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Extreme value analysis of wet and dry periods in Sicily

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With 7 Figures

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Summary

For assessing risk of highly unusual events extreme value statistics needs to be applied, which plays an important role in engineering practices for water resources design and management. In hydrology, the typical application of extreme value theory concerns floods in river basins or landslides. The present paper is, instead, focused on the analysis of extreme wet and dry periods in a sample area (Sicily). First, we have studied monthly precipitation extremes both using the annual maximum and partial duration methods, and return times have been estimated by standard statistical techniques. Next, we studied the extremes of the Standardized Precipitation Index (SPI), which has been proposed as an indicator for monitoring wet and dry conditions. We found considerable differences both in the return periods and in the time location of the extremes. From our study it appears that the SPI better describes wet and dry periods than the precipitation does. Maps of return times for extreme conditions in Sicily are also presented, which cluster the territory into areas of different extreme return periods. Finally, the occurrence of extremes in Sicily has been related to large-scale atmospheric circulation.

1. Introduction

Extreme value theory is concerned with probabilistic and statistical questions related to very high or very low values in sequences of random variables. The subject has a rich mathematical theory and also a long tradition of applications in a variety of areas, especially in applied science like hydrology (Coles, 2001).

The problem setting is quite straightforward: given a set of geophysical measurements, which may be assumed to be drawn out independently from a given distribution (independently and identically distributed, i.i.d.), we wish to determine the nature of the tails of the distribution. Once the distribution fitting the tails is found, some statistical inferences can be made for helping decision makers planning risk management and available options.

A typical application of these methods is the analysis of floods in river basins or of landslides. In this case, the problem is to ascertain the probability that a particular flood event will occur and, thereby, a system of mitigating measures can be designed to avoid the negative impacts of such event. Generally speaking, the basic variables of these assessments are precipitation intensity and duration. However, it is known that such precipitation events tend to cluster in time so that the data lack the required statistical independence for a correct application of the statistical tools. To avoid this shortcoming two approaches have been devised (see for example Beguería, 2005; Kats et al., 2002):

(i) The first method samples the tail of the distribution by choosing as extreme events those separated in time long enough so that any correlation between them cannot be envisioned (for example annual maxima series). A shortcoming of such an approach is that we may miss other extremes that fall within the pre-chosen time span, thus greatly overestimating the probability of occurrence;

(ii) The second method considers as extremes those data that exceed a given threshold (known as partial duration series). The pitfall now is the difficulty in choosing (not parametrically) the threshold and the possibility of getting clustered extremes as well.

In the present paper we wish to apply these two methods for assessing wet and dry periods in a given area. Here the question is: given the precipitation cumulated on a given time scale (we consider monthly precipitation), may we assess the probability of occurrence of wet and dry periods? In doing this, besides problems related to the two sample strategies, we face the fundamental task to quantify the vague definition of *wet* and *dry* episodes and their relationship with extreme value theory.

As an example, consider a single rain-gauge station, which records the monthly precipitation. We wish to define, what is an extreme wet condition. For the annual maximum (AM) precipitation we may form an extreme sample set that satisfies the requirement of statistical independence. In this context, we may choose a threshold and use as extreme sample set the observations above this bound. In this case, the degree of dependence of the data will be very weak, since monthly precipitation is generally uncorrelated. Now we can define as an extremely wet period, an event that has a very low probability of recurrence in one of the two methods and readily move to managing procedures for water resources distribution. However, there is a pitfall of this approach: generally speaking, the main concern is that water resources availability does not depend only on extreme monthly precipitation, but rather on the monthly positive anomaly of precipitation with respect to some undefined average condition. Thus, in quantifying a wet period we should consider *anomaly* extremes rather than precipitation extremes. But, on the other hand, an anomaly calculation requires a definition of a location parameter (of the distribution of events) in respect to which the deviation is computed. Since, monthly precipitation has an inter-annual distribution, which is heavily skewed, any location measure, such as the mean, may be inappropriate for a proper definition of a random variable that describes a relative abundance of precipitation. Likewise, similar considerations apply to the definition of dry periods.

Recently, an index based on monthly precipitation, the Standardized Precipitation Index (SPI), has been proposed to monitor wetness and dryness (McKee et al., 1993) that appears (see next section) to overcome these problems by adding a quantitative meaning to the otherwise loose definition of dry and wet periods.

To support these ideas, we analyse also SPI extremes for a European region: Sicily where precipitation data were at hand. The paper is structured as follows. Section 2 describes data and methods, in particular the SPI assumptions. Results are presented in Section 3. In Section 4, we present the climatological mean flow in the North Atlantic-European Sector associated with few extremes. In the final section conclusions and suggestions for future investigations are discussed.

2. Methods and data

To quantify wet and dry conditions the Standardized Precipitation Index is introduced before describing standard methods of extreme value statistics applied to the index. Finally, the data sets to compute the SPI and to provide the atmospheric flow fields associated to extremes are presented.

2.1 The Standardized Precipitation Index (SPI)

In order to illustrate the ability of the Standardized Precipitation Index to capture both extreme wet and dry conditions, we describe the steps for the index computation. The SPI was first introduced by McKee et al. (1993) and requires, for its computation, only long time series of monthly precipitation. For each month of the year and for a particular location, the observed frequency distribution of monthly precipitation is estimated. Several authors, most recently Guttman (1999), tested several probability density functions to fit the empirical one, concluding that the two-parameter Gamma distribution can be considered the best choice. Although the choice has not a fundamental meaning, we consider, for sake of simplicity, such a distribution as the one that best fits the empirical distribution for each month of the year. Thus, for each month we estimate the two parameters of the Gamma distribution with a standard method, such as the maximum likelihood approach. The parameters are then used to compute the cumulative probability $G(x)$ of the observed precipitation x. Since the Gamma function is undefined at zero, but precipitation distribution may contain zeros, the cumulative probability becomes:

$$H(x) = q + (1 - q)G(x),$$

where q is the probability to have zero precipitation. However, when we use monthly precipitation averaged over stations, rarely $q \neq 0$.

Next, by means of an equal-probability transformation, the empirical probability density distribution is transformed into the Normal one. Thus, this transformation allows considering an anomaly with respect to the Normal distribution, instead of an anomaly with respect to a pre-chosen location parameter of the original distribution (say, mean or median). This quantifies the meaning of relative wetness and dryness. The SPI, in fact, is a Z-score or the number of standard deviations (above or below) that an event deviates from the normalised mean of the month considered. Thus, a very large positive/negative SPI describes an extremely wet/dry event. Further details on the numerical computations of the index may be found in the original paper of McKee et al. (1993) or in Bordi and Sutera (2001).

It is also of common use to group SPI values into different classes of dryness and wetness as listed in Table 1. Note that extreme wet and dry conditions are identified by SPI values greater than 2 and less than -2, respectively. Since the SPI is standardised, it also allows comparing climate conditions of areas governed by different hydrological regimes and monitoring both dry and wet events.

Table 1. Weather classification according to SPI values

SPI value	Class
>2.0	Extremely wet
From 1.5 to 1.99	Very wet
From 1.0 to 1.49	Moderately wet
From −0.99 to 0.99	Near normal
From −1.0 to −1.49	Moderately dry
From −1.5 to −1.99	Severely dry
<−2.0	Extremely dry

Furthermore, the index can be computed for multiple time scales by considering, instead of monthly precipitation, the cumulated precipitation over a selected period; usually 1, 3, 6, 12 and 24 months are used. This allows specifying different effects of wetness/dryness: short time scales affect growing seasons, long time scales characterise hydrologic balances with a net water gain/loss. However, except for the 1-month SPI (SPI-1), long time scales require the use of cumulated precipitation over the period, thereby introducing a correlation into the SPI time series, an unwished property for applying standard extreme value technique. Therefore, we decide to limit the present study to the SPI on 1-month time scale.

2.2 Extreme value analysis: AM and PD approaches

The analysis of extremes is usually based on the estimation of extreme event return times. The return time, or return period, of an event can be defined as the average number of observations to be made to obtain one observation equalling or exceeding its magnitude. This implies the assumption that the extremes are independent random variables described by a probability distribution, which should not change from sample to sample, i.e. the data should be homogeneous. As anticipated in the introduction, two methods are applied to sample the original data: Annual maximum (AM) and partial duration (PD) series (also known as Peaks Over Threshold series).

The AM series consists of the greatest events of each year in a given time period. According to the Fisher-Tippett theorem (1928), the asymptotic distribution of a series of sample maxima (AM series) belongs to one of three basic distributions (Fréchet, Weibull and Gumbel), regardless of the original distribution of the observed data. These three families were combined into a single distribution, which is now known as the Generalised Extreme Value (GEV) distribution (see Jenkinson, 1955).

The PD analysis (or peaks over threshold) considers all the values of the variable that exceed an a priori determined threshold. A theorem by Balkema and de Haan (1974) and Pickands (1975) shows that for sufficiently high threshold, the distribution function of the excess may be approximated by the Generalised Pareto (GP) distribution such that, as the threshold gets large, the excess distribution converges to the GP distribution. A brief summary of the methods is presented in the Appendix. In the present paper we apply these statistical tools both to precipitation and SPI-1 time series. For the PD approach, when we use precipitation as the basic variable, the threshold is chosen by applying standard statistical techniques, while when we use the SPI-1 it is just given by the index classification (± 2). In the following section the main results are shown.

2.3 Data

Results presented in this work are based on monthly precipitation time series from 36 stations in Sicily covering the period 1926–2000. The stations have been extracted from a larger set according to Alecci et al. (2000) criterions, which are mainly the record length, data quality and homogeneous spatial distribution.

To analyse the large-scale atmospheric patterns (see section 4) associated to SPI-1 extremes we use NCEP/NCAR re-analysis data for the period 1948–2000 (see Kalnay et al., 1996 for a detailed description of the data set).

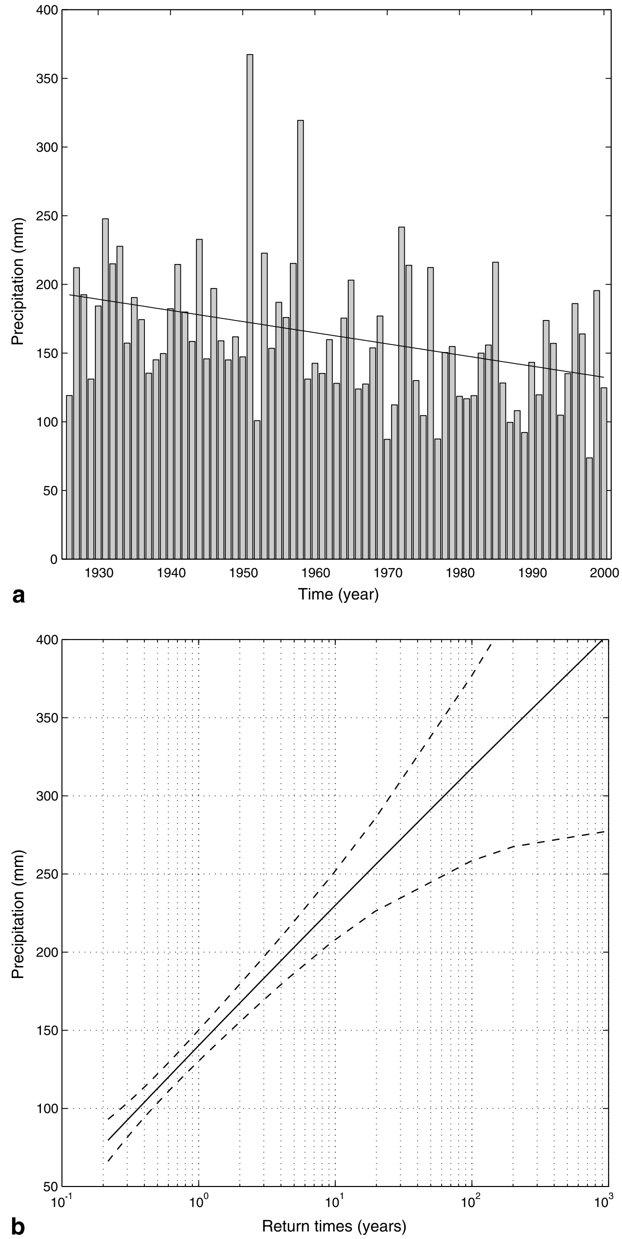


Fig. 1. (**a**) AM series of monthly precipitation averaged over the 36 stations in Sicily. Straight line is the computed linear trend; unit is mm. (**b**) Return times for precipitation AM series. Solid line denotes the fit of return times estimated from observations; dashed lines provide the error band at 95% confidence level. Unit is year

3. Results: Return times and their spatial variability

First, the Sicily monthly precipitation time series averaged over the 36 stations has been computed and the annual maxima series has been extracted from the data for the 75 years considered starting from 1926. The resulting AM series is shown in Fig. 1a. Precipitation annual maxima show values ranging from about 73 mm to about 250 mm with two peaks higher than 300 mm occurring in 1951 and 1958. A trend towards lower values of annual extremes can be noted that characterises the time series: a loss of about 50 mm has occurred in the last 75 years, while the time average of annual maxima is about 160 mm. Whether this trend is related to global change, given the local nature of our data, is uncertain, but we like to stress that its statistical significance is very high to warrant further analysis in the future. We remark also that trends may have impacts on extremes, but we decide to not take into account these two problems in the present paper and postpone a proper analysis to another occasion.

3.1 Return times

Next, return times for precipitation annual maxima are computed and their fit is displayed in Fig. 1b. Also the PD method with two different precipitation thresholds, 76 and 200 mm, is applied to the averaged precipitation time series. The PD sample is shown in Fig. 2a and the return time statistics in Fig. 2b. The two thresholds have been chosen according to the methodology commonly used and discussed in the Appendix. A comparison between results obtained for AM and PD approach (see Fig. 3) suggests that return times for different precipitation amounts are comparable, especially when a low threshold is considered. Precipitation extremes between 200 and 300 mm have return periods ranging from 4 year to 60 years. Within this range of precipitation the return times estimated by AM and PD methods using the two thresholds show a good agreement, while for 300 mm they differ remarkably. However, as expected, for high precipitation values, we get a large error band, thus leaving the corresponding return times not clearly defined.

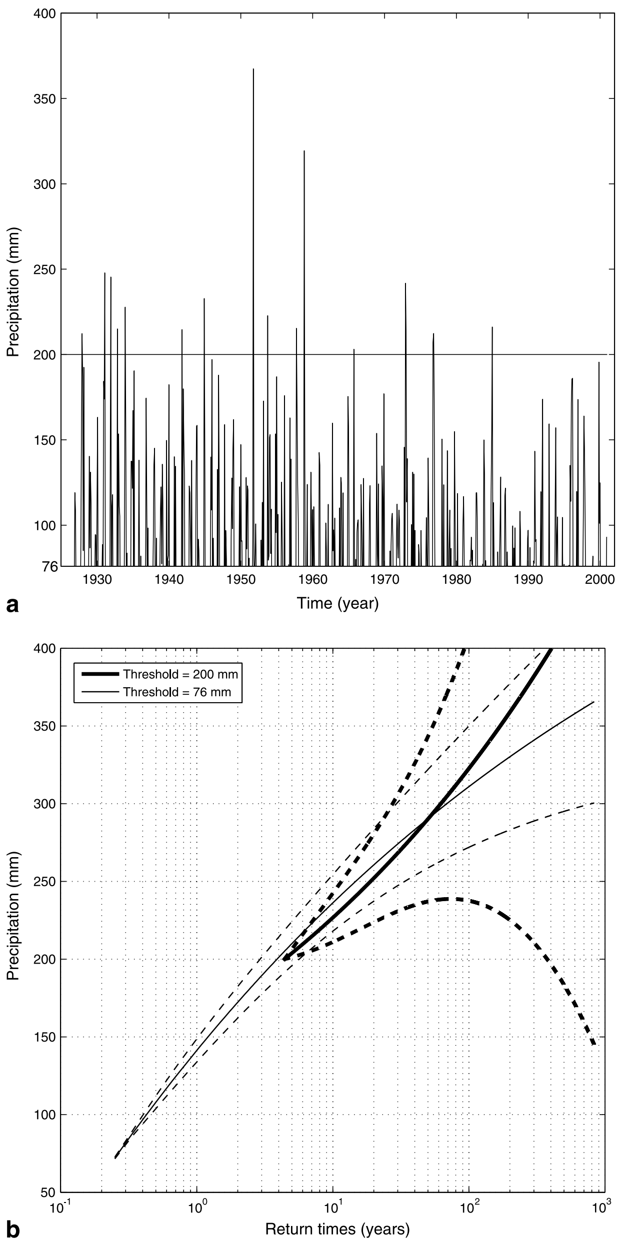


Fig. 2. (**a**) PD series of monthly precipitation for two different thresholds: 76 and 200 mm (denoted by straight line). (**b**) Return times of precipitation extremes selected with PD method for the two thresholds: 76 mm (thin line) and 200 mm (thick line). Solid lines denote the fits of return times estimated from observations; dashed lines provide the error bands at 95% confidence level. Unit is year

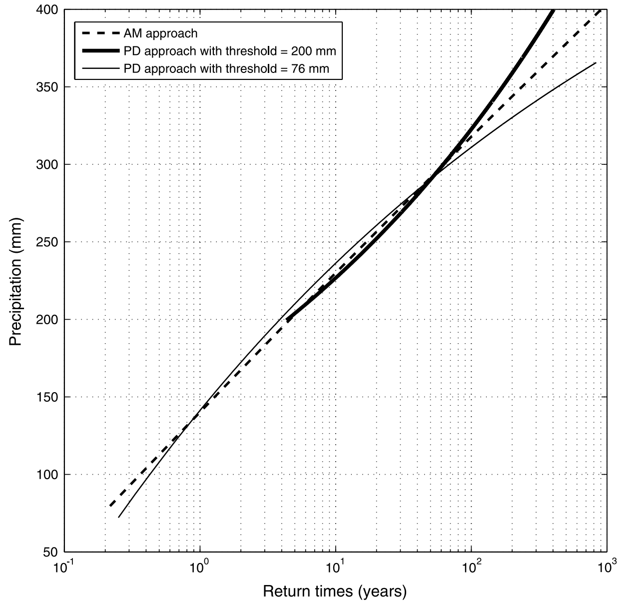


Fig. 3. Return times curves for precipitation extremes in Sicily computed applying the AM (dashed line) and the PD method with the two thresholds (76 mm thin line, 200 mm thick line). Unit is year

At this stage of our analysis, we can raise the question whether an AM extreme, occurring at a particular month of the year, really represents an abnormal departure from some location parameter of the empirical probability distribution of precipitation for that month, so that we can consider it as a wet extreme. At this purpose, we show in Fig. 4 the probability distribution (histogram) of observed precipitation for, say, December. The solid line denotes the Gamma function fitting the distribution and filled circles, on the bottom of the figure, are the annual maxima precipitation

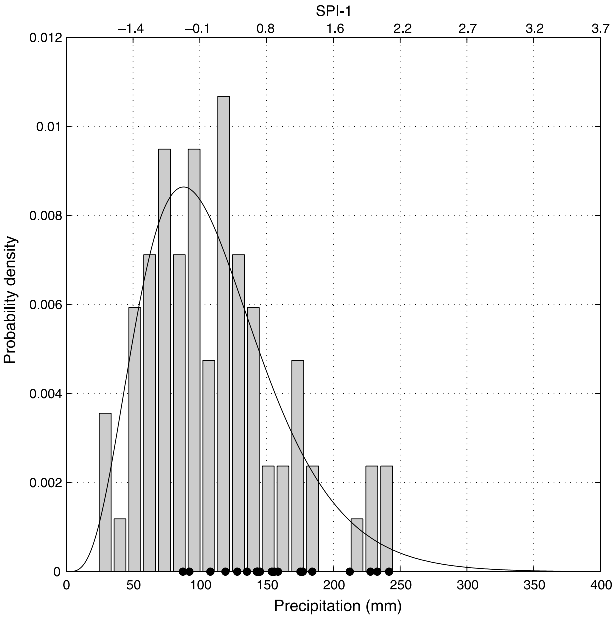


Fig. 4. Probability density distribution of observed precipitation for December. Solid line denotes the Gamma function fitting the distribution and filled circles, on the bottom of the figure, are precipitation annual maxima occurring in December. Unit on the (bottom) x-axis is mm. On the top x-axis there are values of the SPI-1 for different precipitation amounts occurring in December

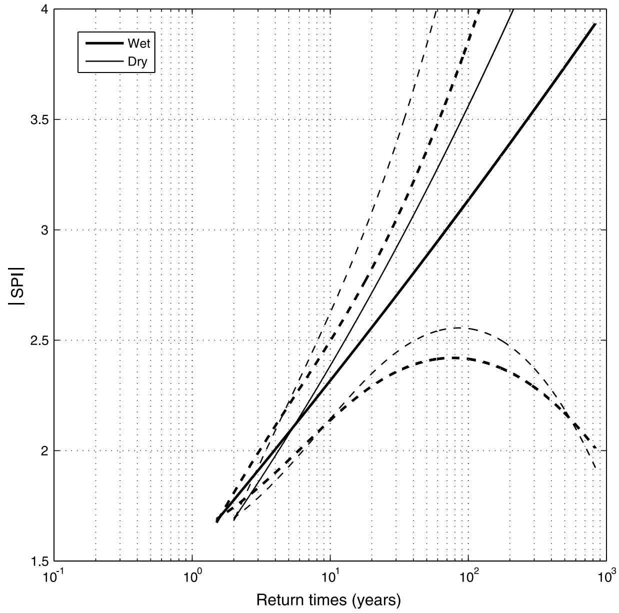


Fig. 5. Return times of extremely wet (thick lines) and dry (thin lines) conditions obtained applying the PD approach to the SPI-1 series with a threshold of ±1.7. Solid lines denote the fits of return times estimated from SPI-1; dashed lines provide the error bands at 95% confidence level. Units are years

occurring in December. As can be seen, only four values greater than 200 mm (corresponding to December 1927, 1933, 1944, 1972 with precipitation amount of 212, 228, 233 and 242 mm respectively) lie on the tail of the distribution, suggesting that only these annual maxima can be considered as extremes. Let us consider, instead, the SPI-1 computed using the monthly precipitation occurring in December. In these cases SPI-1 values (see the x-axis on the top of the figure) show that only two of the four events (December 1944 and 1972), selected with the strategy above described, are extremely wet events since their SPI-1 are 2.0 and 2.1.

As a consequence of this result, we may suppose to move to PD approach selecting extremes as the precipitation values above the threshold of 233 mm. Unfortunately, this will also lead to a misinterpretation, since the distributions of precipitation from month to month change and so their tails. Thus, the threshold should be a function of the particular month when the extreme occurs. This, of course, will bear practical consequences since the return times will differ greatly as well. The same conclusions can be drawn for dry events.

In avoiding these shortcomings, a suitable solution appears to be the use of the SPI-1 for extreme analysis. In this case, in fact, the distribution of the SPI is Gaussian, so that we definitively remove the ambiguity illustrated above. Thus, we compute the SPI-1 time series from the averaged precipitation in Sicily and then, for estimating return times of extremely wet and dry conditions, we apply the PD approach to the index series, with a threshold of ±1.7. Results are displayed in Fig. 5. For absolute SPI values between 1.9 and 2.4, return times of extremely wet and dry events are comparable, ranging from 3 to 10 years, while for higher index values they differ a lot, but the corresponding error bands are remarkably wider. So we may conclude that both wet and dry periods have similar extreme behaviour with a typical return time of few years.

3.2 Spatial variability

Next, we wish to investigate the spatial variability of return times for extreme wet and dry conditions. At this purpose, we compute the SPI-1 time series using monthly precipitation recorded at 36 stations and estimate return times for SPI-

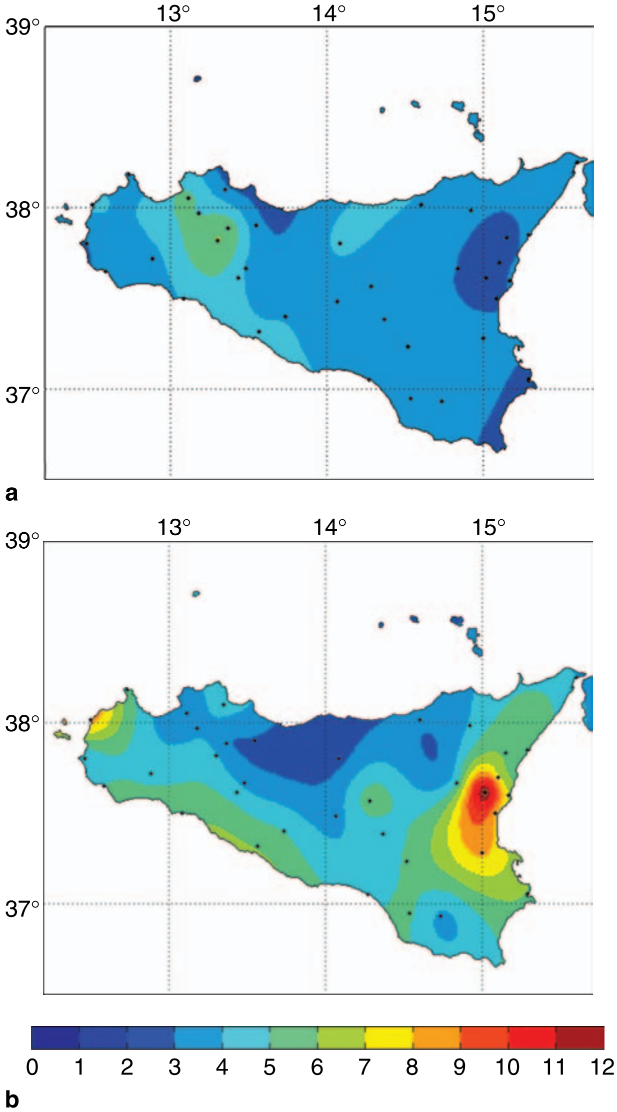


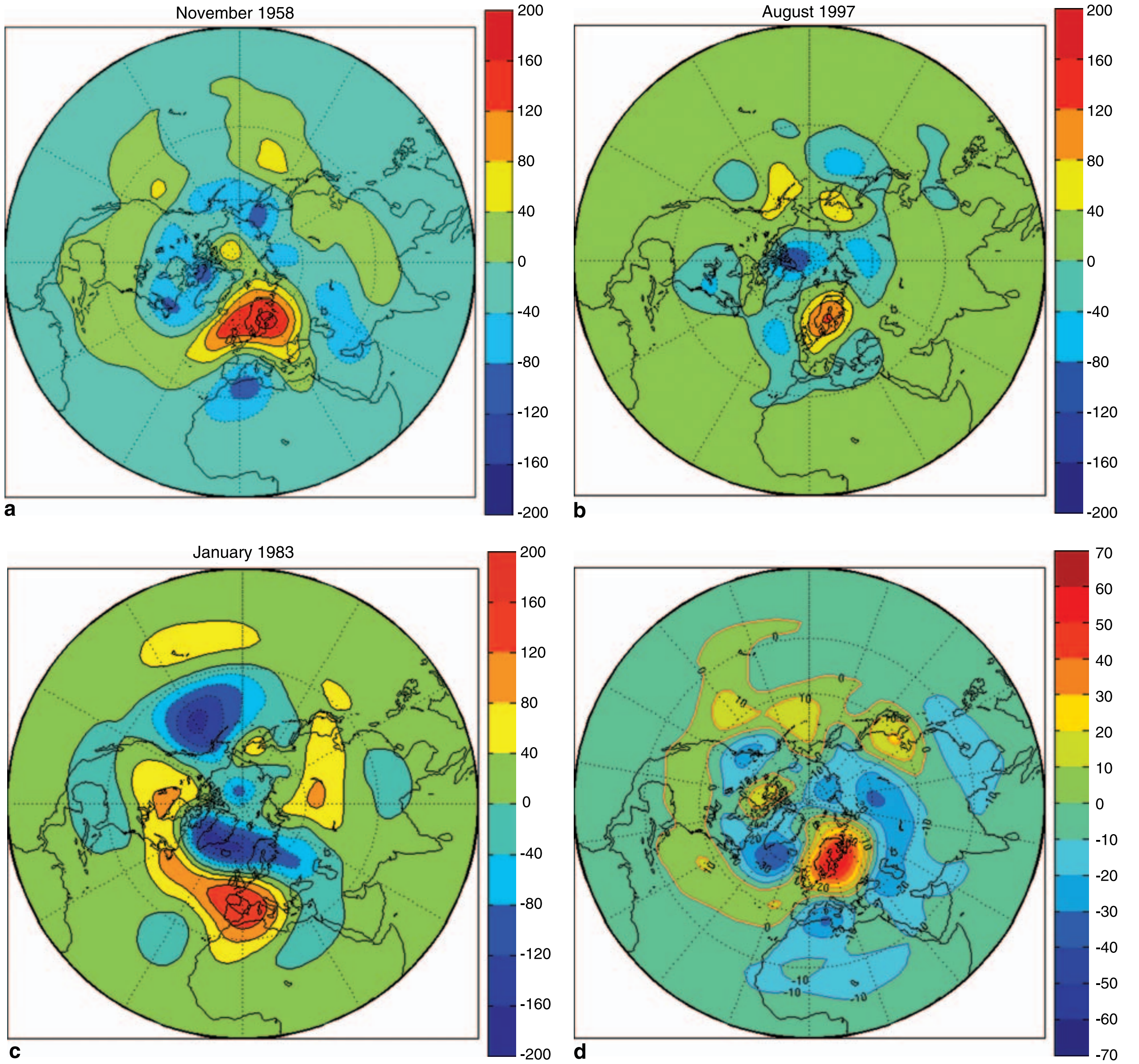
Fig. 6. Spatial distribution of estimated return times for (**a**) SPI-1 > 1.7 and (**b**) SPI-1 < −1.7. Units are years

1 > 1.7 and SPI-1 < −1.7 (see Fig. 6a, b). We have assumed that there is no spatial correlation among the stations, which has been tested to be very low, so that return times may be assumed to be independently distributed. Maps show that return times for extreme wet and dry events have different features. Extreme wet events are more frequent in the eastern part of Sicily (return times between 2 and 3 years), while in the remaining parts of the island such events are less frequent (return times between 3 and 6 years). This is probably related to the orographic effects induced by mountain Etna on the precipitation field. On the other hand, extreme dry events occur more frequently in the Northern part of Sicily with return times ranging from 2 to 4 years. Furthermore, longer return times, from 8 to 12 years, characterise the eastern part of the island. In summary, we may conclude that, in mitigating adverse effects of wet or dry events, planning that accounts for a cyclic time (which, of course random) around ten years should be considered, with emphasis on the geographical distribution above described.

4. Atmospheric patterns

At variance with other studies, we do not try to connect dry or wet conditions to some particular large-scale atmospheric phenomena, like El-Nino Southern Oscillation (ENSO) or North Atlantic Oscillation (NAO) (see for example Piechota and Dracup, 1996; Tadesse et al., 2004). In this section we illustrate, instead, the phenomenology of large-scale atmospheric conditions associated to extreme wet and dry events in Sicily. For this purpose we consider the monthly means extracted from the NCEP/NCAR re-analysis data set covering the period 1948–2000 and the long-term mean for each month of the year. We select from the SPI-1 time series (based on averaged precipitation over stations) the annual maxima (minima) above (below) the threshold 2 (−2) from 1948 to 2000, which leads to 13 extreme events both for wet and dry conditions.

To identify systematic circulation patterns emerging both for wet and dry periods, we use the 500 hPa geopotential height field, which may be considered a good proxy for large-scale precipitation. For each of the 13 cases, the geopotential anomaly map is computed (that is, we consider the difference between the geopotential field for the month with extreme SPI-1 and the long-term mean for that month). The anomalies associated to few extreme cases are displayed in Fig. 7a–c, which correspond to the following events: November 1958 (SPI-1 = 2.7), August 1997 (SPI-1 = 2.1) and January 1983 (SPI-1 = −2.1). The first two episodes present extreme wet conditions, which occur during the winter and the summer season (note that only the first case corresponds also to a precipitation annual maximum), while the third one is an extreme dry condition. As can be seen by Fig. 7a, b, these wet events are characterised at 500 hPa by a dipole structure over the European



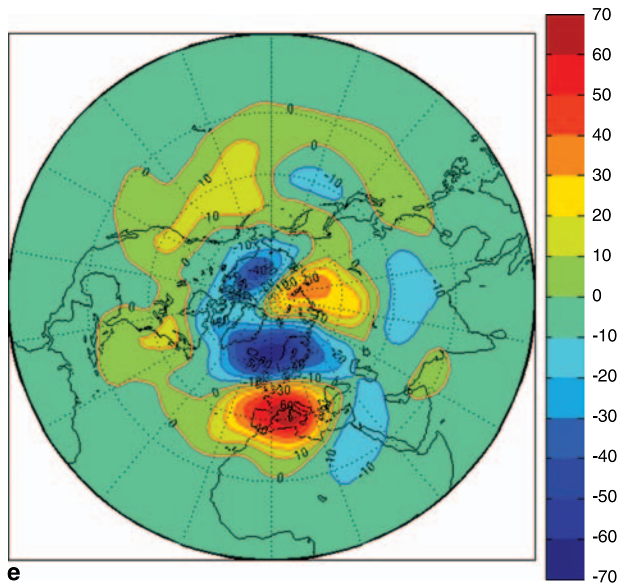


Fig. 7. Anomaly maps of the 500 hPa geopotential height corresponding to extreme events in (**a**) November 1958 (SPI-1 = 2.7), (**b**) August 1997 (SPI-1 = 2.1) and (**c**) January 1983 (SPI-1 = −2.1). Mean anomaly maps averaged over the 13 extreme wet (**d**) and dry cases (**e**). Anomalies are deviations from the long-time monthly mean. Units are m, contour interval is 40 m in (a)–(c) and 10 m in (d)–(e)

area. Such a feature appears to be independent of the season when the extreme occurs; the negative anomaly of geopotential is localised in the Mediterranean basin extending to about 50N, while the positive anomaly characterises Northern Europe. On the other hand, the extreme dry event selected here (Fig. 7c) shows a positive geopotential anomaly in the Mediterranean basin extending to Western Europe and a positive anomaly over North-Eastern Europe. Note that these patterns characterise mid- to high-latitude flow anomalies, while tropical regions do not appear to be of any relevance. The magnitudes of the interannual standard deviations for each month suggest that these features are statistically significant (for example, for the first extreme case in November, the interannual standard deviation of the geopotential eddy component is about 15 m with similar magnitudes observed for the other months of the year). These patterns, in fact, may be considered typical of all extremely wet and dry events identified in the historical record. This is confirmed by the mean anomaly maps averaged over the 13 extreme wet (Fig. 7d) and dry cases (Fig. 7e). As can be seen, the features described above characterise the mean anomaly maps as well. These results suggest that, although we recognise the limitation to have used only precipitation averaged over Sicily, extreme wet and dry events in the Mediterranean basin occur when dipole-like geopotential anomalies characterise the large-scale circulation. Moreover, it appears that both wet and dry extremes are characterised by long persistent features. These signatures, in fact, dominate over the transient component so that they appear even in the monthly mean field. This finding should be corroborated by a deeper analysis, which, however, has a strong limitation in the smallness of the statistical sample.

5. Summary and conclusions

Extreme values analysis of wet and dry periods has been addressed in this paper for a sample area (Sicily). First, we have studied precipitation extremes as deduced from the AM and PD series with two different thresholds (76 and 200 mm), and we have estimated the return periods of such extreme events by using standard statistical tools. A comparison of the results obtained from the application of the two approaches suggests that the return periods range from 4 to 60 years for precipitation extremes between 200 and 300 mm.

Next, we noted that the set of extremes selected according these two methods, rarely are in the tail of the interannual empirical distribution of precipitation for a given month, since this distribution is heavily skewed. Thus, we introduced the SPI as a measure of wet and dry periods. By its construction, the index overcomes the shortcomings of the extreme precipitation sample and allows comparing return periods homogeneously for regions governed by different climatological regimes. When the standard statistical technique is applied to the SPI-1 we found that the return periods both for wet and dry conditions range from 4 to 10 years.

Moreover, we investigate the special distribution of extremes return periods identifying areas characterised by different recurrence times. Finally, we found that extremes over Sicily are connected to large-scale South–North dipole structure covering the European Sector.

In conclusion, we like to mention that trends have been found in the extreme set of AM and PD samples. This leads to support the view that the statistical tools here employed must be adapted to sets of correlated data. This will be the object of a future work.

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We want to express also our gratitude to Prof. Giuseppe Rossi of the University of Catania for the useful discussions and suggestions.

Appendix

The annual maxima (AM) series consists of the greatest events of each year in a given time period. According to the Fisher-Tippett theorem (1928), the asymptotic distribution of a series of sample maxima belongs to one of three basic distributions, regardless of the original distribution of the observed data. These three families were combined into a single distribution (see Jenkinson, 1955), which is now known as the Generalised Extreme Value (GEV) distribution.

The GEV distribution, which best fits the observed distribution of maxima, can be evaluated as:

$$G(x) = \begin{cases} e^{-\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-1/\xi}} & \text{if } \xi \neq 0 \\ e^{-e^{-(x-\mu)/\sigma}} & \text{if } \xi = 0 \end{cases} \tag{A.1}$$

where x is the observed random variable, μ is the location parameter, σ and ξ are respectively the scale and shape parameter. The tail index is defined as $\alpha = 1/\xi$. When $\xi > 0$, the distribution is known as the Fréchet distribution and has a fat-tail. The larger the shape parameter, the more fat-tailed the distribution. If $\xi < 0$, the distribution is known as the Weibull distribution; if $\xi = 0$, it is the Gumbel distribution. Therefore, the tail behaviour of the data series can be estimated from one of these three distributions.

Estimates of extreme quantiles of the annual maxima distribution may be obtained by inverting (A.1) (Coles, 2001):

$$x_p = \begin{cases} \mu - \frac{\sigma}{\xi}\left[1 - y_p^{-\xi}\right] & \text{for } \xi \neq 0 \\ \mu - \sigma \log(y_p) & \text{for } \xi = 0 \end{cases} \tag{A.2}$$

where $G(x_p) = 1 - p$ and $y_p = -\log(1-p)$; x_p is the return level associated with the return period $1/p$ (i.e. x_p is exceeded by the annual maximum in any particular year with probability p). A limitation of the AM approach is that it may ignore annual registrations that can be considered extremes as well.

On the other hand, in PD analysis we are interested in the behaviour of large numbers of observations that exceed a high threshold. A theorem by Balkema and de Haan (1974) and Pickands (1975) shows that for sufficiently high threshold u, the distribution function of the excess may be approximated by the generalised Pareto (GP) distribution such that, as the threshold gets large, the excess distribution converges to the GP distribution, which is:

$$G(y) = \begin{cases} 1 - \left(1 + \xi\frac{y}{\tilde{\sigma}}\right)^{-1/\xi} & \text{if } \xi \neq 0 \\ 1 - e^{-y/\tilde{\sigma}} & \text{if } \xi = 0 \end{cases} \tag{A.3}$$

with $Y = X - u$ the excess of the observed variable X over the threshold u, given that $X > u$ for u sufficiently large, and $\tilde{\sigma} = \sigma + \xi(u - \mu)$. The GP distribution embeds a number of other distributions. When $\xi > 0$, it takes the form of the ordinary Pareto distribution. The GP distribution can be estimated with various methods such as the method of probability weighted moments (PWM, known also as L-moments) or the maximum likelihood (ML) method (Hosking and Wallis, 1987). In hydrologic applications, the PWM technique is more popular than ML because its simplicity and good performance for small samples. Nevertheless, the PWM method has the limitation to be not able to readily take into account covariates (Kats et al., 2002).

To estimate the return levels for PD series, let suppose that a GP distribution is a suitable model for exceedances of a threshold u by a variable X. That is, for $x > u$ and $\xi \neq 0$,

$$\Pr\{X > x \mid X > u\} = \left[1 + \xi\left(\frac{x-u}{\sigma}\right)\right]^{-1/\xi}. \tag{A.4}$$

It follows that (Coles, 2001):

$$\Pr\{X > x\} = \zeta_u\left[1 + \xi\left(\frac{x-u}{\sigma}\right)\right]^{-1/\xi}. \tag{A.5}$$

where $\zeta_u = \Pr\{X > u\}$. Hence, the level x_m that is exceeded on average once every m observations is the solution of:

$$\zeta_u\left[1 + \xi\left(\frac{x_m-u}{\sigma}\right)\right]^{-1/\xi} = \frac{1}{m}. \tag{A.6}$$

Solving (A.6) for x_m we get:

$$x_m = u + \frac{\sigma}{\xi}\left[(m\,\zeta_u)^{\xi} - 1\right] \tag{A.7}$$

provided m is sufficiently large to ensure that $x_m > u$. If $\xi = 0$, we have:

$$x_m = u + \sigma \log(m\,\zeta_u). \tag{A.8}$$

The PD approach has several advantages with respect to the AM approach, because it better adapts to heavy-tailed distributions and permits to consider more extreme cases. Nevertheless, the PD method has two shortcomings concerning the choice of the threshold value and the possible non-independent extreme values considered in the analysis, that is, those related to events correlated in time. Hydrological phenomena, in fact, such as rainfall show a tendency to appear grouped in bunches, introducing a degree of serial dependence in the data series. Usually, a declustering process is applied to the data to solve the problem.

In threshold determination, we face a trade off between bias and variance. If we choose a low threshold, the number of exceedances increases and the estimation becomes more precise. However, choosing a low threshold also introduces some observations from the center of the distribution and the estimation becomes biased. Thus, several techniques, such as the QQ-plot (Quantile-Quantile plot), the Hill-plot (Hill, 1975) and the Mean Excesses Function (MEF or ''mean residual life plot''), have been developed for the threshold determination. In the present work we use the latter procedure. The mean residual plot is defined as the locus of points

$$\left\{\left(u, \frac{1}{n_u}\sum_{i=1}^{n_u}\left(x_{(i)} - u\right)\right) : u < x_{\max}\right\}, \tag{A.9}$$

where $x_{(1)}, x_{(2)}, \ldots, x_{(nu)}$ consist of n_u observations that exceed u and $x_{\max}$ is the largest of the $x_{(i)}$. It can be proved (Coles, 2001) that, above a threshold u_0 at which a GP distribution provides a valid approximation to the excess distribution, the mean residual life plot should be approximately linear in u.

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