# Fast-ion diagnostics in constants-of-motion phase-space

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**Introduction** Assuming a linear dependence of the fast-ion diagnostic signals on the fast-ion distribution function, the expected signal per ion is quantised by so-called weight functions. Since the full fast-ion distribution function can be described by the three constants of motion, the kinetic energy, the magnetic moment and the toroidal canonical angular momentum, respectively,

$$E = \frac{1}{2}mv^2, \qquad \mu = \frac{mv_\perp^2}{2B}, \qquad P_\phi = mR\frac{B_\phi}{B}v_\parallel + q\Psi_p,$$

it is desirable to develop an understanding of where in the constant-of-motion phase-space a given fast-ion diagnostic is able to observe, and thus which information about the fast-ions it is possible to obtain. The weight functions thus quantise the sensitivity of a given diagnostic to each part of phase-space. The constant-of-motion phase-space is essential for stability calculations, and this work describes the relation between phase-space and real position- and velocity-space.

In this work we calculate three-dimensional weight functions in the constant-of-motion phase space using the projected velocity $u$ of the fast ions along the diagnostic lines of sight as a proxy for FIDA diagnostic signals. The projected velocity is a fundamental quantity in several fast-ion diagnostics, such as FIDA spectroscopy [1, 2, 3, 4, 5], collective thomson scattering (CTS) [6], gamma-ray spectroscopy (GRS) [7, 8] and neutron emission spectroscopy (NES) [9], and this allows us to calculate the weight functions analytically and have full transparency in the phase-space sensitivities. It is given by $u = v_\parallel \cos(\varphi) + v_\perp \sin(\varphi)\cos(\gamma)$, where $\varphi$ is the angle between the line of sight and the magnetic field in the measurement volume, and $\gamma$ is the gyro-angle, which is taken to be uniformly distributed in $[0, 2\pi[$. Weight functions have previously been calculated in 2D velocity-space quantifying the velocity-space sensitivity for collective Thomson scattering (CTS) [6, 10, 11, 12], fast-ion $D_\alpha$ spectroscopy [5, 10, 12], one- and two-step gamma-ray spectroscopy (GRS) [8, 7, 9, 10, 11, 12] and neutron emission spectroscopy (NES) [9, 10, 12]. Weight functions have also been calculated numerically in phase-space for FIDA diagnostics in the DIII-D tokamak [13]. Here, we calculate the weight functions in a COMPASS-Upgrade geometry and equilibrium.

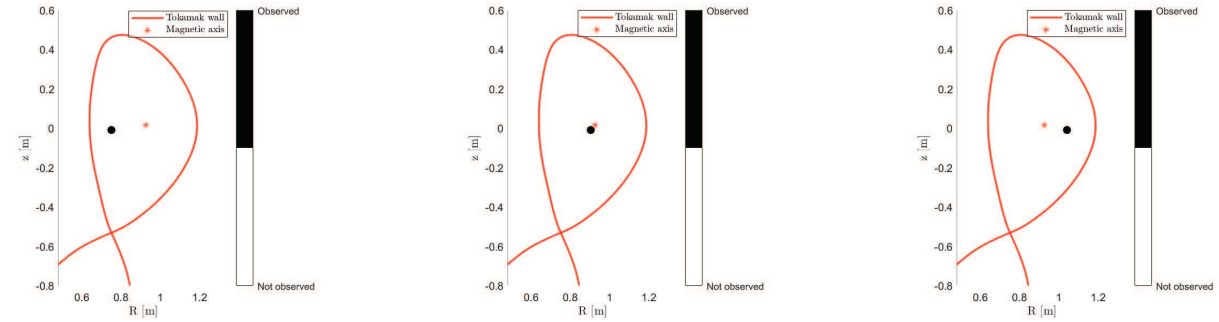
**Weight functions** The linear relation between the fast-ion distribution function and the diagnostic signal is given by a six-dimensional integral over position-space and velocity-space. This can be reduced by utilising the symmetries of the tokamak and averaging over the gyro-motion of the fast ions around the magnetic field lines. The relation is then reduced to a three-dimensional integral expressed in terms of the constants of motion,

$$s(u_1, u_2, \varphi) = \sum_{\sigma=\pm 1} \int_E \int_\mu \int_{P_\phi} \left( w^{(3D)}(u_1, u_2, \varphi, E, \mu, P_\phi; \sigma) f^{(3D)}_{\text{COM}}(E, \mu, P_\phi; \sigma) \right) \mathrm{d}P_\phi \mathrm{d}\mu \mathrm{d}E,$$

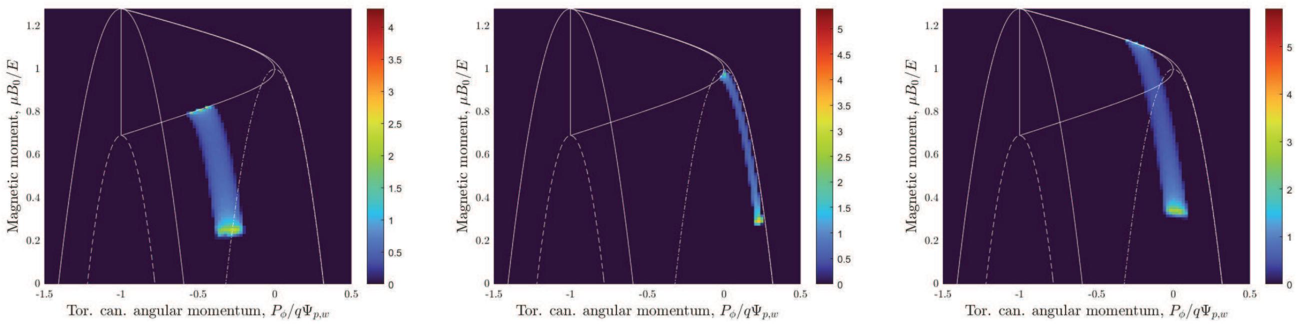
where $s$ is the signal, $w^{(3D)}$ is the weight function and $f^{(3D)}_{\text{COM}}$ is the fast-ion distribution function. The Jacobian of the transformation from the Cartesian coordinates to the constants-of-motion coordinates is absorbed in $f^{(3D)}_{\text{COM}}$. There is a sum over $\sigma = \pm 1$, since there is an ambiguity in the direction of the particles relative to the magnetic field. Co- and counter-passing fast-ion guiding-center orbits can have the same $(E, \mu, P_\phi)$-triplet, such that we need a binary index $\sigma$ to be able to distinguish between the two orbits. The diagnostic sensitivity to a given projected velocity $u_a$, quantified by the weight function, is proportional to the probability that a fast ion, moving through the measurement volume, has the projected velocity $u_a$, which is given by [5]

$$\text{prob}\left(u_1 \le u_a \le u_2 | \varphi, v_\parallel, v_\perp\right) = \frac{1}{\pi}\left(\arccos\left(\frac{u_1 - v_\parallel \cos(\varphi)}{v_\perp \sin(\varphi)}\right) - \arccos\left(\frac{u_2 - v_\parallel \cos(\varphi)}{v_\perp \sin(\varphi)}\right)\right) = \frac{\gamma_1 - \gamma_2}{\pi}.$$

The relation between real space and velocity- and phase-space is found by considering different small measurement volumes in the poloidal projection of the tokamak. The equilibrium and geometry used is that of the COMPASS-Upgrade tokamak, see Figure 1 for an example. Here we consider the weight functions for the observed projected velocity $u$ corresponding to




(a) Observe high-field side (b) Observe around magnetic axis (c) Observe low-field side




(d) $\varphi = 26.4°$, $E = 95keV$ (e) $\varphi = 26.4°$, $E = 95keV$ (f) $\varphi = 26.4°$, $E = 95keV$

*Figure 1: Weight functions for $E_u = 25.44keV$ for $\varphi = 26.4°$. The energy planes shown are all for $E = 95keV$. The values on the colorbars are in arbitrary units.*

$E_u = 1/2mu^2 = 25.44$keV in a plane of constant energy for three different measurement volumes located near the midplane on the high-field side, close to the magnetic axis, and on the low-field side respectively, for a single angle $\varphi = 26.4°$. We can observe how the observable region moves along the trapped-passing boundary in phase-space as the measurement volume is moved from the high-field side to the low-field side. The weight functions show high sensitivity at each end, which is related to the $\cos(\gamma)$, which varies more slowly close to $\gamma = 0°$ and $\gamma = 180°$ than for angles in between. This feature is also observed in the 2D velocity-space weight functions mentioned in the introduction. The finite size of the measurement volume explains the width of the sensitivity region. The larger the measurement volume, the wider the sensitivity region in Figure 1.

**Conclusion** In conclusion, we have calculated three-dimensional weight functions for fast-ion diagnostics such as FIDA, CTS, GRS and NES. The use of projected velocities as a proxy for the diagnostic signal allows us to calculate the weight functions analytically and study the relationship between real space and phase-space. This enables easy, direct identification of observable regions in phase-space.

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