d from the dataset for multiple consecutive time steps (in this case 8 time steps) such that in the majority of time steps each neuron receives receptive and contextual input elicited by the same stimulus. For an overview over the specific parameter settings see Section 5.3. Note that the recurrent connection weights \mathbf{w}_C between the neurons were held fixed to random initial values to prevent an unintended trivial solution for maximizing unique information $I_{\text{unq}}(Y: R)$, which is to minimize overall information in C, i.e. in essence learning to "ignore" other neurons. This particular behaviour will be discussed more in Section 5.

Over the course of training the neurons learn to encode individual bars by maximizing unique information with respect to each other from \mathbf{X}_R (see Fig. 4B-C). The total mutual information of the whole layer $I(T : Y_1, \ldots, Y_8)$ converges to the entropy of the dataset, which means that the neurons are finding a good representation of the dataset, encoding all these generating factors independently. The $I_{unq}(Y : R)$ converges to 0.45 bits rather than the expected 1.0 bits which is due to the nature of I^{sx} that assigns 0.45 bits of $I_{unq}(Y : R)$ and 0.55 bits of negative $I_{unq}(Y : C)$ (misinformation) for pure unique encoding of X_R (see UNQGATE in Makkeh et al., 2021).

An interesting observation in this experiment is the early phase of learning when the neurons have not converged to their encoded bar yet (see weights \mathbf{w}_R Fig. 4D Column 1). In this phase neurons compete on encoding the bars until settling on a certain bar. This self-organization phase is critical for learning and we extended it for better performance by keeping the weights low via a weight decay (linear downscaling of all weights). Using this weight decay neurons are given enough opportunity to pick a bar that is not yet encoded, rather than ending up not encoding any bar or encoding the same bar already encoded by another neuron. Interestingly, in some runs of this experiment certain groups of neurons learned a distributed encoding of their bars, meaning that multiple neurons encoded combinations of the same group of bars. We conclude that an unsupervised learning aimed at extracting independent features from the data can be interpreted as neurons striving to encode unique information about the input that is not yet encoded by other neurons. Put differently, this is a PID-based formulation of efficient coding principles for self-organizing distributed computations.

4.3 Associative memory learning using a Hopfield-like recurrently connected dynamical network

In our final experiments, we apply infomorphic networks to associative memory learning, constructing a Hopfield-like infomorphic network. Simply put, the goal of an auto-associative memory network is to learn memorize a number of patterns. Optimally, the network solves this task by memorizing a large number of patterns in their exact form and being able to reconstruct them when presented with a distorted version or only part of the original pattern (hence the name "auto-associative"). More precisely, auto-associative memory networks are recurrent neural networks that learn to possess a set of so-called "memories", which are the

fixed-point attractors in their recurrent network activity. Training a memory network means systematically changing the recurrent weights such that the desired fixed-point attractors are created. One way to train an auto-associative memory during run time ("online"), is to use external input to force the network into a specific state of repeating activity, and let the recurrent weights change in a way such that this pattern becomes an attractor, i.e. a memory. The trained memory can be retrieved by bringing the network into a state that lies within the respective basin of attraction of the memory such that the network state will converge to the original memory. Thus the network structure is often set up such that each element of the pattern corresponds to one neuron in the network (see Fig. 5A).

Structure For an attractor network, the neurons need to be connected in a recurrent structure in order to communicate amongst each other. Similarly as in the unsupervised learning task, the infomorphic neurons in an associative memory need to distinguish between the external input of new memory patterns and the recurrent input of activity from other neurons. However, *both* of these inputs have to be able to *drive* the output state of each neuron: External input, if present, should force the network to fire in accordance with this input, whereas in the absence of external input, each neuron should be driven purely by the recurrent activity. This consideration led us to choose a symmetric activation function (e.g. $A(r,c) = ||r||_8 + ||c||_8$). Because of this symmetry, in principle the assignment of the terms "receptive" and "contextual" becomes arbitrary. In accordance with the previous experiments, we chose that each neuron receives one element of the pattern as the receptive input \mathbf{X}_R and the recurrent input from other neurons as \mathbf{X}_C .

Goal In addition to the network structure the local goals of the neurons need to be determined. In order to collectively learn the pattern, i.e., make it an attractor of the recurrent dynamics of the network, each neuron's output needs to learn to be coherent with other neurons. The output of each neuron needs thus needs to align with the external input, yet also needs to be driven equally by the collective output of the other neurons. This means that each neuron needs to maximize the encoding of redundant information that its receptive and contextual inputs provide. A canonical goal function of each neuron is thus $G = I_{\rm red}$. Note, that similar to the unsupervised experiments, during training, we presented each pattern for multiple consecutive time steps and presented different patterns in random order (to prevent association with another pattern).

Results In our experiments, neurons learn to extract coherent of their external input and the output received from other neurons by optimizing the encoding of redundant information of the two inputs. The resulting memory capacity of infomorphic networks is larger than that of classical Hopfield networks. The memory capacity is empirically assessed by computing an average accuracy during retrieval as follows: The tested pattern is only presented in the first step and an average cosine similarity of the output of the network and the tested pattern is computed over 20 times steps of free recurrent network activity (see Fig. 5B). We observe that infomorphic networks attain a high level of accuracy (above 0.95) for up to 40 trained patterns per 100 neurons which significantly exceeds that of classical Hopfield networks (up to 14 patterns per 100 neurons).

The enhanced memory capacity for infomorphic networks, however, comes at the expense of slightly less noise tolerance. The noise level is parametrized by β which is the percentage



Figure 5: Infomorphic networks implements associative memorization outperforming classical Hopfield networks. A The architecture of the infomorphic network implementing a simple memorization task of binary patterns of length 100. The networks consists of 100 recurrently connected neurons. Each neuron has an activation function $A(r,c) = ||r||_8 + ||c||_8$ and receives a single element of the input pattern via \mathbf{X}_R while receiving the outputs of all other neurons via \mathbf{X}_{C} . During training, each neuron maximizes $I_{\rm red}(Y:R,C)$. $I_{\rm red}$ is expected to foster coherence with the encoding of other neurons leading to collectively storing the pattern. **B** Relevant observables for a random network run storing 40 patterns. **B**, Top The recurrent weights w_C are not fully symmetric and the patterns are perfectly retrieved for all 20 times steps where the pattern is only presented in the first step. **B**, **Bottom** The neurons learn gradually on average. **C** Comparison between performance of infomorphic network and Hopfield networks while memorizing a given number of patterns. The infomorphic network has a higher memory capacity while its relative noise tolerance is smaller compared to the Hopfield networks. D The average five information quantities over 100 neurons and 25 runs per pattern size (in total 250 networks for 10 pattern sizes). The neurons $I_{\rm red}$ approaches the expected analytical entropy H(Y) of one bit.

of pattern elements randomly set during retrieval. For example, a noise level of $\beta = 1.0$ means that each element of the pattern is assigned a random binary value; $\beta = 0.5$ that half of the pattern are set at randomly. Infomorphic networks are less tolerant to noise than Hopfield networks, but nonetheless still retain a better capacity under small amounts of noise. These results hint at high redundancy being a local information processing goal that leads to associativity and possibly might give new insights into how associative memories are formed.

5 Discussion

In this work we show that a wide spectrum of learning tasks can be addressed using a simple neuron model and flexible information-theoretic local goals. The local goals, learnable through a universal, parameterized learning rule, are a particular strength of this novel framework, with their easy interpretability and unaltered reappearance in the gradients used for learning. In this context, information theory serves as a valuable tool for removing superfluous details, and highlighting fundamental aspects of neural information processing. It facilitates the flexible description of goals, independent of task, substrate, type of signals and encoding of information therein Wibral et al. (2015, 2017). The experiments conducted provide a proof of principle that the level of abstraction gained by an information-centric approach does not compromise the ability of model neural networks to learn and resolve diverse tasks.

The underlying premise of this work is that neurons act as local information processors, encoding information from incoming signals into their output activity. However, the classical channel perspective based on mutual information only provides a coarse insight into the complexity of information processing in biological neurons (Section 2.2). To address this, we have adopted the PID framework (Williams and Beer, 2010), which allows for a more comprehensive view, accounting for the unique, redundant, and synergistic contributions from multiple input classes. This multi-source approach is an indispensable feature of our model, mirroring the multiple functionally distinct groups of inputs that a biological neuron processes.

Importantly, we do not suggest that the learning rules derived for our model neuron are directly implemented in the biological substrate. Instead, the learning in biological neural networks could use different learning rules but eventually indirectly converge to the same information-theoretic goals that can be expressed through PID.

5.1 Experiments

We have demonstrated that a single neuron model, inspired by pyramidal level-5 neurons, can be applied flexibly to address a wide variety of tasks spanning different learning paradigms: supervised learning, unsupervised learning, and associative memory learning. This is achieved by assigning specific information-theoretic local goals along with a general parameterized learning rule.

In the case of **supervised learning**, we find that the direct use of label information as context to individual neurons guides their learning towards extracting task-relevant information from the data. The empirical link that we observe between redundancy maximization and logistic regression could be explored further by establishing a theoretical equivalence between these two learning goals. To improve the performance of the networks, additional hidden layers could be introduced, which will be discussed further in the section on *future work*.

The **unsupervised learning** experiments emphasize the benefits of self-organization in neural populations (e.g. Shu et al. (2003)). The neurons within this network successfully learn an informative representation of mutually independent features in the input through coordination. This coordination is only possible by providing context information to each other and striving to encode additional, unique information with respect to other neurons. We hypothesize that similar mechanisms of self-organization are essential for larger population of neurons that collaborate to accomplish a single task. As pointed out earlier, the lateral connections in this experiment were held fixed in order to prevent a trivial solution of ignoring information from other neurons. This could in principle be avoided by actively forming strong lateral connections with neurons that show a high degree of redundancy. However, investigating such a mechanism further is beyond the scope of this work.

In the last set of experiments we uses an information-theoretic optimization goal of redundancy maximization to form an **associative memory**, exceeding the memory capacity of classical Hopfield networks. We note that this motif of redundancy maximizing local information processing that leads to emergence of associative memory could be verified by studying the local information processing occurring in hippocampus. Hippocampus is crucial for formulating and retrieving of associative memory (Rolls and Treves, 1997; Rolls, 2018). In particular, the hippocampal area CA3 that has a substantial recurrent connectivity amongst its pyramidal neurons (Rolls and Wirth, 2018).

In all experiments conducted here, the ability to intuitively comprehend the local learning principles that guide the overall function proves advantageous for understanding the network function as a whole. This interpretability of learning seems to be absent in conventional artificial neural networks, where a global error minimization goal is broadcasted at the local level and directly adjusts neuron parameters (e.g., LeCun et al. (2015); Samek et al. (2017)).

5.2 Future work

The experiments primarily focused on single-layer neural networks. The great power of neural networks, however, lies in the formation of larger networks with a multilayered structure. To fully embrace the potential of infomorphic networks and applying them to more complex tasks, it becomes necessary to develop methods to construct and train multi-layer networks with numerous individual neurons.

To develop these larger networks, several possible avenues exist. First, it seems feasible that the optimal network structure and information-theoretic objectives could be *derived* in a top-down approach. Second, existing models of neural hierarchies could be implemented using infomorphic neurons, with hierarchical predictive coding as an intriguing possibility (e.g. Huang and Rao (2011); Spratling (2017)). Third, simulating evolutionary mechanisms to discover successful structures and local goal parameters could also be a viable approach.

The infomorphic neuron model introduced in this study is designed with two classes of inputs. However, it may not be enough to reflect the complexity of neuronal information processing observed in biological systems. Augmenting the model to incorporate additional classes of inputs could enable handling more diverse architectures and interactions. We hypothesize that for many tasks, neurons equipped with at least three classes of inputs are needed for effective self-organization: receptive feed-forward, contextual feed-back and lateral connections for coordination with other neurons. Furthermore, the neuron model could be expanded to include a burst-like output state, believed to play a significant role in neural computation Shai et al. (2015).

5.3 Conclusion

Leveraging Partial Information Decomposition, this work has established the infomorphic neuron, a novel neuron model permitting the flexible and direct optimization of interpretable information-theoretic goals. Through several lines of experimentation, the capacity of these neurons to solve diverse tasks has been demonstrated. We propose infomorphic neurons as abstract neuron models that can provide a new foundation for studying local information processing in neural networks, opening up many exciting avenues for future research.

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Appendix A. Learning rules

In this appendix we will present the analytical learning rules of the local infomorphic goal function. We will go through the high level derivation leading to the learning rules and along the way interpret some aspects of these rules.

The infomorphic neuron aims to learn a certain information processing attribute by optimizing (maximizing) a designated local goal function. Such learning is performed via the plasticity of the infomorphic neuron that consists of the two classes of weights \mathbf{w}_R and \mathbf{w}_C . Thus, learning this goal function boils down to finding the set of weights \mathbf{w}_R and \mathbf{w}_C for which the local goal function attains its maximum. The maximization in this framework is achieved using gradient ascent, i.e. an infomorphic neuron follows the direction of the gradient of the local goal function in the space of the receptive and contextual weights towards a maximum value of that function. Therefore, the learning rules are summarized by $\frac{\partial G}{\partial \mathbf{w}_R}$ and $\frac{\partial G}{\partial \mathbf{w}_C}$, the gradients of G w.r.t. to \mathbf{w}_R and \mathbf{w}_C .

gradients of G w.r.t. to \mathbf{w}_R and \mathbf{w}_C . To derive $\frac{\partial G}{\partial \mathbf{w}_R}$ and $\frac{\partial G}{\partial \mathbf{w}_C}$ we need to look into the analytical nature of G. Recall that G has its basis as the four PID terms and the intrinsic noise of the neuron captured by $H(Y \mid \mathbf{X}_R, \mathbf{X}_C)$ as follows:

$$\begin{aligned} G(Y:R,C) &= \Gamma_0 I_{\mathrm{unq},R}(Y:R,C) + \Gamma_1 I_{\mathrm{unq},C}(Y:R,C) \\ &+ \Gamma_2 I_{\mathrm{red}}(Y:R,C) + \Gamma_3 I_{\mathrm{syn}}(Y:R,C) \\ &+ \Gamma_4 H(Y \mid R,C) \;. \end{aligned}$$

These information theoretic parts of G are nonlinear averages of $\theta(r, c) = P(Y = +1 | R = r, C = c)$ and $1 - \theta(r, c) = P(Y = -1 | R = r, C = c)$. So, the learning rules $\frac{\partial G}{\partial \mathbf{w}_R}$ and $\frac{\partial G}{\partial \mathbf{w}_C}$ are in fact the potential change in the value of G w.r.t θ multiplied by the potential change in the value of θ value w.r.t. \mathbf{w}_R and \mathbf{w}_C :

$$\frac{\partial G}{\partial \mathbf{w}_R} = \left\langle \frac{\partial G}{\partial \theta} \frac{\partial \theta}{\partial \mathbf{w}_R} \right\rangle_{r,c}$$
$$\frac{\partial G}{\partial \mathbf{w}_C} = \left\langle \frac{\partial G}{\partial \theta} \frac{\partial \theta}{\partial \mathbf{w}_C} \right\rangle_{r,c}.$$

Intuitively, the change in the value of goal function w.r.t. the weights is dictated by the change in the value of the goal function itself w.r.t. the likelihood of the infomorphic neuron being in the HIGH state and the change of this likelihood w.r.t. the weights.

First we derive $\frac{\partial \theta}{\partial \mathbf{w}_R}$, the change of the likelihood of the infomorphic neuron being in the HIGH state w.r.t. weights. This directly amounts to:

$$\frac{\partial \theta}{\partial \mathbf{w}_R} = \frac{\partial A}{\partial r} \mathbf{x}_R \theta(r, c) (1 - \theta(r, c))$$
$$\frac{\partial \theta}{\partial \mathbf{w}_C} = \frac{\partial A}{\partial c} \mathbf{x}_C \theta(r, c) (1 - \theta(r, c)) .$$

This change of Likelihood is proportional to a tight balance between the likelihood of the HIGH and the LOW state, i.e. the term $\theta(r, c)(1 - \theta(r, c))$, modulated by the potential change in the value of the activation function w.r.t. to the presynaptic input $(\frac{\partial A}{\partial r})$ and the presynaptic input \mathbf{x}_R or \mathbf{x}_C themselves. This is inherited by potential change of the sigmoid, since

 $\theta(r,c) = \sigma(A(\mathbf{w}_R \cdot \mathbf{x}_R, \mathbf{w}_C \cdot \mathbf{x}_C))$, w.r.t. the weights that is (a) large around the inflection point between LOW and HIGH which translates to the infomorphic neurons being in an undecided state $\theta(r,c) = 1/2$ and (b) small along the extremities which translates to the infomorphic neurons encoding the information via the HIGH ($\theta(r,c) = 1$) or the LOW ($1 - \theta(r,c) = 1$) state and modulated in both (a) and (b) by A and the presynaptic inputs \mathbf{x}_R or \mathbf{x}_C accordingly.

Now that the potential change of θ w.r.t. weights is derived, we turn to the potential change of G w.r.t. θ . To facilitate deriving $\frac{\partial G}{\partial \theta}$, we rewrite G as a function of H(Y), $H(Y \mid R)$, $H(Y \mid C)$, $H(Y \mid R, C)$ and $I^{\text{sx}}(Y : R, C)$. This rewriting stems from the fact that the structure of PID from Section 2.3, all four PID terms are defined once the redundant information $I_{\text{red}}(Y : R, C)$ is defined, where in the infomorphic neurons $I_{\text{red}}(Y : R, C) := I_{\cap}^{\text{sx}}(Y : R, C)$. To this end,

$$G(Y:R,C) = \gamma_0 H(Y) + \gamma_1 H(Y \mid R) + \gamma_2 H(Y \mid C) + \gamma_3 H(Y \mid R,C) + \gamma_4 I_{\cap}^{\mathrm{sx}}(Y:R,C) \ .$$

where $\gamma_0 = \Gamma_0 + \Gamma_1 - \Gamma_3$, $\gamma_1 = \Gamma_3 - \Gamma_0$, $\gamma_2 = \Gamma_3 - \Gamma_1$, $\gamma_3 = \Gamma_4 - \Gamma_3$ and $\gamma_4 = \Gamma_2 + \Gamma_3 - \Gamma_0 - \Gamma_1$. The derivation of G in the new basis is simpler since the terms H(Y), $H(Y \mid R)$, $H(Y \mid C)$, $H(Y \mid R, C)$ are simpler averages of θ than I_{unq} or I_{syn} and their learning rules have been previously derived by Kay (1994) where they are part of the multichannel goal function F. This projected G is written in terms of θ as follows:

$$\begin{split} G(\theta) &= \gamma_0 \left[E(\theta) \log E(\theta) + E(1-\theta) \log E(1-\theta) \right) \right] \\ &+ \gamma_1 \left\langle E_r(\theta) \log E_r(\theta) + E_r(1-\theta) \log E_r(1-\theta) \right\rangle_r \\ &+ \gamma_2 \left\langle E_c(\theta) \log E_c(\theta) + E_c(1-\theta) \log E_c(1-\theta) \right\rangle_c \\ &+ \gamma_3 \left\langle \theta(r,c) \log \theta(r,c) + (1-\theta(r,c)) \log(1-\theta(r,c)) \right\rangle_{r,c} \\ &+ \gamma_4 \left[E(\theta) \log E(\theta) + E(1-\theta) \log E(1-\theta) \right] \\ &- \gamma_4 \left\langle \theta(r,c) \log U(\theta) + (1-\theta(r,c)) \log U(1-\theta) \right\rangle_{r,c} , \end{split}$$

where

$$\begin{split} U(\theta) &:= p(Y = +1 \mid R = r \cup C = c) = p(r)E_r(\theta) + p(c)E_c(\theta) - p(r,c)\theta(r,c) \\ U(1-\theta) &:= p(Y = -1 \mid R = r \cup C = c) = p(r \cup c) - U(\theta) \\ E(\theta) &:= p(Y = +1) = \langle \theta(r,c) \rangle_{r,c} \\ E(1-\theta) &:= p(Y = -1) = 1 - E(\theta) \\ E_r(\theta) &:= p(Y = +1 \mid R = r) = \langle \theta(r,c) \rangle_{c|r} \\ E_r(1-\theta) &:= p(Y = -1 \mid R = r) = 1 - E_r(\theta) \\ E_c(\theta) &:= p(Y = +1 \mid C = c) = \langle \theta(r,c) \rangle_{r|c} \\ E_c(1-\theta) &:= p(Y = -1 \mid C = c) = 1 - E_c(\theta) . \end{split}$$

Therefore, the derivation yields:

 \langle

$$\begin{split} \frac{\partial G}{\partial \theta} \bigg\rangle_{r,c} &= \gamma_0 \left\langle \log \frac{E(\theta)}{E(1-\theta)} \right\rangle_{r,c} \\ &+ \gamma_1 \left\langle \log \frac{E_r(\theta)}{E_r(1-\theta)} \right\rangle_{r,c} \\ &+ \gamma_2 \left\langle \log \frac{E_c(\theta)}{E_c(1-\theta)} \right\rangle_{r,c} \\ &+ \gamma_3 \left\langle A(r,c) \right\rangle_{r,c} \\ &+ \gamma_4 \left\langle \log \frac{E_r(\theta)}{E_r(1-\theta)} \right\rangle_{r,c} \\ &- \gamma_4 \left\langle \log \left(\frac{U(\theta)}{U(1-\theta)} \right) + U \left(\frac{\theta}{U(\theta)} - \frac{1-\theta}{U(1-\theta)} \right) \right\rangle_{r,c} , \end{split}$$

We see that $\frac{\partial G}{\partial \theta}$ exhibits a similar tight balance between the likelihoods of HIGH and LOW states. This balance in $\frac{\partial G}{\partial \theta}$, however, is wrapped up with the information functionals such as log and certain averages. A more in-depth understands of the terms of $\frac{\partial G}{\partial \theta}$ is the research topic for another study, however.

We can finally write the learning rule in the following compact form that emphasis the information theoretic component $\frac{\partial G}{\partial \theta}$ and the likelihood component of the infomorphic neuron $\frac{\partial \theta}{\partial \mathbf{w}_R}$:

$$\frac{\partial G}{\partial \mathbf{w}_R} = \left\langle (\gamma_3 A - \widetilde{O}) \frac{\partial A}{\partial r} \theta(r, c) (1 - \theta(r, c)) \mathbf{x}_R \right\rangle_{r,c}
\frac{\partial G}{\partial \mathbf{w}_C} = \left\langle (\gamma_3 A - \widetilde{O}) \frac{\partial A}{\partial c} \theta(r, c) (1 - \theta(r, c)) \mathbf{x}_C \right\rangle_{r,c}$$
(11)

where

$$\widetilde{O} = (\gamma_4 + \gamma_0) \log \frac{E(\theta)}{(1 - E(\theta))} + \gamma_1 \log \frac{E_r(\theta)}{(1 - E_r(\theta))} + \gamma_2 \log \frac{E_c(\theta)}{(1 - E_c(\theta))} - \gamma_4 \log \left(\frac{U(\theta)}{U(1 - \theta)}\right) - \gamma_4 U \left(\frac{\theta(r, c)}{U(\theta)} - \frac{1 - \theta(r, c)}{U(1 - \theta)}\right)$$

$$U(\theta) := p(Y = 1 \mid R = r \cup C = c) = p(r)E_r(\theta) + p(c)E_c(\theta) - p(r,c)\theta(r,c),$$
$$U(1 - \theta) := p(Y = -1 \mid R = r \cup C = c) = p(r \cup c) - U(\theta)$$

and

$$E(\theta) = \langle \theta(r,c) \rangle_{r,c}, E_r(\theta) = \langle \theta(r,c) \rangle_{c|r}, E_c(\theta) = \langle \theta(r,c) \rangle_{r|c}$$

Appendix B. Experiments: parameters and statistics

In what follows we will give a full account of the setup of the experiments. These are finegrained details such as learning parameters which might be interesting and necessary for reproducing the results. The main parameters are those within the implementation of the neuron. These parameters are important for the learning of the neuron. We will first explain each parameter and then for each of three experimental schemes (Supervised, Unsupervised and Memory) tabulate these parameters.

Parameters of the infomorphic neuron.

Each infomorphic neuron has a number of parameters that formulate its identity. These parameters are listed as follows:

• Training:

- Phases: The number of epochs each neuron is trained on per phase. In some experiments the neuron is trained in two different phases where certain parameters are changed from one phase to the other. With multiple phases, the parameter Phases is given as a tuple.
- $-N_{\rm tr}$ and $N_{\rm te}$: The number of training and testing samples per epoch, respectively.
- $-m_{\rm rep}$: The number of times each sample is presented. For instance, in recurrent networks the samples need to be presented repeatedly.

• Learning:

- scaling: The order of magnitude of the initialized weights. For example scaling = 0.1 means that the weights are initialized randomly in the interval [-0.1, 0.1].
- Goal function Γ : The defining parameters of the local goal function that the neuron is supposed to learn. It is a tuple of five values; one value per information basis.
- Learning rate η : The learning rate for the gradient ascent utilized to learn the goal function.
- Pullback rate λ : The weights decay factor. In unsupervised, the weights are pulled back by λ at each step to keep them from growing. This pullback is as follows: $w_R = (1 - 2\lambda)w_R + \eta \nabla(w_R)$ where $\nabla(w_R)$ is the gradient update at this step.

• Input Integration:

- $n_{\text{receptive}}$: The size of the input \mathbf{X}_R or in other words the number of receptive inputs. Evidently, $n_{\text{receptive}}$ is the size of the vector \mathbf{w}_R .
- $n_{\text{contextual}}$: The size of the input \mathbf{X}_C or in other words the number of receptive inputs. Evidently, $n_{\text{contextual}}$ is the size of the vector \mathbf{w}_C .
- The J_R and J_C : The interval that R and C values are limited to. If any realization r or c exceed the limits of their interval they are assigned to the nearest bin.
- The n_R -bins and n_C -bins: The n_{rbins} and n_{cbins} are the number of quantization for R and C respectively. The intervals J_R and J_C are quantized thus formulating the discrete random variables \tilde{R} and \tilde{C} .

Supervised learning

In this experiment we ran 100 networks. These networks are identical in architecture with a single layer of 10 neurons, but having different random initialization of their weights. During training, the 28-by-28 MNIST pixel images are received as vector \mathbf{X}_R where each pixel is in [0, 1] and the label is received as a one-hot representation \mathbf{X}_C . During testing, the network receive the MNIST image as in training, however, a constant vector of ones on \mathbf{X}_C . The full detail of the individual neurons parameters are in summarized in Table 1.

Table 1: Parameters of the infomorphic neurons in the supervised learning experimental scheme. The parameters are organized into the three categorize (Learning, Training, and Integration). *Left:* Parameter and its values. *Right:* A further explanation of the value when neccessary.

Training				
Parameter	Value	Comment		
Phases	800	a single phase of training with 800 epochs		
$N_{ m tr}$	1000	sampled uniformly from the 60000 MNIST testing images		
$N_{ m te}$	1000	sampled uniformly from the 10000 MNIST testing images		
$m_{ m rep}$	1	no repetition is needed		
Learning				
Parameter	Value	Comment		
scaling	0.01	scale of initialization		
Г	(0.1, 0.1, 1, 0.1, 0)	$I_{\mathrm{unq}}(Y:R), I_{\mathrm{unq}}(Y:C), I_{\mathrm{red}}, I_{\mathrm{syn}}, H(Y:R,C)$		
η	1.0	learning rate		
λ	0.0	no pullback needed		
Input Integration				
Parameter	Value	Comment		
$ $ $n_{\rm receptive}$	784	MNIST image pixel size		
$n_{\rm contextual}$	1	a bit of one-hot label representation		
J_R	[-20, 20]	a smaller range might hinder the learning		
J_C	[-20, 20]	a smaller range might hinder the learning		
n_R -bins	200	uniform bin-size is 0.1		
n_C -bins	200	uniform bin-size is 0.1		

Unsupervised Learning

In this experiment we ran 300 networks. These networks are identical in architecture with a single layer of 8 neurons, but having different random initialization of their weights. These neurons are recurrently connected via their \mathbf{w}_C which are fixed throughout training to a randomly chosen value in [5, 10]. During training and testing, an 8-by-8 grid of pixels are received as vector \mathbf{X}_R with discrete pixels in $\{-1, 1\}$. Training is split into two phases: the first has a weight pullback while in the second phase the weight pullback is turned off. The

full detail of the individual neurons parameters are in summarized in Table 2.

Table 2: Parameters of the infomorphic neurons in the unsupervised learning experimental scheme. The parameters are organized into the three categorize (Learning, Training, and Integration). *Left:* Parameter and its values. *Right:* A further explanation of the value when neccessary.

Training				
Parameter	Value	Comment		
Phases	(50, 50)	a single phase of training with 800 epochs		
$N_{ m tr}$	1000	generated randomly from a distribution of independent bars		
$N_{ m te}$	1000	generated randomly from a distribution of independent bars		
$m_{\rm rep}$	8	each sample is fed in 8 consecutive times		
Learning				
Parameter	Value	Comment		
scaling	0.1	scale of initialization		
Г	(1,0,0,0,0)	$I_{\mathrm{unq}}(Y:R), I_{\mathrm{unq}}(Y:C), I_{\mathrm{red}}, I_{\mathrm{syn}}, H(Y:R,C)$		
η	(10.0, 1.0)	Higher learning rate during weight pullback		
λ	(0.28, 0.0)	initial phase of weight pullback		
Input Integration				
Parameter	Value	Comment		
$n_{\text{receptive}}$	64	size of the input		
$n_{\rm contextual}$	7	one per each neuron with no self connections		
J_R	[-25, 25]	a smaller range might hinder the learning		
J_C	[-25, 25]	a smaller range might hinder the learning		
n_R -bins	500	uniform bin-size is 0.05; larger size might hinder the learning		
n_C -bins	500	uniform bin-size is 0.05		

Associative Memory

In this experiment we ran 250 individual networks as follows. We had 10 different number of patterns to be memorized and 25 networks are ran per each group of patterns. Each set of input patterns is randomly chosen from the 2^{100} sample space. These networks are identical in architecture with a single layer of 100 neurons, but having different random initialization of their weights. These neurons are recurrently connected via their \mathbf{w}_R which are to be trained. During training, patterns, vectors of 100 bits, are received as \mathbf{X}_R with bits in $\{-1, 1\}$. During testing, the pattern is presented only in the first time step and then for 19 time steps the responses of the neurons are readout to be compared with the originally presented pattern. The full detail of the individual neurons parameters are in summarized in Table 3.

Table 3: Parameters of the infomorphic neurons in the associative memory experimental scheme. The parameters are organized into the three categorize (Learning, Training, and Integration). *Left:* Parameter and its values. *Right:* A further explanation of the value when neccessary.

Training				
Parameter	Value	Comment		
Phases	200	a single phase of training with 800 epochs		
$N_{ m tr}$	200	sampled from the set of input patterns		
$N_{ m te}$	200	sampled from the set of input patterns		
$m_{ m rep}$	8	each sample is fed in 8 consecutive times		
Learning				
Parameter	Value	Comment		
scaling	0.1	scale of initialization		
Γ	(0.1, 0.1, 1.0, 0.1, 0)	$I_{unq}(Y:R), I_{unq}(Y:C), I_{red}, I_{syn}, H(Y:R,C)$		
η	0.48	an $\eta = 0.5$ won't change the result		
λ	0.0	no weight pullback		
Input Integration				
Parameter	Value	Comment		
n _{receptive}	64	size of the input		
$n_{\rm contextual}$	7	one per each neuron with no self connections		
J_R	[-20, 20]	a smaller range might hinder the learning		
J_C	[-20, 20]	a smaller range might hinder the learning		
n_R -bins	20	uniform bin-size is 1; smaller size doesn't affect the learning		
n_C -bins	20	uniform bin-size is 1; smaller size doesn't affect the learning		

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