

# Competition in Higher Education: Sorting, Ranking and Fees

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# Competition in Higher Education: Sorting, Ranking and Fees

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April 17, 2024

## Abstract

We model student enrollment in markets for higher education where public universities, private non-profit universities, and private for-profit universities compete. Universities differ with respect to their capacity, graduation probability, and profit objective; students differ in ability. The value of a diploma at each university depends on its endogenous ranking based on average student ability.

In every equilibrium, the private for-profit university attracts the least able students. Under additional conditions, the private non-profit university attracts the top students. Paradoxically, a higher capacity at the public university might decrease its equilibrium market share as it incentivizes the for-profit university to compete more aggressively. The for-profit university benefits from an increased enrollment in higher education.

**Keywords:** Higher education, for-profit universities

**JEL-Classification:** C78, I23, I26

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# 1 Introduction

The recent rise of for-profit universities has altered the structure of the higher education market in many countries. In this paper, we develop a theoretical model to study how these new market entrants interact with existing players, including students, public universities, and, in some countries, also private non-profit universities. In particular, we focus on study fees, segregation of the student body based on ability, and market share of the different universities.

The global higher education market was valued at 477.12 billion U.S. dollars in 2022 and is projected to reach 853.28 billion U.S. dollars by 2030.<sup>1</sup> The largest regional markets for higher education are the Asia Pacific region, followed by Europe and the United States. In 2020, China awarded 13.89 million higher education degrees, while Europe and the U.S. issued 4.3 million and 4.2 million degrees, respectively, in 2021.<sup>2</sup>

Traditionally, public universities have dominated the higher education market in most countries. However, recent reductions in public funding, combined with a growing willingness among high school graduates to pursue higher education, the availability of financial aid and student loans, and the convenience of online courses offered by private for-profit universities, have significantly decreased the market share of public universities. For instance, enrollment in private for-profit universities increased by 6.4% (public 0.8%) over the past four years in the U.S.<sup>3</sup>, by 11.3% (public 7.2%) from 2006 to 2011 in India (Angom, 2015), and from non-existence to 764 out of 3013 universities in China (Duan et al., 2023). While some countries reduced government funding (Goodman and Volz, 2020), others have strategically increased capacity to enhance accessibility of public higher education (Benn and Fieldhouse, 1993; Fu, 2014). Yet, the impact of capacity changes at public universities on resulting market outcomes is not well understood.

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<sup>1</sup>Retrieved on January 26, 2024, from <https://www.verifiedmarketresearch.com/product/global-higher-education-market-size-and-forecast/>.

<sup>2</sup>The data are obtained from the Educational Yearbook of China (2020), Eurostat (2023) and the U.S. Department of Higher Education.

<sup>3</sup>Fall 2023 Enrollment National Student Clearinghouse Research Center's Monthly Update on Higher Education Enrollment, April 17, 2024. <https://nscresearchcenter.org/stay-informed/>

To get a better understanding of the interplay of different actors, we consider a model with a public university, a private non-profit university, a private for-profit university, and a continuum of students. Students decide which university to attend—or whether to attend university at all—based on a trade-off between tuition fees, likelihood of graduation, and their expected future earnings which depend on the reputation of the university. A student’s ability, indicated by her high school grade, influences her graduation probability. The quality of a university is given by its rank, which depends on the average ability of its students. The public and private non-profit universities have constrained capacity and cannot cover the entire demand.

In the first stage, the for-profit university chooses a tuition fee to maximize profits. For the non-profit university, the tuition fee is exogenously determined by the requirement to balance their budget, while the public university charges no tuition fee.<sup>4</sup> In stage two, students decide at which university to enroll. If a university has more capacity than applicants, it admits all students. Otherwise, the university admits the top applicants up to their capacity.

We first provide a complete characterization of equilibria at the application stage, where we allow any subset of universities to be active. Across all equilibria, the for-profit university attracts the least able students. If both the non-profit and public university are active, they compete for the top students and, depending on the parameters, either one of these universities may be ranked first. Furthermore, there need not be full segregation of students in terms of their ability, that is, a university may attract students with high and low ability but not with intermediate ability.

Under our equilibrium refinement of group-strategyproofness, which enables coordination among subgroups of students, the private non-profit university attracts the better students on average. Despite only attracting the least able students, the for-profit university has a non-trivial interaction with the other universities. If the capacity at other universities is low, the for-profit university will attract the left-over students and charges a high fee. Otherwise, the for-profit university chooses to not only focus on the left-over

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<sup>4</sup>Most results depend only on the ordering of tuition fees, so the assumption on the public university is made for convenience of exposition.

students, but to compete with the public university for a larger number of students by lowering its own tuition fee. This can induce an interesting non-monotonicity: as the public university increases its capacity, their student numbers might fall in equilibrium.

Our results have direct policy implications. First, by allowing and promoting private non-profit universities, a government tends to lose control over the education of its intellectual elite; the admission of private for-profit universities does not have the same effect. This might explain why we observe all three types of universities in the U.S. while the Chinese market consists mainly of public and private for-profit universities.

Second, an increased capacity at the public university induces more competitive pressure on the private for-profit university, which may react by competing more aggressively with the public university. In particular, the optimal fee at the for-profit university is not continuous in the quota of the public university, and a higher quota at the public university might reduce its equilibrium market share.

Third, if demand for university degrees in a population increases and the average student ability drops, the profit of the for-profit university (weakly) increases; under additional conditions the market share of the for-profit university also increases. Thus, in line with the empirical evidence, private for-profit universities are the main beneficiaries of a growing student body.

**Related Literature.** Our paper is not the first to investigate incentives in higher education markets and their equilibrium behavior. The literature is fragmented along different dimensions, including the types of universities considered, whether quality depends on the ability of enrolled students, deterministic or stochastic acceptance by universities, inclusion of capacity constraints, inclusion of application fees, and analytical versus numerical methods.

Drawing on a seminal paper for primary and secondary education markets (Epple and Romano, 1998), Epple et al. (2006) develop an equilibrium model for the higher education market consisting of ex-ante identical private non-profit universities that maximize their quality. A university's quality is endogenously determined by the mean ability and income of the student body, as well as by expenditures per student. In equilibrium, a strict sorting of student types obtains, where types are two-dimensional consisting of

ability and income. Epple et al. (2017) and Domnisoru and Schiopu (2021) extend the model to include public and private for-profit universities; both papers predominantly employ numerical methods and focus on the U.S. market. In contrast, we focus on a simpler model to obtain analytical solutions for general comparative statics and between-country comparisons.

Rothschild and White (1995) study a model of competition between fee-setting for-profit universities whose costs of producing human capital depend on the students they attract. To the extent that the cost decreases if a university attracts more able students, the value of a diploma from the university may endogenously increase. The authors examine whether competition results in an efficient allocation of students to universities. By contrast, we examine the interaction of for-profit universities with other types of universities.

Chade et al. (2014) and Fu (2014) consider models with application costs for the applicants and stochastic acceptance by the universities. Chade et al. (2014) are able to derive closed-form solutions in a model with two universities which are exogeneously ordered in terms of quality. Their main focus is on the application stage where students face a portfolio choice problem given the application costs and uncertainty of acceptance. In equilibrium, sorting of student applications in terms of ability might fail. In a related setting, Fu (2014) considers four types of universities with a strong focus on numerical methods, estimation techniques, and the U.S. market.

In Chade et al. (2014), an increase in capacity at one university leads to a lower admission threshold at both universities as more students enroll. In our model, the entire market is served by the three universities. An increase in the capacity of the public university implies more competitive pressure for the private for-profit university, which may react by reducing its fee. In consequence, both the market share and the average ability of students at the private for-profit university may increase.

To conclude, our study diverges from prior work by incorporating a private for-profit university perspective and adopting a simpler model structure to derive analytical solutions and generalizable comparative statics. Unlike much of the existing literature, which is predominantly U.S.-centric, our model is designed with the flexibility to accommodate

various international contexts, reflecting the diverse nature of global higher education systems. This adaptability is particularly relevant when considering the differing roles and historical development of public and private universities across countries, such as the U.S. and China.

The rest of the paper is organized as follows. We introduce the formal model in Section 2. Section 3 first provides a full characterization of equilibria on the application stage and then proceeds to analyze the optimal fee of the private for-profit university. Section 4 contains the main economic insights, i.e., comparative statics, including results under equilibrium selection. Section 5 provides a discussion of the model and possible extensions. Section 6 concludes.

## 2 Model

Consider a model with a continuum of students and three universities: a public university, a private non-profit university, and a private for-profit university. The mass of students is normalized to one. Every student has an ability type  $h \in [\underline{h}, \bar{h}] \subseteq \mathbb{R}$ , which represents the student's high-school grade. The mass of students with a type up to  $h$  is given by a strictly increasing and continuous CDF  $G$ , with  $G(\underline{h}) = 0$ . Types are publicly observable. In the following, "type" and "student" mean the same.

Every type chooses at which university to apply.<sup>5</sup> The public university can admit a mass  $0 < q_{pu} < 1$  of students. The private non-profit university can admit a mass  $0 < q_n < 1 - q_{pu}$  of students. The private for-profit has no capacity constraint.<sup>6</sup> If the capacity constraint is binding for a university, then it admits the applicants with the highest types. For a given profile of application decisions across types, we say that a

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<sup>5</sup>While we limit each student to apply at one university, by virtue of the results in Azevedo and Leshno (2016), the equilibrium outcome is equivalent to that of a model where students can apply to multiple universities and are allocated based on a student-proposing Gale-Shapley algorithm (Gale and Shapley, 1962).

<sup>6</sup>The main tradeoffs remain the same if we allow for full market coverage of the public university; the small size of the private non-profit university is crucial for our results. It is in line with empirical evidence and the intuition that such universities rely on exclusive clubs of private donors.



university is *active* if there are types who apply at the university; otherwise the university is *inactive*.

At the public university, a student with type  $h$  obtains a diploma with probability  $\phi(h)$ . The function  $\phi$  is strictly increasing and continuous, with  $\phi(\bar{h}) = 1$ . Students at the private universities obtain a diploma for sure, regardless of the type. The assumption that the private universities have significantly higher graduation probability is motivated by better, smaller classrooms and laxer grading standards.

The value of a diploma depends on the rank of the university. The rank is determined by the ranking of the average types at the active universities. A diploma from a university with rank  $k = 1, 2, 3$  has a value of  $\pi_k$ , where  $\pi_1 > \pi_2 > \pi_3 > 0$ . If the average type is the same at two active universities, ties are broken according to some arbitrary rule that is fixed throughout.

The private for-profit university chooses a fee  $a_f \geq 0$  and the private non-profit university charges an exogenously given fee  $a_n \geq 0$ . The public university charges the lowest fee which we normalize to 0 for convenience of exposition.

The game consists of two stages. First, the private for-profit university chooses its fee  $a_f$ . Next, every student decides at which university to apply. The payoff of a student is the value of the diploma, if any, minus the fee, if any. The payoff of the private for-profit university is its revenue. Everyone is risk neutral and maximizes their expected payoff.

The solution concept is subgame-perfect Nash equilibrium in pure strategies, with two restrictions: First, students cannot deviate to an inactive university. Second, students whose optimal university is not unique apply at the private for-profit university if it is optimal, and otherwise at the public university. The first restriction reflects that a university cannot operate without a sufficient amount of student; the second restriction is a standard tie-breaking assumption.

### 3 Equilibrium Computation

In this section, we compute subgame-perfect equilibria of our game. By backward induction, we start at the second stage where students decide to apply given the fees. We

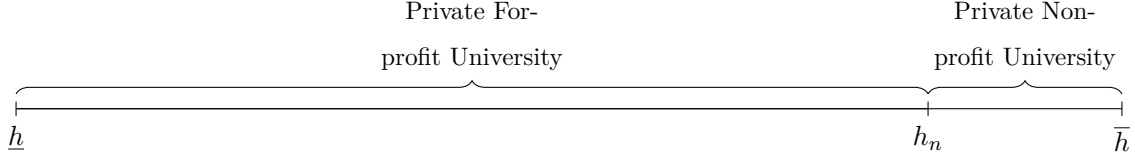


Figure 1: Equilibrium with private non-profit and private for-profit universities

then proceed back to the first stage.

### 3.1 Application Decision

For an arbitrary fixed fee  $a_f \geq 0$  of the private for-profit university, we study the Nash equilibria of the corresponding subgame in stage 2.

Let  $h_{pu}$  be the solution to  $1 - G(h) = q_{pu}$ . Because  $G$  is strictly increasing and continuous,  $h_{pu}$  exists and is unique. Analogously, let  $h_n$  be the unique solution to  $1 - G(h) = q_n$ . Let  $h_r$  be the unique solution to  $1 - G(h) = q_{pu} + q_n$ .

We always have three trivial Nash equilibria in which all students apply at the same university, the other universities being inactive. Next, we derive equilibria with two active universities, starting with the case that just the two private universities are active.

**Proposition 1** (Only private non-profit and private for-profit). *There exists at most one Nash equilibrium in which only the private non-profit university and the private for-profit university are active. In this equilibrium, all types in  $(h_n, \bar{h}]$  apply at the private non-profit university and all types in  $[\underline{h}, h_n]$  apply at the private for-profit university. The equilibrium exists if and only if  $\pi_1 - a_n > \pi_2 - a_f \geq 0$ .*

*Proof.* If the private non-profit university and the private for-profit university are the active universities, then, by our tie-breaking rule, every student who applies at the private non-profit university must strictly prefer it over the private for-profit university. Because the graduation probability at these two universities does not depend on the type, it follows that every student strictly prefers the private non-profit university; hence, this university must get the top students. Thus,  $\pi_1 - a_n > \pi_2 - a_f \geq 0$  is necessary and sufficient for the existence of said equilibrium, and there exists no other equilibrium in

which the private non-profit university and the private for-profit university are the active universities.  $\square$

Proposition 1 is illustrated in Figure 1. We now turn to equilibria in which the public university is active.

**Proposition 2** (Only public and private for-profit). *There exists at most one Nash equilibrium in which only the public university and the private for-profit university are active. In this equilibrium, there is a cutoff  $h' \in (\underline{h}, \bar{h})$  such that all types in  $(h', \bar{h}]$  apply at the public university and all types in  $[\underline{h}, h']$  apply at the private for-profit university. The equilibrium exists if and only if  $\pi_2 - a_f \geq 0$ .*

*Proof.* Suppose that both universities are active. Let  $k$  be the rank of the public university, and  $l$  the rank of the private for-profit university. Because the private for-profit university has no capacity constraint, there exists a type  $h'$  that prefers the public university:

$$\phi(h')\pi_k \geq \pi_l - a_f.$$

Since  $\phi$  is strictly increasing, it follows that

$$\phi(h)\pi_k > \pi_l - a_f, \quad \forall h > h'.$$

This implies that in a Nash equilibrium in which only the public university and the private for-profit university are active, the public university must be ranked first,  $k = 1$  and  $l = 2$ . Hence the equilibrium exists only if  $\pi_2 - a_f \geq 0$  (if  $\pi_2 - a_f < 0$ , then all types would apply at the public university).

Let now  $\pi_2 - a_f \geq 0$ . If

$$\phi(h_{pu})\pi_1 \geq \pi_2 - a_f \geq 0,$$

then the proposition holds with  $h' = h_{pu}$ . If

$$\phi(h_{pu})\pi_1 < \pi_2 - a_f,$$

then the proposition holds with  $h'$  being the solution to

$$\phi(h)\pi_1 = \pi_2 - a_f,$$

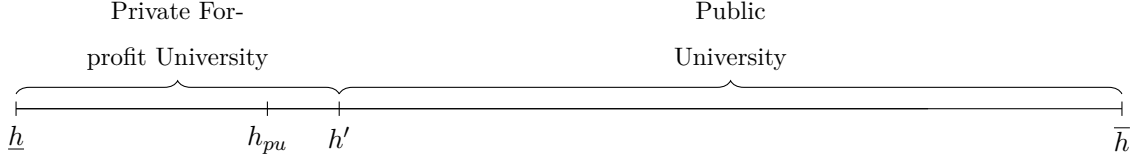


Figure 2: Equilibrium with public and private for-profit universities

which exists and is unique because  $\phi$  is strictly increasing and continuous, with  $\phi(h_{pu})\pi_1 < \pi_2 - a_f$  and  $\phi(\bar{h}) = 1$ .  $\square$

Intuitively, attending the public university is relatively more attractive for better students, as they have a higher probability of graduating. If the top students were to enroll in the private university, the private university would also be optimal for lower ranked students and the public university would have no students. Thus, in any equilibrium with both universities, the top students enroll in the public university. This process stops when the public university reaches its quota or when there is a cutoff type for whom attending the better and free university is offset by a sufficiently high chance of not getting a diploma. In the first case, lower ability students would not be admitted to the private university while in the second case—illustrated in Figure 2—they prefer to enroll in the private university. In both cases, the remaining students below the cutoff type attend the private university unless it charges an excessively high fee. In equilibrium, we obtain complete segregation of the student population.

The next two propositions deal with equilibria when the public and private non-profit universities compete. In this case, the equilibrium reasoning is less straightforward. The next proposition shows that there are equilibria without complete segregation.

**Proposition 3** (Only public and private non-profit; public ranked first).

a) *If*

$$\phi(h_{pu})\pi_1 \geq \pi_2 - a_n > 0,$$

*then there exists a Nash equilibrium in which all types  $h \in [h_{pu}, \bar{h}]$  apply at the public university and all types  $h \in [h_r, h_{pu})$  at the private non-profit university.*

b) If

$$\phi(h_{pu})\pi_1 < \pi_2 - a_n,$$

let  $h'$  be the unique solution to  $\phi(h)\pi_1 = \pi_2 - a_n$ , and  $h''$  the unique solution to  $G(h') - G(h) = q_n$ . If

$$\frac{1}{q_{pu}} \left( \int_{h'}^{\bar{h}} h dG(h) + \int_{h_r}^{h''} h dG(h) \right) > \frac{1}{q_n} \int_{h''}^{h'} h dG(h), \quad (1)$$

then there exists a Nash equilibrium in which all types  $h \in (h'', h')$  apply at the private non-profit university and all types in  $h \in [h_r, \bar{h}] \setminus (h'', h')$  at the public university.

c) There exists no other Nash equilibrium in which only the public university and the private non-profit university are active and the public university is ranked first.

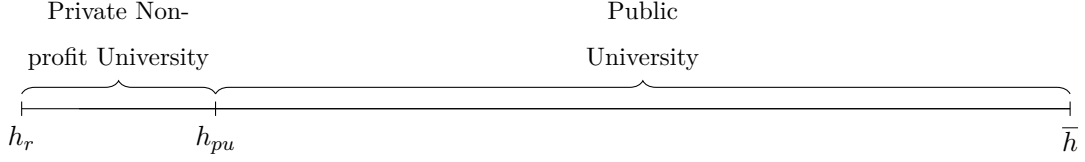
*Proof.* a) Because  $\phi$  is strictly increasing, we have  $\phi(h)\pi_1 > \pi_2 - a_n$  for all  $h > h_{pu}$ .

b) By (1), the public university is ranked first if all types  $h \in (h'', h')$  attend the private non-profit university and all types in  $h \in [h_r, \bar{h}] \setminus (h'', h')$  the public university. The claim then follows because  $\phi(h)\pi_1 > (<)\pi_2 - a_n$  for all  $h > (<)h'$ .

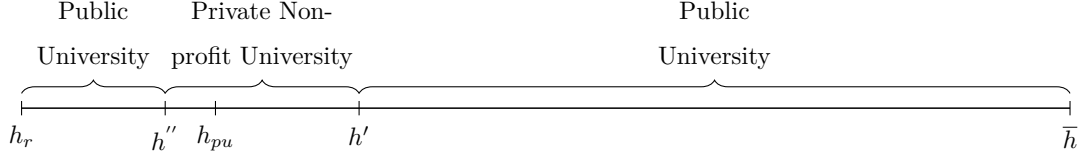
c) If the public university is ranked first, then either  $\phi(h)\pi_1 > \pi_2 - a_n$  for all  $h \in (h_{pu}, \bar{h}]$  or there exists a type  $\tilde{h} \in (h_{pu}, \bar{h}]$  such that  $\phi(\tilde{h})\pi_1 = \pi_2 - a_n$ . In the former case, any Nash equilibrium with these active universities must be as in part a). In the latter case,  $\phi(h)\pi_1 > (<)\pi_2 - a_n$  for all  $h > (<)\tilde{h}$ , whence any Nash equilibrium must be as in part b).  $\square$

Proposition 3 is illustrated in Figure 3. Case a) is similar to the previous proposition. The public university fills its quota with the top students. Even for the weakest of these students, it is still better to attend the public university with a chance of not graduating instead of switching to the private non-profit. There is complete segregation.

In Case b), the best students still prefer the public university which ranks highest. However, for medium ability students, the private non-profit university becomes more attractive as it guarantees graduation. The non-profit university is thus able to fill its quota with medium ability students and the some less able students attend the public one as they would not be admitted by the non-profit university.



(a) Segregation equilibrium



(b) Non-segregation equilibrium

Figure 3: Equilibria with public and private non-profit universities; Public ranks first

In addition, when the public and private non-profit universities compete, there can be equilibria where the private non-profit university attracts more able students.

**Proposition 4** (Only public and private non-profit; public ranked second).

a) If

$$\pi_1 - a_n > \pi_2, \quad (2)$$

then there exists a Nash equilibrium in which all types  $h \in [h_n, \bar{h}]$  apply at the private non-profit university and all types  $h \in [h_r, h_n)$  at the public university.

b) If

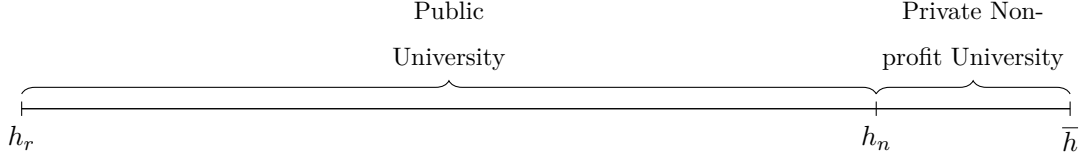
$$0 \leq \pi_1 - a_n \leq \pi_2,$$

let  $h'$  be the unique solution to  $\pi_1 - a_n = \phi(h)\pi_2$ . If there exists a solution  $h'' > h_r$  to  $G(h') - G(h) = q_n$  (which is unique), and if

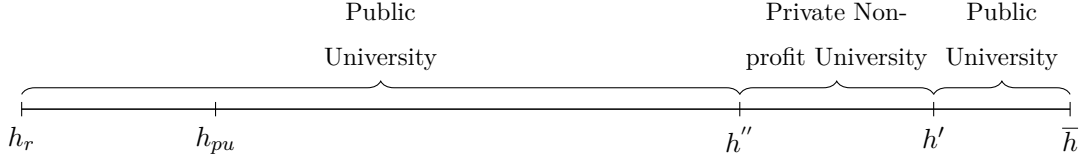
$$\frac{1}{q_{pu}} \left( \int_{h'}^{\bar{h}} h dG(h) + \int_{h_r}^{h''} h dG(h) \right) < \frac{1}{q_n} \int_{h''}^{h'} h dG(h), \quad (3)$$

then there is a Nash equilibrium in which all types  $h \in [h'', h')$  apply at the private non-profit university and all types  $h \in [h_r, \bar{h}] \setminus [h'', h')$  at the public university.

c) There exists no other Nash equilibrium in which only the public university and the private non-profit university are active and the public university is ranked second.



(a) Segregation equilibrium



(b) Non-segregation equilibrium

Figure 4: Equilibria with public and private non-profit universities; Public ranks second

*Proof.* a) If (2) holds, then all types prefer the private non-profit university.

b) By (3), the private non-profit university is ranked first if all types  $h \in [h'', h')$  attend the private non-profit university and all types in  $h \in [h_r, \bar{h}] \setminus [h'', h')$  the public university. The claim then follows because  $\phi(h)\pi_2 > (<)\pi_1 - a_n$  for all  $h > (<)h'$ .

c) If the private non-profit university is ranked first, then either  $\pi_1 - a_n > \phi(h)\pi_2$  for all  $h \in (h_n, \bar{h}]$  or there exists a type  $\tilde{h} \in (h_n, \bar{h}]$  such that  $\pi_1 - a_n = \phi(\tilde{h})\pi_2$ . In the former case, any equilibrium must be as in part a). In the latter case,  $\pi_1 - a_n < (>)\phi(h)\pi_2$  for all  $h > (<)\tilde{h}$ , whence any equilibrium must be as in part b).  $\square$

Proposition 4 is illustrated in Figure 4. Again, there are two possible types of equilibria. In the first case, the top students attend the non-profit university. Weaker students would like to do the same, but this is not possible given the quota. As such, less able students instead attend the public university which has a lower rank than the non-profit university.

In the second case, the difference in diploma values is relatively small. Then, for the very best students, the public university is attractive based on lower tuition, even though its average ability of students is lower. The close-to-the-top students choose the private non-profit based on guaranteed graduation and fill its quota. Less able students have no choice but to go to the public university.

Note that for some parameter values of  $\pi_1$  and  $\pi_2$ , both types of equilibria in 2a) and 3a) co-exist. When the benefit of attending the best university is high, the choice of university becomes a coordination problem among top students; we provide a possible refinement for selection in Section 5.

The next proposition extends the previous logic to the case where all three universities are active. Again, after presenting the proof, we include four sketches to illustrate the four possible equilibria described in Proposition 5.

**Proposition 5** (All three universities). *The following holds in every Nash equilibrium in which all three universities are active:*

- a) *The private for-profit university is ranked last.*
- b) *There exists a cutoff  $h^\dagger \geq h_r$  such that the types that apply at the private for-profit university are the types in  $[\underline{h}, h^\dagger]$ .*
- c) *It holds that  $\pi_3 - a_f \geq 0$ . Let  $k$  be the rank of the public university. Then  $\phi(h^\dagger)\pi_k \geq \pi_3 - a_f$ , and equality holds if  $h^\dagger > h_r$ . Let  $l$  be the rank of the private non-profit university. Then  $\pi_l - a_n > \pi_3 - a_f$ .*
- d) *The Nash equilibria are, and exist under the same conditions (next to the conditions in part c)), as in Proposition 3 and 4, respectively, with  $h^\dagger$  instead of  $h_r$ .*

*Proof.* a) Because both private universities are active and the for-profit university has no capacity constraint, all types strictly prefer the private non-profit university, which is hence ranked higher than the private for-profit university. By the same argument as in the proof of Propositions 2, the public university must be ranked higher as well.

b) As shown in the proof of part a), all types strictly prefer the private non-profit university over the private for-profit university. Because the private for-profit university has no capacity constraint, there exists a type  $\tilde{h}$  that prefers the public university over the private for-profit university:

$$\phi(\tilde{h})\pi_k \geq \pi_3 - a_f,$$



where  $k$  is the rank of the public university. Since  $\phi$  is strictly increasing, it follows that

$$\phi(h)\pi_k > \pi_3 - a_f, \quad \forall h > \tilde{h},$$

concluding the proof of part b).

c) If  $\pi_3 - a_f < 0$ , then no student applies at the private for-profit university.

Because the private for-profit university has no capacity constraint, we have  $\phi(h^\dagger)\pi_k \geq \pi_3 - a_f$ . If  $h^\dagger > h_r$ , then the capacity of the public university is not filled, whence  $\phi(h^\dagger)\pi_k = \pi_3 - a_f$  must hold.

As shown in the proof of part a), all types strictly prefer the private non-profit university over the private for-profit university, so  $\pi_l - a_n > \pi_3 - a_f$ .

d) follows from the arguments in the proofs of Propositions 3 and 4.  $\square$

The equilibria take a similar shape as before, where the for-profit university attracts the least able students. It cannot charge more than the added value of getting a diploma there. There is no clear ranking between the other two universities. Proposition 5 is illustrated in Figure 5.

### 3.2 Optimal Fee at the Private For-Profit University

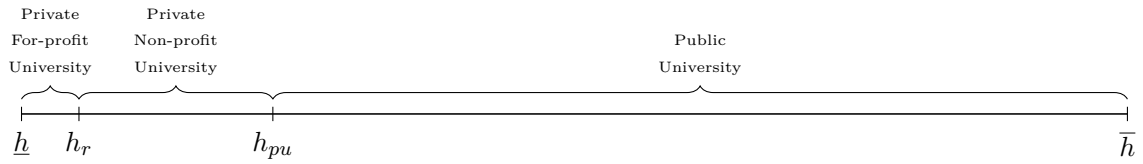
We now turn to stage 1 of the game, in which the private for-profit university chooses its fee  $a_f$ . Every fee induces a subgame in stage 2, where the students make their application decision. We focus on Nash equilibria in which the private for-profit university is not the only active university and, across subgames, competes with the same universities.

Our first result in this section concerns situations in which the private for-profit university competes only with the public university. Recall from Proposition 2 that such stage-2 Nash equilibria exist if and only if the fee of the private for-profit university is  $a_f \leq \pi_2$ .<sup>7</sup>

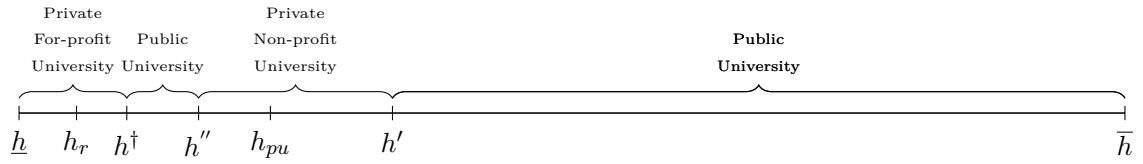
**Proposition 6.** *For every  $a_f \leq \pi_2$ , select the stage-2 Nash equilibrium in which only the public university and the private for-profit university are active. There exists a cutoff*

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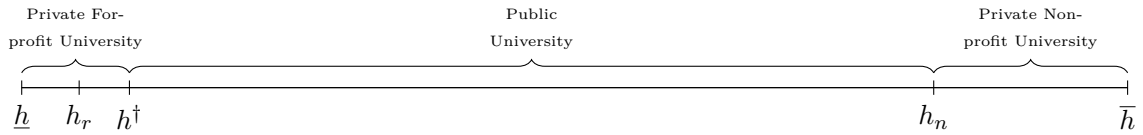
<sup>7</sup>In particular, for fees  $a_f > \pi_2$  the private for-profit university is inactive by Propositions 2 and 5.



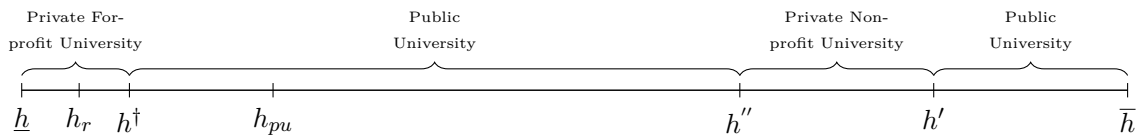
(a) Public ranks first; Segregation equilibrium



(b) Public ranks first; Non-segregation equilibrium



(c) Public ranks second; Segregation equilibrium



(d) Public ranks second; Non-segregation equilibrium

Figure 5: Equilibria with all three universities

$\tilde{q} \in (0, 1)$  such that if  $q_{pu} < \tilde{q}$ , then in a subgame-perfect Nash equilibrium the private for-profit university sets the fee  $a_f = \pi_2$ , and if  $q_{pu} > \tilde{q}$ , it sets a fee

$$a_f \in \arg \max_{a \in [0, \pi_2 - \phi(h_{pu})\pi_1]} G\left(\phi^{-1}\left(\frac{\pi_2 - a}{\pi_1}\right)\right).$$

*Proof.* By Proposition 2 the applicants at the private for-profit university are the types  $h \in [h, h']$  for some cutoff  $h'$ . The proof of the proposition showed that if  $\phi(h_{pu})\pi_1 \geq \pi_2 - a_f$  then  $h' = h_{pu}$ , and if  $\phi(h_{pu})\pi_1 < \pi_2 - a_f$  then  $h' = \phi^{-1}((\pi_2 - a_f)/\pi_1)$ . Hence the revenue of the private for profit-university at fee  $a_f$  is

$$R(a_f) := \begin{cases} G(h_{pu})a_f & \text{if } a_f \geq \pi_2 - \phi(h_{pu})\pi_1, \\ G\left(\phi^{-1}\left(\frac{\pi_2 - a_f}{\pi_1}\right)\right) a_f & \text{if } a_f < \pi_2 - \phi(h_{pu})\pi_1. \end{cases}$$

The function  $R$  is continuous, so it has a maximum on  $[0, \pi_2]$ . The maximum is either attained at the fee  $a_f = \pi_2$  or at a fee

$$a_f \in \arg \max_{a \in [0, \pi_2 - \phi(h_{pu})\pi_1]} G\left(\phi^{-1}\left(\frac{\pi_2 - a}{\pi_1}\right)\right) a.$$

Because  $h_{pu} = G^{-1}(1 - q_{pu})$ , the revenue in the former case is  $G(h_{pu})\pi_2 = (1 - q_{pu})\pi_2$ , which is strictly decreasing in  $q_{pu}$ . The revenue in the latter case is

$$\max_{a \in [0, \pi_2 - \phi(h_{pu})\pi_1]} G\left(\phi^{-1}\left(\frac{\pi_2 - a}{\pi_1}\right)\right) a,$$

which is weakly increasing in  $q_{pu}$  and bounded from above by

$$\max_{a \in [0, \pi_2]} G\left(\phi^{-1}\left(\frac{\pi_2 - a}{\pi_1}\right)\right) a < \pi_2.$$

This concludes the proof. □

The private for-profit university has two options. Either it charges a study fee equivalent to the value of their diploma. In this case, it only attracts leftover students who cannot attend the public university due to quota restrictions. Alternatively, it can compete with the public university by setting a low fee. In this case, the market share of the private for-profit university rises, but the revenue per student decreases.

Next, we show that an analogous result holds in situations in which the private for-profit university competes with both other universities and the private non-profit

university is ranked first. In the following section, we provide a theoretical foundation for this ranking under the assumption that  $\pi_1$  is large.

By Proposition 5, a necessary condition for a stage-2 Nash equilibrium in which all three universities are active and the private non-profit university is ranked first is that the fee of the private for-profit university satisfies  $\pi_3 - \pi_1 + a_n < a_f \leq \pi_3$ . In particular, for large  $\pi_1$  (i.e.,  $\pi_1 - a_n > \pi_2$ ), this condition is also sufficient, and the Nash equilibrium is as in part a) of Proposition 5.

**Proposition 7.** *Let  $\pi_1 - a_n > \pi_2$ . For every  $a_f \leq \pi_3$ , select the stage-2 Nash equilibrium in which all three universities are active and the private non-profit university is ranked first; for every  $a_f > \pi_3$ , select a stage-2 Nash equilibrium in which the private for-profit university is inactive. There exists a cutoff  $\hat{q} \in (0, 1)$  such that if  $q_{pu} < \hat{q}$ , then in a subgame-perfect Nash equilibrium the private for-profit university sets the fee  $a_f = \pi_3$ , and if  $q_{pu} > \hat{q}$ , it sets a fee*

$$a_f \in \arg \max_{a \in [0, \pi_3 - \phi(h_r)\pi_2]} G\left(\phi^{-1}\left(\frac{\pi_3 - a}{\pi_2}\right)\right).$$

*Proof.* The proof is analogous to the proof of Proposition 6; we state it for completeness because some formulas change. By parts b) and c) of Proposition 5 the applicants at the private for-profit university are the types  $h \in [h, h^\dagger]$  for some cutoff  $h^\dagger \geq h_r$ ; if  $\phi(h_r)\pi_2 \geq \pi_3 - a_f$  then  $h^\dagger = h_r$ , and if  $\phi(h_r)\pi_2 < \pi_3 - a_f$  then  $h^\dagger = \phi^{-1}((\pi_3 - a_f)/(\pi_2))$ . Hence the revenue of the private for profit-university at fee  $a_f$  is

$$R(a_f) := \begin{cases} G(h_r)a_f & \text{if } a_f \geq \pi_3 - \phi(h_r)\pi_2, \\ G\left(\phi^{-1}\left(\frac{\pi_3 - a_f}{\pi_2}\right)\right)a_f & \text{if } a_f < \pi_3 - \phi(h_r)\pi_2. \end{cases}$$

The function  $R$  is continuous, so it has a maximum on  $[0, \pi_3]$ . The maximum is either attained at the fee  $a_f = \pi_3$  or at a fee

$$a_f \in \arg \max_{a \in [0, \pi_3 - \phi(h_r)\pi_2]} G\left(\phi^{-1}\left(\frac{\pi_3 - a}{\pi_2}\right)\right)a.$$

Because  $h_r = G^{-1}(1 - q_{pu} - q_n)$ , the revenue in the former case is  $G(h_r)\pi_3 = (1 - q_{pu} - q_n)\pi_3$ , which is strictly decreasing in  $q_{pu}$ . The revenue in the latter case is

$$\max_{a \in [0, \pi_3 - \phi(h_r)\pi_2]} G\left(\phi^{-1}\left(\frac{\pi_3 - a}{\pi_2}\right)\right)a,$$

which is weakly increasing in  $q_{pu}$  and bounded from above by

$$\max_{a \in [0, \pi_3 - \phi(h_n)\pi_2]} G\left(\phi^{-1}\left(\frac{\pi_3 - a}{\pi_2}\right)\right) a < G(h_n)a = (1 - q_n)\pi_3.$$

This concludes the proof. □

## 4 Equilibrium Selection and Comparative Statics

This section contains our main economic insights. First, we state conditions under which the private non-profit university is ranked first, i.e., where the government loses control of the education of their intellectual elite if such universities are allowed. Second, we show that the equilibrium market share of the public university is not monotone in its quota. As such, governments need to carefully examine their choice of quota, where a larger quota incentivizes private for-profit universities to compete more aggressively. Finally, we show that private for-profit universities are the beneficiaries of a larger student body.

### 4.1 Equilibrium Selection

The presence of multiple equilibria is a recurrent theme in this literature; see e.g., Epple et al. (2017) and Fu (2014) for empirical approaches to deal with it. From a theoretical point of view, Barberà et al. (2016) argue that especially in the presence of relatively small coalitions, group strategy-proofness is a particularly attractive refinement. Given the existence of small informal networks of prospective elite students (circles of parents, elite schools, or student olympiads) and the small size of private non-profit universities, we pursue group strategy-proofness as our selection criterion.

We consider Nash equilibria in stage 2 that are immune to deviations by groups of students. A Nash equilibrium is *group strategy-proof* if there exist no set of types  $I \subseteq [\underline{h}, \bar{h}]$  and a deviation by these types to some active university that strictly increases the expected payoff of each type  $h \in I$ . Note that in contrast to a deviation by a single student, a deviation by a group of students may change the ranking of universities.

There are different ways to interpret group strategy-proofness in our model. For instance, one might think about direct coordination of high-ability students via social

media, via a seal of excellence at a university, or via the presence of a Nobel prize winner at the faculty. In either case, group strategy-proofness should be understood as a coordination device for a select small group of students. Under this refinement, we obtain a more clear prediction.

**Theorem 1.** *There exists a cutoff  $\pi_1^*$  such that if  $\pi_1 > \pi_1^*$ , then in every group strategy-proof Nash equilibrium of the subgame in stage 2 in which the public university and the private non-profit university are active, the private non-profit university is ranked first.*

*Proof.* Suppose first that the private for-profit university is inactive. Nash equilibria in which the public university is ranked higher than the private non-profit university are then as in Proposition 3. Let  $h_1(\pi_1)$  be the solution to

$$\phi(h)\pi_1 = \pi_1 - a_n;$$

$h_1(\pi_1)$  exists and is unique because  $\pi_1 - a_n > \pi_2 - a_n > 0$  by Proposition 3. Let  $h_2(\pi_1)$  be the unique solution to  $G(h_1(\pi_1)) - G(h) = q_n$ , which exists when  $h_1(\pi_1)$  is large. Then

$$\lim_{\pi_1 \rightarrow \infty} h_1(\pi_1) = 1 \quad \text{and} \quad \lim_{\pi_1 \rightarrow \infty} h_2(\pi_1) = h_n.$$

By the Dominated Convergence Theorem, it follows that there exists  $\pi_1'$  such that for  $\pi_1 > \pi_1'$ ,

$$\frac{1}{q_n} \int_{h_2(\pi_1)}^{h_1(\pi_1)} h dG(h) > \frac{1}{q_{pu}} \left( \int_{h_r}^{h_2(\pi_1)} h dG(h) + \int_{h_1(\pi_1)}^{\bar{h}} h dG(h) \right). \quad (4)$$

Let  $\pi_1''$  such that  $h_1(\pi_1'') > h_{pu}$ . Then the Nash equilibrium in part a) of Proposition 3 is not group strategy-proof for  $\pi_1 > \max\{\pi_1', \pi_1''\}$ : if the types  $h \in [\max\{h_{pu}, h_2(\pi_1)\}, h_1(\pi_1))$  deviate and apply at the private non-profit university instead of the public university, they would be admitted, make the private non-profit university the top-ranked university by (4), and all have a strictly higher expected payoff:

$$\pi_1 - a_n = \phi(h_1(\pi_1))\pi_1 > \phi(h)\pi_1, \quad \forall h \in [\max\{h_{pu}, h_2(\pi_1)\}, h_1(\pi_1)).$$

The Nash equilibrium in part b) of Proposition 3 is not group strategy-proof for  $\pi_1 > \pi_1'$ : By the definition of  $h'$  in part b) of Proposition 3,

$$\phi(h')\pi_1 = \pi_2 - a_n < \pi_1 - a_n = \phi(h_1(\pi_1))\pi_1.$$

Hence  $h' < h_1(\pi_1)$ . If the types  $h \in [\max\{h', h_2(\pi_1)\}, h_1(\pi_1))$  deviate and apply at the private non-profit university instead of the public university, they would be admitted, make the private non-profit university the top-ranked university by (4), and all have a strictly higher expected payoff:

$$\pi_1 - a_n > \pi_2 - a_n = \phi(h')\pi_1, \quad \forall h \in [\max\{h', h_2(\pi_1)\}, h_1(\pi_1)).$$

Suppose now that the private for-profit university is active. Let  $a_f$  be its fee. Nash equilibria in which the public university is ranked higher than the private for-profit university are then as in Proposition 5. In particular, the types  $h \in (h^\dagger, \bar{h}]$  apply at the public university or the private non-profit university, and the types in  $h \in [\underline{h}, h^\dagger]$  apply at the private for-profit university. As stated in part c) of Proposition 5, it holds that  $h^\dagger \geq h_r$ , and if  $h^\dagger > h_r$  then  $\phi(h^\dagger)\pi_1 = \pi_3 - a_f$ . Hence for  $\pi_1 \geq \pi_3/\phi(h_r) \geq (\pi_3 - a_f)/\phi(h_r)$ , we have  $h^\dagger = h_r$ . The same arguments as above show that Nash equilibria in which the public university is ranked higher than the private non-profit university are not group strategy-proof for  $\pi_1 > \max\{\pi_3/\phi(h_r), \pi'_1, \pi''_1\}$ .  $\square$

Thus, our selection criterion predicts that a small intellectual elite will pool at the non-profit private university. This is line with empirical evidence from the U.S.; For instance, the Ivy league universities are small private institutions attracting the highest ability students.

## 4.2 Implications of Capacity Choice at the Public University

By our results in the previous section, the optimal fee at the private for-profit university depends on the quota at the public university. A higher quota at the public university increases the competitive pressure on the private for-profit university, which may react by lowering its fee. In particular, we shall see that the tuition fee at the private for-profit university is not continuous in the quota of the public university. In turn, the equilibrium market share of the public university might drop when setting a higher quota.

We illustrate the issue in Figure 6. In the example, the private university charges  $a_f = \pi_2$  for  $q_{pu} < \frac{13}{16}$  and a significantly lower fee for  $q_{pu} > \frac{13}{16}$ , i.e., when the competitive

pressure by the public university is sufficiently high to induce price competition by the private for-profit university. The following theorem formalizes the previous insights.

**Theorem 2.** *The optimal tuition fee at the private for-profit university is not continuous in the quota of the public university. Moreover, the equilibrium market share at the public university is not monotone in its quota.*

*Proof.* From the proof of Proposition 6, recall that

$$R(a_f) := \begin{cases} G(h_{pu})a_f & \text{if } a_f \geq \pi_2 - \phi(h_{pu})\pi_1, \\ G\left(\phi^{-1}\left(\frac{\pi_2 - a_f}{\pi_1}\right)\right)a_f & \text{if } a_f < \pi_2 - \phi(h_{pu})\pi_1. \end{cases}$$

As  $h_{pu} = 1 - q_{pu}$ , the (local) optimum of the first part of the piecewise function strictly decreases in  $q_{pu}$ , while the local optimum on the second part is (weakly) increasing in  $q_{pu}$ . The second part is strictly positive and smaller than  $a_f$ , while the first part goes to 0 as  $q_{pu}$  goes to 1 and to  $a_f$  as  $q_{pu}$  goes to 0. By continuity of all involved functions, there is a cutoff such that the maximum of  $R(a_f)$  lies on the first part for any value of  $q_{pu}$  below the cutoff and on the second part for any value above the cutoff. Thus, the optimal tuition fee is not continuous at the cutoff. Moreover, the equilibrium market share at the public university increases up to the cutoff, but has a downward jump at the cutoff, which establishes the claim.  $\square$

Despite the strong segregation results in Section 3 which show that the private for-profit university always attracts the least able students, the interplay of this university type with the public university is far from trivial. By setting the quota at the public university, the government decides on the competitive pressure on the for-profit university. By pushing the quota too high, the government is no longer able to ensure that the public university operates at capacity, resulting in economic inefficiencies.

### 4.3 Growing Student Numbers

As we argued in the introduction, empirical evidence suggests an increased demand for higher education worldwide. Arguably, this reduces average quality of applicants. Thus,



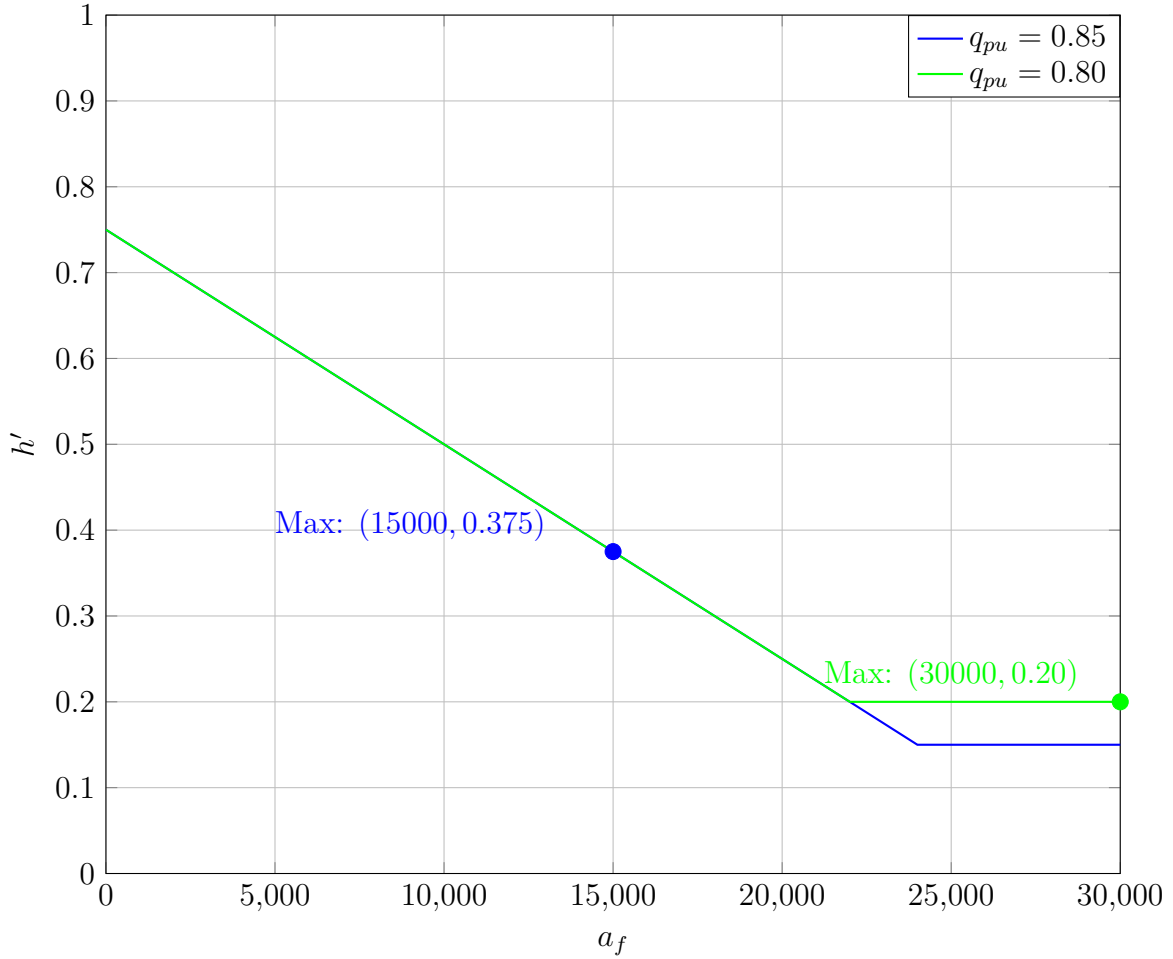


Figure 6: The figure depicts the market share of the for profit university as a function of its tuition fee  $a_f$  for  $G(h) = \phi(h) \sim UNI_{[0,1]}$  and  $q_{pu} = 0.8$  (green) and  $q_{pu} = 0.85$  (blue). Both lines overlap for market shares above 0.2, below which the first quota becomes binding. The dots illustrate the equilibrium choice.

we assume that the old ability distribution of applicants  $G(h)$  first-order stochastically dominates the new ability distribution  $G^n(h)$ .

For simplicity of exposition, we consider the case without a private non-profit university. We keep the quotas of the public and private non-profit university constant and we analyze the impact of a decrease in student ability.

**Theorem 3.** *The private for-profit university makes a (weakly) higher equilibrium profit under  $G^n(h)$ .*

*Proof.* From the proof of Proposition 6, recall that

$$R(a_f) := \begin{cases} G(h_{pu})a_f & \text{if } a_f \geq \pi_2 - \phi(h_{pu})\pi_1, \\ G\left(\phi^{-1}\left(\frac{\pi_2 - a_f}{\pi_1}\right)\right)a_f & \text{if } a_f < \pi_2 - \phi(h_{pu})\pi_1. \end{cases}$$

Stochastic dominance implies  $G^n(h) \geq G(h)$  for all  $h$ . Furthermore,  $R(a_f)$  is weakly monotone increasing in  $G$  which implies the corresponding claim for maximal revenue.  $\square$

Less able students prefer the private for-profit university as it has a higher graduation probability. Thus, for any given fixed fee, the private for-profit university attracts weakly more students. Intuition might suggest that the market share of the private for-profit university also grows in this case. The following example establishes the result holds for the uniform case.<sup>8</sup>

**The Uniform Case:** Suppose  $\phi(h) \sim UNI_{[0,1]}$  and  $G(h) \sim UNI_{[\hat{h},1]}$  with  $\hat{h} \geq 0$ . If  $a_f^* \geq \pi_2 - \phi(h_{pu})\pi_1$ , the private university only attracts the left-over students under the original  $G(h)$ , which is clearly a lower bound on its market share for any  $G(h)$ .

Thus suppose  $a_f^* < \pi_2 - \phi(h_{pu})\pi_1$ . In this case, we get

$$G\left(\phi^{-1}\left(\frac{\pi_2 - a_f}{\pi_1}\right)\right)a_f = G\left(\frac{\pi_2 - a_f}{\pi_1}\right)a_f = \frac{\pi_2 - a_f - \hat{h}}{1 - \hat{h}}a_f$$

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<sup>8</sup>As the other universities are passive in the first stage, the choice of an optimal fee at the private for-profit university is akin to the price choice of a monopolist in demand theory. In general, an increased demand at any price does not imply a higher realized demand of the monopolist, as the optimal price depends on the price elasticity of demand.

Taking the first-order condition and rearranging, we obtain:

$$a_f^* = \frac{\pi_2 - \underline{h}\pi_1}{2} \quad \text{and} \quad G(a_f^*) = \frac{\frac{\pi_2}{2\pi_1} - \frac{\underline{h}}{2}}{1 - \underline{h}}.$$

The revenue of the private for-profit university is given by

$$a_f^*G(a_f^*) = \frac{(\pi_2 - \underline{h}\pi_1)^2}{4\pi_1(1 - \underline{h})}$$

which is decreasing in  $\underline{h}$ . We illustrate the uniform case in Figure 6 for  $\hat{h} = 0$ .

**Theorem 4.** *In the uniform case, a lower student quality leads to a (weakly) higher market share of the private for-profit university.*

Thus, the model predicts the private for-profit university to be the beneficiary of an increased demand for higher education, which is in line with the empirical evidence presented in the introduction. The theoretical prediction in terms of market share relies on a downward shift in the distribution. Our *ceteris paribus* assumption requires a proportionally increased capacity at each university type. For the public universities, this requires additional funding, while private for-profit institutions directly benefit from higher student numbers. Thus, next to the forces inside the model, there is an additional push for shifting additional demand for higher education to private for-profit institutions.

## 5 Discussion

Throughout the paper, we have mostly treated the government as passive. Yet, it is an important actor with the power to regulate higher education. In particular, it has to accredit the diplomas awarded by private universities. Thus, the government has at least some choice in which types of universities it admits. In the U.S., the government admits both for-profit and non-profit universities. As our model predicts, some of the elite universities are small non-profit universities. These universities are heavily subsidized by alumni and other private sponsors. In contrast, the Chinese landscape consists of public universities which attract the top students and private universities who help in

covering the market (He et al., 2020).<sup>9</sup>. The different university systems are closely tied to the political ideologies in the countries. China is arguably more interested in providing public education to its intellectual elite, while the U.S. places higher weight on reducing government expenditure by delegating higher education of the elite to small private schools.

In our model, we consider three pre-determined values of a diploma depending on the rank of the university. If we instead were to assume that the value of a diploma directly depends on the average student ability at the awarding institution, the push for top students to attend the small elitist non-profit universities would be even stronger. Our current setting thus makes our segregation results harder to obtain, but it keeps the computation of optimal fees tractable.

Some of our complete segregation results based on student ability rely on one-dimensional student types. A second dimension which influences study choice is the student's wealth level. Poor students might not be able or willing to afford higher private education due to incomplete credit markets, underconfidence, or incomplete information about their abilities and opportunities, and debt aversion (Kane, 1996; Belley and Lochner, 2007; Cunningham and Santiago, 2008; Hoxby and Turner, 2015; Bradley and Green, 2020). At the same time, empirical evidence suggests that the value to a diploma might be higher for more wealthy students (Blanden et al., 2007; Bukodi and Goldthorpe, 2011a,b; Britton et al., 2019). In such a two-dimensional model, the richer and less able a student is, the more likely she is to prefer the private for-profit university over the public one, i.e., segregation occurs according to a two-dimensional indicator. Otherwise, the intuition remains similar, but the computation becomes less tractable.

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<sup>9</sup>Following the enactment of the Law on the Promotion of Non-Public Schools in China, private universities, while officially categorized as non-profit, were permitted to legally retain a 'reasonable' portion of their financial surplus (Ministry of Education of the People's Republic of China, 2009). The absence of a clear definition for 'reasonable' led to a surge in the number of accredited private universities capable of awarding Bachelor's degrees, from 4 in 2002 to 390 by 2021, predominantly operating as for-profit universities in practice (Yan and Levy, 2005; Liu et al., 2021; Ministry of Education of the People's Republic of China, 2022)

## 6 Conclusion

Motivated by the rise of for-profit universities, we have studied a model of higher education with three types of universities: public, private non-profit, and private for-profit. Compared to the private for-profit university, the public university has a lower graduation rate and the private non-profit university is more selective. The private for-profit university always attracts the least able students in equilibrium, resulting in a diploma of low value. Under a natural equilibrium selection criterion, the private non-profit university attracts the most able students, which may shed some light on the striking difference regarding the roles of non-profit universities in the U.S. and China. Our model predicts that the private for-profit university is the main beneficiary of an increase in student numbers. This result provides an explanation of the rise of for-profit universities in recent years. We also showed that the fee of the private for-profit university is highly sensitive, and negatively related, to the quota of the public university. Thus, increased funding of public universities may have unexpected benefits for the entire higher education market.

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