Supplementary Material: Catalysing Cooperation: The Power of Collective Beliefs in Structured Populations

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Supplementary Methods

Varying hunting party size The model presented by Gokhale et al. [1] assumes that individuals form groups of a constant size. However, social systems often exhibit group-size heterogeneity [2,3]. Group size diversity can impact the qualitative outcomes of the evolutionary games [4,5]. Therefore, we assess how varying the hunting party size affects cooperation in our model.

A mutant strategy can arise in a population consisting of otherwise homogeneous individuals. In the context of human social behaviour, the deviation from the status quo can be caused by a human's natural inclination to explore available options [6]. Suppose the change happens at a low enough rate. In that case, the novel behaviour can either take over the population or go extinct before a new type appears. Hence, it is sufficient to analyse the impact of group-size diversity on dynamics between pairs of strategies. In particular, we are interested in how the fraction of hare hunters ((S,H,2), (H,S,1), (H,H,1) or (H,H,2)) changes when confronted with stag hunters ((S,H,1), (H,S,2), (S,S,1) or (S,S,2)) in the presence of beliefs.

Considering a pair of strategies, (H,H,1) and (S,H,1) is equivalent to a basic N-player stag hunt game with no beliefs involved. We denote the fraction of stag hunters, (S,H,1), as x and the group size as n. Thus, the difference between the average payoff of a stag hunter and a hare hunter,

$f(x, n)$, is given as,

$$f(x, n) = \sum_{j=0}^{n-1} \binom{n-1}{j} x^j (1-x)^{n-1-j} [\Pi_S - \Pi_H]$$

$$= \sum_{j=0}^{n-1} \binom{n-1}{j} x^j (1-x)^{n-1-j} [P_S \theta(j+1-M) - P_H] \tag{1}$$

where $\theta(x) = 1$ for $x \geq 0$ and 0 if $x < 0$, Π_H is a payoff of a hare hunter, Π_S is a payoff of a stag hunter, P_H is a value of hare and P_S is a value of a stag.

As per Jensen's inequality, [4] show that group size does not affect the game if $h(x, n) \equiv nf(x, n)$ is a linear function in n. Since $\frac{\partial^2 h}{\partial n^2} \neq 0$ for $n \geq M$, the stag hunt game will be affected by varying the size of the hunting party. The effect of changing n is unclear as $h(x, n)$ is neither convex nor concave concerning n.

Similarly, analysis can be performed for any pair of strategies $((a_1, a_2, u)$ and $(a_1^*, a_2^*, u^*))$. We denote an action chosen by an individual playing the first strategy in a group choosing their preferred narrative as a_u and in a group choosing a narrative preferred by the other strategy as a_{u^*}. The same principle is also used to denote the action taken by individuals playing the second strategy. The payoff of an individual depends on their action, which is indicated by an appropriate subscript. The difference between the average payoffs can be calculated as

$$f(x, n) = \sum_{j=0}^{n-1} \binom{n-1}{j} x^j (1-x)^{n-1-j} \left[\frac{j+1}{n} \Pi_{a_u} + \frac{n-j-1}{n} \Pi_{a_{u^*}} - \frac{j}{n} \Pi_{a_u^*} - \frac{n-j}{n} \Pi_{a_{u^*}^*} \right],$$

$$\tag{2}$$

with x denoting the fraction of players choosing the first strategy.

In particular, for a pair of strategies (S,H,1)and (S,H,2), $f(x, n)$ takes a form of:

$$f(x, n) = \frac{P_S - P_H}{n}.$$

(3)

Hence, it is straightforward to show that the function $h(x, n) \equiv nf(x, n)$ is linear with respect to n. Analogous analysis can be performed for strategies (H,S,2) and (H,S,1). In those cases, the stag hunting strategy dominates the hare hunting, and the group size does not influence the dynamics (provided that $n \geq M$).

Except for these two strategy pairs, ((S,H,2), (S,H,1) and (H,S,1), (H,S,2)), $h(x, n)$ is not linear in n. Hence, it is reasonable to assume that group size diversity will impact a population's cooperation (stag hunting) level.

Based on animal group-size distributions, we perform a numerical analysis for three group-size distributions - truncated Poisson, geometric, and Waring [7,8]. In particular, the Waring distribution exhibits power-law degree distribution, a characteristic often obtained by preferential attachment [9]. We assume the average group size to be constant across the cases. For $E[N] \approx 8$, $M = 4$, $P_S = 4$, $P_H = 1$ the analysis shows, in Supplementary Figure 1, that varying group size strengthens cooperation, regardless of underlying group size distribution.
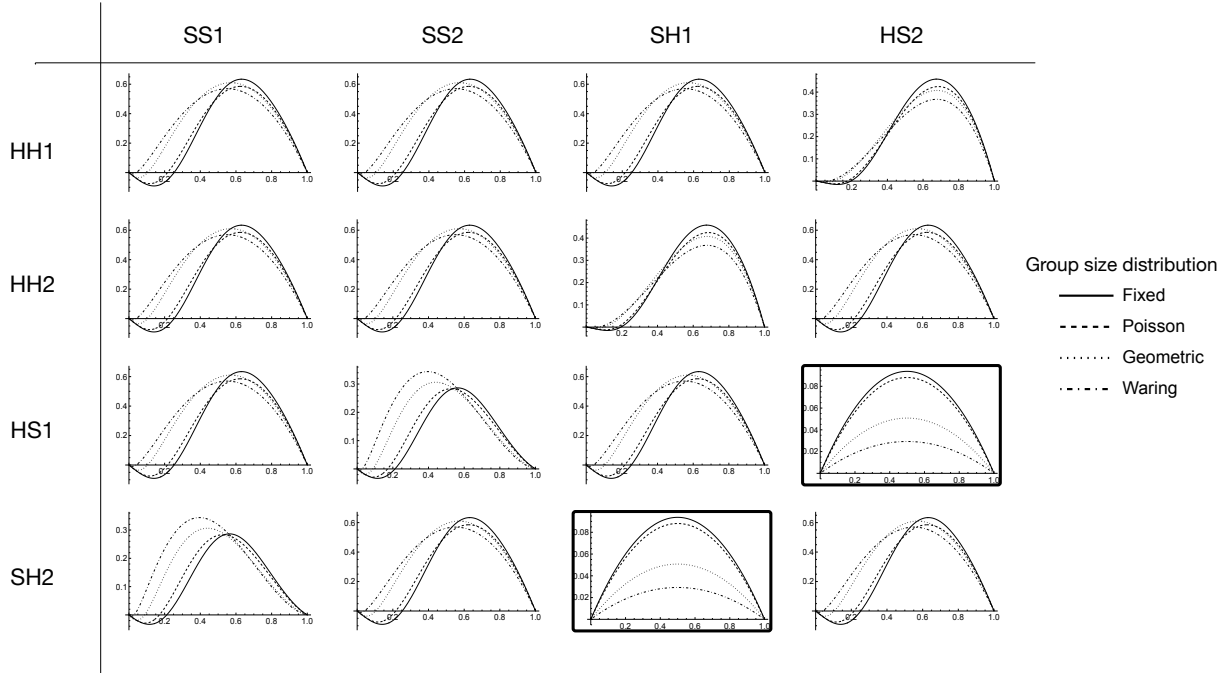
Supplementary Figure 1: **Change in dynamics between two strategies depending on the group size distribution.** Stag hunting strategies (columns) were compared with hare hunting strategies (rows). Each panel of the plot represents a dynamics between the two strategies, with x representing the fraction of stag hunters. Varying the group size moves the internal fixed point of the dynamics to the left, enlarging the basin of attraction of the all stag hunt equilibrium. Bold frame panels: For pairs (H,S,2), (H,S,1) and (S,H,1), (S,H,2) the dynamics lead to all stag hunt equilibrium regardless of the group size distribution. The parameters for each of the considered distributions are $E[N] \approx 8$, $M = 4$, $P_S = 4$, $P_H = 1$. Four distributions were considered: fixed group size 8, Poisson distribution with mean $\mu = 8$, geometric distribution with probability parameter $p = 0.111$ and Waring distribution with $\alpha = 2.25, \beta = 0.5, n = 20$.

Impact of small groups The ER network is generated randomly. Hence, some individuals may end up with fewer neighbours than the minimal size of the successful stag hunt (M). In simulations

presented in this work, such small groups constituted 1.03% of all hunting parties. To check whether the presence of these groups influenced the results, we compared the average takeover time on the networks containing the small groups with the one on the remaining graphs. A two-sample t-test has been used. The results of all the tests are presented in Supplementary Figure 2 The comparison was done within each of the 37 parameter sets considered.
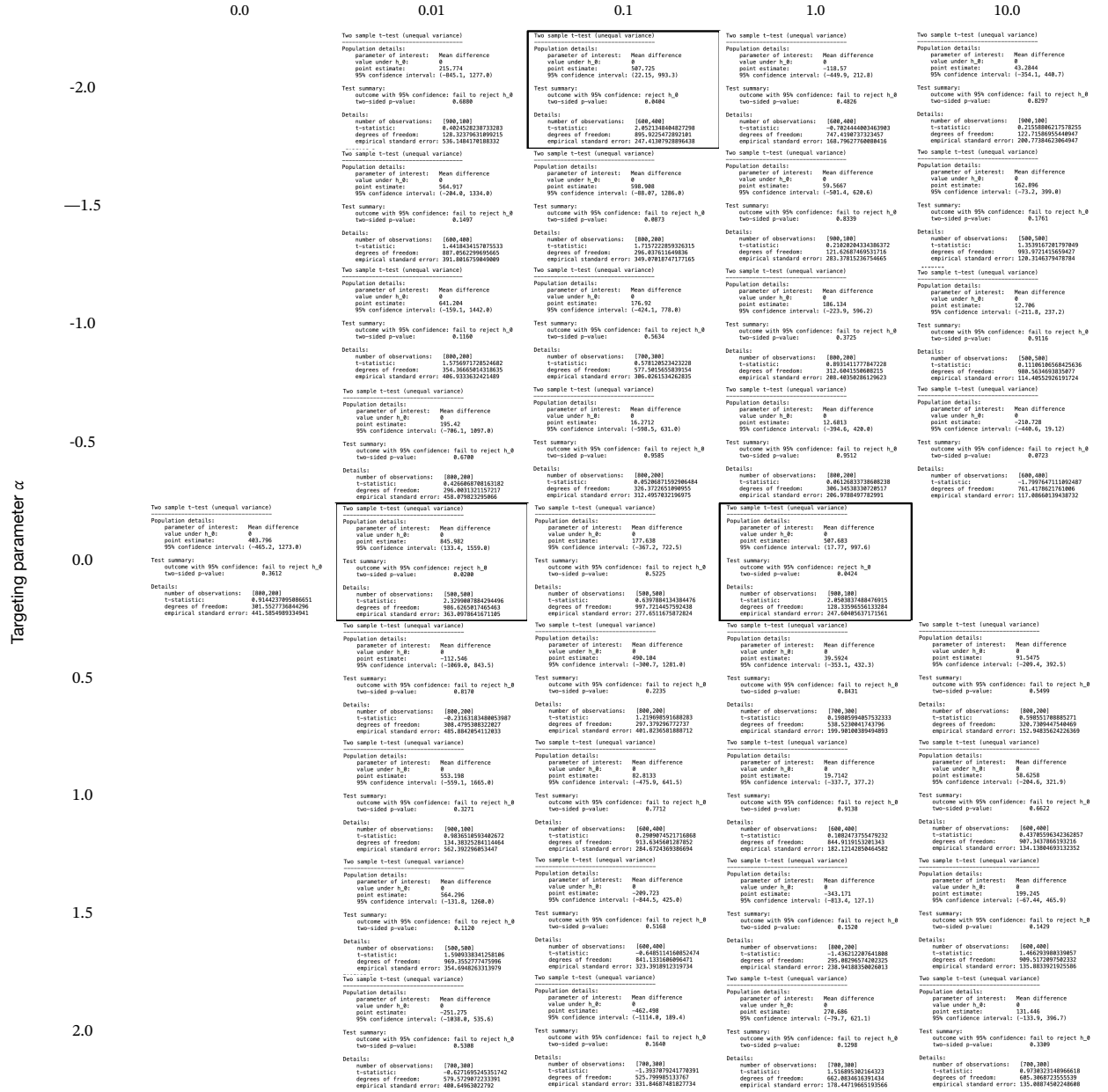
No networks with small groups were present in one of the parameter sets. In three parameter sets, the simulations performed on the networks with small groups were characterized by a faster takeover. In thirty-three parameter sets, we observed no difference between the two types of networks. Hence, we conclude that any differences between the networks with and without small groups are not significant and result from the parameters of the individual networks and stochasticity rather than the attribute of interest.

Different network generating algorithms The main study generated the small world networks using the Newman-Watts-Strogatz algorithm [10]. Similar network properties can also be obtained by using the Watts-Strogatz algorithm [11], where a network is created by deleting a fraction P of links in a d dimensional regular lattice.

The Watts-Strogatz (WS) networks are used as an example of small-world networks in the following sections.

An Erdös-Rényi random network can be generated either by specifying a number of edges [12] or a probability of each edge being generated [13]. Both of the algorithms result in a random network

Ratio of mutation rates μ_B/μ_A

| | 0.0 | 0.01 | 0.1 | 1.0 | 10.0 |

Targeting parameter α

-2.0, -1.5, -1.0, -0.5, 0.0, 0.5, 1.0, 1.5, 2.0

Supplementary Figure 2: **Summaries of two sample t-tests between simulations run on networks with and without hunting parties of size at most 3.** For the parameter set ($\alpha = 0.0$, $\mu_B = 0.01$) no networks with small groups were present. For three parameter sets (indicated with a frame) the simulations run on networks with small hunting parties were characterized with a significantly shorter takeover time. However, the difference is only siginifcant with the confidence level 0.05 and not 0.01.

with similar properties.

The ER algorithm's latter formulation (depending on specifying the probability of edge generation) is used in the following sections.

Moving away from small networks Our main text results focus on a small population of size $Z = 32$. However, we presume that our conclusions from the analysis hold for other small population sizes. To check that we run an additional set of simulations with the following parameters: $Z \in \{52, 73, 92, 112, 132, 152, 172, 192, 212, 232, 252\}$, $\mu_A = 10^{-3}$, $\mu_B \in \{10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}\}$, $P_H = 1$, $P_S = 4$, $M = 4$, $\omega = 1.0$, $\alpha = 0.0$. For each parameter set, ten graphs were generated, and simulations were run ten times on each graph. The results are then averaged over those 100 simulation runs.

In the smallest considered network $Z = 32$, we observe a decrease in takeover time with an increase in belief mutation rate μ_B. The same trend can be observed in the bigger network. However, for a big enough network size, the trend is reversed at $\mu_B = \mu_A = 10^{-3}$, and increasing belief mutation leads to an increase in takeover time. For each network type, the size threshold at which the shift in the direction of the effect of belief mutation varies. For the WS networks, the downward trend can be observed until $Z = 132$. For BA and ER, the size at which high mutation still leads to a decrease in takeover time equals $Z = 92$ and $Z = 152$, respectively. The change in the trend is depicted in Supplementary Figure 3 .

These results suggest that our conclusions are applicable to small networks, where the exact

definition of a small network depends on the network type. The effects of introducing collective beliefs on larger networks remain an interesting future direction of study.
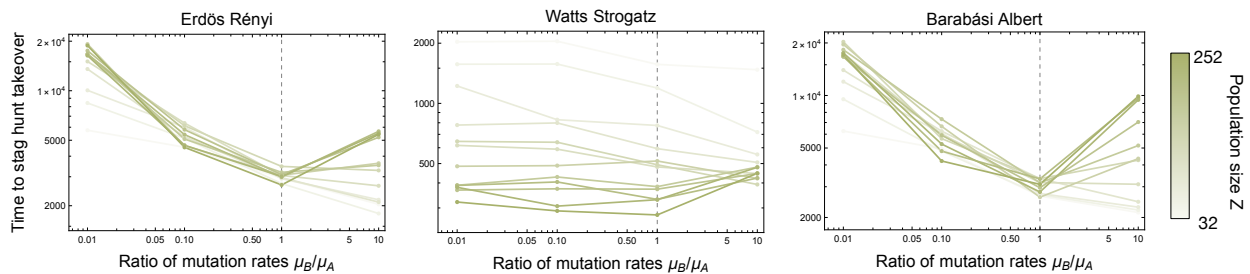
Supplementary Figure 3: **Time to stag takeover depending on the population size Z and the ratio of mutation rates μ_B/μ_A.** For small enough network sizes increase in the mutation rate leads to a decrease in takeover time. However, with an increase in network size, too high mutation rates may cause an increase in takeover time. The population size at which the change of direction occurs depends on the network type. The presented results are averaged over 100 runs per network type and parameter set.

Impact of selection intensity The importance of the game (interaction) on the overall dynamics can be modelled by varying the value of the ω parameter [14]. For $\omega = 0$ all strategies are equivalent, as the hunt does not influence the fitness.

An additional set of simulations was conducted to determine the importance of ω on the dynamics. We use $Z = 32$, $\mu_A = 10^{-3}$, $\mu_B \in \{0.0, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}\}$, $P_H = 1$, $P_S = 4$, $M = 4$, $\omega \in \{0.0, 0.25, 0.5, 0.75\}$, $\alpha = 0.0$. Each simulation is run for 10^5 generations. Presented results are averaged over 10 graphs per network type (ER, WS, BA) and 100 runs per graph.

As expected, for $\omega = 0.0$ (first row in Supplementary Figure 4), the proportion of stag

hunters in the population, in the long run, is equal to approximately 50% for all values of belief mutation, as there is no advantage to any of the strategies. As the parameter's value increases slightly (subsequent rows in Supplementary Figure 4), the proportion of stag hunters grows to 98%. The effect of belief mutation can also be observed, as the percentage of stag hunters decreases with the mutation rate due to increased stochasticity.

The same effect is not so visible for an abundance of different belief followers, represented as grey bars in Supplementary Figure 4, as the belief itself does not impact one's fitness. Additionally, the belief abundance depends highly on the mutation rate at which the second belief is introduced. The proportion of belief followers increases significantly with the increase of μ_B. A slight increment can also be observed with the growing value of ω, suggesting that the second belief is essential for the game dynamic and acts as a coordination device.

Varying value of Stag The takeover of the stag hunters may be caused, or at least aided, by a relatively high value of the stag compared to the hare. To determine whether the high payoff of stag hunters is required for their prevalence, we conduct simulations considering a smaller value of stag. In particular, we use the following parameters: $Z = 32$, $\mu_A = 10^{-3}$, $\mu_B \in \{10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}\}$, $P_H = 1$, $P_S = \in \{1, 2, 3\}$, $M = 4$, $\omega = 1.0$, $\alpha = 0.0$. Each simulation is run for a maximum of 10^5 generations. The results are averaged over 10 graphs per network type and 10 simulation runs on each graph.

Surprisingly, almost all simulations still led to stag hunt takeover, even for $P_S = 1$. Only for the lowest value of belief mutation, $\mu_B = 10^{-5}$ and lowest value of stag $P_S = 1$ stag hunters
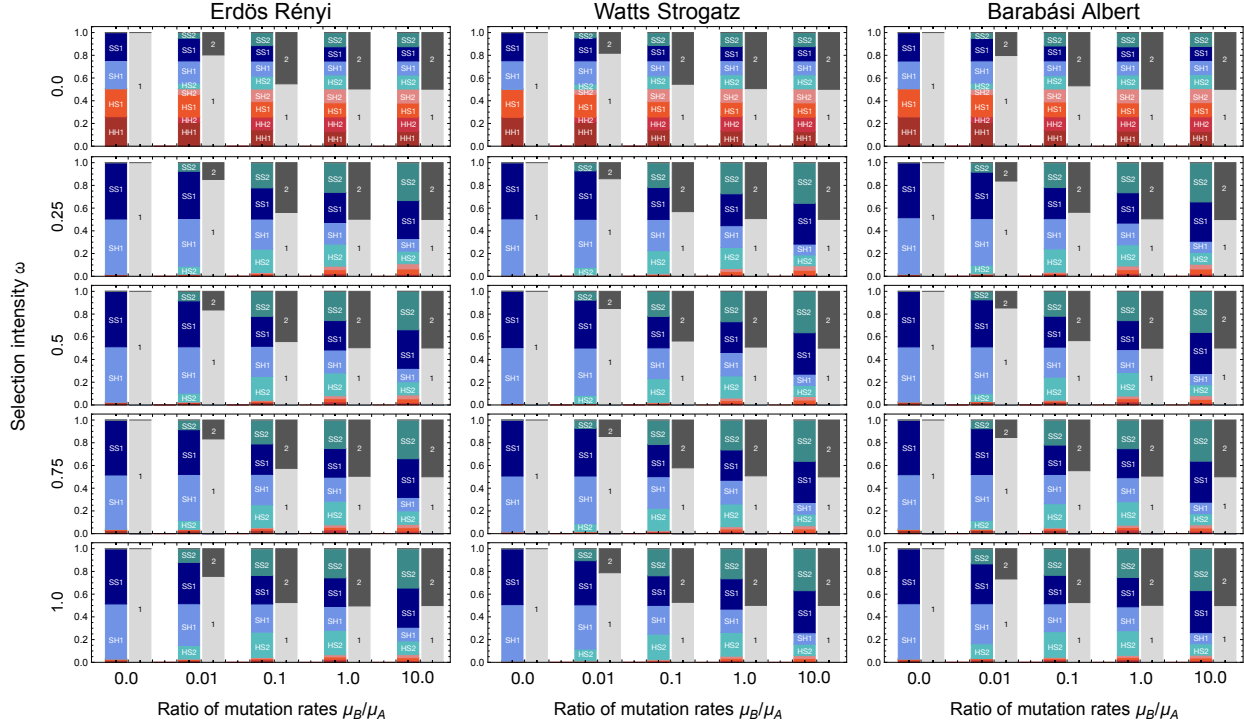
Supplementary Figure 4: **Equilibrium frequency of each strategy and belief in the population depending on the selection intensity ω and the ratio of mutation rates μ_B/μ_A.** While selection intensity ω equals 0, all strategies are equally likely to appear in the population. An increase in the parameter leads to a swift rise in the fraction of stag hunters. The frequency of each of the two narrative believers depends mainly on the value of the mutation rate, showing that the beliefs do not impact the payoff directly. The presented results are averaged over 100 runs per network type and parameter set.

were not able to take over the population in 23% and 17% of cases on the BA and ER networks respectively. On the WS network, all simulations led to a stag hunter takeover. This result clearly shows the power of collective narratives to entail coordination.

Let us analyse how stag hunters can take over a population with their payoff being at most equal to the one of hare hunters. Without the collective beliefs in place, it would be almost impossible for stag hunters to arise in the population - any single stag hunter that would appear due to mutations would have minimal chances of surviving. However, in the presence of beliefs, an individual can be a hare hunter in one narrative and a stag hunter in another, and that way, exist and reproduce while hunting a hare until they are surrounded by like-minded individuals, at which point a change in belief may cause them to hunt stags. In particular, we can imagine a group of (H,S,1) individuals who are hare hunters with the possibility to hunt stags. By introducing a belief mutation to one of them, we can create a (H,S,2)individual. If the hunter with the new belief can convince their hunting party of their belief (which can always happen by chance), stag hunting may emerge. While the takeover of stag hunters in that scenario would be rapid if they had a payoff advantage, it is still possible to observe the same effect by drift. Having the same payoff as hare hunters gives the singular stag hunters the same chance to reproduce as their less cooperative neighbours. Hence, it is still possible for stag hunters to take over the population. However, it is not guaranteed for them to remain in power, as the drift may cause a shift in the opposite direction. Additionally, the takeover is much slower since the change is driven by drift rather than selection (Supplementary Figure 5).

Even so, assuming a here and a stag to be worth the same is unrealistic and negates the game's purpose. Hence, we focus on analysing the results for slightly higher values of a stag. Even for $P_S = 2$, all simulations lead to a stag takeover. Increasing the value of a stag leads to a faster takeover. Moreover, as shown in Supplementary Figure 5, we can see that the difference between

low and high stag values is diminished with increased belief mutations, showing that introducing

collective beliefs leads to a faster and more effective spread of cooperative behaviours.
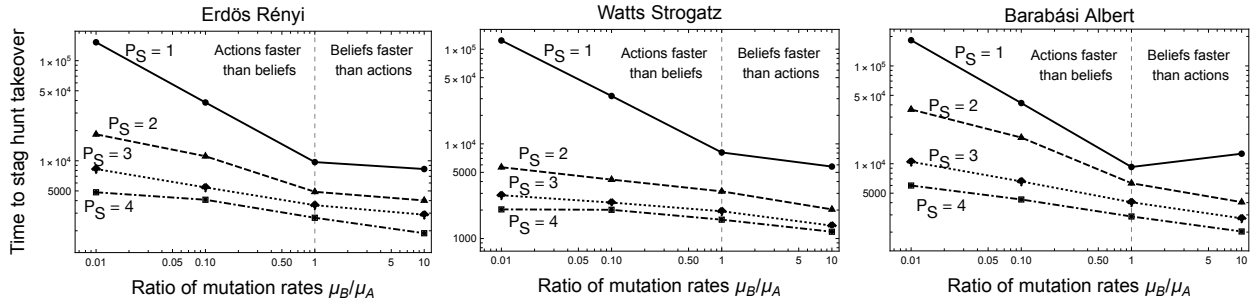
Supplementary Figure 5: **Time to stag hunter takeover depending on the value of Stag P_S**

and the ratio of mutation rates μ_B/μ_A. For $P_S = P_H = 1$, not all simulations resulted in stag

takeover. However, when stag hunters had an advantage over hare hunters, they could always take

over the population. An increase in the value of stag led to a decrease in takeover time. The

presented results are averaged over 100 runs per network type and parameter set.

Varying value of M   Hunting a stag is a group effort. In our model, we assume that at least

M hunters need to partake in the effort for the hunt to be successful. The value of that hunting

threshold may affect the game dynamics, especially in structured populations. As the necessary

number of hunters grows, it is more probable that the group size is smaller than the threshold. To

investigate the effect of the threshold, we run additional simulations while varying the parameter

M. We use $Z = 32$, $\mu_A = 10^{-3}$, $\mu_B \in \{10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}\}$, $P_H = 1$, $P_S = 4$, $M = \in$

$\{2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$, $\omega = 1.0$, $\alpha = 0.0$. For each parameter set, 10 graphs were

generated, and 10 runs were conducted on each graph. The presented results are averaged over 100

runs.

The networks used in the simulations were generated for $M = 4$, with minimal connectivity of the BA network set to that value. Hence, as the threshold value increased, the probability of the group being too small to succeed in the stag hunt increased. Subsequently, the probability of a stag takeover decreased with the increase of M, as exhibited on Supplementary Figure 6. This effect was particularly apparent for $\mu_B = 10^{-5}$, as the takeover was generally the slowest for low belief mutation rate. For the highest value of the threshold considered, $M = 15$, the simulations ended with the stag takeover in 50% on BA networks, 35% on WS and 34% on ER. The relatively better performance of the BA networks may be regarded as the effect of the degree distribution of the networks. The long tail of the degree distribution of the BA networks ensures that at least some stag hunters can appear in the population and meet the threshold value, leading to a higher probability of stag takeover. For higher belief mutation rates, stag hunters are more likely to take over. For $\mu_B = 10^{-2}$, even for $M = 15$, all simulations on all network types ended in stag hunt takeover.

We measure conditional time to stag takeover, i.e. the time to takeover for each threshold value, given that the takeover takes place. The effects of a high hunting threshold are the most apparent for low belief mutation rates. Still, the conditional time to takeover increases with the threshold increase for all belief mutation values, as shown in Supplementary Figure 7.

The effect of the belief mutation rate on takeover time for each value of M varies. There is no trend for low threshold values ($M \in \{2, 3\}$, represented by the lightest colours in Supplementary Figure 7), and differences between takeover times for different mutation rates are minimal. Hence,
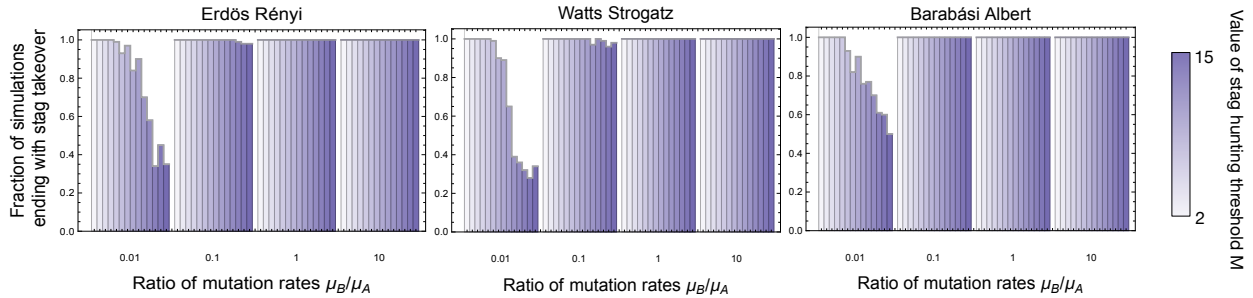
Supplementary Figure 6: **Fraction of simulations ending in stag takeover depending on the vale of M and the ratio of mutation rates μ_B/μ_A.** For low mutation values and high threshold value M only part of simulations led to stag takeover. However, with a lower threshold or higher belief mutation, all simulation runs ended with a stag takeover. The presented results are averaged over 100 runs per network type and parameter set.

if hunting a stag is relatively easy, collective narratives do not have a meaningful effect, as the population would become stag hunters regardless. For intermediate values of M, we observe a decrease in takeover time with the increase in mutation rate, as discussed in the main manuscript. As the value of M increases, we observe a change in the trend and an increase in the takeover time between $\mu_B = 10^{-3}$ and 10^{-2}. This suggests that as stag hunting is more challenging to sustain, too high belief mutation rate may jeopardise the stag takeover. For these parameter values, strategies must spread among more individuals for the population to switch to a stag hunt. If the belief mutation rate is too high, the change in belief happens before enough potential stag hunters are present and the hunt is unsuccessful. However, this effect dissipates if M is increased even more. For a high enough threshold value, we again observe the monotonous decrease in takeover time with increased mutation rates. For high enough values of the hunting threshold, stag hunters

are too disadvantaged and struggle to take over the population. The takeover is then facilitated by a rapid mutation of beliefs, which may randomly cause the takeover. However, since the switch is caused by the random factor rather than the genuine advantage of stag hunters, the stag hunt equilibrium is not guaranteed to be retained. The limiting values of M vary between the network types considered.

The hunting threshold M proves to be an essential parameter of the model. Setting it too low makes stag hunt easy to obtain and renders collective beliefs unimportant. However, higher values of the parameter allow for collective beliefs to showcase their power - first as a coordination device between players and later, when stag hunt is almost impossible to achieve, by introducing an additional level of stochasticity and facilitating the switch. Notably, if the stag hunters are not guaranteed to succeed in their hunt even in a population of all stag hunters, that is if the threshold is larger than the group size, their takeover is a random process and the population can revert to hare hunting.

The effect of clustering We hypothesise that high clustering promotes the spread of cooperative behaviour [15,16]. We use the Watts-Strogatz network to check if that is the case. By varying the rewiring parameter while generating the network, we can produce a set of graphs varying from regular to random. Subsequently, we vary the clustering coefficients of the graphs from the highest in the regular graph to the lowest in the random network. We use $Z = 32$, $\mu_A = 10^{-3}$, $\mu_B \in \{10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}\}$, $P_H = 1$, $P_S = 4$, $M = 4$, $\omega = 1.0$, $\alpha = 0.0$, $p \in \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$, where p is the rewiring probability in the
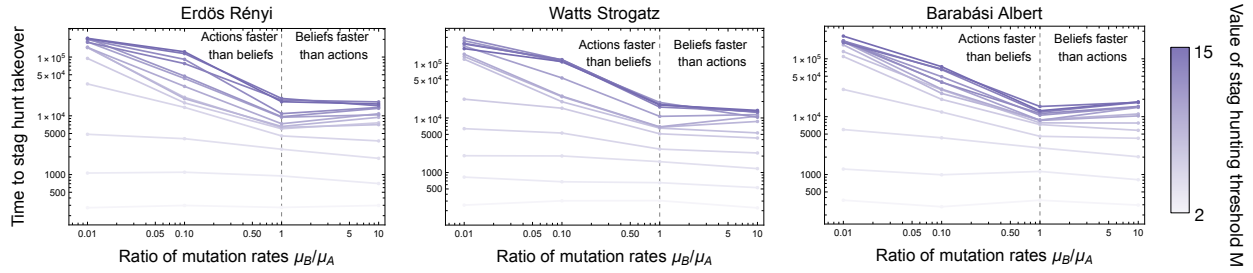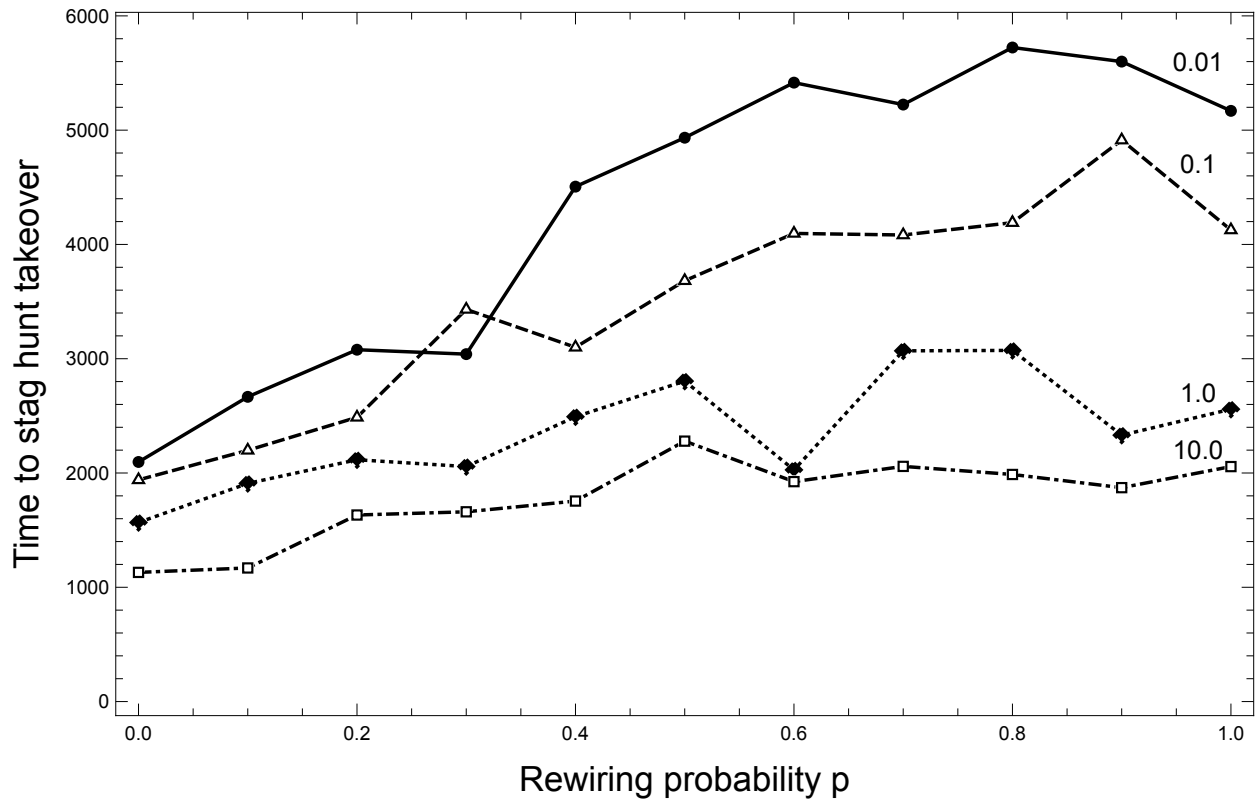
Supplementary Figure 7: **Conditional time to stag takeover depending on the vale of** M **and the ratio of mutation rates** μ_B/μ_A. If hunting a stag is easy (low values of M) narratives are not very important and the time to takeover does not depend on the belief mutation rate. However, as hunting requires more and more participants, beliefs become more important and we observe a decrease in takeover time with an increase in mutation rate. For very high values of M, a slight increase in takeover time can be observed if the mutation is too high - showing that too rapid mutation turnover may hinder cooperation. The presented results are averaged over 100 runs per network type and parameter set.

WS network generating algorithm. For each set of parameters, 10 graphs were generated, and 10 runs of the simulation were run on each of the graphs.

An increase in p leads to decreased clustering in the network [11]. The results of the simulations show that an increase in p leads to an increase in takeover time, as shown in Supplementary Figure 8. In other words, lowering the clustering coefficients leads to a slower cooperator takeover. These results confirm our claim about the beneficial impact of clustering on levels of cooperation.

Supplementary Figure 8: **Time to stag takeover depending on the vale of rewiring probability** p

and the ratio of mutation rates μ_B/μ_A. High rewiring probability leads to low global coefficients

in the network. Simulations run on networks characterized by lower clustering result in higher

takeover time across all mutation rates. The presented results are averaged over 100 runs per

network type and parameter set.

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