Closed-loop fMRI at the mesoscopic scale of columns and layers: Can we do it and why would we want to?

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Supplementary appendix: Calculation of tSNR dependence on voxel size and number of measurements

We base our SNR calculations on the noise model of [1]. Voxel independent physiological σ_p and voxel volume dependent thermal noise σ_0 are given by $\sigma_p = \lambda$ and $\sigma_0 = 1/\kappa V$, with total time-course SNR $tSNR = \frac{1}{\sqrt{\sigma_0^2+\sigma_p^2}} = \frac{\kappa V}{\sqrt{1+\lambda^2\kappa^2 V^2}}$. Triantafyllou et al. [1], measured SNR as a function of voxel volume for different magnetic field strengths which can be used to estimate λ and κ. Chaimow et al. [2] derived how to translate κ for different TRs using grey matter T_1 values. Assuming uncorrelated thermal noise, averaging of N_t consecutive measurements in time and N_v voxels in space results in a reduced thermal noise component $\sigma_0^{N_t,N} = \frac{\sigma_0}{\sqrt{N_v N_t}} = \frac{1}{\kappa V \sqrt{N_v N_t}}$.

Furthermore, we assume physiological noise to be approximately homogeneous in space (at small spatial scales) but autocorrelated in time with autocorrelation

$$E\left[\epsilon_{t_1}^{p}\epsilon_{t_2}^{p}\right] = \frac{\sigma_p^2 \exp(-TR\left|t_1-t_2\right|)}{15} \text{ [3]},$$

where ϵ_t^p is the noise component of measurement number t. Assuming zero mean noise, we obtain a reduced physiological noise level after averaging N_t measurements according to

$$\left(\sigma_p^{N_t}\right)^2 = E\left[\left(\frac{1}{N_t}\sum_{t=1}^{N_t}\epsilon_t^p\right)^2\right] = \frac{1}{N_t^2}\left(\sum_{t_1=1}^{N_t}\sum_{t_2=1}^{N_t} E\left[\epsilon_{t_1}^{p}\epsilon_{t_2}^{p}\right]\right) = \frac{1}{N_t^2}\sum_{t_1=1}^{N_t}\sum_{t_2=1}^{N_t}\frac{\sigma_p^2\exp(-TR\left|t_1-t_2\right|)}{15}.$$

Taken together we can calculate total noise as follows. We start with fitted values of κ and λ that were estimated for different magnetic field strength and a TR of 5.4 s [1,4] and modify κ to reflect a TR of 2 s, which is more typical of a current fMRI study [2].

Combining independent thermal and physiological noise components we obtain

$$SNR^{N_t,N} = \frac{1}{\sqrt{\left(\sigma_p^{N_t}\right)^2 + \left(\sigma_0^{N_t,N}\right)^2}} = \frac{\kappa(TR)V\sqrt{N_v N_t}}{\sqrt{1+\kappa^2(TR)V^2 N_v N_t \lambda^2\left(\frac{1}{N_t^2}\sum_{t_1=1}^{N_t}\sum_{t_2=1}^{N_t}\frac{\exp(-TR|t_1-t_2|)}{15}\right)}}.$$

For 3T we used: $\kappa = 6.6567, \lambda = 0.0129, T_1 = 1.607$; and for 7T: $\kappa = 9.9632$, $\lambda = 0.0113, T_1 = 1.939$ [4].

1. Triantafyllou C, Hoge RD, Krueger G, Wiggins CJ, Potthast A, Wiggins GC, L WL. 2005 Comparison of physiological noise at 1.5 T, 3 T and 7 T and optimization of fMRI acquisition parameters. *Neuroimage* **26**, 243–250. (doi:10.1016/j.neuroimage.2005.01.007)

2.Chaimow D, Yacoub E, Uğurbil K, Shmuel A. 2011 Modeling and analysis of mechanisms underlying fMRI-based decoding of information conveyed in cortical columns. *Neuroimage* **56**, 627–642. (doi:10.1016/j.neuroimage.2010.09.037)

3.Purdon PL, Weisskoff RM. 1998 Effect of temporal autocorrelation due to physiological noise and stimulus paradigm on voxel-level false-positive rates in fMRI. *Human Brain Mapping* **6**, 239–249.

4.Chaimow D, Uğurbil K, Shmuel A. 2018 Optimization of functional MRI for detection, decoding and high-resolution imaging of the response patterns of cortical columns. *NeuroImage* **164**, 67–99. (doi:10.1016/j.neuroimage.2017.04.011)