Letter

Tidal effects and renormalization at fourth post-Minkowskian order

Gustav Uhre Jakobsen,[1,2] Gustav Mogull,[1,2] Jan Plefka,[1] and Benjamin Sauer[1]

[1]*Institut für Physik und IRIS Adlershof, Humboldt-Universität zu Berlin, Zum Großen Windkanal 2, 12489 Berlin, Germany*

[2]*Max Planck Institut für Gravitationsphysik (Albert Einstein Institut), Am Mühlenberg 1, 14476 Potsdam, Germany*

(Received 4 January 2024; accepted 19 January 2024; published 20 February 2024)

We determine the adiabatic tidal contributions to the radiation reacted momentum impulse Δp_i^μ and scattering angle θ between two scattered massive bodies (neutron stars) at next-to-next-to-leading post-Minkowskian order. The state-of-the-art three-loop (fourth post-Mikowskian order) worldline quantum field theory toolkit using dimensional regularization is employed to establish the classical observables. We encounter divergent terms in the gravitoelectric and gravitomagnetic quadrupolar sectors necessitating the addition of postadiabatic counterterms in this classical theory. This leads us to include also the leading postadiabatic tidal contributions to the observables. The resulting renormalization group flow of the associated postadiabatic Love numbers is established and shown to agree with a recent gravitoelectric third post-Newtonian analysis in the nonrelativistic limit.

DOI: 10.1103/PhysRevD.109.L041504

With today's routine detection of gravitational waves by the LIGO-Virgo-Kagra observatories emitted from binary merger events of black holes and neutron stars in our Universe [1–3] we are in the era of gravitational wave astronomy. The upcoming space- and earth-based third generation of observatories will widen the frequency range and dramatically increase the sensitivity of the observations [4–6]. This situation calls for in par precision predictions from theory for the observables in the gravitational two-body problem. To achieve this a combination of analytical and numerical approaches is being pursued actively: from the perturbative, analytical side the post-Newtonian [7–9] and post-Minkowskian (PM) [10–14] expansions cover the inspiral phase where the two bodies are still well separated and weak gravitational fields apply; while the self-force expansion [15–18] assumes a mass hierarchy in the two bodies but works exactly in Newton's coupling G. These perturbative results may be resummed using effective-one-body techniques [19,20] to extend their validity close to merger where numerical relativity (NR) [21–23] techniques become indispensable.

Recently, considerable progress has been made upon importing modern techniques from perturbative quantum field theory (QFT) to the problem in the PM expansion. While the natural habitat for the PM expansion is the scattering of black holes or neutron stars [24–28], the scattering data may nevertheless be used to inform models for the bound-state problem that should become particularly relevant for highly eccentric orbits [29–34]. As long as the objects' separation is large compared to their intrinsic sizes, they have an effective description in terms of a massive point particle coupled to Einstein's theory of gravity that may be systematically corrected for intrinsic degrees of freedom such as spin or tidal effects [35]. Based on this effective worldline approach two-body scattering observables (deflections and Bremsstrahlung waveforms) have recently been computed up to next-to-next-to-next-to-leading order (deflections) and leading order (Bremsstrahlung) [36–55]. In parallel, great leaps in the QFT based PM expansions were achieved using techniques based on scattering amplitudes in which quantum field act as avatars of black holes (BHs) or neutron stars (NSs) [56–89].

Next to the masses and spins of the compact objects, tidal deformations are a significant astrophysical phenomenon and observational goal. NSs develop a quadrupole moment due to the tidal interaction with their companion star or BH [90,91]. The strength of this effect is parametrized by the Love numbers that quantify the magnitude of the induced multipole moment in response to an external gravitational field. Measuring them through gravitational waves provides insights into the strong interaction matter within neutron stars. In fact, the gravitational wave signal GW170817 observing the first NS-NS merger [2] was able to put constraints on the first (gravitoelectric) Love number with consequences for the neutron star equation of state [92–95]. The tidal interactions give rise to oscillation modes of the NS, and in particular the so-called f-mode dynamical tides [96,97] have been argued to be central for inferring the NS equation of state from the emitted gravitational waves in a merger [98]. In the adiabatic limit the tides do not oscillate independently and are locked to the external gravitoelectric and gravitomagnetic fields—the limit we shall consider in this work.

In the PN expansion gravitomagnetic and gravitoelectric tides have been established up to 2PN order [99,100] and recently the state-of-the art has been extended to 3PN for dynamical and adiabatic tides [101,102]. Working in dimensional regularization the authors of Ref. [102] encountered a UV divergence that necessitated the inclusion of a post-adiabatic counterterm, leading to a renormalization group flow of its postadiabatic Love number. Meanwhile, in the PM expansion the two-body scattering observables of the impulse (change of momentum) and scattering angle in the presence of tidal interactions have been determined at 2PM [103–106] and 3PM [38] order in the conservative sector. We updated the 3PM result to include dissipation [51] using the in-in worldline QFT (WQFT) formalism. In addition, the Bremsstrahlung waveform with tidal effects at leading order was established in Refs. [44,51].

In this Letter we lift this tidal precision prediction for the impulse and scattering angle to the 4PM, i.e., next-to-next-to-leading order (NNLO) in the PM expansion—both in the conservative and dissipative sectors and for the gravitoelectric and gravitomagnetic tides. As it turns out, this classical computation suffers from an UV (ultraviolet) divergence that may be attributed to the point-particle approximation of the neutron star, equivalent to what was seen at 3PN order [101,102].[1] We regulate the theory using dimensional regularization in the bulk, i.e., the worldline action remains one dimensional. While Newton's constant G is continued to $D = 4 - 2\epsilon$ dimensions, $G_D = (4\pi e^{\gamma_E} R^2)^{-\epsilon} G$, introducing an arbitrary length scale R, the Love numbers are not dimensionally continued. Removing the UV divergence through a postadiabatic counterterm then induces a renormalization group flow of the associated postadiabatic Love numbers, matching the flow in the gravitoelectric sector in the PN analysis of Ref. [102]. Using our results for the impulse and scattering angle we also establish the linear and angular momentum, at 4PM and 3PM order, respectively, radiated off by the gravitational waves emitted in the encounter of the two NSs.

Worldline effective action. The effective description of nonspinning compact objects (neutron stars) including the leading-order adiabatic tidal couplings takes the form of a point-particle action $S = \sum_{i=1}^{2} S_{\rm pp}^{(i)} + S_{\rm tidal}^{(i)}$, where [36]

$$S_{\rm pp}^{(i)} = -m_i \int \mathrm{d}\tau \left[\frac{1}{2e} g_{\mu\nu} \dot{x}_i^\mu \dot{x}_i^\nu + \frac{e}{2}\right], \tag{1}$$

$$S_{\rm tidal}^{(i)} = -m_i \int \mathrm{d}\tau \left[\frac{c_{E^2}^{(i)}}{e^3} E_{\mu\nu}^{(i)} E^{(i)\mu\nu} + \frac{c_{B^2}^{(i)}}{e^3} B_{\mu\nu}^{(i)} B^{(i)\mu\nu}\right]. \tag{2}$$

Here $x_i^\mu(\tau)$ is the trajectory of the ith body of mass m_i and $e(\tau)$ is the einbein ensuring reparametrization invariance of the worldline theory. The quadrupole Love numbers $c_{E^2}^{(i)}$ and $c_{B^2}^{(i)}$ (Wilson coefficients in an effective field theory nomenclature) are of mass dimension four and couple to the gravitoelectric and gravitomagnetic curvature tensors

$$E_{\mu\nu}^{(i)} \coloneqq R_{\mu\alpha\nu\beta} \dot{x}_i^\alpha \dot{x}_i^\beta, \qquad B_{\mu\nu}^{(i)} \coloneqq R^*_{\mu\alpha\nu\beta} \dot{x}_i^\alpha \dot{x}_i^\beta, \tag{3}$$

with the dual Riemann tensor $R^*_{\mu\alpha\nu\beta} \coloneqq \frac{1}{2}\epsilon_{\nu\beta\rho\sigma} R_{\mu\alpha}{}^{\rho\sigma}$. Introducing $B_{\mu\nu\rho}^{(i)} = R_{\alpha\mu\nu\rho} \dot{x}_i^\alpha \sqrt{\dot{x}_i^2}$ we note the relation

$$B_{\mu\nu}^{(i)} B^{(i)\mu\nu} = E_{\mu\nu}^{(i)} E^{(i)\mu\nu} - \frac{1}{2} B_{\mu\nu\rho}^{(i)} B^{(i)\mu\nu\rho}, \tag{4}$$

that generalizes (2) to D dimensions. These are the first of an infinite series of tidal Love number couplings, capturing the linear response of the compact body to an external gravitational field. For the case of a black hole they are known to vanish [109–111].

The two neutron stars $x_i^\mu(\tau)$ interact gravitationally according to the gauge-fixed Einstein-Hilbert action

$$S_{\rm bulk} = \int \mathrm{d}^D x \left(-\frac{1}{16\pi G_D}\sqrt{-g} R + \left(\partial_\nu h^{\mu\nu} - \frac{1}{2}\partial^\mu h^\nu{}_\nu\right)^2\right) \tag{5}$$

in the bulk, where $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{32\pi G_D} h_{\mu\nu}$ and $h_{\mu\nu}$ is the graviton field. Here, we take $G_D = (4\pi e^{\gamma_E} R^2)^{-\epsilon} G$, the extension of Newton's constant G to $D = 4 - 2\epsilon$ dimensions, working in an $\overline{\rm MS}$ scheme adapted to configuration space. R denotes an intrinsic length scale of the compact object, such as its radius.

Let us briefly comment on our PM counting scheme. For a neutron star, or other compact body whose radius is of the order of its Schwarzschild radius, it is natural to factor the scale of the Schwarzschild radius out of the Love numbers such that

$$c_{E^2}^{(i)} = (Gm_i)^4 \tilde{c}_{E^2}^{(i)}, \qquad c_{B^2}^{(i)} = (Gm_i)^4 \tilde{c}_{B^2}^{(i)}, \tag{6}$$

with dimensionless Love numbers $\tilde{c}_{E^2/B^2}^{(i)}$ of order unity. From that perspective, the results reported in this Letter are enhanced with an additional factor of G^4, which pushes them to the (physical) 8PM order. In this Letter, however, we opt for a (formal) PM counting aligned with all explicit

[1]This divergence was also seen in gravitational radiation from a single compact object with a quadrupole in Refs. [107,108].

instances of G in the action—Eqs. (5) and (7) below. For the adiabatic tidal results this implies that nPM corresponds with $(n-1)$ loops while for the postadiabatic interaction introduced below nPM corresponds with $(n-3)$ loops (see, e.g., also the discussion in Ref. [112] regarding PM counting with spin).

Renormalization. As mentioned above, in the computation of the impulse at the (formal) 4PM level one encounters an UV divergence that is canceled upon including the postadiabatic tidal counterterm

$$S_{\rm ct}^{(i)} = -m_i^3 G^2 \int \mathrm{d}\tau \left[\frac{c_{E^2}^{(i)} \tilde{\kappa}_{\dot{E}^2}^{(i)}}{e^5} \dot{E}_{\mu\nu}^{(i)} \dot{E}^{(i)\mu\nu} + \frac{c_{B^2}^{(i)} \tilde{\kappa}_{\dot{B}^2}^{(i)}}{e^5} \dot{B}_{\mu\nu}^{(i)} \dot{B}^{(i)\mu\nu} \right] \tag{7}$$

to the total action. Note that we need to use the 4D Newton constant G here, as the worldline action remains one dimensional. In the present work, the postadiabatic counter term Eq. (7) is required only at tree level. At leading order in G its last term straightforwardly generalizes to D dimensions as

$$\dot{B}_{\mu\nu}^{(i)} \dot{B}^{(i)\mu\nu} = \dot{E}_{\mu\nu}^{(i)} \dot{E}^{(i)\mu\nu} - \frac{1}{2} \dot{B}_{\mu\nu\rho}^{(i)} \dot{B}^{(i)\mu\nu\rho} + \mathcal{O}(G^3). \tag{8}$$

In Eq. (7) the dimensionless postadiabatic Love numbers $\tilde{\kappa}_{\dot{E}^2/\dot{B}^2}$ take the form [dropping the neutron star label (i)]

$$\tilde{\kappa}_{\dot{E}^2} = -\frac{107}{105}\frac{1}{\epsilon} + \kappa_{\dot{E}^2}, \qquad \tilde{\kappa}_{\dot{B}^2} = -\frac{107}{105}\frac{1}{\epsilon} + \kappa_{\dot{B}^2}, \tag{9}$$

with the counterterms removing the divergences appearing at the 4PM order being given by the $1/\epsilon$ terms. We also include finite postadiabatic Love numbers $\kappa_{\dot{E}^2}$ and $\kappa_{\dot{B}^2}$. They experience a renormalization group flow as follows: the bare gravitational coupling G_D is independent of the scale R, so the flow equation for Newton's constant reads

$$\begin{aligned} 0 &= R\frac{\mathrm{d}}{\mathrm{d}R} G_D = R\frac{\mathrm{d}}{\mathrm{d}R}\left[(4\pi e^{\gamma_E} R^2)^{-\epsilon} G\right], \\ &= \left[-2\epsilon G + R\frac{\mathrm{d}}{\mathrm{d}R} G\right](4\pi e^{\gamma_E} R^2)^{-\epsilon}, \end{aligned} \tag{10}$$

i.e., there is no flow of G in $D=4$ dimensions. The bare couplings in (2) and (7) do not depend on the scale R, hence

$$\begin{aligned} 0 &= R\frac{\mathrm{d}}{\mathrm{d}R} c_{E^2}, \\ 0 &= R\frac{\mathrm{d}}{\mathrm{d}R}(c_{E^2} G^2 \tilde{\kappa}_{\dot{E}^2}), \\ &= c_{E^2} G^2 R \frac{\mathrm{d}}{\mathrm{d}R}\kappa_{\dot{E}^2} - \frac{107}{105}\frac{c_{E^2}}{\epsilon} R\frac{\mathrm{d}}{\mathrm{d}R} G^2. \end{aligned} \tag{11}$$

Together with (10), this then yields the β functions for the renormalized couplings $\kappa_{\dot{E}^2}$ and $\kappa_{\dot{B}^2}$ [101,102]

$$\beta_{\kappa_{\dot{E}^2}} = R\frac{\mathrm{d}\kappa_{\dot{E}^2}}{\mathrm{d}R} = \frac{428}{105}, \qquad \beta_{\kappa_{\dot{B}^2}} = R\frac{\mathrm{d}\kappa_{\dot{B}^2}}{\mathrm{d}R} = \frac{428}{105}. \tag{12}$$

These induce a logarithmic flow of the renormalized postadiabatic Love numbers as

$$\kappa_{\dot{E}^2}(R) = \kappa_{\dot{E}^2}(R_0) + \frac{428}{105}\log\left[\frac{R}{R_0}\right], \tag{13a}$$

$$\kappa_{\dot{B}^2}(R) = \kappa_{\dot{B}^2}(R_0) + \frac{428}{105}\log\left[\frac{R}{R_0}\right], \tag{13b}$$

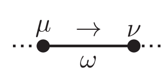
where R_0 is an arbitrary length scale.[2]

Computation. Our 4PM computation is performed using the WQFT three-loop workflow as described in [113,114], which we briefly review. The full tidal effective field theory is given by the sum of the bulk Eq. (5) and worldline actions of Eqs. (1) and (7). For the worldline trajectories we perform a background field expansion about straight line trajectories

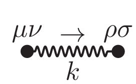
$$x_i^\mu(\tau) = b_i^\mu + v_i^\mu \tau + z_i^\mu(\tau), \tag{14}$$

with perturbative deflections $z_i^\mu(\tau)$. This setup reflects the scattering scenario parametrized by the impact parameters b_i^μ and incoming velocities v_i^μ. In addition we define the physical impact parameter $b^\mu = |b|\hat{b}^\mu = (b_2 - b_1)^\mu$ and we impose $v_i \cdot b = 0$. Moreover, in the PM expansion the metric is taken to be $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{32\pi G_D} h_{\mu\nu}$. The goal is to construct perturbative-in-G solutions to the equations of motion for the deflections $z_i^\mu(\tau)$, which is efficiently generated in a diagrammatic fashion in the WQFT formalism—see Ref. [14] for a recent review.

The propagating fields $z_i^\mu(\tau)$ and $h_{\mu\nu}(x)$ have the retarded propagators



$$= \frac{-i\eta^{\mu\nu}}{m_i(\omega + i0^+)^2}, \tag{15a}$$



$$= \frac{i(\eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{1}{D-2}\eta_{\mu\nu}\eta_{\rho\sigma})}{k^2 + \mathrm{sgn}(k^0) i0^+}, \tag{15b}$$

[2]Note that the 3PN reference [102] works with different conventions than we do. Their postadiabatic coupling is minus one-half ours, $\kappa_{PN} = -\kappa/2$, their adiabatic coupling $\lambda_{PN} = 4mc_{E^2}$, and they take $D-1 = 3 + \varepsilon_{PN}$, i.e., $\varepsilon_{PN} = -2\epsilon$. Taking this into account, we agree with their findings.

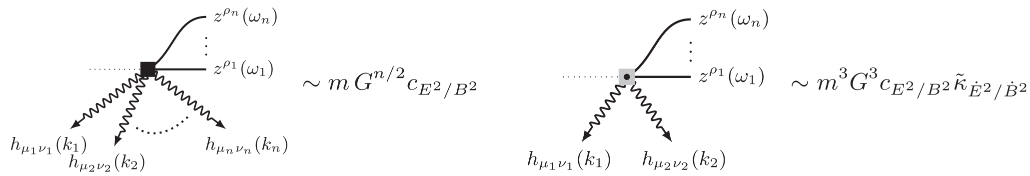


FIG. 1. The tidal interaction vertices needed for the 4PM (order κ^8) computation, originating from $S_{\rm tidal}$ (2) and the counterterm vertices originating from $S_{\rm ct}$ (7).

using the Schwinger-Keldysh in-in formalism [51,115]. The worldline vertex rules originating from $S_{\rm pp}$ in Eq. (1) at lower multiplicities have been exposed explicitly in [49,113]: the vertex couples one graviton to m worldline deflections and conserves the energy on the worldline. The vertices originating from the tidal terms Eq. (2) involve $n \geq 2$ gravitons and m worldline deflections—for the 4PM order computation we need the vertices with up to four graviton legs $n = 2, 3, 4$. The counterterm (7) on the other hand, gives rise to an $n \geq 2$ graviton and m-worldline vertex, that we only need for $n = 2$ and $m = 0, 1$, cf. Fig. 1. The bulk graviton vertices are standard, yet involved. Using these Feynman rules the WQFT tree-level one-point functions $\langle z_i^\mu(\tau)\rangle$ may be systematically constructed. They solve the classical equations of motion [14,116].

WQFT workflow at 4PM. For the computation of the impulse, i.e., the change of momentum under the scattering process,

$$\Delta p_i^\mu = -m_i \omega^2 \langle z_i^\mu(\omega)\rangle|_{\omega=0} = \sum_{n>0} \Delta p_i^{(n)\mu}. \qquad (16)$$

At the 4PM level $\Delta p_i^{(4)\mu}$ we employ an in-house FORM- [117] and *Mathematica*-based code that employs a Berends-Giele-type recursion for the integrand construction. At this order we face three-loop Feynman integrals that depend on the momentum transfer q^μ and the relativistic $\gamma = v_1 \cdot v_2$ factor. The $|q|$ may be scaled out and we subsequently reduce the single-scale tensor integrals to scalars. The integration-by-part reduction and projection on the 4PM master integral basis was obtained in [113,114]. The 4PM master integrals in turn are three-loop single scale integrals (depending on γ) that have been solved employing the differential canonical equation method [118,119] and the method of regions [120] in the conservative and dissipative domains [113,114], see also [84,121–123].

The final step is a Fourier transform of the momentum transfer q^μ to impact parameter space. Here one novelty to the spinning 4PM computation of [113,114] is the appearance of $\log|Rq|$ terms:

$$\begin{aligned}\int_q e^{iq\cdot b}\delta(q\cdot v_1)\delta(q\cdot v_2)|q|^\nu \log|Rq| \\ = \frac{2^{\nu-1}}{\pi^{(D-2)/2}\sqrt{\gamma^2-1}} \frac{\Gamma\left(\frac{D-2+\nu}{2}\right)}{\Gamma\left(-\frac{\nu}{2}\right)}\left(-2\log\left|\frac{b}{2R}\right|\right. \\ \left. +\psi\left(-\frac{\nu}{2}\right)+\psi\left(\frac{D-2+\nu}{2}\right)\right)|b|^{2-D-\nu}, \qquad (17)\end{aligned}$$

with the digamma function $\psi(z) \coloneqq \Gamma'(z)/\Gamma(z)$. This may be most easily derived from the Fourier transform of $|q|^\nu$ via a derivative on the exponent ν. At 4PM order the impulse separates into the test-body contributions with linear mass dependence, $m_1 m_2^4$ or $m_1^4 m_2$, and the comparable-mass contributions, $m_1^2 m_2^3$ or $m_1^3 m_2^2$. In total we face 258 graphs contributing to the 4PM tidal effects—see Fig. 2 for some examples. The divergences arise in the comparable mass sectors. The postadiabatic contributions involving the counterterm are depicted in Fig. 3 and amount to a one-loop integration. Due to the m^3 factor in the counterterm vertices (Fig. 1) they contribute to $m_1^2 m_2^3$ and $m_1^3 m_2^2$ terms as well, thereby canceling the $1/\epsilon$ poles.

Impulse. Tidal contributions to the 4PM impulse may be split into a conservative $\Delta p_{i,\rm cons}^{(4)\mu}$ and dissipative contribution $\Delta p_{i,\rm diss}^{(4)\mu}$ due to the presence of radiative (R) or potential (P) bulk gravitons [113,114]. At the 4PM level only two gravitons may go on-shell and can become radiative. The conservative sector is given by the (PP) region and also receives contributions from the (RR) part—these may be identified upon using Feynman propagators for the bulk gravitons. Dissipative contributions, on the other hand,

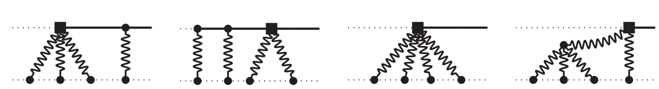


FIG. 2. Examples of 4PM graphs linear in tidal coefficients contributing to the test-body $m_1 m_2^4$ sector.

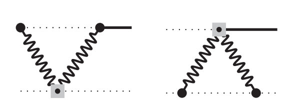


FIG. 3. The postadiabatic graphs proportional to $\tilde{\kappa}_{\dot{E}^2}$ and $\tilde{\kappa}_{\dot{B}^2}$ that also cancel the divergence.

emerge from the mixed (PR) contribution and the remainder of the (RR) part, reflecting the number of radiative gravitons.

Let us begin with the postadiabatic contributions to the 4PM impulse, proportional to $\kappa^{(i)}_{\dot{E}^2/\dot{B}^2}(R)$ and involving one-loop integrals, of Fig. 3, that, for NS 1, take the form

$$\Delta p^{(4)\mu}_{1,\mathrm{tidal}'} = \hat{b}^\mu \frac{1575 G^4 m_1^2 m_2^2}{512|b|^8}\pi\gamma v \sum_{X,i} f_{X^2} m_i c^{(i)}_{X^2}\kappa^{(i)}_{\dot{X}^2}(R), \quad (18)$$

and which we label by a prime on the tidal subscript. The index i runs over the two particles and X over E and B with functions $f_{X^2}(\gamma)$ given by

$$f_{E^2} = 21\gamma^4 - 14\gamma^2 + 9, \quad f_{B^2} = 7(3\gamma^3 - 2\gamma^2 - 1), \quad (19)$$

The adiabatic tidal contributions to the conservative impulse of NS 1 takes the form

$$\Delta p^{(4)\mu}_{1,\mathrm{tidal,cons}} = \frac{G^4 m_1^2 m_2^2}{|b|^8}\sum_{l=1}^{3}\rho^\mu_l\left[\frac{m_2^2}{m_1}C_l(\gamma) + \frac{m_2^2}{m_1}\bar{C}_l(\gamma) + \sum_{\alpha=1}^{19}F_\alpha(\gamma)(m_2 D_{\alpha,l}(\gamma) + m_1\bar{D}_{\alpha,l}(\gamma))\right] + \hat{b}^\mu\frac{1605 G^4 m_1^2 m_2^2}{128|b|^8}\pi\sqrt{\gamma^2-1}\log\left|\frac{2b}{R}\right| \times \sum_{i,X} f_{X^2} m_i c^{(i)}_{X^2}, \quad (20)$$

where $\rho^\mu_l = \{\hat{b}^\mu, v_1^\mu, v_2^\mu\}$. The coefficient functions $C, \bar{C}, D$, and $\bar{D}$ are linear in the tidal Love numbers and rational functions of γ, up to integer powers of $\sqrt{\gamma^2-1}$

$$\begin{aligned} C_l(\gamma) &= \sum_{i=1,2} c^{(i)}_{E^2}C^{(i)}_{E,l}(\gamma) + c^{(i)}_{B^2}C^{(i)}_{B,l}(\gamma), \\ D_{\alpha,l}(\gamma) &= \sum_{i=1,2} c^{(i)}_{E^2}D^{(i)}_{E,\alpha,l}(\gamma) + c^{(i)}_{B^2}D^{(i)}_{B,\alpha,l}(\gamma). \end{aligned} \quad (21)$$

Analogous relations hold for the barred quantities. We find 19 basis functions from the three loop integrals at the 4PM order, which we choose to be even in $v = \sqrt{1-\gamma^{-2}}$, of the following form:

$$\begin{aligned} F_{1,\ldots,5} &= \left\{1, \frac{\log[x]}{\sqrt{\gamma^2-1}}, \log\left[\frac{\gamma_+}{2}\right], \log^2[x], \frac{\log[x]\log\left[\frac{\gamma_+}{2}\right]}{\sqrt{\gamma^2-1}}\right\}, \\ F_{6,\ldots,9} &= \left\{\log[\gamma], \log^2\left[\frac{\gamma_+}{2}\right], \mathrm{Li}_2\left[\frac{\gamma_-}{\gamma_+}\right], \mathrm{Li}_2\left[-\frac{\gamma_-}{\gamma_+}\right]\right\}, \\ F_{10,\ldots,13} &= \left\{\frac{\log[x]}{\sqrt{\gamma^2-1}}\log[\gamma], \frac{1}{\sqrt{\gamma^2-1}}\chi_2\left[\sqrt{\frac{\gamma_-}{\gamma_+}}\right], \mathrm{Li}_2[-x^2] - 4\mathrm{Li}_2[-x] - \log[4]\log[x] - \frac{\pi^2}{4}, \right. \\ &\qquad \left. \frac{\mathrm{Li}_2[-x] - \mathrm{Li}_2\left[-\frac{1}{x}\right] + \log[4]\log[x]}{\sqrt{\gamma^2-1}}\right\}, \\ F_{14,15,16} &= \left\{\mathrm{E}^2\left[\frac{\gamma_-}{\gamma_+}\right], \mathrm{K}^2\left[\frac{\gamma_-}{\gamma_+}\right], \mathrm{E}\left[\frac{\gamma_-}{\gamma_+}\right]\ \mathrm{K}\left[\frac{\gamma_-}{\gamma_+}\right]\right\}, \\ F_{17,18,19} &= \left\{\log\left[\frac{\gamma_-}{2}\right], \frac{\log\left[\frac{\gamma_-}{2}\right]\log[x]}{\sqrt{\gamma^2-1}}, \log\left[\frac{\gamma_-}{2}\right]\log\left[\frac{\gamma_+}{2}\right]\right\}, \end{aligned} \quad (22)$$

where $\gamma_\pm = \gamma \pm 1$, $x = \gamma - \sqrt{\gamma^2-1}$ and $\chi_\nu[z] = \frac{1}{2}(\mathrm{Li}_\nu[z] - \mathrm{Li}_\nu[-z])$ is the Legendre chi function. Note the appearance of complete elliptic integrals of the first and second kind in entries $F_{14,15,16}$. In the conservative impulse the contributions to the v_i^μ directions, i.e., $l = 2, 3$ in (20), only pick up the basis functions F_1 and F_2.

For the dissipative sector we find

$$\Delta p^{(4)\mu}_{1,\mathrm{tidal,diss}} = \frac{G^4 m_1^2 m_2^2}{|b|^8}\sum_{l=1}^{3}\rho_l^\mu \times \sum_{\alpha=1}^{13} F_\alpha(\gamma)(m_2 E_{\alpha,l}(\gamma) + m_1 \bar{E}_{\alpha,l}(\gamma))], \quad (23)$$

which only uses the first 13 basis functions of (22) and does not have a probe-limit $m_1 m_2^4$ or $m_1^4 m_2$ contribution. As mentioned above, the (RP) and (RR) regions contribute here, which we denote in the following as rad^1 and rad^2, respectively. In analogy to (21) the coefficient functions read

$$E_{\alpha,l}(\gamma) = \sum_{i=1,2} c^{(i)}_{E^2} E^{(i)}_{E,\alpha,l}(\gamma) + c^{(i)}_{B^2} E^{(i)}_{B,\alpha,l}(\gamma), \quad (24)$$

and similarly for the barred one. The explicit form of (21) and (24) are collected in the ancillary file included in the `arXiv.org` submission of this Letter.

Scattering angle. A relative, dissipative scattering angle θ may be defined as follows. First, we define the relative momentum,

$$p^\mu = \frac{\nu}{\Gamma^2}\left(\frac{\gamma m_1 + m_2}{m_1} p_1^\mu - \frac{\gamma m_2 + m_1}{m_2} p_2^\mu\right), \quad (25)$$

with $\nu = m_1 m_2/M^2$, total mass $M = m_1 + m_2$, and $\Gamma = E/M$, such that in the initial center of mass frame total momentum $P^\mu = p_1^\mu + p_2^\mu$, and we have $p_1^\mu = (E_1, \boldsymbol{p})$ and $p_2^\mu = (E_2, -\boldsymbol{p})$ with $p^\mu = (0, \boldsymbol{p})$ and $E = |P^\mu|$. The relative scattering angle is now defined as the angle between the initial and final value of p^μ taken in the initial center of mass frame. For planar scattering one finds the formula

$$\tan(\theta) = -\frac{\hat{b}\cdot\Delta p}{p_\infty - \hat{p}\cdot\Delta p}, \quad (26)$$

which for conservative scattering reduces to

$$\sin\left(\frac{\theta_{\mathrm{cons}}}{2}\right) = \frac{|\Delta p^\mu_{i,\mathrm{cons}}|}{2p_\infty}. \quad (27)$$

The angle is PM expanded, $\theta = \sum_{n=1}^\infty \theta^{(n)}$, and expanded in the tidal couplings

$$\theta^{(n)} = \theta^{(n)}_{\mathrm{pp}} + \theta^{(n)}_{\mathrm{tidal}'} + \sum_{X=E,B}\left(\theta^{(n,+)}_{X^2} c^{(+)}_{X^2} + \theta^{(n,-)}_{X^2}\delta c^{(-)}_{X^2}\right). \quad (28)$$

The first term describes the point-particle tidal-free part, the second term postadiabatic tidal effects and the final terms adiabatic tidal corrections. The relative mass difference is $\delta = (m_1 - m_2)/M$ and we use symmetric finite-size couplings defined by

$$c^{(\pm)}_{E^2} = c^{(1)}_{E^2} \pm c^{(2)}_{E^2}, \qquad c^{(\pm)}_{B^2} = c^{(1)}_{B^2} \pm c^{(2)}_{B^2}. \quad (29)$$

The leading adiabatic tidal effects appear at second PM order and the leading postadiabatic effects at fourth PM order (in our formal PM counting).

The (leading-order) 4PM postadiabatic angle is derived from one-loop integrals, and reads

$$\theta^{(4)}_{\mathrm{tidal}'} = \Gamma\frac{1575\pi\nu}{512}\frac{(GM)^4}{|b|^8}\sum_X f_{X^2}\kappa^{(+)}_{\dot{X}^2}(R), \quad (30)$$

with

$$\kappa^{(+)}_{\dot{X}^2}(R) = \frac{m_1 c^{(1)}_{X^2}\kappa^{(1)}_{\dot{X}^2}(R) + m_2 c^{(2)}_{X^2}\kappa^{(2)}_{\dot{X}^2}(R)}{M}. \quad (31)$$

The 4PM tidal contributions take a similar form as the tidal-free angle:

$$\theta^{(4,\pm)}_{X^2} = \Gamma\frac{(GM)^4}{|b|^8}\left[\theta^{(\pm)}_{\mathrm{X^2,cons},\nu^0} + \nu\theta^{(\pm)}_{\mathrm{X^2,cons},\nu^1} + \frac{\nu}{\Gamma^2}\left(\theta^{(\pm)}_{\mathrm{X^2,diss},\nu^1} + \nu\theta^{(\pm)}_{\mathrm{X^2,diss},\nu^2}\right)\right]. \quad (32)$$

The angle coefficients of this expansion depend only on γ and $\log|2b/R|$, and may be expanded on the function basis $F_\alpha(\gamma)$ in terms of polynomials of γ (up to $\sqrt{\gamma^2-1}$).

The angle satisfies the same tail relation as pointed out in Ref. [113]:

$$\theta^{(4)}_{\mathrm{tidal,tail}} = GE\frac{\partial E^{(3)}_{\mathrm{tidal,rad}}}{\partial L}\log\left(\frac{\gamma-1}{2}\right), \quad (33)$$

where we define $\theta^{(4)}_{\mathrm{tidal,tail}}$ as the part of $\theta^{(4)}_{\mathrm{tidal}}$ in the direction of the tail functions F_α with $\alpha = 17, 18, 19$ that depend on $\log[\gamma_-/2]$, E_{rad} being the radiated energy.

Linear response. The dissipative angle obeys the same linear response relation as derived in Ref. [114]

$$\theta^{(4)}_{\mathrm{tidal,rad}^1} = -\frac{1}{2}\left(\frac{\partial\theta^{(1)}_{\mathrm{pp}}}{\partial L}L^{(3)}_{\mathrm{tidal,rad}} + \frac{\partial\theta^{(1)}_{\mathrm{pp}}}{\partial E}E^{(3)}_{\mathrm{tidal,rad}} + \frac{\partial\theta^{(2)}_{\mathrm{tidal}}}{\partial L}L^{(2)}_{\mathrm{pp,rad}}\right), \quad (34)$$

where the "pp" subscript refers to point-particle (and so tidal-free) contributions. We note that neither the 1PM angle nor the 2PM loss of angular momentum have a tidal contribution. On the left-hand side, the subscript "rad^1"

refers to the part of the angle including a single radiative graviton. This part may also be identified from its odd scaling under $v \to -v$. Knowledge of the 4PM rad[1] angle completely determines the 3PM tidal loss of angular momentum, and vice versa (assuming knowledge of other relevant 3PM observables). The 3PM tidal loss of angular momentum was previously derived in Ref. [71] with which we fully agree. The result for $L^{(3)}_{\rm rad}$ reads

$$L^{(3)}_{\rm tidal,rad} = \frac{\pi G^3 M^4 \nu^2}{|b|^6 \Gamma^3} \sum_{\alpha=1}^{3} F_\alpha(\gamma) \sum_{X=E,B} \times \left(H^{(+)}_{\alpha,X^2}(\gamma) c^{(+)}_{X^2} + H^{(-)}_{\alpha,X^2}(\gamma) \delta c^{(-)}_{X^2}\right), \quad (35)$$

where the functions $H^{\pm}_{i,X^2}(\gamma)$ are polynomial (up to $\sqrt{\gamma^2-1}$) in γ and is a linear function in ν. This result, together with point-particle result, is provided in the ancillary file on `arXiv.org`.

Checks. The results in this paper build on the 3PM results of Ref. [51], and naturally agree with them. In addition, we have checked that the post-Newtonian limit $v \to 0$ of the conservative scattering angle reproduces the 2PN and 3PN scattering angles reported in Refs. [102,124].[3] They also obey the nontrivial checks Eqs. (33) and (34) (reproducing the 3PM loss of angular momentum of Ref. [71]) and in general we have checked that the impulse obeys the constraints $(p_i + \Delta p_i)^2 = p_i^2$, which provides a further internal consistency check.

[3]Modulo a typo in Eq. (6.14) of that paper (v1): the mass ratios should be dropped. We thank the authors for communication.

Conclusions. In this Letter we have applied the worldline quantum field theory formalism to tidal effects at 4PM order, demonstrating the power of our technology. It is worth stressing that the work flow was identical (if not simpler) than in the case of spin [113,114]. We established tidal effects in the impulse in the conservative and dissipative sectors at 4PM and derived the conservative and dissipative scattering angle. A new feature appearing at this NNLO order is the need to include postadiabatic couplings in order to cancel a divergence in this classical field theory computation which results in a renormalization group flow of the postadiabatic Love numbers. We also confirmed the tidal contributions to the radiated angular momentum previously derived by Heissenberg via very different methods [71]. Our findings will be potentially useful for improving future waveform models to include tidal effects [125]. They represent yet another mosaic stone in our steadily improving picture of highest-precision gravitational wave physics.

Acknowledgments. We thank Tanja Hinderer, Rafael Porto, Muddu Saketh, and especially Carlo Heissenberg, Raj Patil, and Jan Steinhoff for discussions. This work was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) Projektnummer 417533893/GRK2575 "Rethinking Quantum Field Theory" and by the European Union through the European Research Council under Grant No. ERC-AdG-101097219 (GraWFTy). Views and opinions expressed are however those of the authors only and do not necessarily reflect those of the European Union or European Research Council Executive Agency. Neither the European Union nor the granting authority can be held responsible for them.

[1] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Observation of gravitational waves from a binary black hole merger, Phys. Rev. Lett. **116**, 061102 (2016).
[2] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), GW170817: Observation of gravitational waves from a binary neutron star inspiral, Phys. Rev. Lett. **119**, 161101 (2017).
[3] R. Abbott *et al.* (LIGO Scientific, Virgo, and KAGRA Collaborations), GWTC-3: Compact binary coalescences observed by LIGO and Virgo during the second part of the third observing run, Phys. Rev. X **13**, 041039 (2023).
[4] P. Amaro-Seoane *et al.* (LISA Collaboration), Laser interferometer space antenna, arXiv:1702.00786.
[5] M. Punturo *et al.*, The Einstein Telescope: A third-generation gravitational wave observatory, Classical Quantum Gravity **27**, 194002 (2010).
[6] S. W. Ballmer *et al.*, Snowmass2021 Cosmic Frontier White Paper: Future gravitational-wave detector facilities, in *Snowmass 2021* (2022), 3, arXiv:2203.08228.
[7] L. Blanchet, Gravitational radiation from post-Newtonian sources and inspiralling compact binaries, Living Rev. Relativity **17**, 2 (2014).
[8] R. A. Porto, The effective field theorist's approach to gravitational dynamics, Phys. Rep. **633**, 1 (2016).
[9] M. Levi, Effective field theories of post-Newtonian gravity: A comprehensive review, Rep. Prog. Phys. **83**, 075901 (2020).
[10] D. A. Kosower, R. Monteiro, and D. O'Connell, The SAGEX review on scattering amplitudes Chapter 14: Classical gravity from scattering amplitudes, J. Phys. A **55**, 443015 (2022).
[11] N. E. J. Bjerrum-Bohr, P. H. Damgaard, L. Plante, and P. Vanhove, The SAGEX review on scattering amplitudes

Chapter 13: Post-Minkowskian expansion from scattering amplitudes, J. Phys. A **55**, 443014 (2022).
[12] A. Buonanno, M. Khalil, D. O'Connell, R. Roiban, M. P. Solon, and M. Zeng, Snowmass White Paper: Gravitational waves and scattering amplitudes, in *Snowmass 2021* (2022), 4, arXiv:2204.05194.
[13] P. Di Vecchia, C. Heissenberg, R. Russo, and G. Veneziano, The gravitational eikonal: From particle, string and brane collisions to black-hole encounters, arXiv:2306.16488.
[14] G. U. Jakobsen, Gravitational scattering of compact bodies from worldline quantum field theory, other thesis, 8, 2023.
[15] Y. Mino, M. Sasaki, and T. Tanaka, Gravitational radiation reaction to a particle motion, Phys. Rev. D **55**, 3457 (1997).
[16] E. Poisson, A. Pound, and I. Vega, The motion of point particles in curved spacetime, Living Rev. Relativity **14**, 7 (2011).
[17] L. Barack and A. Pound, Self-force and radiation reaction in general relativity, Rep. Prog. Phys. **82**, 016904 (2019).
[18] S. E. Gralla and K. Lobo, Self-force effects in post-Minkowskian scattering, Classical Quantum Gravity **39**, 095001 (2022).
[19] A. Buonanno and T. Damour, Effective one-body approach to general relativistic two-body dynamics, Phys. Rev. D **59**, 084006 (1999).
[20] A. Buonanno and T. Damour, Transition from inspiral to plunge in binary black hole coalescences, Phys. Rev. D **62**, 064015 (2000).
[21] F. Pretorius, Evolution of binary black hole spacetimes, Phys. Rev. Lett. **95**, 121101 (2005).
[22] M. Boyle *et al.*, The SXS Collaboration catalog of binary black hole simulations, Classical Quantum Gravity **36**, 195006 (2019).
[23] T. Damour, F. Guercilena, I. Hinder, S. Hopper, A. Nagar, and L. Rezzolla, Strong-field scattering of two black holes: Numerics versus analytics, Phys. Rev. D **89**, 081503 (2014).
[24] S. J. Kovacs and K. S. Thorne, The generation of gravitational waves. 4. Bremsstrahlung, Astrophys. J. **224**, 62 (1978).
[25] K. Westpfahl and M. Goller, Gravitational scattering of two relativistic particles in postlinear approximation, Lett. Nuovo Cimento **26**, 573 (1979).
[26] L. Bel, T. Damour, N. Deruelle, J. Ibanez, and J. Martin, Poincaré-invariant gravitational field and equations of motion of two pointlike objects: The postlinear approximation of general relativity, Gen. Relativ. Gravit. **13**, 963 (1981).
[27] T. Damour, High-energy gravitational scattering and the general relativistic two-body problem, Phys. Rev. D **97**, 044038 (2018).
[28] S. Hopper, A. Nagar, and P. Rettegno, Strong-field scattering of two spinning black holes: Numerics versus analytics, Phys. Rev. D **107**, 124034 (2023).
[29] C. Cheung, I. Z. Rothstein, and M. P. Solon, From scattering amplitudes to classical potentials in the post-Minkowskian expansion, Phys. Rev. Lett. **121**, 251101 (2018).
[30] G. Kälin and R. A. Porto, From boundary data to bound states, J. High Energy Phys. 01 (2020) 072.
[31] G. Kälin and R. A. Porto, From boundary data to bound states. Part II. Scattering angle to dynamical invariants (with twist), J. High Energy Phys. 02 (2020) 120.
[32] M. V. S. Saketh, J. Vines, J. Steinhoff, and A. Buonanno, Conservative and radiative dynamics in classical relativistic scattering and bound systems, Phys. Rev. Res. **4**, 013127 (2022).
[33] R. Gonzo and C. Shi, Boundary to bound dictionary for generic Kerr orbits, Phys. Rev. D **108**, 084065 (2023).
[34] G. Cho, G. Kälin, and R. A. Porto, From boundary data to bound states. Part III. Radiative effects, J. High Energy Phys. 04 (2022) 154.
[35] W. D. Goldberger and I. Z. Rothstein, An effective field theory of gravity for extended objects, Phys. Rev. D **73**, 104029 (2006).
[36] G. Kälin and R. A. Porto, Post-Minkowskian effective field theory for conservative binary dynamics, J. High Energy Phys. 11 (2020) 106.
[37] G. Kälin, Z. Liu, and R. A. Porto, Conservative dynamics of binary systems to third post-Minkowskian order from the effective field theory approach, Phys. Rev. Lett. **125**, 261103 (2020).
[38] G. Kälin, Z. Liu, and R. A. Porto, Conservative tidal effects in compact binary systems to next-to-leading post-Minkowskian order, Phys. Rev. D **102**, 124025 (2020).
[39] C. Dlapa, G. Kälin, Z. Liu, and R. A. Porto, Dynamics of binary systems to fourth Post-Minkowskian order from the effective field theory approach, Phys. Lett. B **831**, 137203 (2022).
[40] C. Dlapa, G. Kälin, Z. Liu, and R. A. Porto, Conservative dynamics of binary systems at fourth post-Minkowskian order in the large-eccentricity expansion, Phys. Rev. Lett. **128**, 161104 (2022).
[41] Z. Liu, R. A. Porto, and Z. Yang, Spin effects in the effective field theory approach to post-Minkowskian conservative dynamics, J. High Energy Phys. 06 (2021) 012.
[42] S. Mougiakakos, M. M. Riva, and F. Vernizzi, Gravitational Bremsstrahlung in the post-Minkowskian effective field theory, Phys. Rev. D **104**, 024041 (2021).
[43] M. M. Riva and F. Vernizzi, Radiated momentum in the post-Minkowskian worldline approach via reverse unitarity, J. High Energy Phys. 11 (2021) 228.
[44] S. Mougiakakos, M. M. Riva, and F. Vernizzi, Gravitational bremsstrahlung with tidal effects in the post-Minkowskian expansion, Phys. Rev. Lett. **129**, 121101 (2022).
[45] M. M. Riva, F. Vernizzi, and L. K. Wong, Gravitational bremsstrahlung from spinning binaries in the post-Minkowskian expansion, Phys. Rev. D **106**, 044013 (2022).
[46] G. Mogull, J. Plefka, and J. Steinhoff, Classical black hole scattering from a worldline quantum field theory, J. High Energy Phys. 02 (2021) 048.
[47] G. U. Jakobsen, G. Mogull, J. Plefka, and J. Steinhoff, Classical gravitational bremsstrahlung from a worldline quantum field theory, Phys. Rev. Lett. **126**, 201103 (2021).
[48] G. U. Jakobsen, G. Mogull, J. Plefka, and J. Steinhoff, Gravitational bremsstrahlung and hidden supersymmetry of spinning bodies, Phys. Rev. Lett. **128**, 011101 (2022).

[49] G. U. Jakobsen, G. Mogull, J. Plefka, and J. Steinhoff, SUSY in the sky with gravitons, J. High Energy Phys. 01 (2022) 027.
[50] G. U. Jakobsen and G. Mogull, Conservative and radiative dynamics of spinning bodies at third post-Minkowskian order using worldline quantum field theory, Phys. Rev. Lett. **128**, 141102 (2022).
[51] G. U. Jakobsen, G. Mogull, J. Plefka, and B. Sauer, All things retarded: Radiation-reaction in worldline quantum field theory, J. High Energy Phys. 10 (2022) 128.
[52] C. Shi and J. Plefka, Classical double copy of worldline quantum field theory, Phys. Rev. D **105**, 026007 (2022).
[53] F. Bastianelli, F. Comberiati, and L. de la Cruz, Light bending from eikonal in worldline quantum field theory, J. High Energy Phys. 02 (2022) 209.
[54] F. Comberiati and C. Shi, Classical double copy of spinning worldline quantum field theory, J. High Energy Phys. 04 (2023) 008.
[55] T. Wang, Binary dynamics from worldline QFT for scalar QED, Phys. Rev. D **107**, 085011 (2023).
[56] D. Neill and I. Z. Rothstein, Classical space-times from the S matrix, Nucl. Phys. **B877**, 177 (2013).
[57] A. Luna, I. Nicholson, D. O'Connell, and C. D. White, Inelastic black hole scattering from charged scalar amplitudes, J. High Energy Phys. 03 (2018) 044.
[58] D. A. Kosower, B. Maybee, and D. O'Connell, Amplitudes, observables, and classical scattering, J. High Energy Phys. 02 (2019) 137.
[59] A. Cristofoli, R. Gonzo, D. A. Kosower, and D. O'Connell, Waveforms from amplitudes, Phys. Rev. D **106**, 056007 (2022).
[60] N. E. J. Bjerrum-Bohr, J. F. Donoghue, and P. Vanhove, On-shell techniques and universal results in quantum gravity, J. High Energy Phys. 02 (2014) 111.
[61] N. E. J. Bjerrum-Bohr, P. H. Damgaard, G. Festuccia, L. Planté, and P. Vanhove, General relativity from scattering amplitudes, Phys. Rev. Lett. **121**, 171601 (2018).
[62] Z. Bern, C. Cheung, R. Roiban, C.-H. Shen, M. P. Solon, and M. Zeng, Scattering amplitudes and the conservative Hamiltonian for binary systems at third post-Minkowskian order, Phys. Rev. Lett. **122**, 201603 (2019).
[63] Z. Bern, C. Cheung, R. Roiban, C.-H. Shen, M. P. Solon, and M. Zeng, Black hole binary dynamics from the double copy and effective theory, J. High Energy Phys. 10 (2019) 206.
[64] N. E. J. Bjerrum-Bohr, L. Planté, and P. Vanhove, Post-Minkowskian radial action from soft limits and velocity cuts, J. High Energy Phys. 03 (2022) 071.
[65] C. Cheung and M. P. Solon, Classical gravitational scattering at $\mathcal{O}(G^3)$ from Feynman diagrams, J. High Energy Phys. 06 (2020) 144.
[66] N. E. J. Bjerrum-Bohr, P. H. Damgaard, L. Planté, and P. Vanhove, The amplitude for classical gravitational scattering at third Post-Minkowskian order, J. High Energy Phys. 08 (2021) 172.
[67] P. Di Vecchia, C. Heissenberg, R. Russo, and G. Veneziano, Universality of ultra-relativistic gravitational scattering, Phys. Lett. B **811**, 135924 (2020).
[68] P. Di Vecchia, C. Heissenberg, R. Russo, and G. Veneziano, The eikonal approach to gravitational scattering and radiation at $\mathcal{O}(G^3)$, J. High Energy Phys. 07 (2021) 169.
[69] P. Di Vecchia, C. Heissenberg, R. Russo, and G. Veneziano, Radiation reaction from soft theorems, Phys. Lett. B **818**, 136379 (2021).
[70] P. Di Vecchia, C. Heissenberg, R. Russo, and G. Veneziano, Classical gravitational observables from the Eikonal operator, Phys. Lett. B **843**, 138049 (2023).
[71] C. Heissenberg, Angular momentum loss due to tidal effects in the post-Minkowskian expansion, Phys. Rev. Lett. **131**, 011603 (2023).
[72] T. Damour, Radiative contribution to classical gravitational scattering at the third order in G, Phys. Rev. D **102**, 124008 (2020).
[73] E. Herrmann, J. Parra-Martinez, M. S. Ruf, and M. Zeng, Radiative classical gravitational observables at $\mathcal{O}(G^3)$ from scattering amplitudes, J. High Energy Phys. 10 (2021) 148.
[74] P. H. Damgaard, K. Haddad, and A. Helset, Heavy black hole effective theory, J. High Energy Phys. 11 (2019) 070.
[75] P. H. Damgaard, L. Plante, and P. Vanhove, On an exponential representation of the gravitational S-matrix, J. High Energy Phys. 11 (2021) 213.
[76] P. H. Damgaard, E. R. Hansen, L. Planté, and P. Vanhove, The relation between KMOC and worldline formalisms for classical gravity, J. High Energy Phys. 09 (2023) 059.
[77] R. Aoude, K. Haddad, and A. Helset, On-shell heavy particle effective theories, J. High Energy Phys. 05 (2020) 051.
[78] M. Accettulli Huber, A. Brandhuber, S. De Angelis, and G. Travaglini, From amplitudes to gravitational radiation with cubic interactions and tidal effects, Phys. Rev. D **103**, 045015 (2021).
[79] A. Brandhuber, G. Chen, G. Travaglini, and C. Wen, Classical gravitational scattering from a gauge-invariant double copy, J. High Energy Phys. 10 (2021) 118.
[80] Z. Bern, D. Kosmopoulos, A. Luna, R. Roiban, and F. Teng, Binary dynamics through the fifth power of spin at O (G2), Phys. Rev. Lett. **130**, 201402 (2023).
[81] R. Aoude, K. Haddad, and A. Helset, Classical gravitational spinning-spinless scattering at O(G2S∞), Phys. Rev. Lett. **129**, 141102 (2022).
[82] R. Aoude, K. Haddad, and A. Helset, Classical gravitational scattering amplitude at O(G2S1∞S2∞), Phys. Rev. D **108**, 024050 (2023).
[83] Y. F. Bautista, Dynamics for super-extremal Kerr binary systems at O(G2), Phys. Rev. D **108**, 084036 (2023).
[84] P. H. Damgaard, E. R. Hansen, L. Planté, and P. Vanhove, Classical observables from the exponential representation of the gravitational S-matrix, J. High Energy Phys. 09 (2023) 183.
[85] A. Brandhuber, G. R. Brown, G. Chen, S. De Angelis, J. Gowdy, and G. Travaglini, One-loop gravitational bremsstrahlung and waveforms from a heavy-mass effective field theory, J. High Energy Phys. 06 (2023) 048.

[86] A. Herderschee, R. Roiban, and F. Teng, The sub-leading scattering waveform from amplitudes, J. High Energy Phys. 06 (2023) 004.
[87] A. Georgoudis, C. Heissenberg, and I. Vazquez-Holm, Inelastic exponentiation and classical gravitational scattering at one loop, J. High Energy Phys. 06 (2023) 126.
[88] A. Elkhidir, D. O'Connell, M. Sergola, and I. A. Vazquez-Holm, Radiation and reaction at one loop, arXiv:2303.06211.
[89] S. Caron-Huot, M. Giroux, H. S. Hannesdottir, and S. Mizera, What can be measured asymptotically?, arXiv:2308.02125.
[90] E. E. Flanagan and T. Hinderer, Constraining neutron star tidal Love numbers with gravitational wave detectors, Phys. Rev. D **77**, 021502 (2008).
[91] T. Hinderer, Tidal Love numbers of neutron stars, Astrophys. J. **677**, 1216 (2008).
[92] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Properties of the binary neutron star merger GW170817, Phys. Rev. X **9**, 011001 (2019).
[93] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), GW170817: Measurements of neutron star radii and equation of state, Phys. Rev. Lett. **121**, 161101 (2018).
[94] K. Chatziioannou, Neutron star tidal deformability and equation of state constraints, Gen. Relativ. Gravit. **52**, 109 (2020).
[95] B. K. Pradhan, A. Vijaykumar, and D. Chatterjee, Impact of updated multipole Love numbers and f-Love universal relations in the context of binary neutron stars, Phys. Rev. D **107**, 023010 (2023).
[96] C. M. Will, Tidal gravitational radiation from homogeneous stars, Astrophys. J. **274**, 858 (1983).
[97] J. Steinhoff, T. Hinderer, A. Buonanno, and A. Taracchini, Dynamical tides in general relativity: Effective action and effective-one-body Hamiltonian, Phys. Rev. D **94**, 104028 (2016).
[98] G. Pratten, P. Schmidt, and N. Williams, Impact of dynamical tides on the reconstruction of the neutron star equation of state, Phys. Rev. Lett. **129**, 081102 (2022).
[99] D. Bini, T. Damour, and G. Faye, Effective action approach to higher-order relativistic tidal interactions in binary systems and their effective one body description, Phys. Rev. D **85**, 124034 (2012).
[100] Q. Henry, G. Faye, and L. Blanchet, Tidal effects in the equations of motion of compact binary systems to next-to-next-to-leading post-Newtonian order, Phys. Rev. D **101**, 064047 (2020).
[101] M. V. S. Saketh, Z. Zhou, and M. M. Ivanov, Dynamical tidal response of Kerr black holes from scattering amplitudes, arXiv:2307.10391.
[102] M. K. Mandal, P. Mastrolia, H. O. Silva, R. Patil, and J. Steinhoff, Renormalizing Love: Tidal effects at the third post-Newtonian order, arXiv:2308.01865.
[103] D. Bini, T. Damour, and A. Geralico, Scattering of tidally interacting bodies in post-Minkowskian gravity, Phys. Rev. D **101**, 044039 (2020).
[104] C. Cheung and M. P. Solon, Tidal effects in the post-Minkowskian expansion, Phys. Rev. Lett. **125**, 191601 (2020).
[105] K. Haddad and A. Helset, Tidal effects in quantum field theory, J. High Energy Phys. 12 (2020) 024.
[106] Z. Bern, J. Parra-Martinez, R. Roiban, E. Sawyer, and C.-H. Shen, Leading nonlinear tidal effects and scattering amplitudes, J. High Energy Phys. 05 (2021) 188.
[107] L. Blanchet, Gravitational wave tails of tails, Classical Quantum Gravity **15**, 113 (1998).
[108] W. D. Goldberger and A. Ross, Gravitational radiative corrections from effective field theory, Phys. Rev. D **81**, 124015 (2010).
[109] H. Fang and G. Lovelace, Tidal coupling of a Schwarzschild black hole and circularly orbiting moon, Phys. Rev. D **72**, 124016 (2005).
[110] T. Damour and A. Nagar, Relativistic tidal properties of neutron stars, Phys. Rev. D **80**, 084035 (2009).
[111] T. Binnington and E. Poisson, Relativistic theory of tidal Love numbers, Phys. Rev. D **80**, 084018 (2009).
[112] P. Rettegno, G. Pratten, L. Thomas, P. Schmidt, and T. Damour, Strong-field scattering of two spinning black holes: Numerical Relativity versus post-Minkowskian gravity, Phys. Rev. D **108**, 124016 (2023).
[113] G. U. Jakobsen, G. Mogull, J. Plefka, B. Sauer, and Y. Xu, Conservative scattering of spinning black holes at fourth post-Minkowskian order, Phys. Rev. Lett. **131**, 151401 (2023).
[114] G. U. Jakobsen, G. Mogull, J. Plefka, and B. Sauer, Dissipative scattering of spinning black holes at fourth post-Minkowskian order, Phys. Rev. Lett. **131**, 241402 (2023).
[115] G. Kälin, J. Neef, and R. A. Porto, Radiation-reaction in the effective field theory approach to post-Minkowskian dynamics, J. High Energy Phys. 01 (2023) 140.
[116] D. G. Boulware and L. S. Brown, Tree graphs and classical fields, Phys. Rev. **172**, 1628 (1968).
[117] B. Ruijl, T. Ueda, and J. Vermaseren, FORM version 4.2, arXiv:1707.06453.
[118] T. Gehrmann and E. Remiddi, Differential equations for two loop four point functions, Nucl. Phys. **B580**, 485 (2000).
[119] J. M. Henn, Multiloop integrals in dimensional regularization made simple, Phys. Rev. Lett. **110**, 251601 (2013).
[120] M. Beneke and V. A. Smirnov, Asymptotic expansion of Feynman integrals near threshold, Nucl. Phys. **B522**, 321 (1998).
[121] Z. Bern, J. Parra-Martinez, R. Roiban, M. S. Ruf, C.-H. Shen, M. P. Solon, and M. Zeng, Scattering amplitudes, the tail effect, and conservative binary dynamics at O(G4), Phys. Rev. Lett. **128**, 161103 (2022).
[122] C. Dlapa, G. Kälin, Z. Liu, J. Neef, and R. A. Porto, Radiation reaction and gravitational waves at fourth post-Minkowskian order, Phys. Rev. Lett. **130**, 101401 (2023).
[123] C. Dlapa, G. Kälin, Z. Liu, and R. A. Porto, Bootstrapping the relativistic two-body problem, J. High Energy Phys. 08 (2023) 109.
[124] M. K. Mandal, P. Mastrolia, H. O. Silva, R. Patil, and J. Steinhoff, Gravitoelectric dynamical tides at second post-Newtonian order, J. High Energy Phys. 11 (2023) 067.
[125] T. Hinderer *et al.*, Effects of neutron-star dynamic tides on gravitational waveforms within the effective-one-body approach, Phys. Rev. Lett. **116**, 181101 (2016).