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Exploring new frameworks for electromagnetic charge non-conservation

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Abstract

Extended scalar sectors and the spontaneous breaking of symmetries form the baseline of many theories beyond the standard model to explain, e.g., neutrino masses, dark matter or baryogenesis. An assumption almost ubiquitous in all literature is the neutrality of the vacuum and the conservation of electric charge. We provide a new perspective on this assumption, which, although experimentally well-founded, is not a theoretical necessity. General scalars that obtain vacuum expectation values in their charged components are considered, and we will see how they bring about corrections to the gauge boson masses, the weak mixing angle, expected fermion and boson charges, and many more. We develop a framework to sensibly talk about electric charge in the broken phase and apply it to the case of minicharged particles. Scalars with special representations under the standard model gauge group that allow for new interactions are examined, and we show how they lead to the mixing of charged and uncharged leptons. To this end, we introduce a new basis with corresponding Feynman rules and demonstrate their application with a few examples. Throughout this discussion, we relate our results to the latest experimental limits and give general bounds on parameters and observables.

Zusammenfassung

Erweiterte skalare Sektoren und spontane Symmetriebrechung sind die Grundlage vieler Theorien, die über das Standardmodell hinausgehen und z.B. Neutrinomassen, dunkle Materie, oder Baryogenese erklären. Eine beinahe allgegenwärtige Annahme in sämtlicher Literatur ist hierbei die Neutralität des Vakuums und die Erhaltung elektrischer Ladung. Wir eröffnen neue Perspektiven auf diese Annahme, die, obwohl experimentell gut fundiert, aus theoretischer Sicht nicht zwingend notwendig ist. Allgemeine Skalare, welche Vakuumerwartungswerte in ihren geladenen Komponenten entwickeln, werden betrachtet, und wir werden feststellen, wie diese zu Korrekturen auf Massen der Eichbosonen, des elektroschwachen Mischungswinkels, der zu erwartenden Teilchenladungen und vielem weiteren führen. Wir entwickeln eine Herangehensweise, um sinnvoll über elektrische Ladung in der gebrochenen Phase sprechen zu können, und wenden sie auf Teilchen sehr geringer Ladung an. Skalare mit speziellen Representationen der Standard-Modell-Eichgruppe, die neue Interaktionen erlauben, werden untersucht und wir zeigen, wie sie zur Mischung von geladenen und ungeladenen Leptonen führen. Dafür führen wir eine neue Basis mit den dazugehörigen Feynman-Diagrammen ein und demonstrieren ihre Anwendung anhand einiger Beispiele. Während unserer Diskussion vergleichen wir unsere Resultate durchweg mit den aktuellsten Experimenten und geben allgemeine Begrenzungen auf Parameter und Observablen an.

Contents

Chapter

1. Introduction

1.1. History and Motivation

Electromagnetic phenomena have captured the interest of humans all throughout history; from lightning strikes observed by our early ancestors all the way up to contemporary sophisticated electric circuits and machinery.
The first attempts to systematically study them are commonly attributed to Thales of Miletus (ca. 624–546 BCE), who reportedly observed the electrostatic charge of amber after rubbing it with a piece of cloth or fur. Although modern analyses show that these effects were almost certainly known already for some time and Thales likely only referenced them to make a point about the soul of inanimate objects (cf. [1, 2]), this popular story still serves to demonstrate the pull this mysterious force had on the curious minds of the last millennia.
During the industrial revolution, the invention of light bulbs, telegraphs, electric motors, etc. brought electricity into the public consciousness, which in turn inspired more research in the field and provided the opportunity for many today well-known scientists, such as Michael Faraday (1791-1867), Carl F. Gauß (1777-1855), and Wilhelm E. Weber (1804-1891), to earn their place in scientific history. One of the most important of these physicists was James C. Maxwell (1831-1879), who was the first to quantitatively describe electricity and magnetism as a unified theory in his work *A Treatise on Electricity and Magnetism* (1873) [3].
The 20^{th} century saw the development of gauge theories, paving the way to a further combination of Maxwell's electromagnetism with the weak and strong nuclear force into what is today known as the standard model of particle physics (SM) (cf. [4] and [5] for a more detailed history).

A crucial feature of the SM is the breaking of the gauge group $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ at a certain energy scale into the group $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{U}(1)_\mathrm{Q}$ by use of the *Higgs mechanism*[1]. It introduces a scalar particle that obtains a non-zero vacuum expectation value (vev). After this phase transition occurs, the new vacuum of the theory appears to carry the quantum numbers of the scalar, so they look broken in this phase. For example, the left-handed (LH) electron e_L has a weak hypercharge of

[1] We use here the popular name Higgs mechanism, but also do not want to neglect the important contributions by many other physicists. Unfortunately, the name *ABEGHHK'tH mechanism* (for Anderson, Brout, Englert, Guralnik, Hagen, Higgs, Kibble, and 't Hooft), as suggested by Peter Higgs [6], somewhat impedes the readability of the work.

$y = -\frac{1}{2}$, while its right-handed (RH) counterpart e_R has $y = -1$. The electron mass term $\overline{e_\mathrm{L}} e_\mathrm{R}$ therefore has a leftover hypercharge of $y = -\frac{1}{2}$ and is hence not invariant under $\mathrm{U}(1)_Y$.
This begs an important question: we know that the SM is not the complete picture, as it does not account for neutrino masses, dark matter, or the matter-antimatter asymmetry of the universe, and we know that many of the models trying to address these shortcomings rely on extended scalar sectors. How sure are we then that there are no additional scalars obtaining vevs that break the remaining $\mathrm{U}(1)_\mathrm{Q}$ of the SM?

Since we already have a broken U(1) gauge theory in the SM, we know that, at least in theory, it must be possible. However, as we have outlined above, electromagnetism is of long-standing interest and has, with time, become one of the best-tested theories we have in all of physics. We find ourselves in the era of precision measurements, and although there is some work on electric charge breaking (see [7] for a modern review), the theoretical landscape has not been studied in similar detail.
In this work, we want to make an effort to explore this landscape and, at the very least, bring some overlooked and underappreciated consequences of a broken electromagnetic gauge group to the readers' attention. We will be careful to be general in our approach but also compare with experimental limits wherever possible.
In chapter 2, we will extend the standard Higgs mechanism to additional scalars with arbitrary representations under the SM gauge group and investigate in what way they contribute to the gauge boson masses. The constraints that can be derived from this will lead us to examine two categories for representations: minicharged particles (chapter 3), which do not have any new interactions with SM fermions but still contribute to charge corrections for all particles, and representations that allow for new renormalizable interactions (chapter 4), which lead to new mass eigenstates of the fermions that we examine in the leptonic sector.

1.2. Theoretical Background

Before we begin our discussion of electric charge non-conservation, we want to briefly review some important aspects, namely the Higgs mechanism of the SM. More detailed information can be found in any introductory book on SM physics [8, 9, 10, 11].
The Higgs field H is a scalar with the representation[2] $(\mathbf{1}, \mathbf{2})_{1/2}$, i.e. we can write it as a doublet

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} . \tag{1.1}$$

[2]When we write *representation*, we always mean under the SM gauge group. It is given in the form (SU(3), SU(2))$_{\mathrm{U}(1)}$.

It is the only scalar in the SM, so the most general scalar potential we can write down is

$$V(H^\dagger H) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2 \,. \tag{1.2}$$

Any higher order term would need coupling constants with negative mass dimensions to keep the action $S = \int \mathrm{d}^4x\mathcal{L}$ dimensionless, and would hence spoil the renormalizability of the theory. We have two parameters here, μ and λ. We know that we must have $\lambda \geq 0$, otherwise the potential would be unbounded from below. The sign of μ^2, however, could be either positive or negative.

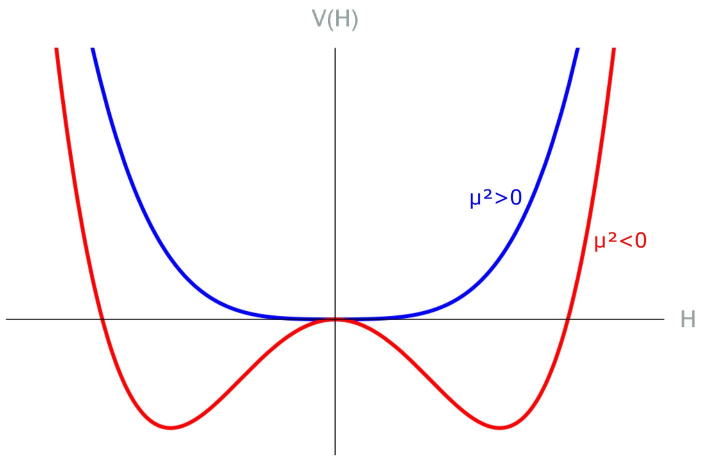


Figure 1.1.: Schematic plot of the Scalar potential with $\mu^2 > 0$ (blue) and $\mu^2 < 0$ (red).

These two cases are depicted schematically in Figure 1.1 for a scenario where H is a real scalar. In the case of $\mu^2 > 0$, the potential has a minimum at $H = 0$, as we expect from a regular quantum field. But when $\mu^2 < 0$, we can see that two distinct minima develop. Their position can be obtained by taking the derivative of the potential with respect to the field:

$$\frac{\partial}{\partial H} V(H^\dagger H)\bigg|_{H^\dagger H = v^2} = \mu^2 H^\dagger + 2\lambda H^\dagger (H^\dagger H) \stackrel{!}{=} 0 \quad \rightarrow \quad v^2 = \pm \frac{(-\mu^2)}{2\lambda} \,, \tag{1.3}$$

where v is the vev of the scalar. Although we can see from Figure 1.1 that the potential is still symmetric, the field will have to choose one of the minima at low energies and hence the symmetry will appear broken when close to the minimum. This scenario is easily generalized to the case of a complex scalar H. The two minima then become a circular trough around the central maximum, as we can see from the minimum condition

$$\sqrt{v^2} = \sqrt{\frac{-\mu^2}{2\lambda}} e^{i\theta} \equiv v e^{i\theta} \tag{1.4}$$

for some angle θ. There are two possible modes here: the field can move along the trough and stay in the minimum, corresponding to a massless particle, the Goldstone boson. The other is a movement in radial direction, where the potential changes, corresponding to a massive particle. We put this into formulae in the following way. We choose one of the components of the doublet to write the vev into. This choice is essentially arbitrary, as we can use a SU(2)×U(1) transformation to change this however we like. In this component, we expand around the vev as $H^0 = v + h$. The excitation h is identified with the massive radial mode and is a real field, the complex phase π is is naturally understood as an angle and hence identified with the massless angular mode. All of this can be put together nicely as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} e^{2i\frac{\pi^a \tau^a}{v}} . \tag{1.5}$$

The factors $\sqrt{2}$ and 2 appear for normalization of the real fields and $\pi^a\tau^a$ is the equivalent of the angular mode in SU(2)×U(1)-space with $\tau^a = \sigma^a/2$ the SU(2) generators.

Let us take a closer look at what this vev means for the gauge group structure. We reiterate that the Higgs has the SM representation $(\mathbf{1}, \mathbf{2})_{1/2}$, i.e. it transforms as

$$H \to e^{i\alpha^a\tau^a} e^{i\beta/2} H \tag{1.6}$$

for some free parameters α and β. But when we only look at the vev, this transformation looks like

$$\begin{aligned} \langle H \rangle \to & \begin{pmatrix} \cosh\frac{\alpha^1}{2} & \sinh\frac{\alpha^1}{2} \\ \sinh\frac{\alpha^1}{2} & \cosh\frac{\alpha^1}{2} \end{pmatrix} \begin{pmatrix} \cosh\frac{\alpha^2}{2} & \sin\frac{\alpha^2}{2} \\ -\sin\frac{\alpha^2}{2} & \cosh\frac{\alpha^2}{2} \end{pmatrix} \begin{pmatrix} e^{\alpha^3/2} & \\ & e^{-\alpha^3/2} \end{pmatrix} \begin{pmatrix} e^{\beta/2} & \\ & e^{\beta/2} \end{pmatrix} \langle H \rangle \\ &= \frac{v}{\sqrt{2}} e^{(\beta-\alpha^3)/2} \begin{pmatrix} \cosh\frac{\alpha^1}{2}\sin\frac{\alpha^2}{2} + \sinh\frac{\alpha^1}{2}\cosh\frac{\alpha^2}{2} \\ \sinh\frac{\alpha^1}{2}\sin\frac{\alpha^2}{2} + \cosh\frac{\alpha^1}{2}\cosh\frac{\alpha^2}{2} \end{pmatrix} , \end{aligned} \tag{1.7}$$

which is only invariant if $\alpha^1 = \alpha^2 = 0$ and $\alpha^3 = \beta$. In other words, there is a particular combination of generators $\tau^3 + \mathbb{1}$, that leaves the vev invariant. This generator is diagonal with one degree of freedom and hence belongs to the gauge group U(1). We have therefore seen that

$$\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y \ \to\ SU(3)_C \times U(1)_Q} . \tag{1.8}$$

What happens to the gauge bosons of these groups? To find out, let us examine the kinetic term of the Higgs

$$(D_\mu H)^\dagger (D^\mu H) . \tag{1.9}$$

We ignore the gluons for now, such that the covariant derivative is

$$D_\mu H = \left(\partial_\mu - ig\tau^a W_\mu^a - i\frac{g'}{2} B_\mu \right) H , \tag{1.10}$$

where W_μ^a are the gauge bosons of SU(2) and B_μ the one of U(1). When the Higgs field develops it's vev, this derivative becomes

$$\begin{aligned} D_\mu \langle H \rangle &= -\tfrac{1}{\sqrt{2}} \tfrac{i}{2} \begin{pmatrix} g'B_\mu + gW_\mu^3 & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & g'B_\mu - gW_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \\ &= -\tfrac{v}{\sqrt{2}} \tfrac{i}{2} \begin{pmatrix} g(W_\mu^1 - iW_\mu^2) \\ g'B_\mu - gW_\mu^3 \end{pmatrix} . \end{aligned} \tag{1.11}$$

We see here the combinations $W_\mu^1 - iW_\mu^2$ and $W_\mu^1 + iW_\mu^2$. Since all interactions of gauge bosons stem from some covariant derivatives, which in turn only contain the W bosons together with their corresponding generator, this means that W^1 and W^2 will only appear in one of those two combinations. We hence define

$$W_\mu^+ \equiv \frac{1}{\sqrt{2}}(W_\mu^1 - iW_\mu^2) \quad \text{and} \quad W_\mu^- \equiv \frac{1}{\sqrt{2}}(W_\mu^1 + iW_\mu^2)\ . \tag{1.12}$$

We now insert everything back into Equation 1.9 and find

$$(D_\mu\langle H\rangle)^\dagger(D^\mu\langle H\rangle) = \frac{v^2}{8}\left(2g^2W_\mu^+W^{-\,\mu} + (g'B_\mu - gW_\mu^3)^2\right) \tag{1.13}$$

The first term is a mass term of a complex particle, as $(W^+)^\dagger = W^-$ with mass $m_W^2 = \frac{1}{4}g^2v^2$. The second term, however, contains not only mass terms, but also terms of the form $\sim v^2W_\mu^3B^\mu$. They can be interpreted as an interaction with the vacuum that converts the two bosons into each other, that occurs all the time. This makes it difficult to speak of either of the two bosons, as their is no useful timescale on which we can say they are present for any process. We can remedy this by using the same trick we already employed for the W bosons. If we introduce two orthogonal fields as combinations of W^3 and B,

$$A_\mu = \sin\theta_w W_\mu^3 + \cos\theta_w B_\mu \quad \text{and} \quad Z_\mu = \cos\theta_w W_\mu^3 - \sin\theta_w B_\mu\ , \tag{1.14}$$

we find that the terms $\sim A_\mu Z^\mu$ disappear when the mixing angle satisfies

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \quad \text{and} \quad \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}\ . \tag{1.15}$$

Equation 1.9 now finally reduces to

$$(D_\mu\langle H\rangle)^\dagger(D^\mu\langle H\rangle) = \frac{v^2}{4}g^2W_\mu^+W^{-\,\mu} + \frac{v^2}{8}(g^2+g'^2)Z_\mu Z^\mu\ . \tag{1.16}$$

Note that the A_μ has vanished entirely, i.e. it corresponds to a massless boson, which we identify with the photon. The other massive particle is the familiar Z-boson.

We will briefly review and contextualize the most important aspects of this section. In the very early and hot universe, temperature corrections have to be taken into account (an introduction to finite-temperature field theory can be found in [12]). These corrections go with T^2 and contribute to the "mass term" of the Higgs potential (Equation 1.2) $\sim(\mu^2+\alpha T^2)$, for some α. We conjecture that μ^2 is negative, but with large enough corrections this term may be positive, such that the potential has a clear minimum, which the field assumes.
As the universe cools, the mass-squared term of the potential truly becomes negative and the potential changes it's form: the minimum now becomes a local maximum surrounded by equivalent minima, which break the symmetry when the field chooses one of them. Their is some residual symmetry left, which is the $\mathrm{U}(1)_\mathrm{Q}$ electromagnetic gauge group:

$$\mathrm{SU}(3)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y} \ \rightarrow\ \mathrm{SU}(3)_\mathrm{C}\times\mathrm{U}(1)_\mathrm{Q}\ . \tag{1.17}$$

This breaking of symmetry determines the properties of the gauge bosons and their mass comes from their interaction with the scalar Higgs field. The gauge bosons corresponding to the unbroken groups, the gluon for the $SU(3)_C$ and the photon for the $U(1)_Q$, remain massless.
This is everything we need to know to understand the relevant effects that arise when we add more scalars to the standard model.

Chapter 2. Charged vevs

2.1. Extension to multiple scalars

Let us begin by considering the electroweak symmetry group $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ and their associated gauge bosons W^i_μ and B_μ. We extend this case by a set of N complex Lorentz-scalars $\Phi_{i=1\ldots N}$ with n_i components each and label their eigenvalues under the 3rd generator of $\mathrm{SU(2)_L}$ and the generator of $\mathrm{U(1)_Y}$ as $m_{i\,a}$ and y_i, respectively (no summation over i):

$$\left(\hat{T}^3\,\Phi_i\right)_a = m_{i\,a}\,\Phi_{i\,a} \qquad , \qquad \hat{Y}\,\Phi_i = y_i\,\Phi_i\;. \tag{2.1}$$

The hypercharge of any n_i-plet is always the same for all components (i.e. $\hat{Y} \propto \mathbb{1}$), while every component has a different eigenvalue under $\hat{T}^3$.
According to representation theory we can always find an eigenbasis to $\hat{T}^3$ in which it acts as $\hat{T}^3 = \mathrm{diag}(j, j-1, \ldots, -j)$ (see e.g. chapter 5 of [13]). The j here is connected to the multiplicity $n = 2j+1$ of the $\mathrm{SU(2)_L}$-representation in analogy to spin systems of ordinary quantum mechanics, where it is usually referred to as the total angular momentum.
For example, we have $\hat{T}^3 = \mathrm{diag}(\frac{1}{2}, -\frac{1}{2})$ for a doublet **2** and $\hat{T}^3 = \mathrm{diag}(1,0,-1)$ for a triplet **3**, which we can again recognize as containing the allowed values for the spin in regular quantum mechanics.
We shall call this eigenbasis e_m, labelled by it's eigenvalue m. The different eigenvectors can be transformed into each other with the usual ladder operators

$$\hat{T}^+ = \hat{T}^1 + i\hat{T}^2 \qquad \text{and} \qquad \hat{T}^- = \hat{T}^1 - i\hat{T}^2\;; \tag{2.2}$$

they act analogously to their angular momentum counterparts:

$$\begin{aligned} \hat{T}^+ e_m &= \sqrt{j(j+1) - m(m+1)}\, e_{m+1} \\ \text{and}\quad \hat{T}^- e_m &= \sqrt{j(j+1) - m(m-1)}\, e_{m-1}\;. \end{aligned} \tag{2.3}$$

Indeed, the SM definitions of W^+_μ / W^-_μ give rise to

$$\begin{aligned} \hat{T}^1 W^1_\mu + \hat{T}^2 W^2_\mu &= \tfrac{1}{2}(\hat{T}^+ + \hat{T}^-)W^1_\mu + \tfrac{1}{2i}(\hat{T}^+ - \hat{T}^-)W^2_\mu \\ &= \tfrac{1}{2}(W^1_\mu - iW^2_\mu)\hat{T}^+ + \tfrac{1}{2}(W^1_\mu + iW^2_\mu)\hat{T}^- \\ &\equiv \tfrac{1}{\sqrt{2}}(W^+_\mu \hat{T}^+ + W^-_\mu \hat{T}^-)\;, \end{aligned} \tag{2.4}$$

showing that the ladder operators are the generators that correspond to the charged W-bosons, as they should be [8].

Our new scalar bosons may be written in this particular basis as $\Phi_i = \phi_{im} e_m$ for some appropriate functions ϕ_{im}. In the most general case and at sufficiently low energies, all of our scalar bosons may develop vevs, i.e. we can find minima satisfying $\Phi_i \Phi_i^\dagger = v_i^2/2$ for some real constant v_i (the factor 2 is conventional and chosen such that it matches the canonical normalization of the bosons fluctuating around the minima).
Crucially, the vev is constraining only the absolute value of the n_i-plets Φ_i and therefore allows for a global O$(2n_i)$ symmetry that allows us to freely choose which components to write our vevs in when broken to a O$(2n_i - 1)$; *a priori* nothing forbids us from writing them into electrically charged components.
Let us pick a component $\hat{m}$ to obtain a vev, such that we can write $\langle \Phi_i \rangle = \frac{v_i}{\sqrt{2}} e_{\hat{m}_i}$. Under the SM electric charge operator, given by the Gell-Mann-Nishijima formula $\hat{Q} = \hat{T}^3 + \hat{Y}$, any vev has an eigenvalue $\hat{m}_i + y_i$. If this value is non-zero for the vacuum, the corresponding gauge symmetry, i.e. the U(1)$_{\text{EM}}$, must have been spontaneously broken; much like in the SM Higgs mechanism, we expect to see this reflected in the mass of the corresponding gauge boson: the photon.
Similarly, if the vacuum is not an electroweak singlet, we also expect contributions to the W- and Z-boson masses.
In the following, we will derive just what these contributions to the masses look like and how much they constrain the model building.

2.2. Gauge boson masses

The masses of the gauge bosons originate from the kinetic terms of the scalar bosons. The Lagrangian under consideration here is

$$\mathcal{L}_{\text{Scalar, kin}} = (D_\mu \Phi_i)^\dagger (D^\mu \Phi_i) \ , \tag{2.5}$$

with the covariant derivative

$$\begin{aligned} D_\mu &= \partial_\mu - ig\hat{T}^i W_\mu^i - ig'\hat{Y} B_\mu \\ &= \partial_\mu - i\tfrac{g}{\sqrt{2}}(\hat{T}^+ W_\mu^+ + \hat{T}^- W_\mu^-) - ig\hat{T}^3 W_\mu^3 - ig'\hat{Y} B_\mu \ . \end{aligned} \tag{2.6}$$

Gauge boson mass terms arise naturally when the scalar bosons obtain non-zero vevs. In this case, the Lagrangian becomes (ignoring the fluctuations around the minima):

$$\mathcal{L}_{\text{Scalar, kin}} = \frac{v_i^2}{2} e_{\hat{m}_i}^\dagger \left(\tfrac{g}{\sqrt{2}}(\hat{T}^+ W_\mu^+ + \hat{T}^- W_\mu^-) + g\hat{T}^3 W_\mu^3 + g'\hat{Y} B_\mu \right)^2 e_{\hat{m}_i} \ . \tag{2.7}$$

Since our eigenbasis is orthonormal by construction, only combinations of generators that leave $e_{\hat{m}_i}$ invariant survive:

$$\mathcal{L}_{\text{Scalar, kin}} = \frac{v_i^2}{2} e_{\hat{m}_i}^\dagger \left(\tfrac{g^2}{2}(\hat{T}^+\hat{T}^- + \hat{T}^-\hat{T}^+) W_\mu^+ W_\mu^- + (g\hat{T}^3 W_\mu^3 + g'\hat{Y} B_\mu)^2 \right) e_{\hat{m}_i} \ . \tag{2.8}$$

The first term is responsible for contributions to the W mass, while the second term will result in photon and Z masses. Let us first look at the former, where the action of the generators given in Equation 2.3 can be inserted to obtain

$$\mathcal{L}_{\text{Scalar, kin}} \supset \frac{g^2 v_i^2}{2} \left(j_i(j_i+1) - \hat{m}_i^2 \right) W_\mu^+ W^{\mu -} . \tag{2.9}$$

For any SU(2)$_\text{L}$-non-singlet we add, we will have a contribution to the W mass

$$M_W^2 = \frac{g^2 v_i^2}{2} \left(j_i(j_i+1) - \hat{m}_i^2 \right) . \tag{2.10}$$

If we consider only the SM Higgs ($j = \frac{1}{2}$ and $\hat{m} = -\frac{1}{2}$), this indeed yields the correct result of $m_W^2 = \frac{1}{4} g^2 v^2$ [14]. Since the W^+/W^--bosons are comprised of only SU(2)$_\text{L}$ gauge bosons, their masses do not depend on the hypercharges of the scalars and we need to know only their representations and what component the vev lies in to compute them. This is important as we expect the term in brackets to be roughly $\mathcal{O}(1)$ and we can hence suppress any contributions in addition to the SM only via the vev. We will reflect on this more at the end of this section.

For now, let us continue by examining the remaining neutral gauge boson masses. They are determined by

$$\begin{aligned} \mathcal{L}_{\text{Scalar, kin}} &\supset \frac{v_i^2}{2} e_{\hat{m}_i}^\dagger \left(g \hat{T}^3 W_\mu^3 + g' \hat{Y} B_\mu \right)^2 e_{\hat{m}_i} \\ &= \frac{v_i^2}{2} \left(g^2 \hat{m}_i^2 (W_\mu^3)^2 + 2 g g' \hat{m}_i y_i W_\mu^3 B_\mu + g'^2 y_i^2 (B_\mu)^2 \right) . \end{aligned} \tag{2.11}$$

It is advantageous to write this expression in matrix form:

$$\begin{aligned} \mathcal{L}_{\text{Scalar, kin}} &\supset \frac{v_i^2}{2} \begin{pmatrix} B_\mu & W_\mu^3 \end{pmatrix} \begin{pmatrix} g'^2 y_i^2 & g g' \hat{m}_i y_i \\ g g' \hat{m}_i y_i & g^2 \hat{m}_i^2 \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} \\ &\equiv \frac{1}{2} \begin{pmatrix} B_\mu & W_\mu^3 \end{pmatrix} M_{\text{Gauge}} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} ; \end{aligned} \tag{2.12}$$

this way we will always have a 2×2 real symmetric matrix, which we can diagonalize with an orthogonal rotation matrix

$$R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} .$$

We can solve for the angle analytically by simply demanding that the off diagonal elements of $R M_{\text{Gauge}} R^T$ be zero. A more refined approach can be found e.g. in [15]. Applying the author's solution to our problem leaves us with:

$$\sin\theta = \frac{1}{\sqrt{2}} \left(1 + \sqrt{\frac{\kappa_1^2}{\kappa_1^2 + \kappa_2^2}} \right)^{1/2} , \quad \cos\theta = \frac{1}{\sqrt{2}} \left(1 - \sqrt{\frac{\kappa_1^2}{\kappa_1^2 + \kappa_2^2}} \right)^{1/2} , \tag{2.13}$$

where we have defined the model dependent parameters

$$\kappa_1 = \sum_i v_i^2 (g'^2 y_i^2 - g^2 \hat{m}_i^2) \quad \text{and} \quad \kappa_2 = 2gg' \sum_i v_i^2 y_i \hat{m}_i \, . \tag{2.14}$$

We made the sum explicit in this definition to stress that the terms in Equation 2.13 do not simplify due to the presence of a binomial formula.
If we only add electrically neutral scalars (i.e. $\hat{m}_i = -y_i \, \forall i$) the parameters κ_1 and κ_2 have a common factor $\sum_i v_i^2 y_i^2$ that drops out. We then get the usual SM formula for the sine and cosine of the Weinberg angle.
Another important observation is that we no longer have the SM relation $e = g \sin\theta = g' \cos\theta$ for the electric charge if we add charged vevs. Of course, this is to be expected since the $\mathrm{U(1)_{EM}}$ is now broken; the best we can do is to define two separate charges

$$e_1 \equiv g' \cos\theta \qquad \text{and} \qquad e_2 \equiv g \sin\theta \tag{2.15}$$

for the $\mathrm{U(1)_Y}$ and $\mathrm{SU(2)_L}$ part of the electric charge, respectively.
To keep our Lagrangian unchanged we also need to rotate our fields. With our above definitions, this amounts to

$$R \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} = \begin{pmatrix} \sin\theta \, W_\mu^3 + \cos\theta \, B_\mu \\ \cos\theta \, W_\mu^3 - \sin\theta \, B_\mu \end{pmatrix} . \tag{2.16}$$

We recognize the top and bottom components as the photon and Z-boson, respectively. They differ from their SM definitions only in the value of θ. The corresponding masses are given by the entries of the diagonal matrix obtained by the above rotation (i.e. the eigenvalues of the mass matrix), they can be written compactly in terms of the rotation angles:

$$M_A^2 = \sum_i v_i^2 \left(g' y_i \cos\theta + g \, \hat{m}_i \sin\theta \right)^2 , \tag{2.17}$$

$$M_Z^2 = \sum_i v_i^2 \left(g' y_i \sin\theta - g \, \hat{m}_i \cos\theta \right)^2 . \tag{2.18}$$

We can see that in the absence of any charge breaking vevs, where $\hat{m}_i = -y_i$ and $\theta = \theta_\mathrm{W}$, the photon mass correctly vanishes and the Z mass becomes $M_Z^2 = v_i^2 y_i^2 (g^2 + g'^2)$.

All of the quantities we have looked at up to now are of course very precisely measured. Chief amongst them is the photon mass, determined currently to be $M_A < 10^{-18}$ eV [16]. This puts a hefty constraint on our model and we need to explore the available parameter space carefully.
Figure 2.1 shows the allowed parameter space for the photon mass predicted by Equation 2.17 for the SM Higgs boson and an additional $\mathrm{SU(2)_L}$ singlet with a set hypercharge and vev. We see immediately that if the hypercharge is $\mathcal{O}(1)$ or larger, the vev has to be suppressed heavily to be roughly 10^{-18} eV or smaller, but if the hypercharge happens to be very small ($y \ll 1$), we may lift the vev to higher energies. For larger multiplets the parameter space looks analogous, only shifted

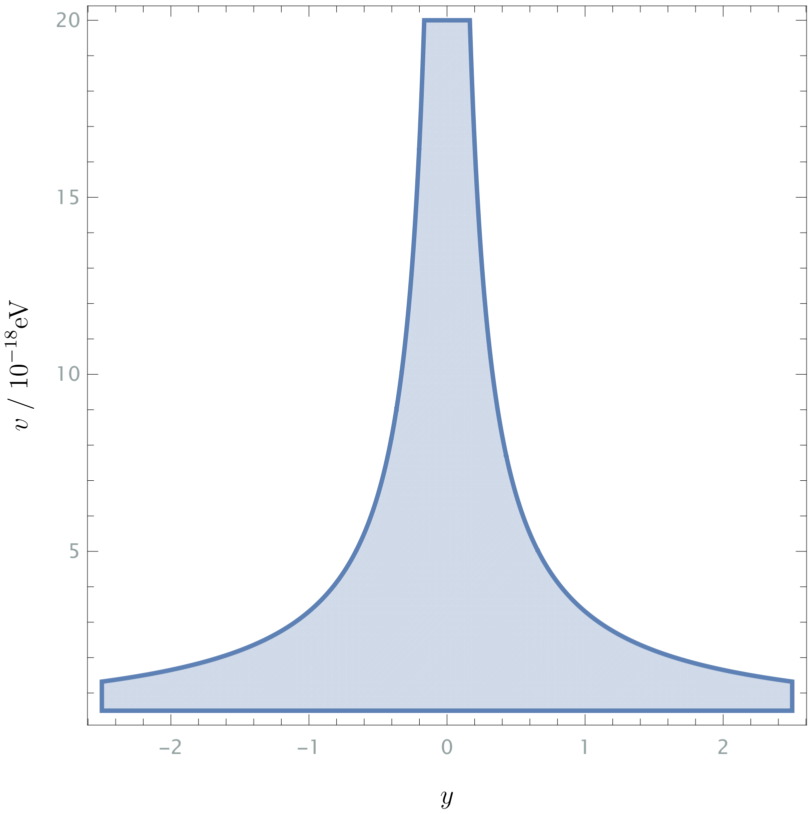


Figure 2.1.: Allowed parameter space for the photon mass in the case of the SM Higgs plus a scalar $SU(2)_L$ singlet with vev v and hypercharge y. The shaded region is compatible with the bounds from [16].

slightly lower and with the y-pole such that $y = -\hat{m}$. This is because here the component with the vev becomes neutral and the contributions to the photon mass vanish; in fact we should be able to make our vevs as large as we want if we also send $y \to -\hat{m}$.

Unfortunately there is a catch to this; we have already seen in Equation 2.10 that the W mass is independent of the hypercharge. Since all contributions to it's mass are necessarily positive and proportional to v^2, there is an absolute upper limit on any additional vevs that are not $SU(2)_L$ singlets. Figure 2.2 shows the maximum vev for any single scalar added to the SM Higgs that gives a W-mass still within one sigma of experimental results (not including the 2022 CDF result [17]). We see that, as a general trend, the larger the n-plet, the smaller the vev can be at most. In the most generous case of adding just another doublet (marked green in the figure), the vev still needs to be smaller than $\sim 50\,$GeV. We have to conclude that even if the hypercharge is very small, there still cannot be any charge breaking on scales higher than Higgs or even GUT, except if it is mediated by a singlet, which does not contribute to the W-mass. It is to be noted, though, that throughout this discussion we have assumed that the vev of the SM Higgs v_H keeps its value of $\sim 246\,$GeV. One might be able to raise the scale of symmetry breaking further by decreasing v_H, but

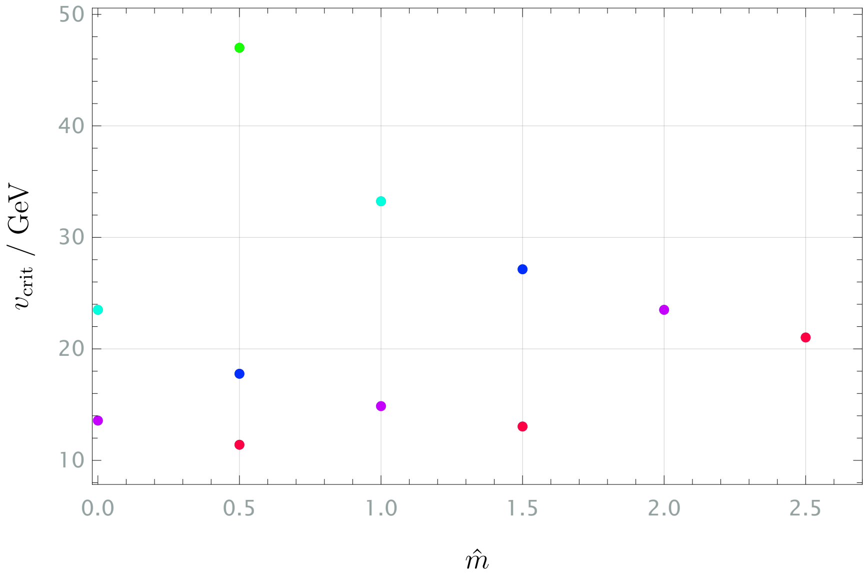


Figure 2.2.: The critical vev for a scalar multiplet (in addition to the SM Higgs) at which the contributions to the W mass exceed the one sigma range of current experimental bounds [14] ($M_W = 80.377(12)$ GeV, not including the more recent but significantly deviating CDF result [17]). The x-axis shows the $\hat{T}^3$ eigenvalue of the component the vev lies in and the color signifies what multiplet we have added (green – doublet, light blue – triplet, dark blue – 4-plet, purple – 5-plet, red – 6-plet). Results for negative $\hat{m}$ are mirrored.

this would also mean changing the Yukawa couplings to the fermions to keep their masses the same. We will not explore this case further here, but leave the possibility open to be explored in future work.

2.3. The ρ-parameter

We want to spend some time investigating the ρ-parameter at this point. It is defined by the equation

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta} \tag{2.19}$$

and determines the ratio of the W- and Z-masses [18]. In the standard model, this parameter is equal to one at tree level, where it is protected by the custodial symmetry; at loop-level this symmetry is broken and small corrections appear [19]. They match very well with the current experimental bound $\rho = 1.00038(20)$ [14]. In our model, we may write ρ as

$$\rho = \frac{g^2}{2\cos^2\theta} \frac{\sum_i v_i^2 \left(j_i(j_i+1) - \hat{m}_i^2\right)}{\sum_i v_i^2 \left(g' y_i \sin\theta - g\hat{m}_i \cos\theta\right)^2} . \tag{2.20}$$

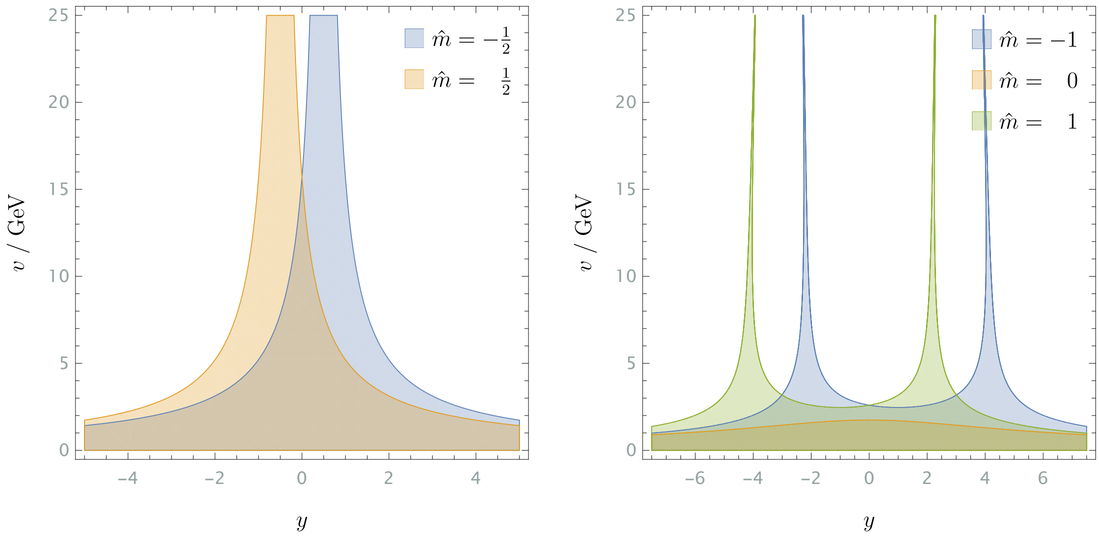


Figure 2.3.: The ρ-parameter for the SM Higgs and one additional scalar doublet (left) or triplet (right).

The question at hand is whether this leads to any further constraints on our new vevs. If all additional scalars are $SU(2)_L$ singlets, the ρ-parameter becomes

$$\rho = \frac{g^2}{\cos^2\theta}\left[(g'\sin\theta + g\cos\theta)^2 + 4g'^2\sin^2\theta\sum_i \frac{v_i^2}{v_H^2}y_i^2\right]^{-1} . \tag{2.21}$$

As before, any new vev can be arbitrarily large as long as the corresponding hypercharge is small enough. Since the limits on the photon mass are much stronger than the ones on the ρ-parameter, there are no further restrictions arising from precision measurements in this area.

What about scalar multiplets? Figure 2.3 shows the allowed parameter space for one additional doublet (left) or triplet (right). In the doublet case, there is a noticeable pole around $y = \pm\frac{1}{2}$ with the sign being opposite to the sign of the $\hat{T}^3$ eigenvalue. These cases occur in the well known Two-Higgs-doublet models (for a recent review see e.g. [20]). We can see in Equation 2.20 that for this particular combination of parameters (and indeed any N-Higgs-doublet model) the vevs drop out of the equation. In the limit of $v \gg v_H$ the ρ-parameter becomes that of a single doublet; the size of the parameter space then increases again as larger y can be compensated with $\cos\theta \to 0$ in the same limit.
This behaviour is only possible if the quantum numbers of the vev are such that the ρ-parameter is equal to one irrespective of the hypercharge. Equation 2.20 allows us to extract the condition

$$3\hat{m}^2 = j(j+1) . \tag{2.22}$$

The lowest order solution to this equation is the doublet with $\hat{m} = \pm\frac{1}{2}$ (**7**-plet with $\hat{m} = \pm 2$, **26**-plet with $\hat{m} = \pm\frac{15}{2}$, ...). Of course, with the constraint on the vevs from the W-mass we have discussed earlier, these cases are not particularly viable for us. Other configurations do not have any open parameter space for large vevs and at most have poles that asymptotically approach a specific hypercharge value (e.g. $y \approx \pm 3.234$ in the triplet case of Figure 2.3).

Although there is some interesting behaviour to be found in the ρ-parameter, we have to conclude that it does not provide any relevant constraints for our discussion. The $\mathrm{SU}(2)_\mathrm{L}$ singlet is limited much more by the photon mass and the ρ-parameter also allows for large vevs with small hypercharge. For higher multiplets the W-mass still blocks large vevs altogether.
It is clear that these are the two directions we will need to investigate from here: hypercharges very close to electric neutrality with reasonably high vevs (i.e. $v = \mathcal{O}(\mathrm{GeV})$) and hypercharges that allow for renormalizable interactions with other particles and very small vevs.

Chapter 3. Minicharges

In our construction of the multiplet basis in section 2.1 we have already seen that the eigenvalues of the $\mathrm{SU}(2)_{\mathrm{L}}$'s third generator $\hat{T}^3$ are discrete (integer multiples of $\frac{1}{2}$). The hypercharge, on the other hand, is the eigenvalue of the continuous $\mathrm{U}(1)_{\mathrm{Y}}$ group, where this is *per se* not the case. Invoking again the Gell-Mann-Nishijima formula $\hat{Q} = \hat{T}^3 + \hat{Y}$, we see that this means – even in the SM – that the electric charge may also take any continuous value.
It is therefore puzzling that nature seems to only allow for charges that are integer multiples of the down-quark charge. There are many models addressing this apparent inconsistency, the most prominent of which are Dirac's famous magnetic monopoles [21] and the embedding of the hypercharge $\mathrm{U}(1)_{\mathrm{Y}}$ into a larger gauge group such as the SU(5) GUT model [22].
However, as long as we lack experimental evidence for any of these mechanisms, we need to consider the possibility of particles with very small fractional charges, hereinafter referred to as minicharged particles (mCP). There already is a plethora of work, both experimentally and theoretically, on limits for such particles, especially in the context of dark matter candidates (for recent reviews see e.g. [23, 24]). We want to highlight that specifically the parameter space we are interested in here, i.e. $y \ll 1$ and $v = \mathcal{O}(\mathrm{GeV})$, is still largely unexplored.

In the following sections we will consider the phenomenological implications of both the minicharged scalar bosons we add to break the $\mathrm{U}(1)_{\mathrm{EM}}$, as well as the small charge corrections to the SM fermions that will arise from the symmetry breaking.

3.1. Electric charge in the broken phase

Before we continue we need to acknowledge that we cannot rigorously talk about the electric charge of particles in the phase where the electromagnetic gauge group is broken, as it is not a good quantum number anymore. We can, however, still talk about eigenvalues under the broken generator $\hat{Q}'$ of the $\mathrm{U}(1)_{\mathrm{EM}}$; when we refer to the electric charge of a particle from here on, we mean this coupling strength to the photon.
To put this into more quantitative terms, let us investigate the interaction between a photon and a generic fermion ψ, which stems from the action of the covariant derivative:

$$D_\mu \psi \supset \partial_\mu \psi - ig' B_\mu \hat{Y} \psi - ig W^3_\mu \hat{T}^3 \psi \,. \tag{3.1}$$

We have defined the photon and Z-boson in Equation 2.16 as certain mixed states of the B- and W^3-field. Thus, they in turn can be expressed as mixed states of the photon and Z. This allows us to rewrite the above definition of the covariant derivative into a more useful form in the broken phase:

$$D_\mu\psi \supset \partial_\mu\psi \, - i\left(g'\cos\theta\,\hat{Y} + g\sin\theta\,\hat{T}^3\right)A_\mu\psi \, - i\left(-g'\sin\theta\,\hat{Y} + g\cos\theta\,\hat{T}^3\right)Z_\mu\psi \,. \tag{3.2}$$

From this, we can read off the definition of the new electric charge

$$e'\hat{Q}' = g'\cos\theta\,\hat{Y} + g\sin\theta\,\hat{T}^3 \equiv e_1\hat{Y} + e_2\hat{T}^3 \,. \tag{3.3}$$

This is true regardless of the assumptions we make for our additional scalars, but we can make some good approximations if we assume only minicharged particles and $v \leq \mathcal{O}(\text{GeV})$ to get a better feeling about what orders of magnitude we deal with for the charge corrections.
We earlier defined the model dependent parameters κ_1 and κ_2 (cf. Equation 2.14) that determine the weak mixing angle. In the minicharge limit, where $y = -\hat{m} + \varepsilon$ for some small $\varepsilon > 0$, these parameters can be approximated as

$$\begin{aligned}\kappa_1 &\approx \left(\frac{v_H^2}{4} + \sum_i v_i^2\hat{m}_i^2\right)(g'^2 - g^2) - 2g'^2\sum_i v_i^2\varepsilon_i\hat{m}_i \,, \\ \kappa_2 &\approx -2gg'\left(\frac{v_H^2}{4} + \sum_i v_i^2(\hat{m}_i^2 - \varepsilon\hat{m}_i)\right) \,.\end{aligned} \tag{3.4}$$

Since $\varepsilon = y + \hat{m}$ is the deviation from the electrically neutral case, we may interpret it as the SM electric charge q of the scalar (in units of e). To first order in q, we can then approximate

$$\begin{aligned}\cos\theta &\approx \cos\theta_W\left(1 + 4\sin^2\theta_W\sum_i\frac{v_i^2}{v_H^2}q_i\hat{m}_i\right) \,, \\ \sin\theta &\approx \sin\theta_W\left(1 - 4\cos^2\theta_W\sum_i\frac{v_i^2}{v_H^2}q_i\hat{m}_i\right) \,.\end{aligned} \tag{3.5}$$

We explicitly differentiate between the *weak mixing angle* θ as derived in the previous chapter and the *Weinberg angle* θ_W, which is the SM limit of θ. The quantity measured in experiments is θ, although its value to current precision coincides with θ_W. Inserting back into Equation 3.3 gives us a good estimate for the corrected charge:

$$\begin{aligned}e'\hat{Q}' &\approx e\hat{Q} + 4e\sum_i\frac{v_i^2}{v_H^2}q_i\hat{m}_i(\sin^2\theta_W\hat{Y} - \cos^2\theta_W\hat{T}^3) \\ &\equiv e\left[\hat{Q} + \kappa_Q(\sin^2\theta_W\hat{Y} - \cos^2\theta_W\hat{T}^3)\right] \,.\end{aligned} \tag{3.6}$$

To keep our formulae clean, we have introduced another model-dependent parameter $\kappa_Q = 4\sum_i\frac{v_i^2}{v_H^2}q_i\hat{m}_i$.

As an example, consider a doublet scalar with $v = 3\,\text{Gev}$, $\hat{m} = -\frac{1}{2}$ and $q = 0.1\,e$ (which would naturally have a mass and charge such that it will likely be possible to exclude it with upcoming run-3 LHC data (specifically the MoeDAL-MAPP experiment [25, 26]), cf. Figure 3.1); this particle would be on the upper border of where our above approximations are valid and yield $\kappa_Q \approx -3.0 \cdot 10^{-5}$.
Since this represents the larger end of the not-quite excluded parameter space, we generically expect from the above numerical example that the charge corrections $|\Delta q|$ (i.e. the eigenvalue of $|e\hat{Q} - e'\hat{Q}'|$) are roughly of order $\mathcal{O}(10^{-4})\,e$. Although larger corrections can be constructed, e.g. by adding more scalars, they lead to too large deviations from the fermion charges, as we will discuss in the following section.

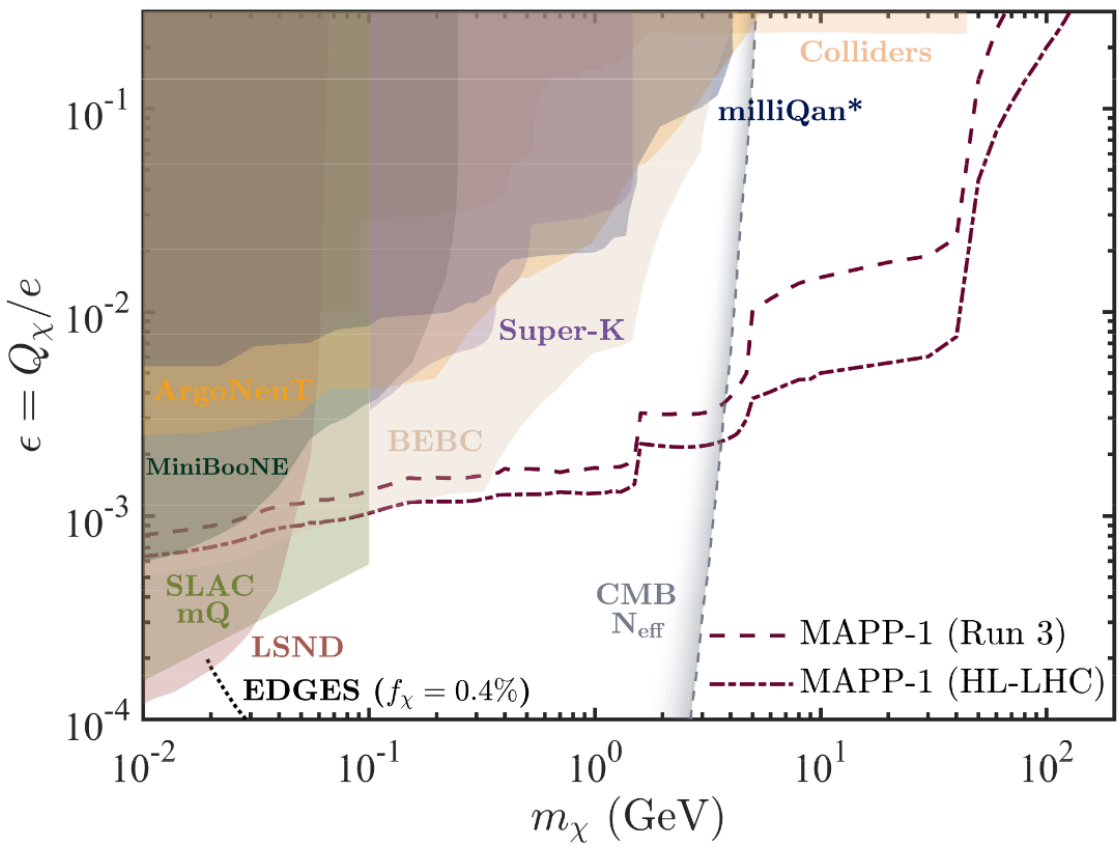


Figure 3.1.: Parameter-space for minicharged particles (χ), figure taken from [26]. The red dashed lines show the projected 95% confidence level exclusion limits for minicharged particles produced in various channels in the MAPP–1 detector.

3.2. Charge corrections to fermions

In 2019, the International System of Units (SI) was redefined to have all units derived from seven defining constants. These constants, in turn, are defined to have the value of their current best measurements. One of these constants is the elementary charge [27]

$$e = 1.602\,176\,634 \cdot 10^{-19}\,\text{C}\,.$$

This value has been measured numerous times over the years, first and most famously by Robert Andrews Millikan [28], for which he received the 1923 Nobel Price. In our framework, these measurements still hide a splitting between e_1 and e_2, or, equivalently, a difference in charge of left- and right-handed fermions. We therefore want to take the spirit of the above definitions of SI units and simply *define* the measured charge to be the value of the unbroken charge e and view e_1 and e_2 as deviations from this value. Note, however, that the elementary charge $e = g \sin\theta_W = g' \cos\theta_W$ and the charge of the electron are two conceptually different things. Despite that, it is still a sensible definition to make, as it allows us to easily compare our results with literature. Besides, the precision of e is such that $e \approx q'(e^-) \approx e_1 \approx e_2$ is a very good approximation.

Particle name	SM charge q $[e]$	Charge q' under $\hat{Q}'$	Approx. Δq $[\kappa_Q e]$
LH Electron e_L^-	-1	$-\frac{1}{2}(e_1 + e_2)$	$\frac{1}{2}(\cos^2\theta_W - \sin^2\theta_W)$
RH Electron e_R^-	-1	$-e_1$	$\cos^2\theta_W$
LH Neutrino ν_L	0	$-\frac{1}{2}(e_1 - e_2)$	$\frac{1}{2}$
LH u-Quark u_L	$+\frac{2}{3}$	$+\frac{1}{6}(e_1 + 3e_2)$	$-\frac{1}{6}(\cos^2\theta_W - 3\sin^2\theta_W)$
RH u-Quark u_R	$+\frac{2}{3}$	$+\frac{2}{3}e_1$	$-\frac{2}{3}\cos^2\theta_W$
LH d-Quark d_L	$-\frac{1}{3}$	$+\frac{1}{6}(e_1 - 3e_2)$	$-\frac{1}{3}\sin^2\theta_W$
RH d-Quark d_R	$-\frac{1}{3}$	$-\frac{1}{3}e_1$	$+\frac{1}{3}\cos^2\theta_W$

Table 3.1.: 1st generation SM fermions and their SM electric charge, their charge in the broken phase (i.e. under $\hat{Q}'$), and the correction to the electric charge Δq in the minicharge approximation of section 3.1.

In Table 3.1 we show the charges of the fermions in the SM, the exact charges in the broken phase, and the correction to the SM charge in units of the model parameter κ_Q as we have determined in the previous section.
This table clearly shows the difference between the left- and right-handed fermions as the latter get their charge only from the $\text{U}(1)_\text{Y}$. Another big takeaway is the charge of the previously neutral neutrino:

$$q'_\nu = -\frac{1}{2}(e_1 - e_2) \approx \frac{\kappa_Q}{2}\, e\,. \tag{3.7}$$

This puts a much stronger constraint on κ_Q than the direct minicharge searches we saw in the previous section, as the current best limits on the neutrino charge are $|q'_\nu| < 3.3 \cdot 10^{-12}\, e$ [29] (as measured by the CONUS collaboration; there exist stronger limits from CMB asymmetries [30], but we do not take them into account as a phase transition into the charge breaking phase could have taken place much more recently). In other words, we must have $|\kappa_Q| \lesssim 6.6 \cdot 10^{-12}$ to fit experimental data.
There are even wider reaching consequences, as many precision measurements of fermions are tied to the electric charge. Take, for example, the (spin-)magnetic moment.

On tree-level, it is a direct result from the coupling to the photon, as depicted in Figure 3.2. In the SM, the magnetic moment is a vector quantity determined by [8]

$$\vec{\mu} = g\frac{q}{2m}\vec{S} \; , \tag{3.8}$$

with g the Landé g-factor and $\vec{S}$ the spin of the particle. The electron, for example, has $|\mu_e| \approx -2\frac{e}{2m_e}\frac{\hbar}{2} = -\mu_\mathrm{B}$. In the broken phase, this gets a correction $|\mu'_e| = -\frac{q'_e}{e}\mu_\mathrm{B}$ by a small factor (the charge q'_e will be determined later and can be found in Table 3.2). Even more strikingly, the neutrino also has a tree-level contribution. Here, we now expect

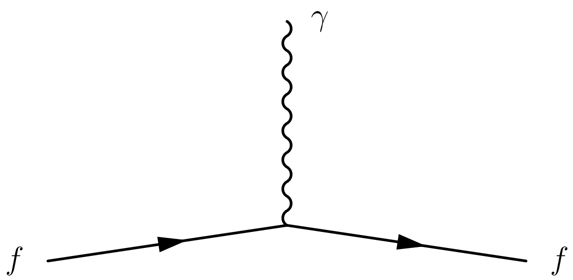


Figure 3.2.: Feynman-Diagram that gives rise to the tree-level magnetic moment of a fermion f.

$$|\mu_\nu| \approx -\frac{q'_\nu}{e}\frac{m_e}{M_\nu}\mu_\mathrm{B} \approx -\frac{\kappa_Q}{2}\frac{m_e}{M_\nu}\mu_\mathrm{B}. \tag{3.9}$$

M_ν here is the (effective) mass of the neutrino, regardless of the mass-giving mechanism. Currently the best limits on the neutrino mass fixes the mass ratio to $m_e/M_\nu \gtrsim 4.6 \cdot 10^6$ [14]. Overall, the best limit on the neutrino magnetic moment is $|\mu_\nu| < 6.4 \cdot 10^{-12}\,\mu_\mathrm{B}$ by XENONnT [31], which implies that more likely we have $|\kappa_Q| \lesssim 1.4 \cdot 10^{-18}$.
Another way to interpret Figure 3.2 with $f = \nu$ is the decay of the photon to neutrinos. Indeed, only the neutrino is a possible decay product in this interaction. We know from oscillation experiments that the second lightest neutrino has a mass of at least $8.7 \cdot 10^{-3}$ eV [32], which further kinematically disqualifies every particle except for the lightest neutrino (which could still be massless) as a possible photon decay product.
The Feynman rule of this vertex is the familiar $-iq\gamma^\mu$ with the charge given by Equation 3.7. This results in the matrix element

$$\mathcal{M} = \frac{1}{2}(e_1 - e_2)\epsilon_\mu(k)\,\overline{u}(p)\gamma^\mu v(p-k) \; , \tag{3.10}$$

where k is the momentum of the incoming photon and p that of the outgoing neutrino (cf. Figure 3.3).

We can extract observables from this using the usual techniques, i.e. squaring the matrix element, averaging over the spin-states of the incoming photon, summing over the spin states of the outgoing neutrinos, accounting for the phase space, etc. It is important not to forget that the photon now has mass ($k_\mu k^\mu = M_A^2$). This has an important consequence: there is an additional longitudinal polarization, similar to the other massive gauge bosons. The sum of polarizations therefore changes to

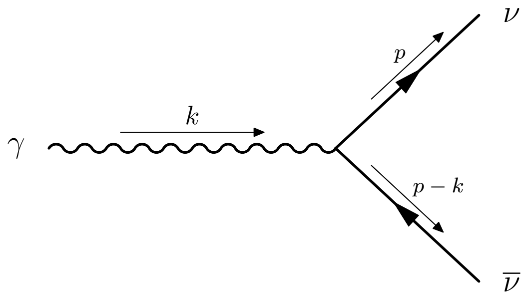


Figure 3.3.: Feynman-Diagram for photon decay into a neutrino anti-neutrino pair.

$$\epsilon_\mu(k)\,\epsilon^*_\nu(k) = -\eta_{\mu\nu} \longrightarrow \left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{M_A^2}\right) . \tag{3.11}$$

Crucially, this means that there is in general no smooth limit from the model in which the photon has mass to the SM. With these caveats in mind, we can determine the decay width for this process:

$$\Gamma = \frac{S}{48\pi} M_A (e_1 - e_2)^2 \sqrt{1 - 4\frac{M_\nu^2}{M_A^2}}(1 + 2\frac{M_\nu^2}{M_A^2}) . \tag{3.12}$$

The S accounts for the additional factor $\frac{1}{2}$ in the case of indistinguishable outgoing particles (i.e. $S = 1$ for Dirac-neutrinos and $S = \frac{1}{2}$ for Majorana-neutrinos). Naturally, Equation 3.12 is only defined when $M_A \geq 2M_\nu$.
Let us for simplicity assume that the lightest neutrino is massless. This lets us extract a very simple equation for the lifetime of the photon:

$$\tau_\gamma = \frac{48\pi}{S}\frac{1}{M_A(e_1 - e_2)^2} \approx \frac{48\pi}{S\,M_A}\frac{1}{\kappa_Q^2 e^2} . \tag{3.13}$$

With our previously determined limits of $|\kappa_Q| \lesssim 1.4 \cdot 10^{-18}$ and $M_A < 10^{-18}$ eV, we have at least $\tau_\gamma \gtrsim 1.75 \cdot 10^{25}$ Gyr in this decay channel. This is far above the current model-independent bounds for a photon in this mass-range $\tau_\gamma \gtrsim 2.7$ yr [33]. The fact that this limit is so small is because of the huge time dilation ($\mathcal{O}(10^{15})$), which makes the lifetime observed in the laboratory frame much larger. This highlights that the photon lifetime is not a useful quantity for us, in the sense that it is far off anything we can measure in the near future.

There is another important aspect we have yet to address. In Table 3.1 we have listed the charges of the left- and right-handed fermions separately, but the particles we see in nature are superpositions of both states. Of course the resulting state is not a charge eigenstate and it therefore does not have a well-defined charge as a system. We can nevertheless define an average charge and assign it to the particle in question. This can be motivated in the following way:

The transition between the LH and RH state of the particle occurs due to the mass term, e.g. $m_e(\overline{e_L}e_R + \overline{e_R}e_L)$. We can interpret this term as a continuous interaction with the vacuum that flips the particle's chirality. Both interactions have the same "coupling constant" m_e, i.e. neither of the two states should be preferred over the other.

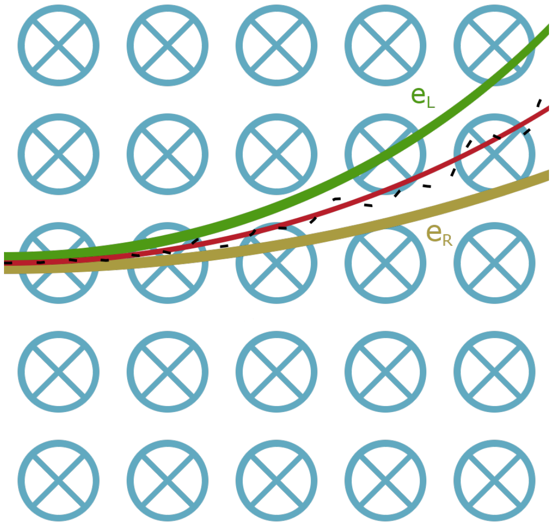


Figure 3.4.: Depiction of the path of an exclusively left-handed (green line) and right-handed (yellow line) electron in a magnetic field. The real path (black dashed line), that is determined by the interaction with the magnetic field, lies in-between these two cases and approaches the average between the two cases (red line).

Consider an electron in a magnetic field (cf. Figure 3.4). If it were purely left- or right-handed, we could predict the distinct path it would follow. When the chirality flips, the path changes to be in-between these two cases. With an increasing number of flips, the path naturally approaches the average between the exclusively left- and right-handed path. Conversely, this path is the one followed by a particle with a charge that is the average of the edge-cases.
Applying this to the electron, we get

$$q'(e^-) = \frac{1}{2}\Big(q'(e_L) + q'(e_R)\Big) = -\frac{1}{4}(3e_1 + e_2). \tag{3.14}$$

In the SM limit $e_1, e_2 \to e$, this does indeed give the expected result. In this manner we can also define the charges for the other fermions, as can be seen in Table 3.2.
A couple of comments are in order. Firstly, the charge of the neutrino in the table is the same as the charge of the purely LH neutrino. This is because we do not consider RH neutrinos, as they have not been experimentally confirmed thus far. However, the charge of the neutrino from this mechanism is actually dependent on the nature of the neutrino; the given value of $q'_\nu = -\frac{1}{2}(e_1 - e_2)$ is for Majorana neutrinos (by which we mean only the LH part exists). For a Dirac neutrino, as the RH part is a full singlet and remains uncharged even in the broken phase,

Particle name	SM charge q $[e]$	Charge q' under $\hat{Q}'$
Electron e^-	-1	$-\frac{1}{4}(3e_1 + e_2)$
Neutrino ν	0	$-\frac{1}{2}(e_1 - e_2)$
u-Quark u	$+\frac{2}{3}$	$\frac{1}{12}(5e_1 + 3e_2)$
d-Quark d	$-\frac{1}{3}$	$-\frac{1}{12}(e_1 + 3e_2)$

Table 3.2.: 1^{st} generation SM fermions, their SM electric charge, and their charge in the broken phase.

our procedure of assigning an average charge would give an additional factor $\frac{1}{2}$, resulting in $q'_\nu = -\frac{1}{4}(e_1 - e_2)$.
We can also use these charges to estimate e_1 and e_2. Comparing the electron and neutrino charges leaves us with $-e_1 = q'(e^-) + q'(\nu)$ and $-e_2 = q'(e^-) - 3q'(\nu)$ for Dirac neutrinos. This justifies our definition of the measured elementary charge e as the unbroken electric charge, as indeed the best limits on the neutrino charge (cf. [29]) tell us that $|e_2 - e_1| = 4|q'(\nu)| < 1.3 \cdot 10^{-11}\, e$. The splitting of electric charge into its higher gauge group components therefore occurs some orders of magnitude below current experimental precision. In the Majorana case, this limit is even tighter (by a factor of 2).

Some further quantities that can be obtained from Table 3.2 have to do with neutrality of matter. The most abundant element in the universe is hydrogen, consisting of a proton and an electron. We should check that it is neutral (or at least have a small enough charge). The charge of the proton is a result of the charge of its constituent quarks:

$$q'(p) = 2q'(u) + q'(d) = \frac{1}{4}(3e_1 + e_2). \tag{3.15}$$

Luckily, this is indeed the opposite charge of the electron, ensuring the large-scale neutrality of elementary hydrogen. However, the situation changes once we introduce neutrons (from isotopes or heavier elements). For neutrons we have

$$q'(n) = 2q'(d) + q'(u) = \frac{1}{4}(e_1 - e_2). \tag{3.16}$$

Interestingly, this is the opposite charge to Dirac neutrinos. The current bounds on the neutron charge[1] are $q'_n = (-0.4 \pm 1.1) \cdot 10^{-21}\, e$ [34]. Since properties of the neutron are far easier to measure than the neutrino's, this limit is much more stringent than the neutrino charge. But since analytically they are given by the same expression (up to a potential factor $\frac{1}{2}$), we can use this to give a much stronger bound also on q'_ν. Concerning our model dependent parameter κ_Q, this translates to even stronger limits than from the neutrino magnetic moment:

$$-3.158 \cdot 10^{-21} \leq \kappa_Q \leq 1.474 \cdot 10^{-21}\ . \tag{3.17}$$

[1] There are more recent and precise bounds (cf. [14]), but they generally assume charge conservation in some form or another, most often in the form of neutrality of β-decays.

Additionally, there are a lot of astrophysical and cosmological consequences to consider, like the overall charge of the universe, electromagnetic contributions to dark energy and dark matter, changes in electromagnetic fields of stars etc., but they are beyond the scope of this work.

3.3. Other charge corrections

Having established how the charges of our fermions change when introducing charged vevs, we can further establish charges for other objects like gauge bosons. We do this by inference from their interactions with other particles. For example, the W^--boson interacts with pairs of $\overline{u}$- and d-Quarks or electrons and anti-neutrinos, respectively (cf. Figure 3.5).

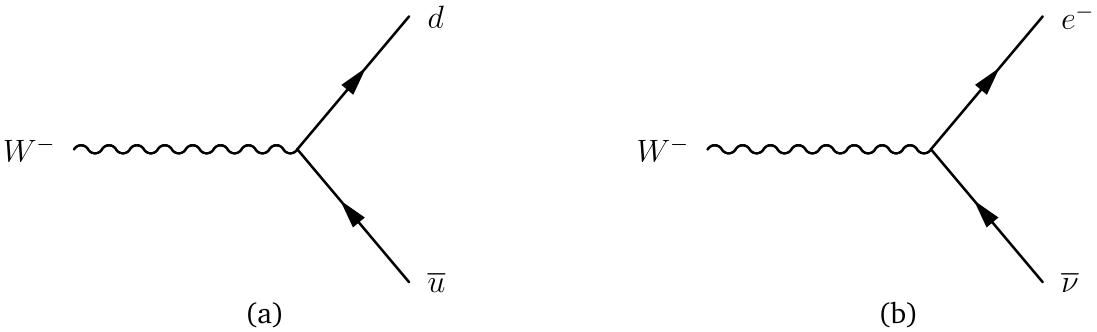


Figure 3.5.: Interactions involving the W^--boson. On tree-level and with only 1st generation fermions, there are two distinct vertices: quark-interactions (a) and lepton-interactions (b).

This allows us define $q'(W^-)$ by reconstructing the flow of charge in the diagram; in this way Figure 3.5a yields

$$q'(W^-) = q'(d_\mathrm{L}) - q'(u_\mathrm{L}) = -e_2 \ , \tag{3.18}$$

where we need to take into account that the W-bosons only interact with the LH fermions. As we would expect, the charge is therefore only dependent on e_2, i.e. the $\mathrm{SU}(2)_\mathrm{L}$ part of the electric charge. Figure 3.5b confirms this:

$$q'(W^-) = q'(e_\mathrm{L}) - q'(\nu_\mathrm{L}) = -e_2 \ . \tag{3.19}$$

By the same logic we will also get $q'(W^+) = +e_2$. All bosons that mediate a neutral current stay uncharged, as their interactions can be written as the annihilation of a fermion and an anti-fermion.

We have collected all the gauge bosons and their charges in Table 3.3. The only remaining object we need to determine the charge of is the vacuum itself. We attempt do this in the same manner; previously we noted that the fermions interact with the vacuum to flip their chirality and therefore define it's charge to be the

Particle name	SM charge q $[e]$	Charge q' under $\hat{Q}'$
Photon γ	0	0
Z-Boson Z	0	0
W^--Boson W^-	-1	$-e_2$
W^+-Boson W^+	$+1$	$+e_2$
Gluon g	0	0

Table 3.3.: SM gauge bosons, their SM electric charge, and their charge in the broken phase.

difference between the two charges of the fermions' LH and RH state. For instance, we have

$$q'(\Omega) = q'(e_{\mathrm{L}}) - q'(e_{\mathrm{R}}) = \tfrac{1}{2}(e_1 - e_2) \;, \tag{3.20}$$

where Ω represents the vacuum. Of course, this process can also take place in the opposite direction. By our logic, this would give the vacuum the opposite charge. Both interactions need to be able to take place, so we cannot give one definite charge to the vacuum. Although this means that we have to treat the vacuum as a charge non-eigenstate, we can still determine the magnitude of the charge breaking effects it induces as an effective charge in interactions. Indeed, this number comes out consistently for all fermions:

$$|q'(\Omega)| = \frac{1}{2}(e_1 - e_2) \;. \tag{3.21}$$

To conclude this section, we want to highlight again that in contrast to the LH and RH chiral fermions, whose charges can be determined from their representations directly, the charges of the fermions in Dirac-representation and the gauge bosons are definitions that we have given to states that are not necessarily charge-eigenstates. They are still useful quantities, especially in the context of phenomenology, but it is important to keep in mind that, fundamentally, we are talking about the quantum number of a broken symmetry. But our approach is still valid, as even in the broken phase charge is still conserved at the vertex, if you include interactions with the vacuum (like with the $\mathrm{U}(1)_{\mathrm{Y}}$ in the SM).
We will continue to use the charges we have determined in this chapter; they will serve as a baseline when we deal with new interactions. For example, as we will see in the next chapter, choosing certain representations can lead to interactions like e.g. $e^- \to \gamma\,\nu$. This throws a wrench into our previous logic of inferring charges from interactions, because they depend on the vevs in more complex ways, e.g. through lepton mixing. We will *define* the charges of the involved particles as the ones listed in Table 3.1-Table 3.3, where our way of determining charges makes sense, and label any interaction where the influence of the vev is obscured through other mechanisms and charge conservation is visibly broken at the vertex as *strongly charge breaking*. We are free to do this, as in the broken phase matters pertaining to electric charge amount to semantics and, as long as treated consistently, do not come into conflict with physical observations.

Chapter 4. New Interactions

In chapter 2 we have seen that in order for our charge breaking model to be compatible with experimental measurements, there are two possible directions to explore:

1. Large vevs ($v = \mathcal{O}(\text{GeV})$) with very small charges ($q \ll e$)

2. Very small vevs ($v \ll$ eV) and large charges ($q \sim e$)

The first option was examined in chapter 3. We have seen how we can define charges in the broken phase and some of the phenomenological effects that are to be expected. All of these consequences will still hold in what follows, but we are able to expand upon them. By turning to the second of the two above points, it becomes possible to introduce genuinely new interactions, instead of just making corrections to the SM. For example, by introducing a scalar with a certain representation and charge $q = +e$, it is possible for the electron to interact directly with the neutrino via the vev. This is different from vertices like $\sim \gamma\bar{\nu}\nu$, which appear from corrections that prevent $g \sin\theta$ and $g' \cos\theta$ from cancelling exactly. The price we have to pay for this is the abandonment of eV scale vevs.
But as we will see, the richness of possible phenomenology makes this a trade-off worth considering. For example, the mixing of charged and uncharged leptons that becomes possible with accordingly chosen vevs allows for all kinds of interactions between fermions and gauge bosons that would have been forbidden before.
Before we get to the phenomenological part though, we will begin in the following section by systematically examining all kinds of scalars we can add that lead to new interactions with fermions.

4.1. Representations that lead to new interactions

The standard model fermions consist of the two left-handed SU(2) doublets and three right-handed SU(2) singlets shown in Figure 4.1. We want to add new interactions between these fermions and potential new scalars. In order to do so, it is of some importance to first establish what our demands on such interactions are.
Our model is an extension to the SM with light degrees of freedom and as such we want to treat it as fundamental at this energy scale, as opposed to an effective theory. This means that we should only consider renormalizable interactions; fermions have mass dimension $3/2$ and scalars have 1, hence the relevant objects for us to study are Yukawa-type fermion-fermion-scalar interactions. Furthermore, we need

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_{\mathrm{L}} : (\mathbf{1},\mathbf{2})_{-1/2} \quad , \quad Q = \begin{pmatrix} u \\ d \end{pmatrix}_{\mathrm{L}} : (\mathbf{3},\mathbf{2})_{1/6}$$

$$e_{\mathrm{R}} : (\mathbf{1},\mathbf{1})_{-1} \quad , \quad u_{\mathrm{R}} : (\mathbf{3},\mathbf{1})_{2/3} \quad , \quad d_{\mathrm{R}} : (\mathbf{3},\mathbf{1})_{-1/3}$$

Figure 4.1.: SM fermions and their gauge transformation behaviour denoted as $(\mathrm{SU}(3), \mathrm{SU}(2))_{\mathrm{U}(1)}$.

to be able to write down invariant terms in the unbroken Lagrangian, where we demand that all SM gauge group charges are conserved. However, we do allow for the breaking of lepton and baryon number, as they stem from accidental symmetries that need not necessarily be exact. Finally, we will assume that Lorentz invariance holds. Although there has been considerable work on Lorentz breaking theories in the past and present, see e.g. [35] for a recent introduction and review, there is little thematic overlap with our discussion of a broken $\mathrm{U}(1)_{\mathrm{EM}}$.

In summary, we are interested in Yukawa-type interactions that preserve the SM gauge group and Lorentz-invariance in the unbroken phase, but not necessarily lepton and baryon number. There are only finitely many fermions and therefore finitely many combinations of two of them. All we need to do then is to choose a representation for a scalar that makes the combination of the three particles a full singlet. As an example, consider two copies of the doublet L. Together, they transform as

$$(\mathbf{1},\mathbf{2})_{-1/2} \otimes (\mathbf{1},\mathbf{2})_{-1/2} = (\mathbf{1},\mathbf{1})_{-1} \oplus (\mathbf{1},\mathbf{3})_{-1} \; . \tag{4.1}$$

For now, let us only consider singlets and fundamental representations of SU(2) and SU(3). Then we need to add a scalar that transforms as $(\mathbf{1},\mathbf{1})_1$, which we call χ_1, to form an invariant.
Using the Lorentz-transformation properties of left-handed and right-handed spinors (cf. [9]),

$$\begin{aligned} \psi_{\mathrm{L}} &\mapsto e^{\frac{1}{2}(i\theta_j - \beta_j)\sigma_j}\, \psi_{\mathrm{L}} \\ \psi_{\mathrm{R}} &\mapsto e^{\frac{1}{2}(i\theta_j + \beta_j)\sigma_j}\, \psi_{\mathrm{R}} \; , \end{aligned} \tag{4.2}$$

it is easy to show that the only allowed combinations are

$$\overline{\psi_{\mathrm{L}}}\psi_{\mathrm{R}} \quad , \quad \overline{\psi_{\mathrm{R}}}\psi_{\mathrm{L}} \quad , \quad \psi_{\mathrm{L}}^T\sigma_2\psi_{\mathrm{L}} \quad , \quad \psi_{\mathrm{R}}^T\sigma_2\psi_{\mathrm{R}} \; . \tag{4.3}$$

To clear up our notation, we define the inner product between doublets

$$LL = L_\alpha L_\beta \epsilon^{\alpha\beta} = iL^T\sigma_2 L = \nu_{\mathrm{L}} e_{\mathrm{L}} - e_{\mathrm{L}}\nu_{\mathrm{L}} = 2\nu_{\mathrm{L}} e_{\mathrm{L}} \; , \tag{4.4}$$

which identical to the inner product of Weyl spinors, which guarantees the Lorentz-invariance of the term $\nu_{\mathrm{L}} e_{\mathrm{L}}$. We can hence see that $\chi_1 LL$ is indeed an invariant.

Table 4.1 shows all scalars and interactions that can be obtained this way. We have given the names ϕ, ρ, η, and ξ to the scalars that give rise to multiple different

Scalar	Representation	Charge q $[e]$	Interaction Terms
χ_1	$(\mathbf{1},\mathbf{1})_1$	$+1$	$\chi_1 LL$
χ_2	$(\mathbf{1},\mathbf{1})_2$	$+2$	$\chi_2 e_\mathrm{R} e_\mathrm{R}$
χ_3	$(\mathbf{3},\mathbf{1})_{2/3}$	$+2/3$	$\chi_3 d_\mathrm{R} d_\mathrm{R}$
χ_4	$(\mathbf{3},\mathbf{2})_{1/6}$	$\begin{pmatrix}+2/3\\-1/3\end{pmatrix}$	$\chi_4 \overline{d_\mathrm{R}} L$
ϕ	$(\mathbf{1},\mathbf{2})_{1/2}$	$\begin{pmatrix}+1\\0\end{pmatrix}$	$\phi Q \overline{u_\mathrm{R}}$ $\tilde{\phi} Q \overline{d_\mathrm{R}}$ $\tilde{\phi} L \overline{e_\mathrm{R}}$
ρ	$(\overline{\mathbf{3}},\mathbf{2})_{7/6}$	$\begin{pmatrix}+5/3\\+2/3\end{pmatrix}$	$\rho L \overline{u_\mathrm{R}}$ $\tilde{\rho} Q \overline{e_\mathrm{R}}$
η	$(\overline{\mathbf{3}},\mathbf{1})_{1/3}$	$+1/3$	ηLQ $\eta e_\mathrm{R} u_\mathrm{R}$ $\tilde{\eta} QQ$ $\tilde{\eta} u_\mathrm{R} d_\mathrm{R}$
ξ	$(\overline{\mathbf{3}},\mathbf{1})_{4/3}$	$+4/3$	$\xi e_\mathrm{R} d_\mathrm{R}$ $\tilde{\xi} u_\mathrm{R} u_\mathrm{R}$

Table 4.1.: All scalars with fundamental or singlet representations under SU(2) and SU(3) that couple to SM fermions, along with their charge in the unbroken phase and the interaction terms they induce. The inner product in SU(2) and SU(3) space, where needed, is implied. The tilde ˜ signifies the conjugate representation.

interactions and call the representations that are involved in only one interaction $\chi_1 - \chi_4$. Note that ϕ has the same representation as the SM Higgs and gives mass to the up- and down-quark, as well as the electron, when it's vev lies in the neutral component. We can also put the vev into the charged component, which results in several mixing terms between quarks and leptons, respectively.
We want to stress again that this is not an exhaustive list of all possible scalars that fit the requirements we have laid out earlier; there are many higher dimensional representations that work just fine and have merely been omitted for the sake of brevity. For example, Equation 4.1 already tells us that there exists a scalar with representation $\Sigma : (\mathbf{1},\mathbf{3})_1$ that matches our criteria. This particle, which is famously added in the type-2 seesaw model, can generate Majorana masses for not only the neutrino (when forgoing lepton number conservation), but also for the electron with the doubly-charged component.

The scalars from Table 4.1 are well-known and have a vast amount of phenomenology and literature associated with them, especially in the context of non-standard neutrino interactions (see e.g. [36, 37]). Their role in charge non-conserving interactions is, however, rarely considered.
In this chapter, we will be studying the effects of these scalars developing vevs. We will demonstrate them in a simplified environment by only considering the electroweak sector, i.e. we limit ourselves to the scalars χ_1, χ_2, ϕ, and Σ. The quark sector, where we can surely expect limits from proton and neutron stability, among other effects, will be left for a future work.

4.2. Mass Eigenbasis

We now deal with the following Yukawa terms:

$$\mathcal{L}_Y \supset y_1\,\chi_1 LL + y_2\,L\Sigma L + y_3\,\chi_2 e_\mathrm{R} e_\mathrm{R} + y_4\,L\tilde{\phi}\overline{e_\mathrm{R}} + h.c. \tag{4.5}$$

We neglect non-diagonal interactions in flavour-space and hence, without loss of generality, only consider one generation. This means that the Yukawa couplings are complex numbers $y_i \in \mathbb{C}$ instead of 3×3 matrices. When the temperature drops low enough and the scalars develop vevs, we can see that some of these terms become mass terms. For example, when ϕ has a vev in the neutral component we get the electron Dirac mass term as in the SM; when χ_2 obtains a vev we get a Majorana mass term for the RH electron. But other terms give rise to mixing between the leptons: $y_1\chi_1 LL \mapsto 2y_1 v_\chi^+\,\nu_\mathrm{L} e_\mathrm{L}$[1].

As with a regular Dirac mass term, this can be interpreted as a continuous interaction with the vacuum that switches between the two leptons. In order to have objects that we can work with, we need to rotate into a basis where this is no longer the case, called the *mass eigenbasis* (cf. Figure 4.2).

Before we perform this rotation, let us spend some time thinking about how best to work with mass terms. In the standard model, we have a Dirac electron e_D with a mass term $m\,\overline{e_\mathrm{D}}e_\mathrm{D}$. It has a left-handed and a right-handed part $e_\mathrm{D} = (e_\mathrm{L}, e_\mathrm{R})$, such that we can write

$$m\,\overline{e_\mathrm{D}}e_\mathrm{D} = m(\overline{e_\mathrm{L}}e_\mathrm{R} + \overline{e_\mathrm{R}}e_\mathrm{L}) = \begin{pmatrix}\overline{e_\mathrm{L}} & \overline{e_\mathrm{R}}\end{pmatrix}\begin{pmatrix} & m \\ m & \end{pmatrix}\begin{pmatrix}e_\mathrm{L} \\ e_\mathrm{R}\end{pmatrix} \equiv \overline{e_\mathrm{D}}\mathcal{M}e_\mathrm{D}\,. \tag{4.6}$$

We can conclude that in this form the masses are given by the off-diagonal elements of the mass matrix. However, there is a problem with this representation: the diagonal terms correspond to objects like $\overline{e_\mathrm{L}}e_\mathrm{L}$, which are not Lorentz invariant. Therefore, it is not obvious how to add the Majorana mass terms we need in our formalism.

[1]We label the vevs here according to the following convention: the subscript denotes the representation as shown in Table 4.1, while the superscript gives the charge of the component the vev lies in.

Instead, let us borrow notation from the type-1 seesaw mechanism. Here, we define the left-handed object $e'_{\mathrm{D}} = (e_{\mathrm{L}}, {e_{\mathrm{R}}}^c)$. With this notation, we can write the mass term as

$$
\begin{aligned}
-\mathcal{L}_{\text{mass}} \supset\ & \frac{1}{2}\overline{e'^{\,c}_{\mathrm{D}}}\mathcal{M}e'_{\mathrm{D}} + h.c. \\
=\ & \frac{1}{2}\begin{pmatrix}\overline{e_{\mathrm{L}}{}^c} & \overline{e_{\mathrm{R}}}\end{pmatrix}\begin{pmatrix} & m \\ m & \end{pmatrix}\begin{pmatrix} e_{\mathrm{L}} \\ {e_{\mathrm{R}}}^c\end{pmatrix} + h.c. \\
=\ & \frac{1}{2}m(\overline{e_{\mathrm{L}}{}^c}{e_{\mathrm{R}}}^c + \overline{e_{\mathrm{R}}}e_{\mathrm{L}}) + h.c. \\
=\ & \frac{1}{2}m(-{e_{\mathrm{L}}}^T\mathcal{C}^{-1}\mathcal{C}\overline{e_{\mathrm{R}}}^T + \overline{e_{\mathrm{R}}}e_{\mathrm{L}}) + h.c. \\
=\ & \frac{1}{2}m(\overline{e_{\mathrm{R}}}e_{\mathrm{L}} + \overline{e_{\mathrm{R}}}e_{\mathrm{L}}) + h.c. = \overline{e_{\mathrm{D}}}\mathcal{M}e_{\mathrm{D}}\ .
\end{aligned}
\tag{4.7}
$$

In this calculation we have made use of the particle-antiparticle conjugation operator $\psi^c = \mathcal{C}\overline{\psi}^T$ and it's identities that can be found along with physical background e.g. in [38]. Equation 4.7 makes it obvious that both formulations of the mass terms are equivalent for standard Dirac fermions. But now, the diagonal terms are of the form $\overline{e_{\mathrm{L}}{}^c}e_{\mathrm{L}}$, which has the structure of a Majorana mass term.

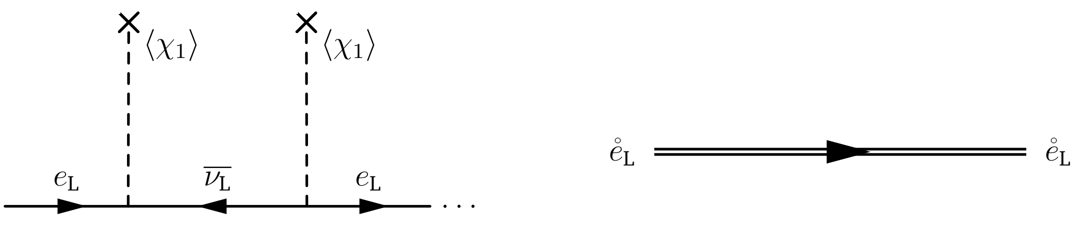
This formalism can now be extended to encompass the entire leptonic sector by including the neutrino. We define the left-handed trispinor

$$
\Psi = \begin{pmatrix} e_{\mathrm{L}} \\ {e_{\mathrm{R}}}^c \\ \nu_{\mathrm{L}} \end{pmatrix}
\tag{4.8}
$$

and use it to construct a general mass term

$$
-\mathcal{L}_{\text{mass}} = \tfrac{1}{2}\overline{\Psi^c}\mathcal{M}\Psi + h.c. = \tfrac{1}{2}\overline{\Psi^c}\begin{pmatrix} M_{\mathrm{L}} & m_{\mathrm{D}} & m_{\mathrm{LL}} \\ m_{\mathrm{D}} & M_{\mathrm{R}} & m_{\mathrm{RL}} \\ m_{\mathrm{LL}} & m_{\mathrm{RL}} & M_\nu \end{pmatrix}\Psi + h.c.
\tag{4.9}
$$

Keep in mind that we are still considering only one generation, so the entries of the matrix are scalars and not matrices. Furthermore, including the possible CP-violating phases would once again go beyond the scope of this work, so we take the



(a) (b)

Figure 4.2.: The mixing term $\sim e_{\mathrm{L}}\nu_{\mathrm{L}}$ represents the continuous interaction with the vacuum (a). After rotating into the new basis, we can introduce a particle $\mathring{e}_{\mathrm{L}}$ that is a superposition of e_{L} and ν_{L}. This fermion is now a mass eigenstate and can propagate without changing it's nature (b).

entries to be real. In turn, this also limits the Yukawa couplings of our Lagrangian in Equation 4.5 to be real numbers. The entries of the mass matrix (which we will refer to individually as masses, even though they do not represent the mass of any particle as such) arise naturally in the maximally broken phase from $\mathcal{L}_Y$:

$$\begin{aligned} M_{\mathrm{L}} &= y_2\, v_\Sigma^{++} \\ m_{\mathrm{D}} &= y_4\, v_\phi^0 \\ m_{\mathrm{LL}} &= 2y_1\, v_\chi^+ \\ M_{\mathrm{R}} &= y_3\, v_\chi^{++} \\ m_{\mathrm{RL}} &= y_4\, v_\phi^+ \\ M_\nu &= y_2\, v_\Sigma^0 \,. \end{aligned} \tag{4.10}$$

We want to note again that higher dimensional or multiple copies of certain representations can also contribute to these masses, the above only represents the minimal case for which the mass matrix is full. All the mass terms for our fermions are now given conveniently in form of a real symmetric 3×3 matrix. Such a matrix is diagonalizable by an orthogonal rotation matrix $\mathcal{R}$, which in three dimensions is a combination of rotations around the $x-$, $y-$, and $z-$direction:

$$\mathcal{R} = \begin{pmatrix} c_y c_z & c_y s_z & s_y \\ -c_x s_z - s_x s_z c_z & c_x c_z - s_x s_y s_z & s_x c_y \\ s_x s_z - c_x s_y c_z & -s_x c_z - c_x s_y s_z & c_x c_y \end{pmatrix} \tag{4.11}$$

with $c_x = \cos\theta_x$, $s_x = \sin\theta_x$ and so on. The whole rotation is parameterized by the three angles θ_x, θ_y, and θ_z. The diagonalized mass matrix is then $\mathcal{M}' = \mathcal{R}^T \mathcal{M} \mathcal{R}$. In principle, all we need to do now is to compute this product and set the off-diagonal elements to zero. This gives us three equations to determine our three angles. In practice, however, this means solving a highly non-trivial system of three trigonometric equations. We can make our lives simpler by making some approximations: Except for the Dirac mass of the electron, which corresponds to m_{D}[2], none of the effects connected to the other elements of $\mathcal{M}$ have been observed experimentally so far. We hence can assume that $M_{\mathrm{L}}, M_{\mathrm{R}}, M_\nu, m_{\mathrm{LL}}, m_{\mathrm{RL}} \ll m_{\mathrm{D}}$. It then also follows that the rotation angles must be very small as well, since larger angles would mean more noticeable effects.

In this approximation, we can expand the trigonometric functions and easily solve the diagonalization equations to first order. We obtain the rotation angles

$$\theta_x = -\frac{m_{\mathrm{RL}} + m_{\mathrm{LL}}}{\sqrt{2}\, m_{\mathrm{D}}} \quad , \quad \theta_y = -\frac{m_{\mathrm{RL}} - m_{\mathrm{LL}}}{\sqrt{2}\, m_{\mathrm{D}}} \quad , \quad \theta_z = \frac{\pi}{4} + \frac{M_{\mathrm{L}} - M_{\mathrm{R}}}{4\, m_{\mathrm{D}}} \,. \tag{4.12}$$

[2]This assumes, of course, that the mass of the electron is actually given (almost) exclusively by m_{D}. One could imagine a scenario in which the mass of the electron has non-negligible contributions from the Majorana mass terms, but this would also bring with it noticeable lepton number and electric charge violation in processes such as $\gamma \to e^- e^-$. Although there is, to the best knowledge of the author, no experiment specifically measuring this kind of interaction, a non-negligible number of these processes would have surely been seen in collider signatures of various other experiments.

Note that θ_z has been expanded around $\frac{\pi}{4}$ instead of zero. At this angle the rotation puts the electron Dirac masses on the diagonal, which we know to be at least approximately true. We introduce $\tilde{\theta}_z = \theta_z - \frac{\pi}{4}$ and absorb the constant $\pi/4$ with trigonometric identities for the sin and cos whenever needed. This is then the rotated matrix we obtain to first order in any new vevs:

$$\mathcal{M}' = \begin{pmatrix} -m_\mathrm{D} + \frac{1}{2}(M_\mathrm{L} + M_\mathrm{R}) & & \\ & m_\mathrm{D} + \frac{1}{2}(M_\mathrm{L} + M_\mathrm{R}) & \\ & & M_\nu \end{pmatrix} ; \tag{4.13}$$

the masses m_LL and m_RL drop out completely at first order. This matrix is certainly diagonal, but there is a problem with it: we have assumed that $M_\mathrm{L}, M_\mathrm{R} \ll m_\mathrm{D}$, which makes the first entry negative. But this can be fixed quite easily, we only need to introduce a diagonal phase matrix $\eta = \mathrm{diag}(-1, 1, 1)$. Instead of the orthogonal rotation matrix we then have a unitary matrix $U_\mathrm{rot} = \mathcal{R} \cdot \sqrt{\eta}$, such that

$$\begin{aligned} \mathcal{M}_\mathrm{diag} &= U_\mathrm{rot}^T \mathcal{M} U_\mathrm{rot} = \sqrt{\eta}^T \mathcal{R}^T \mathcal{M} \mathcal{R} \sqrt{\eta} = \sqrt{\eta}^T \mathcal{M}' \sqrt{\eta} = \mathcal{M}'\eta \\ &= \begin{pmatrix} m_\mathrm{D} - \frac{1}{2}(M_\mathrm{L} + M_\mathrm{R}) & & \\ & m_\mathrm{D} + \frac{1}{2}(M_\mathrm{L} + M_\mathrm{R}) & \\ & & M_\nu \end{pmatrix} . \end{aligned} \tag{4.14}$$

We now have a diagonal mass matrix with real positive eigenvalues. They make sense, too, as two of them are close to the electron mass, loosely corresponding to the two electron chiral states we started with, and one eigenvalue is very small in comparison, as we know the neutrino masses are in nature.

Of course we cannot simply rotate the mass matrix however we like and leave it at that. In order to compensate, we have to also rotate the states Ψ themselves:

$$\begin{aligned} -\mathcal{L}_\mathrm{mass} &= \tfrac{1}{2}\overline{\Psi^c}\mathcal{M}\Psi + h.c. = \tfrac{1}{2}\overline{\Psi^c} U_\mathrm{rot}^* U_\mathrm{rot}^T \mathcal{M} U_\mathrm{rot} U_\mathrm{rot}^\dagger \Psi + h.c. \\ &\equiv \tfrac{1}{2}\overline{\Psi_\mathrm{m}^c}\mathcal{M}_\mathrm{diag}\Psi_\mathrm{m} + h.c. \end{aligned} \tag{4.15}$$

with $\Psi_\mathrm{m} = U_\mathrm{rot}^\dagger \Psi$. This product can be determined to first order as well:

$$\begin{aligned} \Psi_\mathrm{m} = U_\mathrm{rot}^\dagger \Psi &= \frac{1}{\sqrt{2}} \begin{pmatrix} -i(1-\tilde{\theta}_z) & i(1+\tilde{\theta}_z) & i\sqrt{2}\theta_y \\ 1+\tilde{\theta}_z & 1-\tilde{\theta}_z & -\sqrt{2}\theta_x \\ \theta_x+\theta_y & \theta_x-\theta_y & \sqrt{2} \end{pmatrix} \begin{pmatrix} e_\mathrm{L} \\ {e_\mathrm{R}}^c \\ \nu_\mathrm{L} \end{pmatrix} \\ &= \begin{pmatrix} \frac{i}{\sqrt{2}}\tilde{\theta}_z({e_\mathrm{R}}^c + e_\mathrm{L}) + \frac{i}{\sqrt{2}}({e_\mathrm{R}}^c - e_\mathrm{L}) + i\theta_y\nu_\mathrm{L} \\ \frac{1}{\sqrt{2}}({e_\mathrm{R}}^c + e_\mathrm{L}) - \frac{1}{\sqrt{2}}\tilde{\theta}_z({e_\mathrm{R}}^c - e_\mathrm{L}) - \theta_x\nu_\mathrm{L} \\ \frac{1}{\sqrt{2}}\theta_x({e_\mathrm{R}}^c + e_\mathrm{L}) - \frac{1}{\sqrt{2}}\theta_y({e_\mathrm{R}}^c - e_\mathrm{L}) + \nu_\mathrm{L} \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} \left[\begin{pmatrix} i\tilde{\theta}_z \\ 1 \\ \theta_x \end{pmatrix} ({e_\mathrm{R}}^c + e_\mathrm{L}) + \begin{pmatrix} i \\ -\tilde{\theta}_z \\ -\theta_y \end{pmatrix} ({e_\mathrm{R}}^c - e_\mathrm{L}) \right] + \begin{pmatrix} i\theta_y \\ -\theta_x \\ 1 \end{pmatrix} \nu_\mathrm{L} . \end{aligned} \tag{4.16}$$

As expected, we have three mass eigenstates that get small corrections from each other. What is less easy to spot is, that the states $\frac{1}{\sqrt{2}}({e_\mathrm{R}}^c + e_\mathrm{L})$ and $\frac{i}{\sqrt{2}}({e_\mathrm{R}}^c - e_\mathrm{L})$ are actually possible expressions for the mass eigenstates of the electron in the standard model without charge breaking. In that sense, although our results here check out, they are somewhat unwieldy to perform calculations with and there is a much more intuitive base that we can choose.

4.3. The quasi-SM basis

We have spend the last section working out the mass eigenstates of the leptons, i.e. the eigenvalues of the mass matrix $\mathcal{M}$. However, we already know from Equation 4.6 and Equation 4.7 that a non-diagonal mass matrix can be more easy to use in the right basis. Since we only perform a small rotation away from the standard model, it makes sense to aim for a block-diagonal matrix instead, where we have a 2×2 sub-matrix corresponding to the electron and a single entry for the neutrino.

For a 2×2 diagonal matrix with degenerate eigenvalues, there exists a unitary matrix that rotates the entries onto the off-diagonal:

$$\frac{1}{2}\begin{pmatrix} -i & i \\ 1 & 1 \end{pmatrix}^T \begin{pmatrix} m & \\ & m \end{pmatrix} \begin{pmatrix} -i & i \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} & m \\ m & \end{pmatrix} . \tag{4.17}$$

We will employ the exact same transformation to our mass matrix. In contrast to the two dimensional example, $\mathcal{M}_{\text{diag}}$ is not proportional to the unit matrix, which results in only the common term m_{D} being moved off the main diagonal, which happens to be exactly what we are after. The unitary matrix we used in Equation 4.17 is easily extended to three dimensions:

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} -i & i & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} . \tag{4.18}$$

We use this matrix to rotate $\mathcal{M}_{\text{diag}}$, so our diagonalization in the last section was not in vain:

$$\mathcal{M}_{\text{qSM}} = V^T \mathcal{M}_{\text{diag}} V = \begin{pmatrix} \frac{1}{2}(M_{\text{L}} + M_{\text{R}}) & m_{\text{D}} & \\ m_{\text{D}} & \frac{1}{2}(M_{\text{L}} + M_{\text{R}}) & \\ & & M_\nu \end{pmatrix} . \tag{4.19}$$

We now have arrived at the mass matrix we set out to find: the 2×2 matrix in the top left has the Dirac masses on the off-diagonal like we had in Equation 4.6, but with added Majorana terms on the diagonal. In the bottom right we have the small Majorana mass for the neutrino-like eigenstate. The higher order terms have been dropped, as before.
Naturally, we also need to rotate our states:

$$\Psi_{\text{qSM}} = V^\dagger \Psi_{\text{m}} = \begin{pmatrix} e_{\text{L}} - \tilde{\theta}_z {e_{\text{R}}}^c - \frac{1}{\sqrt{2}}(\theta_x + \theta_y)\nu_{\text{L}} \\ {e_{\text{R}}}^c + \tilde{\theta}_z e_{\text{L}} - \frac{1}{\sqrt{2}}(\theta_x - \theta_y)\nu_{\text{L}} \\ \nu_{\text{L}} + \frac{1}{\sqrt{2}}(\theta_x + \theta_y) e_{\text{L}} + \frac{1}{\sqrt{2}}(\theta_x - \theta_y){e_{\text{R}}}^c \end{pmatrix} \equiv \begin{pmatrix} \mathring{e}_{\text{L}} \\ {\mathring{e}_{\text{R}}}^c \\ \mathring{\nu}_{\text{L}} \end{pmatrix} . \tag{4.20}$$

We can now see clearly how the new states are mainly the leptons of the standard model, but with first order corrections from each other due to the mixing. With this in mind, we can define new particles $\mathring{e}$ and $\mathring{\nu}$ that incorporate these corrections. Another advantage of our block-diagonal mass matrix is that we can now easily separate the terms corresponding to our new particles. We have the purely left-handed $\mathring{\nu} = (\mathring{\nu}_{\mathrm{L}}, 0)^T$ and the bispinor $(\mathring{e}_{\mathrm{L}}, {\mathring{e}_{\mathrm{R}}}^c)^T$. Applying the same logic as in Equation 4.7, we know that we can rewrite the Lagrangian in terms of $\mathring{e} = (\mathring{e}_{\mathrm{L}}, \mathring{e}_{\mathrm{R}})^T$ in Dirac form and an accompanying Majorana mass term. Putting everything together, we now have the mass-Lagrangian

$$-\mathcal{L}_{\text{mass}} = m_{\mathrm{D}}\,\overline{\mathring{e}}\mathring{e} + \frac{M_{\mathrm{L}} + M_{\mathrm{R}}}{4}(\overline{\mathring{e}^c}\mathring{e} + h.c.) + \frac{M_\nu}{2}(\overline{\mathring{\nu}^c}\mathring{\nu} + h.c.) \tag{4.21}$$

We call this basis the *quasi-standard model basis* (qSM basis).

This is of course not the end of the story. We have changed the definitions of our leptons and must apply them consistently throughout the entire Lagrangian. Other than the mass terms, we must hence also consider the kinetic terms. Before the charge breaking phase transition, they are given by

$$\mathcal{L} \supset i\overline{L}\not{D}L + i\,\overline{e_{\mathrm{R}}}\not{D}e_{\mathrm{R}}\ . \tag{4.22}$$

We have defined the neutral part of the covariant derivative in the broken phase in Equation 3.2. If we also include the charged current interactions, we can expand the above Lagrangian as

$$\begin{aligned}\mathcal{L} \supset \quad & i\overline{e_{\mathrm{L}}}\not{\partial}e_{\mathrm{L}} + i\overline{\nu_{\mathrm{L}}}\not{\partial}\nu_{\mathrm{L}} + i\overline{e_{\mathrm{R}}}\not{\partial}e_{\mathrm{R}} \\ & - \tfrac{1}{2}(e_1 + e_2)\overline{e_{\mathrm{L}}}\not{A}e_{\mathrm{L}} - e_1\overline{e_{\mathrm{R}}}\not{A}e_{\mathrm{R}} - \tfrac{1}{2}(e_1 - e_2)\overline{\nu_{\mathrm{L}}}\not{A}\nu_{\mathrm{L}} \\ & + \tfrac{1}{2}(z_1 - z_2)\overline{e_{\mathrm{L}}}\not{Z}e_{\mathrm{L}} + z_1\overline{e_{\mathrm{R}}}\not{Z}e_{\mathrm{R}} + \tfrac{1}{2}(z_1 + z_2)\overline{\nu_{\mathrm{L}}}\not{Z}\nu_{\mathrm{L}} \\ & + \tfrac{g}{\sqrt{2}}\left[\overline{e_{\mathrm{L}}}\not{W}^-\nu_{\mathrm{L}} + \overline{\nu_{\mathrm{L}}}\not{W}^+e_{\mathrm{L}}\right]\ ,\end{aligned} \tag{4.23}$$

where we have introduced $z_1 = g'\sin\theta$ and $z_2 = g\cos\theta$ in analogy to e_1 and e_2 and everything neatly takes the familiar form in the SM limit. In matrix form, this Lagrangian can be expressed as

$$\begin{aligned}\mathcal{L} \supset \begin{pmatrix}\overline{e_{\mathrm{L}}} & \overline{{e_{\mathrm{R}}}^c} & \overline{\nu_{\mathrm{L}}}\end{pmatrix} \Bigg[i\not{\partial} - & \begin{pmatrix}\frac{1}{2}(e_1+e_2) & & \\ & -e_1 & \\ & & \frac{1}{2}(e_1-e_2)\end{pmatrix}\not{A} \\ + & \begin{pmatrix}\frac{1}{2}(z_1-z_2) & & \\ & -z_1 & \\ & & \frac{1}{2}(z_1+z_2)\end{pmatrix}\not{Z} \\ + & \frac{g}{\sqrt{2}}\begin{pmatrix} & & \not{W}^- \\ & & \\ \not{W}^+ & & \end{pmatrix} \Bigg] \begin{pmatrix}e_{\mathrm{L}} \\ {e_{\mathrm{R}}}^c \\ \nu_{\mathrm{L}}\end{pmatrix}\ .\end{aligned} \tag{4.24}$$

The identities

$$\overline{\psi_{\mathrm{L,R}}}\,\not{\partial}\,\psi_{\mathrm{L,R}} = \overline{\psi_{\mathrm{L,R}}^{\,c}}\,\not{\partial}\,\psi_{\mathrm{L,R}}^{\,c} \quad \text{and} \quad \overline{\psi_{\mathrm{L,R}}}\,\not{A}\,\psi_{\mathrm{L,R}} = -\overline{\psi_{\mathrm{L,R}}^{\,c}}\,\not{A}\,\psi_{\mathrm{L,R}}^{\,c} \tag{4.25}$$

were used to bring everything in the form $\sim\overline{\Psi}G\Psi$ with some interaction matrix G. When we perform the transformation to the qSM basis, G will naturally also transform:

$$\overline{\Psi}G\Psi = \overline{\Psi}\,U_{\text{rot}}V\,V^{\dagger}U_{\text{rot}}^{\dagger}\,G\,U_{\text{rot}}V\,V^{\dagger}U_{\text{rot}}^{\dagger}\,\Psi = \overline{\Psi}_{\text{qSM}}\,(V^{\dagger}U_{\text{rot}}^{\dagger}\,G\,U_{\text{rot}}V)\,\Psi_{\text{qSM}}\,. \tag{4.26}$$

We can immediately see that $\overline{\Psi}\partial\!\!\!/\Psi$ does not change under unitary transformations. For future reference, let us give the individual interaction matrices of each gauge boson a unique name: $\overline{\Psi}G\!\!\!/\Psi = \overline{\Psi}(G^{\text{A}}A\!\!\!/ + G^{\text{Z}}Z\!\!\!/ + G^{\text{W}}W\!\!\!/^{-} + G^{\text{W}\dagger}W\!\!\!/^{+})\Psi$. We can hence identify in the new basis:

$$\begin{aligned}
G^{\text{A}}_{\text{qSM}} &= -\begin{pmatrix} \frac{1}{2}(e_1+e_2) & \frac{1}{2}\tilde{\theta}_z(3e_1+e_2) & \frac{\theta_y+\theta_x}{\sqrt{2}}e_2 \\ \frac{1}{2}\tilde{\theta}_z(3e_1+e_2) & -e_1 & \frac{\theta_y-\theta_x}{\sqrt{2}}\frac{(3e_1-e_2)}{2} \\ \frac{\theta_y+\theta_x}{\sqrt{2}}e_2 & \frac{\theta_y-\theta_x}{\sqrt{2}}\frac{(3e_1-e_2)}{2} & \frac{1}{2}(e_1-e_2) \end{pmatrix} \\
G^{\text{Z}}_{\text{qSM}} &= \begin{pmatrix} \frac{1}{2}(z_1-z_2) & \frac{1}{2}\tilde{\theta}_z(3z_1-z_2) & -\frac{\theta_y+\theta_x}{\sqrt{2}}z_2 \\ \frac{1}{2}\tilde{\theta}_z(3z_1-z_2) & -z_1 & \frac{\theta_y-\theta_x}{\sqrt{2}}\frac{(3z_1+z_2)}{2} \\ -\frac{\theta_y+\theta_x}{\sqrt{2}}z_2 & \frac{\theta_y-\theta_x}{\sqrt{2}}\frac{(3z_1+z_2)}{2} & \frac{1}{2}(z_1+z_2) \end{pmatrix} \\
G^{\text{W}}_{\text{qSM}} &= \frac{g}{2}\begin{pmatrix} \theta_x+\theta_y & \theta_x-\theta_y & -\sqrt{2} \\ 0 & 0 & -\sqrt{2}\tilde{\theta}_z \\ 0 & 0 & -(\theta_x+\theta_y) \end{pmatrix}\,.
\end{aligned} \tag{4.27}$$

This is a rather remarkable result: through the mixing of the leptons, all of their interactions with electroweak gauge bosons have become possible[3]. Unfortunately, the non-diagonal nature of these matrices forbid us from using the identities from Equation 4.25 and we cannot write the interactions purely in terms of $\mathring{e}$ and $\mathring{\nu}$. We can nevertheless improve the aesthetics of our Lagrangian by noting that $\Psi_{\text{qSM}} = P_{\text{L}}(\mathring{e}, \mathring{e}^c, \mathring{\nu})$. With this we can finally write the entire Lagrangian containing leptons in a concise form:

$$\begin{aligned}
\mathcal{L}_{\text{f}} &= \overline{\mathring{e}}\,(i\partial\!\!\!/ - m_{\text{D}})\mathring{e} + i\overline{\mathring{\nu}}\partial\!\!\!/\mathring{\nu} - \tfrac{1}{4}(M_{\text{L}}+M_{\text{R}})(\overline{\mathring{e}^c}\mathring{e} + \overline{\mathring{e}}\mathring{e}^c) - \tfrac{1}{2}M_{\nu}(\overline{\mathring{\nu}^c}\mathring{\nu} + \overline{\mathring{\nu}}\mathring{\nu}^c) \\
&\quad + \overline{\mathring{e}}(G\!\!\!/_{11}P_{\text{L}} - G\!\!\!/_{22}P_{\text{R}})\mathring{e} + \overline{\mathring{e}}\,G\!\!\!/_{12}P_{\text{L}}\mathring{e}^c + \overline{\mathring{e}^c}\,G\!\!\!/_{21}P_{\text{L}}\mathring{e} + \overline{\mathring{\nu}}\,G\!\!\!/_{33}P_{\text{L}}\mathring{\nu} \\
&\quad + \overline{\mathring{e}}\,G\!\!\!/_{13}P_{\text{L}}\mathring{\nu} + \overline{\mathring{\nu}}\,G\!\!\!/_{31}P_{\text{L}}\mathring{e} + \overline{\mathring{e}^c}\,G\!\!\!/_{23}P_{\text{L}}\mathring{\nu} + \overline{\mathring{\nu}}\,G\!\!\!/_{32}P_{\text{L}}\mathring{e}^c\,.
\end{aligned} \tag{4.28}$$

The indices of the interaction coefficients $G\!\!\!/_{ij}$ mark the i,j-th entry of the corresponding matrix. We have further dropped the label ”qSM” for better readability; when we write $G\!\!\!/_{ij}$ from here on it is always implied that we mean

$$G\!\!\!/_{ij} \mathrel{\hat{=}} \left[G^{\text{A}}_{\text{qSM}}A\!\!\!/ + G^{\text{Z}}_{\text{qSM}}Z\!\!\!/ + G^{\text{W}}_{\text{qSM}}W\!\!\!/^{-} + (G^{\text{W}}_{\text{qSM}})^{\dagger}W\!\!\!/^{+}\right]_{ij}\,.$$

We can read off the Feynman rules for the interactions from this Lagrangian, which we have listed in Appendix A. We are now in a position where we can quantitatively

[3] The attentive reader may have noticed the lack of $\overline{\mathring{e}}_{\text{R}}\mathring{e}_{\text{R}}W^{+/-}$ interactions in Equation 4.27. This vertex does indeed exist on tree-level, but being the only RH-RH interaction without contribution of $\text{U}(1)_{\text{Y}}$, it is doubly suppressed by new vevs and hence dropped in our first order approximation.

examine the phenomenology of our model, but before we turn to this we want to close this section with a few remarks.

At some points in this work we have referred to the particle content and the equations that govern its behaviour, for lack of a better term, as ”model”. We want to make clear, however, that our discussion so far has not been restricted to just one specific set of new scalars. Everything we have derived in this chapter is valid for an arbitrary scalar sector. Any higher dimensional representation can still only contribute to the masses listed in Equation 4.10 and any scalar that is unwanted in a given theory can simply be eliminated from what we have discussed by setting the according vev to zero.
We also want to mention that the mixing effects do not change the charges of our leptons we have derived in chapter 3. In fact, we can find our assigned charges for the electron and neutrino in the matrix $G^{\text{A}}_{\text{qSM}}$, further justifying our methods employed in the previous chapter.

4.4. Phenomenology in the qSM basis

The experimental signature most often linked to electric charge breaking is that of electron decay into a photon and a neutrino, so we will investigate it first.

Kinematically, this is an allowed decay and only the conservation of electric charge prevents it from occurring naturally within the SM. We proceed similarly to the photon decay from section 3.2 and first write down the matrix element corresponding to the Feynman diagram of this process (cf. Figure 4.3):

$$\mathcal{M} = \frac{1}{2}\overline{u}(p-k)G^{\text{A}}_{13}\gamma^{\mu}(1-\gamma^5)\epsilon^*_{\mu}(k)u(p) \ . \tag{4.29}$$

As before, we need the spin averaged squared matrix element

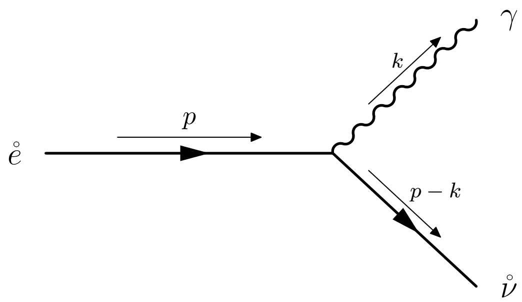


Figure 4.3.: Feynman-Diagram for electron decay into a photon and a neutrino.

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{8}|G^{\text{A}}_{13}|^2(-\eta_{\mu\nu}+\frac{k_\mu k_\nu}{M^2_{\text{A}}})\,\text{Tr}\left[(\not{p}-\not{k}+m_{\mathring{\nu}})\gamma^{\mu}(1-\gamma^5)(\not{p}+m_{\mathring{e}})(1+\gamma^5)\gamma^{\nu}\right] \ , \tag{4.30}$$

where again we have used that for the massive photon the extra term $k_\mu k_\nu/M^2_{\text{A}}$ appears in the polarization sum. The masses $m_{\mathring{e}}$ and $m_{\mathring{\nu}}$ we encounter are the physical masses of the particles, which we have only determined to first order in this chapter.
We can simplify this expression by using standard trace techniques (see e.g. [9])

along with the identities

$$\begin{aligned} (p-k)\cdot p &= -\tfrac{1}{2}\left[(p-k\,-\,p)^2-(p-k)^2-p^2\right] = \tfrac{1}{2}(m_{\mathring{e}}^2+m_{\mathring{\nu}}^2-M_{\text{A}}^2) \\ k\cdot p &= -\tfrac{1}{2}\left[(k-p)^2-k^2-p^2\right] \qquad\qquad\quad = \tfrac{1}{2}(m_{\mathring{e}}^2-m_{\mathring{\nu}}^2+M_{\text{A}}^2) \\ k\cdot(p-k) &= \ \ \tfrac{1}{2}\left[(p-k\,+\,k)^2-(p-k)^2-k^2\right] = \tfrac{1}{2}(m_{\mathring{e}}^2-m_{\mathring{\nu}}^2-M_{\text{A}}^2)\,. \end{aligned} \tag{4.31}$$

In the end, we arrive at

$$\langle|\mathcal{M}|^2\rangle = \frac{e_2^2}{4}(\theta_x+\theta_y)^2\left[m_{\mathring{e}}^2+m_{\mathring{\nu}}^2-2M_{\text{A}}^2+\frac{(m_{\mathring{e}}^2-m_{\mathring{\nu}}^2)^2}{M_{\text{A}}^2}\right]\,. \tag{4.32}$$

As the electron decay is just a simple 2-body decay and independent of the scattering angle, the decay width is proportional to the averaged squared matrix element without any integration:

$$\Gamma = \frac{e_2^2}{32\pi}\frac{|\vec{k}|}{m_{\mathring{e}}^2}(\theta_x+\theta_y)^2\left[m_{\mathring{e}}^2+m_{\mathring{\nu}}^2-2M_{\text{A}}^2+\frac{(m_{\mathring{e}}^2-m_{\mathring{\nu}}^2)^2}{M_{\text{A}}^2}\right]\,, \tag{4.33}$$

where $|\vec{k}|$ is fully determined by the masses of the involved particles:

$$|\vec{k}| = \frac{1}{2m_{\mathring{e}}}\sqrt{(m_{\mathring{e}}^2-(m_{\mathring{\nu}}+M_{\text{A}})^2)(m_{\mathring{e}}^2-(m_{\mathring{\nu}}-M_{\text{A}})}\,. \tag{4.34}$$

Two points are especially important here. Firstly, note how the decay width is suppressed by the mixing angles θ_x and θ_y. If we remind ourselves of their definition (cf. Equation 4.12), we see that their sum corresponds to just $\theta_x+\theta_y = -\sqrt{2}m_{\text{RL}}/m_{\text{D}}$ If we think back to our original mass matrix, where m_{RL} determines the mixing between the right-handed SM electron and the left-handed SM neutrino, this tells us what is going on behind the scenes in this interaction.
Imagine an incoming SM electron; it's right-handed part allows it to turn into a (left-handed) SM neutrino. The photon can couple in this process either to the electron before the mixing, or to the neutrino afterwards (if this interaction is allowed). Accounting for all chiralities, this decay would need at least 2 diagrams if we restrict ourselves to only one mass insertion; with the formalism we have developed so far we can take into account all mass insertions with just one diagram.

The other point we want to highlight here is that the decay width includes a term that is proportional to the inverse square of the photon mass. Because we know the photon mass to be tiny, this must be the dominating term. On the other hand, electron decay has never been measured, so Γ cannot be too large. Let us estimate the magnitudes involved by using the approximation $m_{\mathring{e}} \approx m_e \gg m_{\mathring{\nu}}, M_{\text{A}}$ [4] and $e_2 \approx e$. With this the decay width reduces to

$$\Gamma \approx \frac{e^2}{32\pi}m_e\frac{m_{\text{RL}}^2}{M_{\text{A}}^2} \quad\Rightarrow\quad \tau \approx \frac{32\pi}{e^2 m_e}\frac{M_{\text{A}}^2}{m_{\text{RL}}^2} \approx 1.4\cdot 10^{-18}\,\text{s}\cdot\frac{M_{\text{A}}^2}{m_{\text{RL}}^2}\,, \tag{4.35}$$

[4] The difference between $m_{\mathring{e}}$ and m_e is somewhat semantic, in the sense that the SM value could be made to fit any small correction by slightly tweaking the corresponding Yukawa coupling, which is not yet independently measurable (but will be in future colliders, see e.g. [39]). Conceptually, however, these masses are indeed different.

where $\tau = 1/\Gamma$ is the corresponding lifetime. In the SM limit this is an undefined quantity, showing once more that we cannot simply take this limit when the new third polarization of the photon is involved. The current best measurement of the electron lifetime comes from the Borexino experiment and sets a bound at $\tau \geq 6.6 \cdot 10^{28}\,\text{yr}$ ($\sim 2.1 \cdot 10^{36}\,\text{s}$) [40]. This gives us the inequality

$$m_{\text{RL}} \leq 8.2 \cdot 10^{-28}\, M_{\text{A}} \tag{4.36}$$

Even in the best case scenario, when the photon mass is equal to the current bound of 10^{-18} eV, this would still mean that $m_{\text{RL}} \leq 8.2 \cdot 10^{-46}$ eV. Although this is not an impossibility, a lot of fine-tuning is required to make this value small enough.

But Figure 4.3 is not the only diagram that can contribute to the electron decay. If we let go of lepton number conservation, we also have the decay $\mathring{e} \to \gamma\overline{\mathring{\nu}}$. We can repeat the above process for this diagram, we get

$$\Gamma = \frac{1}{32\pi}\frac{(3e_1 - e_2)^2}{4}\frac{|\vec{k}|}{m_{\mathring{e}}^2}(\theta_x - \theta_y)^2 \left[m_{\mathring{e}}^2 + m_{\mathring{\nu}}^2 - 2M_{\text{A}}^2 + \frac{(m_{\mathring{e}}^2 - m_{\mathring{\nu}}^2)^2}{M_{\text{A}}^2} \right] . \tag{4.37}$$

The mixing of the left-handed electron with the left-handed anti-neutrino is determined by m_{LL}, so in the same approximation as before the lifetime becomes

$$\tau \approx \frac{32\pi}{e^2 m_e}\frac{M_{\text{A}}^2}{m_{\text{LL}}^2 + m_{\text{RL}}^2} . \tag{4.38}$$

This gives the combined stronger limit $\sqrt{m_{\text{LL}}^2 + m_{\text{RL}}^2} \leq 8.2 \cdot 10^{-46}$ eV. The masses m_{LL} and m_{RL} stem from the scalars χ_2 and ϕ (and their corresponding Yukawa couplings), respectively. Models which include these particles, like for example SUSY, the Babu-Zee model, or the two Higgs doublet model, must take special care that their scalars do not obtain vevs (or otherwise introduce very large fine-tuning).

Another interesting interaction for us to look at is that of the W-boson with two electrons as in the figure on the right. For one, we now have a vertex that breaks lepton number conservation, in addition to the electric charge. As we note in Appendix A, these vertices have an additional particle-antiparticle conjugation matrix $\mathcal{C}$, which is cancelled (in this case) by use of the identity

$$\overline{u}^T(p) = \mathcal{C}^{-1} v(p) . \tag{4.39}$$

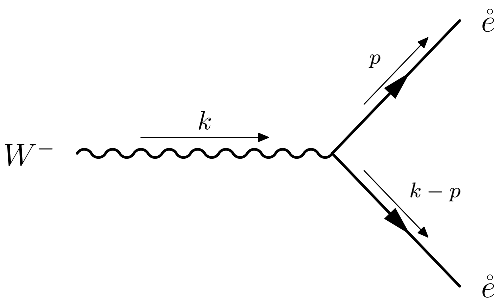


Figure 4.4.: Feynman-Diagram for the decay of a W boson into two electrons.

With this, we can use the usual procedure to obtain to find the lifetime of the W in this channel:

$$\tau \approx \frac{32\pi}{g^2}\frac{m_e^2}{m_{\text{LL}}^2}\frac{m_W}{m_W^2 - m_e^2} . \tag{4.40}$$

Numerically, this amounts to $\tau \approx 793.3\,\text{eV}\,/m_{\text{LL}}^2$. The limit on m_{LL} we have extracted from Equation 4.38 yields

$$\tau \gtrsim 1.18 \cdot 10^{93}\,\frac{1}{\text{eV}} \approx 2.46 \cdot 10^{70}\,\text{yr}\,. \tag{4.41}$$

This is an extraordinarily big number and shows that such a process will not be observed in nature; it serves as an example to demonstrate how much we can expect any of the interactions in our Lagrangian that depend on the mixing angles θ_x and θ_y (cf. Equation 4.27 and Equation 4.28 or Appendix A) to be suppressed.

We have so far seen two classes of vertices: those of SM interactions with slight corrections and new interactions depending on the mixing angles θ_x and θ_y. But there exists a third class: new interactions that depend on $\tilde{\theta}_z$. This is the most difficult class to handle from a phenomenological viewpoint, because it does not contain any vertices that are of much experimental interest. Converting e.g. a photon into two electrons is "obviously" very suppressed in the sense that it offers a clear signature of which we have seen no signs of, so there is little point in constructing theoretical models that introduce a mechanism for this. As a result, no experiments focusing on these signatures are actually undertaken, since there is no theoretical basis for them.
We highlight this viscous cycle here not as criticism, but merely as an explanation for why it has proven so difficult to give quantitative limits on the clearly small parameters M_{L} and M_{R} that are contained in $\tilde{\theta}_z$.

We will give an example regardless, in the form of lepton violating β-decay. We assume that the hadronic part of the β-decay stays the same as the SM and only the leptonic part changes (cf. Figure 4.5).

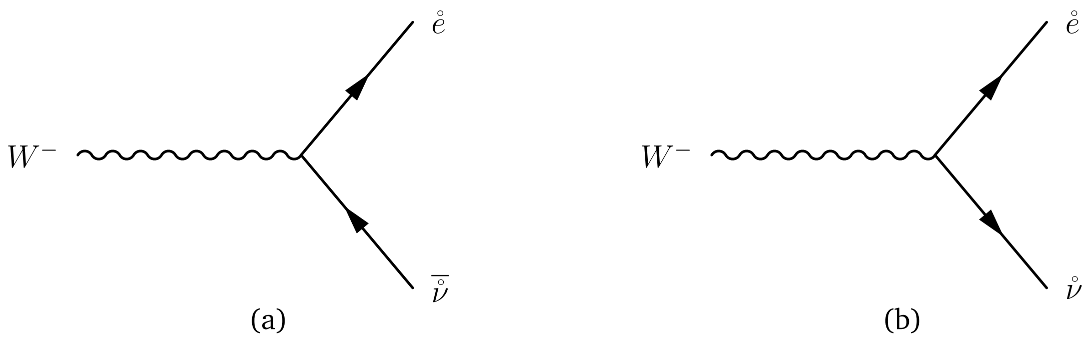


Figure 4.5.: The leptonic part of the familiar β-decay (a) and it's lepton violating variant (b).

The relevant vertex rule in the normal β-decay has not changed in the qSM basis and therefore the averaged matrix element will be the same as well:

$$\langle|\mathcal{M}_a|^2\rangle = \frac{g^2}{4}\epsilon_\mu^*(k)\epsilon_\nu(k)\,\text{Tr}\left[\not{p}\,\gamma^\mu(\not{k}-\not{p})\gamma^\nu\right] \tag{4.42}$$

with the same conventions for the names of the momenta as in the previous diagrams. The same calculation for Figure 4.5b reveals that all differences to the SM case actually drop out and only the dependence on $\tilde{\theta}_z$ remains:

$$\langle|\mathcal{M}_b|^2\rangle = \frac{g^2}{4}\tilde{\theta}_z^2\epsilon_\mu^*(k)\epsilon_\nu(k)\,\mathrm{Tr}\left[\not{p}\,\gamma^\mu(\not{k}-\not{p})\gamma^\nu\right] \ . \tag{4.43}$$

Because $\langle|\mathcal{M}|^2\rangle$ is proportional to the decay width, we can use this to determine the branching ratio for this event in dependence on the mixing angle:

$$\mathrm{BR}(W^- \to \mathring{e}\,\mathring{\nu}) = \tilde{\theta}_z^2\,\frac{\Gamma(W^- \to \mathring{e}\,\overline{\mathring{\nu}})}{\Gamma(W^-)} \approx 0.107\,\tilde{\theta}_z^2 \ , \tag{4.44}$$

where we have used that $\Gamma(W^- \to e\,\overline{\nu})/\Gamma(W^-) = (10.71 \pm 0.16)\%$ [14]. Of course, this alone does not mean much without an experimental limit on this branching ratio. The benefit of this specific interaction is, however, that the corresponding SM process $W^- \to e\nu$ is charge conserving and only lepton-number violating. It is therefore of some interest also outside of our discussion and we might be able to extract useful limits from e.g. reactor (anti)neutrino experiments in the future.

Chapter

5. Discussion and Outlook

Let us briefly summarize what we have achieved so far. We began by extending the scalar sector of the SM in a general way without specifying a specific representation and investigated the generic consequences for the electroweak sector. Specifically, we saw in what way the gauge boson masses

$$\begin{aligned} M_A^2 &= \sum_i v_i^2 \left(g' y_i \cos\theta + g\,\hat{m}_i \sin\theta\right)^2 \;, \\ M_Z^2 &= \sum_i v_i^2 \left(g' y_i \sin\theta - g\,\hat{m}_i \cos\theta\right)^2 \;, \\ M_W^2 &= \frac{g^2 v_i^2}{2} \left(j_i(j_i+1) - \hat{m}_i^2\right) \;, \end{aligned} \tag{5.1}$$

depend on the representations of the new scalars and how the weak mixing angle deviates from the SM Weinberg angle:

$$\sin\theta = \frac{1}{\sqrt{2}} \left(1 + \sqrt{\frac{\kappa_1^2}{\kappa_1^2 + \kappa_2^2}}\right)^{1/2} \quad , \quad \cos\theta = \frac{1}{\sqrt{2}} \left(1 - \sqrt{\frac{\kappa_1^2}{\kappa_1^2 + \kappa_2^2}}\right)^{1/2} \;, \tag{5.2}$$

where κ_1 and κ_2 are model-dependent parameters defined in Equation 2.14. This change away from the Weinberg angle θ_W also means that the relation $g \sin\theta = g' \cos\theta = e$ for the electric charge is not valid anymore; we hence separately introduced $e_1 = g' \cos\theta$ and $e_2 = g \sin\theta$ as the $\mathrm{U}(1)_\mathrm{Y}$ and $\mathrm{SU}(2)_\mathrm{L}$ part of e, respectively, with their difference being $|e_2 - e_1| < 1.3 \cdot 10^{-11}\, e$. This lead us to define an equivalent of the Gell-Mann-Nishijima formula for the broken phase:

$$\hat{Q}' = \tfrac{e_1}{e} \hat{Y} + \tfrac{e_2}{e} \hat{T}^3 \,. \tag{5.3}$$

The constraints by measurements of the gauge boson masses had us conclude that there are two directions for the realization of charge breaking that are worth investigating:

- Large vevs $v \cong \mathcal{O}(\mathrm{GeV})$ and suppression of new effects by a small charge $q \ll e$ (chapter 3)
- Large charges $q \sim e$ and suppression of new effects by small vevs $v \ll \mathcal{O}(\mathrm{eV})$ (chapter 4)

These categories are not mutually exclusive, but if both the vev and the charge of a scalar are small, it is still considered a minicharged particle, only it will be even harder to detect because its interactions are additionally suppressed.
In the first case, we were able to expand the weak mixing angle around the neutral vacuum for small charges q:

$$\begin{aligned}\cos\theta &= \cos\theta_W \left(1+\kappa_Q \sin\theta_W^2\right) + \mathcal{O}(q^2)\\ \sin\theta &= \sin\theta_W \left(1-\kappa_Q \cos\theta_W^2\right) + \mathcal{O}(q^2)\end{aligned} \tag{5.4}$$

with the model dependent parameter $\kappa_Q = 4\sum_i \frac{v_i^2}{v_{\mathrm{H}}^2} q_i \hat{m}_i$. κ_Q features the vevs, charge, and SU(2) eigenvalues of the involved scalars and, although not the only free parameter that arises in this approximation, it can serve as a characteristic quantity for a theory that features minicharged particles as a result of electric charge breaking. Some constraints on this parameter have been derived (cf. Table 5.1).

q'_ν	μ_ν	q'_n
$\lvert\kappa_Q\rvert \leq 6.6\cdot 10^{-12}$	$\lvert\kappa_Q\rvert \leq 1.4\cdot 10^{-18}$	$\kappa_Q \leq 1.5\cdot 10^{-21}$ $\kappa_Q \geq -3.2\cdot 10^{-21}$

Table 5.1.: Limits on κ_Q from the neutrino charge q'_ν, the neutrino magnetic moment μ_ν, and the neutron charge q'_n.

If we are given the parameters of a theory with minicharged scalars we can hence determine its agreement with experiment by simply determining κ_Q. The above table contains only some limits we saw fit to include for illustrative purposes in chapter 3, there may be more stringent ones yet to explore.
A further consequence of the modified Gell-Mann-Nishijima formula is a change in the electric charge of fermions as determined through the photon coupling. We averaged over the different chiralities and examined the couplings of the particles to each other to arrive at the new charges listed in Table 5.2.

Next, we expanded on the minicharge effects by investigating scalars with charges of $\mathcal{O}(1)$ that lead to new renormalizable interactions with fermions. We systematically wrote down allowed interaction terms and found that the lowest dimensional scalars that are of interest for us are

$$\chi_1 : (\mathbf{1},\mathbf{1})_1 \quad , \quad \chi_2 : (\mathbf{1},\mathbf{1})_2 \quad , \quad \phi : (\mathbf{1},\mathbf{2})_{1/2} \quad , \quad \Sigma : (\mathbf{1},\mathbf{3})_1 \, . \tag{5.5}$$

We have neglected any interactions with quarks so far and only looked at the leptonic sector. Here, we saw how the above representations induce mixing between the different leptonic states. We introduced the new quasi-SM basis via a unitary transformation with the mixing-angles θ_x, θ_y, and θ_z; here we found the fermions $\mathring{e}$ and $\mathring{\nu}$, which we take to be the measurable leptons in the current phase of the universe, and their interactions with the gauge bosons (Appendix A).

Particle name	SM charge q $[e]$	Charge q' under $\hat{Q}'$
Electron e^-	-1	$-\frac{1}{4}(3e_1 + e_2)$
Neutrino ν	0	$-\frac{1}{2}(e_1 - e_2)$
u-Quark u	$+\frac{2}{3}$	$\frac{1}{12}(5e_1 + 3e_2)$
d-Quark d	$-\frac{1}{3}$	$-\frac{1}{12}(e_1 + 3e_2)$
Photon γ	0	0
Z-Boson Z	0	0
$W^\pm$-Boson $W^\pm$	±1	$\pm e_2$
Gluon g	0	0

Table 5.2.: 1$^{\text{st}}$ generation SM fermions and gauge bosons, their SM electric charge, and their charge in the broken phase.

Finally, we applied our qSM Feynman rules to some new processes and showed how they simplify calculations compared to the SM base. Experimental observations, or rather a lack thereof, allowed us to put some very stringent bounds on the parameters from which these interactions arise. One example we looked at was the decay of electrons to photons and neutrinos. We noticed that the additional longitudinal polarization of the photon leads to new non-trivial effects that have no smooth limit to the standard model and gives huge contributions to the decay width. This allowed us to determine the strong limit

$$\sqrt{m_{\text{LL}}^2 + m_{\text{RL}}^2} \leq 8.2 \cdot 10^{-46}\,\text{eV}\,, \tag{5.6}$$

where m_{LL} and m_{RL} directly correspond to the vevs of χ_1 and the positive component of ϕ, respectively, as well as higher dimensional representations. This corresponds to interactions dependent on θ_x and θ_y occurring generically on timescales of ~ 10^{70} yr.

The Higgs boson is the most recently discovered fundamental particle of the SM and the only example of a fundamental scalar we have found as of yet. With so many unresolved issues like dark matter, neutrino masses, or baryogenesis, an extended scalar sector is a reasonable area to expect new physics. The spontaneous breaking of symmetries to endow scalars with vevs is also no rarity in beyond the standard model physics. But traditionally, the breaking of the $\text{U}(1)_\text{Q}$ has been ruled out categorically because of lacking experimental evidence, despite there being no known mechanism to explicitly forbid it.
In this work, we have examined some previously unexplored repercussions of electric charge non-conservation; we saw how the particle charges and gauge boson masses get corrections and provided a characteristic quantity to compare and rule out models. We also showed how certain representations lead to mixing of the

fermions and derived Feynman rules in order to make it easier to work in the broken phase.
We want to conclude by highlighting areas building upon this work that require more attention and analysis:

- An obvious starting point is the structure of the scalar sector itself; we have taken for granted that there must be some set of parameters for which the scalars can obtain (simultaneous) vevs, but it is worth exploring just how much of such parameter space exists. Furthermore, quantifiable knowledge of how much fine-tuning is needed to make such a model possible (or even a mechanism which explains it in a natural way) would go a long way towards evaluating the plausibility of electric charge breaking in nature.

- Connected to the above point, a placement of a charge breaking in the thermal history of the universe would be helpful. We have avoided most large-scale and cosmological bounds on the grounds that a (comparatively) small vev is naturally expected to appear at later times in the universe. This also opens the possibility for observable topological effects of the breaking, like e.g. cosmic strings.

- We have computed observables like charges and life-times mostly to illustrate new effects that appear in a possible broken phase, and have only done so at tree-level. A more exhaustive review of the phenomenology needs to be done to find out just how far the parameters in models featuring charge breaking (e.g. κ_Q) can be constraint even now. For example, one could determine the dependence of the Peskin-Takeuchi parameters [42] on the new vevs or calculate the energy loss of electron beams due to conversion to neutrinos. In the same vein, we can also extend our Yukawa couplings to the complex domain and study its CP-violating behaviour.

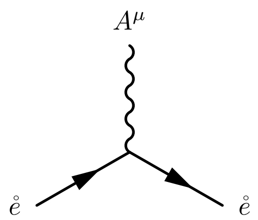
- The quark sector has been neglected almost entirely in this discussion. It would allow for a much bigger mass matrix with the right representations (cf. Table 4.1) and result in new mass eigenstates that are, to a degree, strongly interacting with each other and bring with them all kinds of interesting new problems and phenomena.

Appendix A. Feynman Rules in the Broken Phase

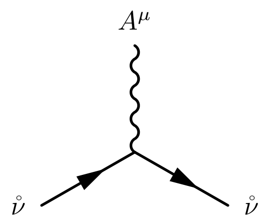
We collect here the Feynman-diagrams and -rules for the interaction vertices of the lepton-lepton-gauge boson interactions that can be derived from Equation 4.28. Importantly, like our derivation of the Lagrangian, they only feature terms to first order in the new vevs. As a result of the appearance of $\mathring{e}^c$, the Feynman rules for the lepton number violating vertices feature the particle-antiparticle conjugation matrix $\mathcal{C}$. In the calculation of matrix elements these matrices will always drop out, either by cancellation with further $\mathcal{C}$s appearing in the Majorana-like propagator or with the help of the identities

$$v_s(\mathbf{p}) = \mathcal{C}\,\overline{u}_s^T(\mathbf{p}) \quad \text{and} \quad u_s(\mathbf{p}) = \mathcal{C}\,\overline{v}_s^T(\mathbf{p}) \tag{A.1}$$

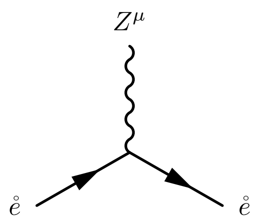
when including the external fermionic legs. For more details and references, see the excellent treatment of [38].



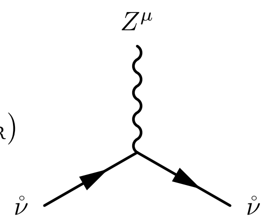
$$: \; i\gamma_\mu\left(\tfrac{1}{2}(e_1+e_2)P_\mathrm{L} + e_1 P_\mathrm{R}\right)$$



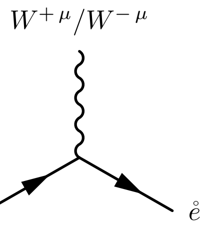
$$: \; \tfrac{i}{2}(e_1-e_2)\gamma_\mu P_\mathrm{L}$$



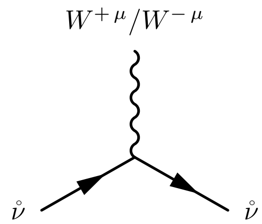
$$: \; -i\gamma_\mu\left(\tfrac{1}{2}(z_1-z_2)P_\mathrm{L} + z_1 P_\mathrm{R}\right)$$



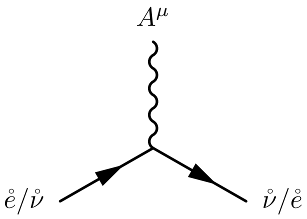
$$: \; -\tfrac{i}{2}(z_1+z_2)\gamma_\mu P_\mathrm{L}$$



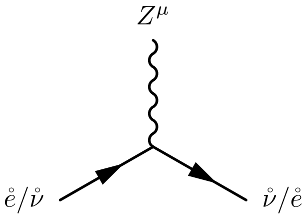
$$: \; -ig\frac{\theta_x+\theta_y}{2}\gamma_\mu P_\mathrm{L}$$



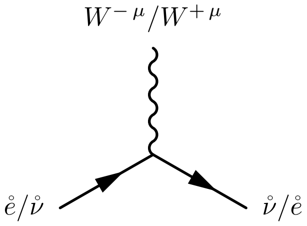
$$: \; ig\frac{\theta_x+\theta_y}{2}\gamma_\mu P_\mathrm{L}$$



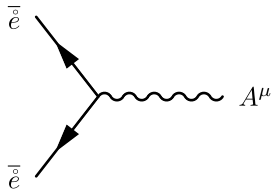
$: \; i\frac{e_2}{\sqrt{2}}(\theta_x+\theta_y)\gamma_\mu P_{\mathrm{L}}$



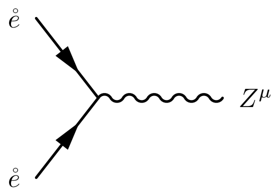
$: \; i\frac{z_2}{\sqrt{2}}(\theta_x+\theta_y)\gamma_\mu P_{\mathrm{L}}$



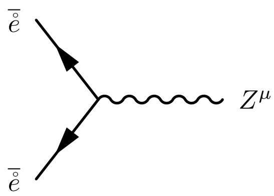
$: \; i\frac{g}{\sqrt{2}}\gamma_\mu P_{\mathrm{L}}$



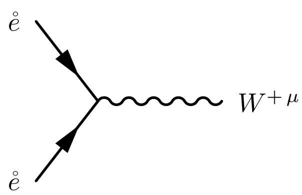
$: \; \frac{i}{2}\tilde{\theta}_z(3e_1+e_2)\gamma_\mu \mathcal{C} P_{\mathrm{R}}$



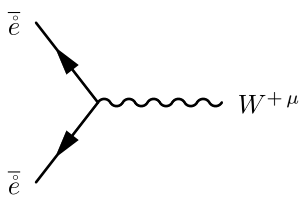
$: \; -\frac{i}{2}\tilde{\theta}_z(3z_1-z_2)\mathcal{C}\gamma_\mu P_{\mathrm{L}}$



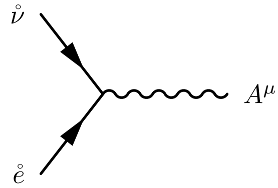
$: \; -\frac{i}{2}\tilde{\theta}_z(3z_1-z_2)\gamma_\mu \mathcal{C} P_{\mathrm{R}}$



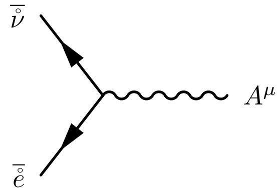
$: \; -ig\frac{\theta_x-\theta_y}{2}\mathcal{C}\gamma_\mu P_{\mathrm{L}}$



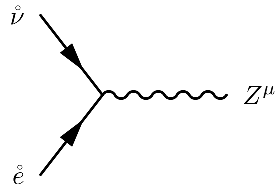
$: \; -ig\frac{\theta_x-\theta_y}{2}\gamma_\mu \mathcal{C} P_{\mathrm{R}}$



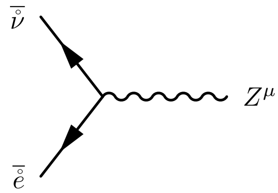
$: \; -i\frac{\theta_x-\theta_y}{2\sqrt{2}}(3e_1-e_2)\mathcal{C}\gamma_\mu P_{\mathrm{L}}$



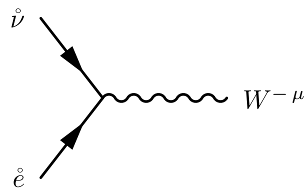
$: \; -i\frac{\theta_x-\theta_y}{2\sqrt{2}}(3e_1-e_2)\gamma_\mu \mathcal{C} P_{\mathrm{R}}$



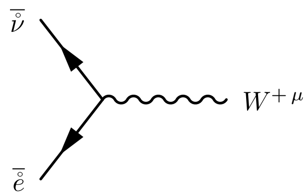
$: \; i\frac{\theta_x-\theta_y}{2\sqrt{2}}(3e_1-e_2)\mathcal{C}\gamma_\mu P_{\mathrm{L}}$



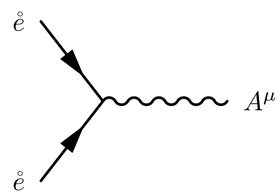
$: \; i\frac{\theta_x-\theta_y}{2\sqrt{2}}(3e_1-e_2)\gamma_\mu \mathcal{C} P_{\mathrm{R}}$



$: \; i\frac{g}{\sqrt{2}}\tilde{\theta}_z\mathcal{C}\gamma_\mu P_{\mathrm{L}}$



$: \; i\frac{g}{\sqrt{2}}\tilde{\theta}_z\gamma_\mu \mathcal{C} P_{\mathrm{R}}$



$: \; \frac{i}{2}\tilde{\theta}_z(3e_1+e_2)\mathcal{C}\gamma_\mu P_{\mathrm{L}}$

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