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Thermodynamics of climate change: generalized sensitivities

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Abstract. Using a recent theoretical approach, we study how global warming impacts the thermodynamics of the climate system by performing experiments with a simplified yet Earth-like climate model. The intensity of the Lorenz energy cycle, the Carnot efficiency, the material entropy production, and the degree of irreversibility of the system change monotonically with the CO_2 concentration. Moreover, these quantities feature an approximately linear behaviour with respect to the logarithm of the CO_2 concentration in a relatively wide range. These generalized sensitivities suggest that the climate becomes less efficient, more irreversible, and features higher entropy production as it becomes warmer, with changes in the latent heat fluxes playing a predominant role. These results may be of help for explaining recent findings obtained with state of the art climate models regarding how increases in CO_2 concentration impact the vertical stratification of the tropical and extratropical atmosphere and the position of the storm tracks.

1 Introduction

The most basic way to characterize the climate system is describing it as a non-equilibrium thermodynamic system, generating entropy by irreversible processes and – if time-dependent forcings can be neglected – keeping a steady state by balancing the input and output of energy and entropy with the surrounding environment.

A primary goal of climate science is to understand how the statistical properties of the climate system change as a result of variations in the value of external or internal parameters. Rigorous mathematical foundations to this problem can be traced to the Ruelle response theory for non equilibrium steady state systems (Ruelle, 1998, 2009). Such an approach has been recently proved to have formal analogies with the usual Kubo response theory for quasi-equilibrium systems (Lucarini, 2008a) and to be amenable to numerical investigation (Lucarini, 2009a).

It has long been recognized that a comprehensive view on the climate system can be attained by adopting a thermodynamic perspective. Two main approaches can be envisioned along this line.

In the first approach, the focus is on the dynamical mechanisms and physical processes responsible for the transformation of energy from one form to the other. The concept of the energy cycle of the atmosphere due to Lorenz (1955) allowed for defining an effective climate machine, driven by the temperature difference between a warm and a cold thermal pool. The atmospheric and oceanic motions at the same time result from the mechanical work (then dissipated in a turbulent cascade) produced by the engine, and re-equilibrate the energy balance of the climate system (Peixoto and Oort, 1992). More recently, Johnson (2000) introduced a rather convincing Carnot engine-equivalent picture of the climate by defining robustly the warm and the cold reservoirs and their temperatures.

In the second approach, the emphasis lies on the analysis of the irreversibility of the climate system, and, especially, of its entropy production. This largely results from the intellectual stimulation coming from the maximum entropy production principle (MEPP), which proposes that an

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out-of-equilibrium nonlinear system adjusts in such a way to maximize the production of entropy (Paltridge, 1979). Even if the general validity of MEPP is unclear (Dewar, 2005; Grinstein and Linsker, 2007), its heuristic adoption in climate science has been quite fruitful (Kleidon and Lorenz 2005; Kleidon et al., 2006; Kunz et al., 2008), and has stimulated a detailed re-examination of the importance of entropy production in the climate system (Peixoto and Oort, 1992; Goody, 2000; Ozawa et al., 2003). Moreover, this has resulted into a drive for adopting of a new generation of diagnostic tools based on the 2nd law of thermodynamics for auditing climate models (Fraedrich and Lunkeit, 2008; Pascale et al., 2009; Lucarini et al., 2010) and for outlining a set of parameterisations to be used in conceptual and intermediate complexity models, or for the reconstruction of the past climate conditions (Fraedrich, 2001). Recently a link has been found between the Carnot efficiency, the entropy production and the degree of irreversibility of the climate system (Lucarini, 2009b). This has made possible a new fruitful exploration of the onset and decay of snowball conditions as parametrically controlled by variations in the solar irradiance (Lucarini et al., 2010). In that analysis, the two branches of cold and warm climate stationary states have been found to feature very distinct macro-thermodynamical properties, well beyond the obviously expected large difference in the surface average temperature. The changeovers between ice-covered and ice-free planet occur as spontaneous transitions accompanied by a reduction in the efficiency of the climate machine. Moreover, for a given value of the solar constant, the two possible states feature average total material entropy production rates which differ by a factor of about 4.

In this paper we revise and revive the classic problem of analyzing the climate sensitivity to CO_2 concentration changes by adding on top of the usual, IPCC-like (IPCC, 2007) analysis of globally averaged surface temperature changes the investigation of how the global thermodynamics of the system is influenced by the atmospheric composition, so that a wider physically-based set of generalized sensitivities are introduced. Our investigation is performed using the simplified and portable climate model Planet Simulator (PLASIM) (Fraedrich et al., 2005; Fraedrich and Lunkeit, 2008). We believe our work contributes to presenting reliable metrics to be used in the validation of climate models of various degrees of complexity.

2 Efficiency and entropy production in the climate system

In this section, for the benefit of the reader, we briefly recapitulate some of the main results recently obtained regarding the description of the non-equilibrium thermodynamical properties of the climate system (Lucarini, 2009a; Lucarini et al., 2010).

We define the total energy of the Ω-domain of the climatic system by $E(\Omega) = P(\Omega) + K(\Omega)$, where P represents the moist static potential energy, given by the thermal – inclusive of the contributions due to water phase transitions – and potential contributions, and K is the total kinetic energy. The time derivative of the total kinetic and potential energy can be expressed as $\dot{K} = -D + W$, $\dot{P} = \Psi + D - W$, where we have dropped Ω-dependence for convenience, D is the (positive definite) integrated dissipation, W is the instantaneous work performed by the system (or, in other words, the total intensity of the Lorenz energy cycle), Ψ represents the heating due to convergence of heat fluxes (which can be split into the radiative, sensible, and latent heat components), such that $\dot{E} = \Psi$. We denote the total heating rate as $\Phi = \Psi + D$. Under the hypothesis of a non-equilibrium steady state system, we have $\bar{\dot{E}} = \bar{\dot{P}} = \bar{\dot{K}} = 0$, where the upper bar indicates time averaging over a long time scale. At any instant, we can partition the domain Ω into Ω^+ and Ω^-, such that the intensive total heating rate J is positive in Ω^+ and negative in Ω^-:

$$\dot{P} + W = \int\limits_{\Omega^+} dV \rho J^+ + \int\limits_{\Omega^-} dV \rho J^- = \Phi^+ + \Phi^- = \Phi, \quad (1)$$

where the upper index refers to the sign of the function, consistently with the definition of the domain of integration.

Since $\bar{D} > 0$, we obtain $\overline{W} = \overline{D} = \overline{\dot{\Phi}^+} + \overline{\dot{\Phi}^-} > 0$. Assuming local thermodynamic equilibrium – which applies well everywhere except in the upper atmosphere, which has a negligible mass – and, neglecting the impact of mixing processes (Lucarini, 2009b) we have that locally $\partial s / \partial t = J / T$, where s is the entropy density of the medium. Therefore, at any instant the entropy time derivative has locally the same sign as the heating rate. The time derivative of the total entropy $\dot{S}$ of the system is:

$$\dot{S} = \int\limits_{\Omega} dV \rho \partial s / \partial t = \int\limits_{\Omega} dV \frac{\rho J}{T} = \int\limits_{\Omega^+} dV \frac{\rho J^+}{T} + \int\limits_{\Omega^-} dV \frac{\rho J^-}{T}$$
$$= \int\limits_{\Omega^+} dV \rho \left| [\partial s / \partial t]^+ \right| - \int\limits_{\Omega^-} dV \rho \left| [\partial s / \partial t]^- \right| = \dot{\Sigma}^+ + \dot{\Sigma}^-, \quad (2)$$

where at all times $\dot{\Sigma}^+ > 0$ and $\dot{\Sigma}^- < 0$. At steady state we have $\bar{\dot{S}} = \overline{\int\limits_{\Omega} dV \rho \partial s / \partial t} = 0$, so that no net trend in the total entropy of the system is present, and $\overline{\dot{\Sigma}^+} = -\overline{\dot{\Sigma}^-}$. Therefore, $\overline{\dot{\Sigma}^+}$ measures the absolute value of the entropy fluctuations in the unit time integrated throughout the domain, because $2\overline{\dot{\Sigma}^+} = \overline{\int\limits_{\Omega} dV \rho |\partial s / \partial t|}$.

Therefore, we obtain $\overline{\dot{\Sigma}^+} = \overline{\Phi^+} / \Theta^+$ and $\overline{\dot{\Sigma}^-} = \overline{\Phi^-} / \Theta^-$, where $\Theta^+ (\Theta^-)$ can be defined as the time and space averaged value of the temperature where absorption (release) of heat occurs. Since $\left| \overline{\dot{\Sigma}^+} \right| = \left| \overline{\dot{\Sigma}^-} \right|$ and $\left| \overline{\Phi^+} \right| > \left| \overline{\Phi^-} \right|$, we

derive that $\Theta^+ > \Theta^-$ and we characterize the climate system as a Carnot engine such that $\overline{W} = \eta \overline{\Phi^+}$, with efficiency $\eta = (\Theta^+ - \Theta^-)/\Theta^+ = (\overline{\Phi^+} + \overline{\Phi^-})/\overline{\Phi^+}$ [6,16].

The 2nd law of thermodynamics imposes that the long-term average of the material entropy production inside the system $\overline{\dot{S}_{in}}(\Omega)$ (this excludes the contributions due to the "degradation" of the solar radiation into terrestrial longwave radiation) is bounded from below by $\overline{\dot{S}_{min}}(\Omega) \approx \overline{W}/\langle\Theta\rangle \approx \eta \overline{\dot{\Sigma}^+}$, where $\langle\Theta\rangle \approx (\Theta^+ + \Theta^-)/2$. Therefore, η sets also the scale relating the minimal material entropy production of the system to the absolute value of the entropy fluctuation per unit time inside the system. If the system is isothermal and at equilibrium, the internal entropy production is zero, since the efficiency η is vanishing: the system has already attained the maximum entropy state. While $\overline{\dot{S}_{min}}$ is related to the dissipation of kinetic energy, the *excess* of material entropy production with respect to the minimum, $\overline{\dot{S}_{exc}} = \overline{\dot{S}_{in}} - \overline{\dot{S}_{min}}$, is due to the heat transport down the gradient of the temperature field. Therefore, we can define:

$$\alpha = \overline{\dot{S}_{exc}} \Big/ \overline{\dot{S}_{min}} \approx \int_\Omega dV \overline{\boldsymbol{H} \cdot \boldsymbol{\nabla}\left(\frac{1}{T}\right)} \Big/ (\bar{W}/\langle\Theta\rangle)$$

$$\approx \int_\Omega dV \overline{\boldsymbol{H} \cdot \boldsymbol{\nabla}\left(\frac{1}{T}\right)} \Big/ \left(\eta \overline{\dot{\Sigma}^+}\right) \geq 0 \qquad (3)$$

as a parameter of irreversibility of the system (Lucarini, 2009b). The parameter α is basically equivalent to the Bejan number (Paoletti et al., 1989) which measures the irreversibility of the system as the ratio $\overline{\dot{S}_{in}} \big/ \overline{\dot{S}_{min}} = \alpha + 1$. When $\alpha = 0$ (and the Bejan number is unity), the system features the smallest rate of material entropy production compatible with the presence of a Lorenz energy cycle of intensity $\overline{W} = \bar{D} = \int \overline{\varepsilon^2} dV$, as no contributions to entropy production come from fluxes transporting heat down the gradient of the temperature. Since $\overline{\dot{S}_{in}} \approx \eta \overline{\dot{\Sigma}^+}(1+\alpha)$, we have that material entropy production is maximized if we have a joint optimization of heat transport downgradient the temperature field and of production of mechanical work.

3 Methods

PLASIM, a simplified yet Earth-like climate model freely available at http://www.mi.uni-hamburg.de/plasim, has been used in a configuration featuring T21 horizontal resolution with five sigma levels in the vertical. The ocean is represented by a 50m slab ocean (with energy transport set to 0), including a 0-dimensional thermodynamic sea ice model. Slab ocean climate models are well suited for providing an accurate steady state climate response (Danabasoglu and Gent, 2009).

In steady state conditions, the global climate energy budget should vanish when long term averages are considered. If this is not the case, the spurious energy imbalance has to be attributed to unphysical energy sources or sinks. As thoroughly discussed by Lucarini and Ragone (2010), this actually is a serious problem of physical consistency for most state-of-the-art climate models included in 4th Assessment Report of IPCC.

The global atmospheric energy balance is greatly improved with respect to previous versions of the model by adding the kinetic energy losses due to surface friction and horizontal and vertical momentum diffusion (Becker, 2003; Lucarini and Fraedrich, 2009). The average global energy imbalance of the system, computed as long term average of the integrated net radiative flux at the top of the atmosphere, is in all simulations is $\leq 0.2\,\mathrm{Wm}^{-2}$, which is about one order of magnitude smaller than most state-of-the-art climate models (Lucarini and Ragone, 2010). The imbalances in the global energy and entropy budgets can be written as $\bar{\bar{S}} = \overline{\dot{\Sigma}^+} + \overline{\dot{\Sigma}^-} = \Delta_S$, with $\Delta_S \ll \left|\overline{\dot{\Sigma}^+}\right|, \left|\overline{\dot{\Sigma}^-}\right|$, and $\bar{\bar{E}} = \bar{P} + \bar{K} = \Delta_E$, with $\Delta_E \ll \left|\overline{\Phi^+}\right|, \left|\overline{\Phi^-}\right|$. Moreover, the Lorenz energy cycle has a spurious term, with $\overline{W} = \overline{\Phi^+} + \overline{\Phi^-} - \Delta_E$. Therefore, the thermodynamic efficiency is ill defined and, similarly, the estimates for entropy production contributions are, in principle, inconsistent. If the numerical errors in the material entropy budget discussed in (Johnson, 2000) are, as in this case, negligible, the 2nd law of thermodynamics imposes that $\Delta_E \approx \langle\Theta\rangle \Delta_S$. Thanks to that, as thoroughly discussed by Lucarini et al. (2010), the two thermodynamic temperatures Θ^+ and Θ^- are still well defined, as the expression $(\Theta^+ - \Theta^-)/\Theta^+$ provides a good first order approximation to the true efficiency $\eta = \overline{W}\big/\overline{\Phi^+} = (\overline{\Phi^+} + \overline{\Phi^-} - \Delta_E)\big/\overline{\Phi^+}$. Similarly, the material entropy production rate is computed as $\overline{\dot{S}_{in}} + \Delta_S$, and the irreversibility factor is evaluated as $\alpha = \left(\overline{\dot{S}_{in}} + \Delta_S\right)\Big/\left(\eta\left(\overline{\dot{\Sigma}^+} + \left|\overline{\dot{\Sigma}^-}\right|\right)\big/2\right)$, where we introduce a correction in the denominator to account for the fact that $\overline{\dot{\Sigma}^+} \neq \left|\overline{\dot{\Sigma}^-}\right|$. With these corrections, all proposed formulas apply with a high degree of approximation.

4 Results

In the usual operative definition, *climate sensitivity* Λ_{T_S} is the increase of the globally averaged mean surface temperature $\overline{T_S}$ between the preindustrial CO_2 concentration steady state and the steady state conditions realized when CO_2 concentration is doubled. As $\overline{T_S}$ is almost linear with respect to the logarithm of the CO_2 concentration on a large range, it is actually easy to generalize the definition of Λ_{T_S} as the impact on $\overline{T_S}$ of CO_2 doubling so that $\Lambda_{T_S} = d\overline{T_S}/d\log_2([CO_2]_{\mathrm{ppm}})$. Such a linear behaviour basically results from the fact that up to a good degree of approximation, the CO_2-related radiative
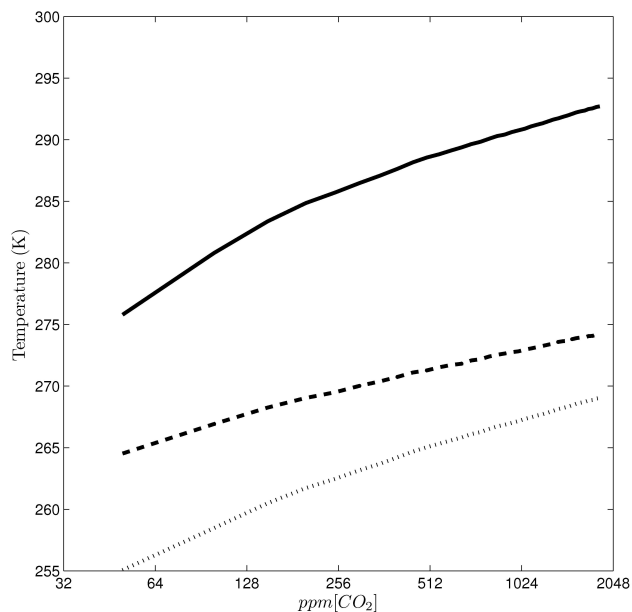
Fig. 1. Time average of the global mean surface temperature T_s (solid line) and of the temperature of the warm (Θ^+) and cold (Θ^-) reservoirs (dashed and dotted lines, respectively).

Table 1. Generalized Sensitivities.

Definition	Value
$\Lambda_{\overline{T_S}}$	2.55 K[a]
Λ_{Θ^+}	1.65 K
Λ_{Θ^-}	2.35 K
Λ_η	-0.002
$\Lambda_{\bar{W}}$	-0.06 Wm^{-2}
$\Lambda_{\overline{\dot{S}_{\mathrm{in}}}}$	0.0004 Wm^{-2} K^{-1}
Λ_α	0.7

[a] The value refers to CO_2 concentrations larger than 150 ppm.

forcing is linear with the logarithm of the CO_2 concentration (Myhre et al., 1998; IPCC, 2007).

The main goal of this work is to check whether it is possible to define, in a similar fashion, generalized sensitivities Λ_X to describe the steady state response to CO_2 concentration changes of the thermodynamical properties X of the climate system. At this scope, we have performed climate simulations for CO_2 concentrations ranging from 50 to 1850 ppm by 50 ppm steps, thus totalling 37 runs. Each simulation has a length of 50 years and the statistics are computed on the last 30 years of the simulations in order to rule out the presence of transient effects. In order to fully characterize the non-equilibrium properties of the climate system, we have analysed the most important thermodynamic variables of the system introduced in the previous section:

- the time average of the temperatures of the warm (Θ^+) and cold (Θ^-) reservoir, and, as reference, the time average of the global mean surface temperature $\overline{T_S}$ (Fig. 1);

- the thermodynamic efficiency η (Fig. 2a);

- the average intensity of the Lorenz energy cycle $\bar{W}$ (Fig. 2b);

- the time average of the rate of material entropy production $\overline{\dot{S}_{\mathrm{in}}}$ (Fig. 2c);

- the degree of irreversibility of the system α (Fig. 2d).

In addition to the surface temperature, all of these thermodynamic variables feature a monotonic behaviour with respect to the CO_2 concentration. In particular, all of the variables feature an approximate linear behaviour with respect to the logarithm of the CO_2 concentration. Therefore, we can safely attribute a robust value (with an uncertainty of at most 10%) to the generalized sensitivities defined as $\Lambda_X \equiv dX/d\log_2([CO_2]_{\mathrm{ppm}})$. Results are summarized in Table 1. Note that the variable with the strongest systematic deviations from linearity is the globally averaged surface temperature, which features a markedly linear dependence only for CO_2 concentrations larger than 150 ppm. Instead, the graph of the rate of material entropy production is noisier than the other variables, but linearity with respect to the logarithm of CO_2 concentration is on the average quite accurate.

The three temperature indicators (Fig. 1) feature, as expected, positive sensitivities: the surface temperature sensitivity (evaluated in the, more realistic, upper range of CO_2 concentrations) is well within the range of what is simulated by the climate models included in 4th IPCC report (IPCC, 2007), whereas the two bulk thermodynamic temperatures have smaller sensitivities. In order to interpret these results, we consider the changes in the heating and temperature patterns between the 1000 and 100 ppm CO_2 concentration runs. In the 100 ppm CO_2 concentration run, on the average, positive heating patterns (which are relevant for defining Θ^+) include the whole equatorial troposphere and the lower levels (below 700 hPa) in the wide latitudinal belt between the Northern and Southern Mid-latitudes (Fig. 3a). When looking at the vertically integrated patterns (Fig. 4a), we discover a major zonal asymmetry, namely that in the mid-latitudes the air aloft the continental masses experiences negative heating rates (as opposed to the ocean areas), thus suggesting that over land the low level positive heating pattern is shallower and/or weaker. In Fig. 3b it is shown that, in the 1000 ppm CO_2 concentration run, the extent of the zonally averaged positive heating region is wider than in the 100 ppm case, as it includes higher latitudes and lower pressure portions of the atmosphere. Looking at the vertically integrated patterns (Fig. 4b), we consistently find a northward shift of the
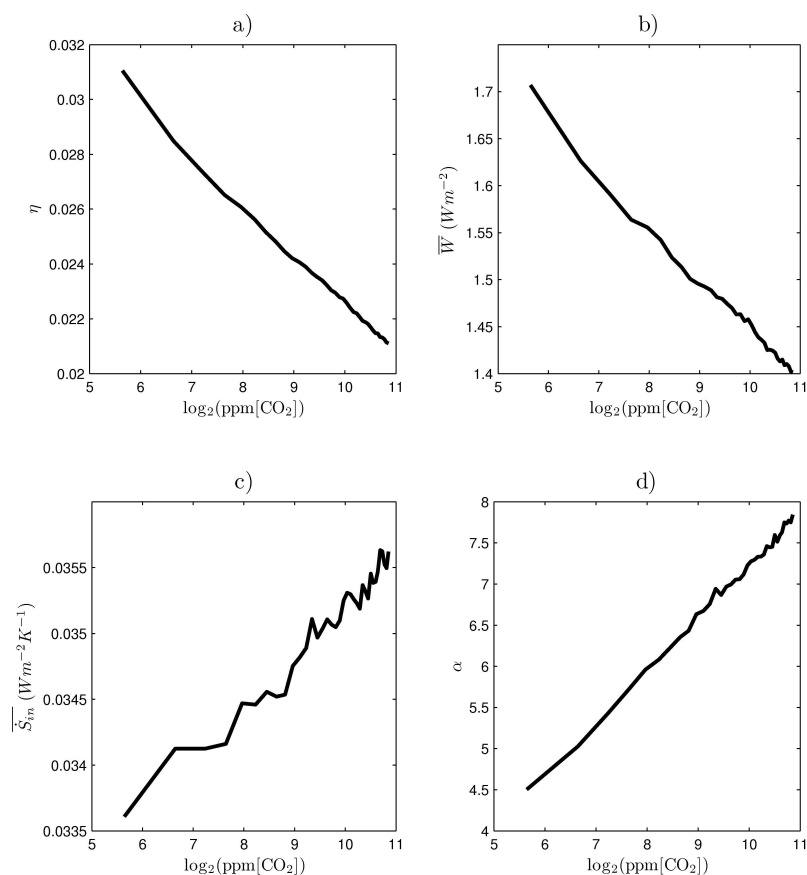
Fig. 2. Generalised climate sensitivities. CO_2 concentration dependence of macroscopic thermodynamic variables: **(a)** efficiency; **(b)** intensity of the Lorenz energy cycle (or, equivalently, total dissipation); **(c)** material entropy production; **(d)** degree of irreversibility. See text for details.

positive heating regions in the mid-latitudes. This is closely related to the poleward migration of the storm track and of the peak of meridional enthalpy transport in both hemispheres under warmer climate conditions, which has been carefully described in state-of-the-art climate models (Yin, 2005; Lucarini and Ragone, 2010). Instead, the air aloft the continental masses experiences negative heating patterns on the average as in the 100 ppm CO_2 concentration case. The fact that relatively colder regions of the atmosphere are considered in the integrals leading to the definition of Θ^+ for increasing values of CO_2 concentrations seems to explain why the sensitivity of Θ^+ is smaller than that of Θ^-.

The reason why the sensitivity of Θ^- is smaller than that of the surface temperature can be ascertained by looking at the actual patterns of average temperature change between the two runs (Fig. 5a). We observe that the most of the region with negative net heating balances – which concurs to defining Θ^- – experiences a larger increase of the vertical temperature gradient, as temperature increases aloft are smaller than those close to surface. Therefore, its average temperature will increase less than the average surface temperature for increasing CO_2 concentrations.

We observe that, whereas the mid-latitude atmosphere becomes more stable, in agreement with what observed in more realistic climate models (Frierson, 2006), the equatorial region the situation is reversed, as the largest temperature increases are observed in the upper troposphere, since the large increase in water vapour content manages to decrease substantially the moist adiabatic lapse rate. Also the latter result is in agreement with what found in state-of-the-art climate models (Chou et al., 2009).
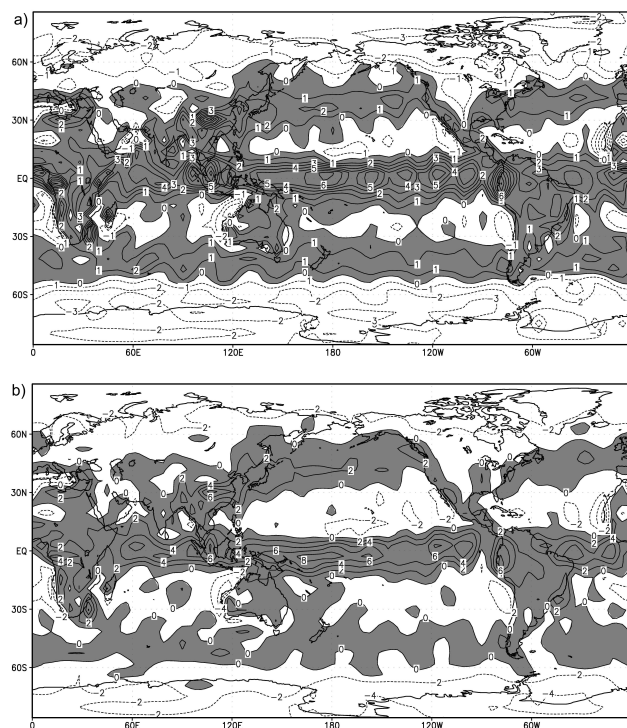
We can rephrase the fact that with higher CO_2 concentrations the temperature of the cold bath increases faster than that of the warm bath by saying that on the average we have a more isothermal atmosphere. In fact, by looking at Fig. 5a, we find that the atmospheric region with the highest average temperature (the tropical lower troposphere) features the smallest temperature increase, whereas large temperature increases are observed in cold regions such as the lower polar troposphere and the whole upper troposphere. The main driver of the reduction of the planetary thermal differences is the large enhancement of the convergence of latent heat with increasing CO_2 concentration. The pattern of change in the zonally averaged field of latent heat heating between the
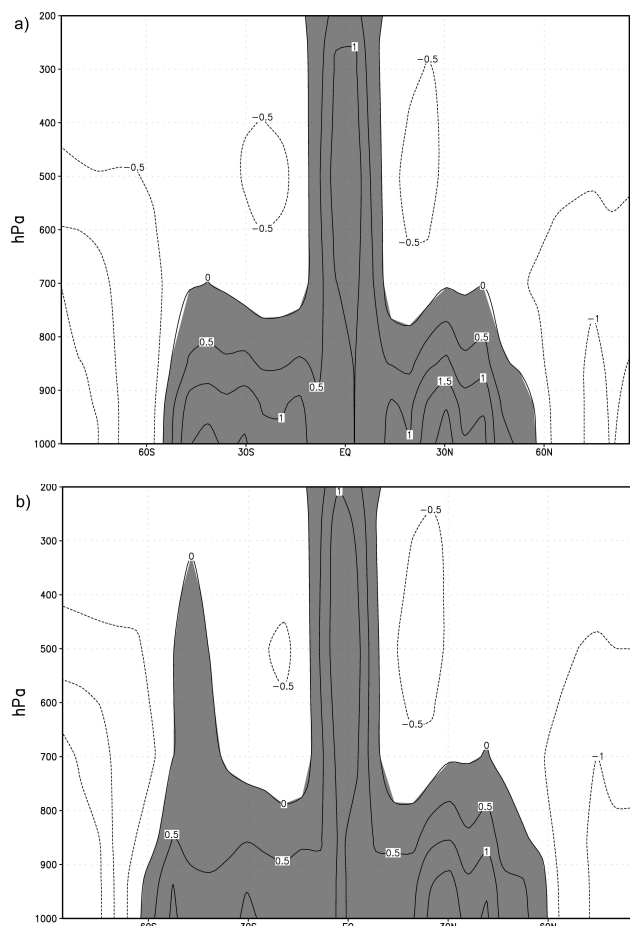
Fig. 3. Zonally averaged mean heating rates (in K/day) for the 100 (**a**) and 1000 (**b**) ppm CO_2 concentration runs.

Fig. 4. Vertically averaged mean heating rates (in K/day) for the 100 (**a**) and 1000 (**b**) ppm CO_2 concentration runs.

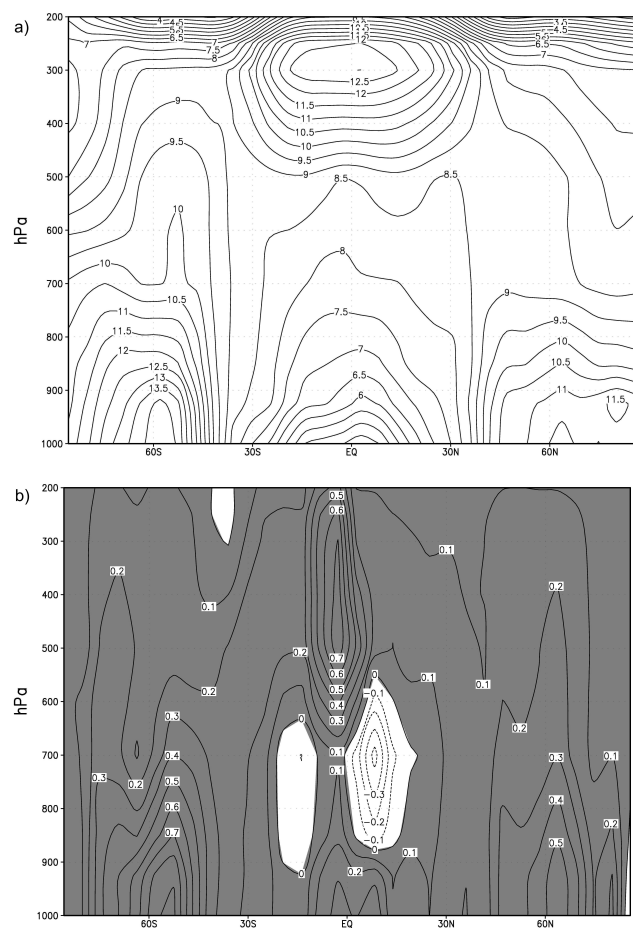1000 and 100 ppm CO_2 concentration runs resembles closely that of mean temperature change depicted in Fig. 5a.

Consequently, increases in the CO_2 concentration cause a steep decrease in the efficiency of the climate system, as shown in Fig. 2a. In the explored range, the efficiency decreases by about 35%, with a relative change of about −7% per CO_2 doubling. In a thicker (and warmer) atmosphere, the total absorbed heat $\overline{\Phi^+}$ is larger, so that the actual strength of the Lorenz energy cycle changes as the result of the competing effects of increasing energy input and decreasing efficiency. The intensity of the Lorenz cycle decreases in a warmer climate (Fig. 2b), with an approximate change of −4% per CO_2 doubling. By energy conservation, the same applies to the total dissipation, so that in a warmer climate weaker surface winds are expected, as found by Hernandez-Deckers and Von Storch (2010) is a recent paper. This is confirmed by inspecting Fig. 6a, where changes in the average square velocity in the lowest sigma level of the model are reported between the 1000 and the 100 ppm CO_2 concentration runs. Interestingly, while changes are small over most latitudes, the negative contributions are concentrated in
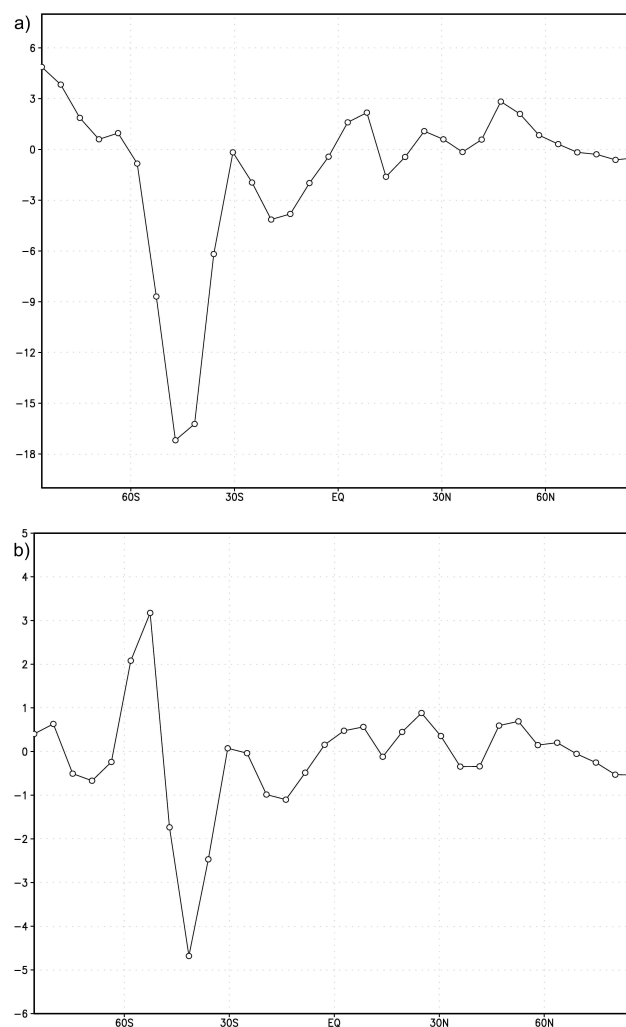
the mid-to-high latitudes of the Southern Hemisphere. Note that the pattern shown in Fig. 6a resembles qualitatively the difference between the 350 and the 100 ppm CO_2 concentration runs, whereas, when observing the changes between the 1000 ppm and the 350 ppm concentration runs (Fig. 6b), we have the negative contributions between 40° and 50° S to be partially compensated by a positive change in the latitudinal belt 50° S–60° S, in qualitative agreement with what shown, e.g. by Kushner et al. (2001) in a typical global warming experiment. This latter feature suggests that PLASIM, in spite of its simplifications and the absence of a coupled dynamic ocean, is able to capture the well known trend of strengthening of the wind stress in the region of the Antarctic Circumpolar Current in a warmer climate. The discrepancies between the zonal profiles depicted in Fig. 6a and Fig. 6b suggest that, latitude by latitude, the changes is the dissipation are not linear (or even monotonic) with the logarithm of the CO_2 concentration. This is not in contradiction with the approximately linear dependence found for the total dissipation (or, equivalently, for intensity of the global Lorenz energy cycle) and depicted in Fig. 2b. Locally, there is no guarantee of linearity, since climate patterns change in a complex fashion as CO_2 concentration is altered.

As with increasing CO_2 concentration the average temperature increases and the total dissipation decreases, the quantity $\overline{\dot{S}_{min}}$, which is uniquely related to mechanical dissipation, must be a decreasing function of CO_2 concentration.

Fig. 5. (a) Zonally averaged mean temperature difference between the 1000 ppm CO_2 and 100 ppm CO_2 concentration runs. **(b)** Zonally averaged difference between the 1000 ppm CO_2 and 100 ppm CO_2 concentration runs for the mean heating rates due to the convergence of latent heat fluxes (in K/day).

Fig. 6. (a) Zonally averaged difference between the 1000 ppm CO_2 and 100 ppm CO_2 runs for the mean squared velocity (in $m^2 s^{-2}$) at the sigma level closest to surface. **(b)** Same as (a), but the difference between the 1000 ppm CO_2 and 350 ppm CO_2 runs is shown.

Instead, as shown in Fig. 2c, the actual average total rate of material entropy production $\overline{\dot{S}_{in}} = \overline{\dot{S}_{min}} + \overline{\dot{S}_{exc}}$ has the opposite behaviour, with an approximate relative increase of 2% per CO_2 doubling. This implies that $\overline{\dot{S}_{exc}}$, giving the entropy production due to the heat transport down the gradient of the temperature field, is much higher in warmer climates, the reason being that the convergence of latent heat fluxes becomes extremely effective in transporting heat from warm to cold temperature regions (Fig. 5b). Therefore, the degree of irreversibility of the system α increases steeply with CO_2 concentration (Fig. 2d). In the considered range, the fraction of entropy production due to mechanical energy dissipation $1/(\alpha+1)$ drops from about 22% to about 12%. This behaviour is specific for climate conditions analogous to the present ones, whereas under snowball conditions, where latent heat fluxes are negligible, the value of α is about unity and only slightly affected by temperature (Lucarini et al., 2010).

5 Conclusions

Using a simplified yet Earth-like climate model, we have re-examined the classical problem of the steady-state response of the climate system to CO_2 concentration variations and have focused on the changes in the non-equilibrium thermodynamical properties of the climate system, building upon a recently introduced theoretical framework (Lucarini, 2009b). The novel approach presented here allows for setting in common framework the changes global properties of the climate system such as the intensity of the Lorenz energy cycle and the material entropy production. As a result, we have introduced a comprehensive set of generalized climate sensitivities.

Table 2. Parameterisations[a].

Definition	Value
$d\Theta_1 \big/ d\overline{T_s}$	0.65
$d\Theta_2 \big/ d\overline{T_s}$	0.92
$d\eta \big/ d\overline{T_s}$	-0.0008 K^{-1}
$d\dot{W} \big/ d\overline{T_s}$	-0.024 Wm^{-2} K^{-1}
$d\dot{S}_{\mathrm{in}} \big/ d\overline{T_s}$	0.00016 Wm^{-2} K^{-2}
$d\alpha \big/ d\overline{T_s}$	0.275 K^{-1}

[a] Values are valid for .globally averaged surface temperatures higher than 284 K.

In addition to the average surface temperature, the intensity of the Lorenz energy cycle, the Carnot efficiency, the material entropy production and the degree of irreversibility of the system change monotonically with the CO_2 concentration. The behaviour is approximately linear with the logarithm of the CO_2 concentration, basically as a result of the fact that the net radiative forcing exerted by CO_2 has an analogous dependence.

It is rather interesting to observe that, among the considered variables, the largest systematic deviations from linearity (relevant for values of the CO_2 concentration smaller than 150 ppm) are observed for the globally averaged surface temperature, which is, somewhat ironically, the quantity that motivated the introduction of the climate sensitivity. This is actually not so surprising because, whereas the other thermodynamical quantities describe bulk properties of the atmosphere which are directly impacted by the changes in the CO_2-related radiative forcing, the surface temperature is more of a boundary property.

The resulting generalized sensitivities proposed here (whose values are reported in Table 1) demonstrate that the climate system becomes less efficient (and more isothermal), more irreversible, and features higher entropy production as it becomes warmer. Changes in the intensity of the latent heat fluxes are the dominating ingredients, thus showing, at a fundamental level, how important it is to address correctly the impact of climate change on the hydrological cycle.

Thanks to the monotonic dependence (and not, specifically, to the linearity) of the diagnosed variables with respect to the logarithm of the CO_2 concentration, it is possible to re-parameterise efficiently all the variables with respect to just one. As an example of this procedure, in Table 2 we provide the linear coefficients of all the thermodynamic macro-variables of the system with respect to the average surface temperature. These coefficients are valid for CO_2 concentrations larger than 150 ppm and, correspondingly, for globally averaged surface temperatures larger than 284 K. These data may be of use when devising simplified yet comprehensive climate models or estimating unknown quantities in comprehensive models or from observational data.

We believe that the investigation proposed here may stimulate the re-examination, from a more fundamental point of view, of the problem of climate change, for studying the impacts of changing the atmospheric composition on the properties of the large scale atmospheric circulation, of convection, and of the processes responsible for coupling the atmosphere with land and ocean.

In this study it is shown that the heating patterns, which are crucial for constructing all of the non-equilibrium thermodynamical properties of the system discussed here, respond to increasing CO_2 concentration consistently with how state of the art climate models describe changes in the stratification in the tropical (Chou et al., 2006) and extratropical atmosphere (Frierson, 2006) as well as in the properties of the meridional atmospheric enthalpy transport (Yin, 2005; Lucarini and Ragone, 2010).

Following the results an earlier study on the onset and the decay of snowball conditions (Lucarini et al., 2010), we propose that the approach presented here may be of help for addressing problems of paleoclimatological relevance, such as the interplay between solar irradiance and atmospheric composition variations in determining slow climate modulations as well as tipping points (Lenton et al., 2008), as, e.g., the onset and decay of ice ages or rapid changes in the large scale properties of the ocean circulation.

The present study calls for several improvements. Firstly, it would be important to address the role of a dynamic ocean, which in the present analysis acts an infinite source of water and is relevant for increasing the thermal inertia of the system. Secondly, it would be very interesting to treat time-dependent changes in the CO_2 concentration and study the adjustment process of the system in terms of the proposed global thermodynamical quantities.

We expect that extensive application of the thermodynamically-based tools adopted here may, in general, help closing the *Gap between Simulation and Understanding in Climate Modeling* (Held, 2005) and may provide the basis for a new generation of metrics aimed at the validation of climate models (Lucarini, 2008b).

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