Phase Space Zonal Structures and equilibrium distribution functions in ORB5

A. Bottino[1], **M.V. Falessi**[2], **T. Hayward-Schneider**[1], **A. Biancalani**[3], **S. Briguglio**[2], **Ph. Lauber**[1], **A. Mishchenko**[4], **E. Poli**[1], **B. Rettino**[1], **G. Vlad**[2], **X. Wang**[1], **F. Zonca**[2]
[1]Max-Planck-Institut für Plasmaphysik, 85748 Garching, Germany
[2]ENEA, Fusion and Nuclear Safety Department, 00044 Frascati (Roma), Italy
[3]Léonard de Vinci Pôle Universitaire, Research Center, 92916 Paris la Défense, France
[4]Max-Planck-Institut für Plasmaphysik, 17491 Greifswald, Germany

E-mail: `alberto.bottino@ipp.mpg.de`

Abstract. Phase space zonal structures (PSZS), obtained by averaging out dependencies on angle-like variables in the energetic particle (EP) distribution function, play a fundamental role in regulating EP transport induced by Alfvén instabilities in burning plasmas, acting as a slowly varying nonlinear equilibrium state. Therefore, they are of great interest for the development of reduced models for the description of EP heat and particle transport on long time scales, comparable with the energy confinement time, for future burning plasma experiments. The information provided by the finite element projection of the distribution function could also be used for significantly improving the quality of δf PIC simulations, by adjusting and updating the plasma reference state (background distribution function), during the nonlinear evolution of the system, consistently with PSZS dynamics. In this work, we discuss the implementation of a PSZS based background distribution function in the global gyrokinetic code ORB5, in experimentally relevant conditions.

Energetic particles (EPs) created by fusion reactions represent the key ingredient for heating magnetically confined plasmas at ignition. Energetic particles also appear in present day experiments as products of auxiliary heating sources. In Tokamaks, the typical EP velocity, v_{EP}, sits between the thermal bulk ion velocity $v_{\mathrm{th,i}}$ and the thermal electron velocity $v_{\mathrm{th,e}}$. Therefore, the EP characteristic dynamical frequencies, associated with their guiding-center motion (transit, bounce and precessional) are often comparable with the frequency of shear Alfvén waves (SAWs) (see e.g. Ref. [1]). Quantitative predictions of the interplay between SAW instabilities and EPs require a kinetic treatment since resonances with ions substantially modify MHD predictions. Nonlinear gyrokinetic (GK) theory provides the theoretical framework for calculating such interplay between SAWs and EPs and their impact on turbulent and collisional transport (see e.g. Ref. [2]).

Phase space structures, obtained by averaging out dependencies on angle-like variables in the energetic particle distribution function, are not rapidly damped by collisionless processes such as Landau damping. They play a fundamental role in regulating EP transport induced by Alfvén instabilities in burning plasmas (see Ref. [3] and references therein). These phase space structures are generally referred to as Phase Space Zonal Structures (PSZSs), by analogy with the meso-scale configuration space structures spontaneously generated by drift-wave turbulence;

i.e., zonal flows and zonal fields. PSZSs act as a slowly evolving nonlinear plasma equilibrium. Recently, evolution equations for PSZSs have been derived from nonlinear GK theory [4, 3]. Those equations allow the development of reduced transport models [5] describing the evolution of macroscopic plasma profiles on long time scales [6], comparable with the energy confinement time, in burning plasma experiments [7]. In this context, it is important to be able to compare the predictions of the time evolution of PSZSs with the results of existing GK code. Therefore a diagnostics for the PSZS has been implemented in the code ORB5 [8] based on a finite element based projection of the EP distribution function [9] . Note that similar diagnostics already exist in GK-hybrid codes, such as XHMGC [10]. The information provided by the finite element projection of the distribution function could be used for improving the quality of δf PIC simulations as a control-variate, i.e. by adjusting and updating the plasma reference state (background distribution function), during the nonlinear evolution of the system, consistently with PSZS dynamics.

Following the definitions of [3, 4, 7], the PSZS is defined as the angle average (orbit average) of the gyrocenter distribution function

$$\hat{F}_{0,sp}(P_\phi,\mu,\epsilon) = \tau_b^{-1}\oint \frac{\mathrm{d}\theta}{\dot{\theta}} F_{z,sp} \ , \quad \tau_b = \oint \frac{\mathrm{d}\theta}{\dot{\theta}} \ , \quad F_{z,sp} = \frac{1}{2\pi}\int_0^{2\pi} F_{sp}(P_\phi,\mu,\epsilon,\theta,\phi)\,\mathrm{d}\phi \ , \qquad (1)$$

with

$$\dot{\theta} = -\frac{v_\parallel}{eB_\parallel^* J_{\psi,\theta,\phi}}\frac{\partial P_\phi}{\partial \psi} \ , \quad J_{\psi,\theta,\phi}^{-1} = \nabla\phi\cdot(\nabla\psi\times\nabla\theta) \ ,$$

where $F_{sp}(P_\phi,\mu,\epsilon,\theta,\phi)$ is the gyrocenter distribution function of the particle species sp, μ is the magnetic moment, $\epsilon = 1/2(mv_\parallel^2 + 2\mu B)$ is the kinetic energy and

$$P_\phi = m_{sp} v_\parallel \frac{T(\psi)}{q_{sp}B} + \psi \ , \qquad (2)$$

is the toroidal canonical angular momentum, associated with the toroidal symmetry of an axisymmetric equilibrium magnetic field; ψ is the poloidal magnetic flux profile; m_{sp} and q_{sp} are mass and charge of the particle species sp, respectively. Note that both ϵ and P_ϕ are constants of motion on the unperturbed trajectories, while μ is an adiabatic invariant for the GK system of equations. Therefore, Eq. (1) can be interpreted, at a given time t and in a given phase-space position (P_ϕ,μ,ϵ), as the average value of the distribution function on the unperturbed trajectories described by (P_ϕ,μ,ϵ). The description of the derivation and implementation of the PSZS diagnostics in ORB5 can be found in [9]. An example of PSZS measured in ORB5 is given in Figure 1, for the so-called called NLED-AUG case [11]. This scenario corresponds to the magnetic equilibrium and profiles measured at $t = 0.84$ s in the discharge number 31213 performed in the ASDEX Upgrade (AUG) Tokamak. This experimental case is particularly relevant for EP physics. In particular the NBI generated fast-ions exhibit an MHD β (ratio of the plasma pressure to magnetic pressure) comparable to that of the bulk plasma, while the EP average kinetic energy is approximately 100 times higher than the bulk species temperature, comparable to the expected ratios of plasma parameters that are going to be met in future fusion machines such as ITER and DEMO. In this scenario, an intense EP-driven activity is observed, the stabilizing effects of the bulk plasma being minimized. The NLED-AUG case has been widely studied and simulated by several codes, including ORB5, see e.g. Refs. [12, 13]. Energetic particles are simulated using an analytical equilibrium distribution function corresponding to a pitch-angle dependent slowing down function (see Eq. (3) of [12]). For such a distribution function, EPs can also excite an energetic-particle driven geodesic acoustic mode (EGAM) via inverse Landau damping with the resonant EPs, which causes a redistribution of the EPs in phase space from higher to lower energies. In Ref. [14] the effects of such redistribution on the nonlinear stability of EGAMs in the presence of zonal flows and SAWs are studied in detail. Figure 1 illustrates a snapshot in time of the PSZS diagnostics during the early nonlinear phase for one of those simulations. It shows that the distribution function already departed from its initial state.

As a proof of principle of the use of a numerically reconstructed equilibrium distribution function as control variate for gyrokinetic particle-in-cell simulations, we have reconstructed a flux-surface averaged distribution function from the saturated state of a cyclone-base case ITG simulation using adiabatic electrons (see Ref. [15] for magnetic equilibrium, profiles and numerical parameters). In the original simulation the background distribution function was an analytical Maxwellian $f_M(\psi, E_k)$, here it has been replaced at $t \simeq 0.65 \times 10^4$ by a projection of the distribution function on a $(\psi, E_k, v_{||})$ space, $f_B(\psi, E_k, v_{||})$, constructed using the PSZS numerical infrastructure. The result is shown in Fig. 2. As a next step, the PSZS of Fig. 1 (at a later nonlinear case) will be used as control-variate for an electromagnetic simulation of the NLED-AUG case.

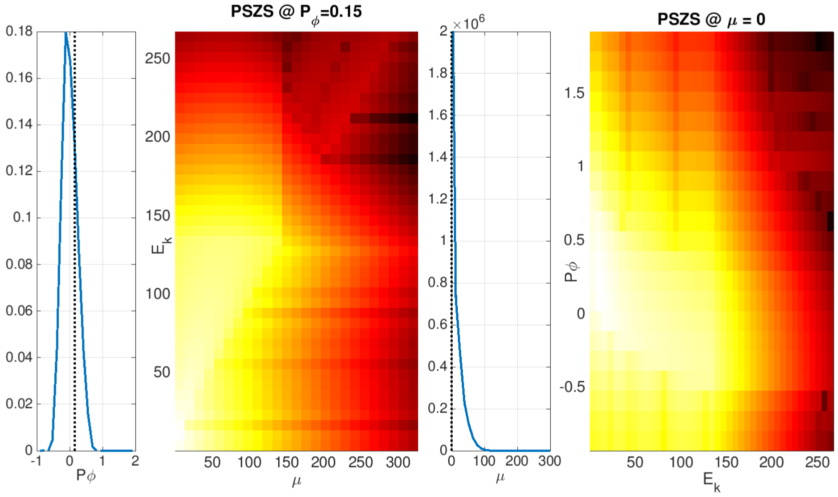

Figure 1: From left to right: 1) integral of the PSZS on the two velocity variables; 2) 2D cut at at fixed $P\phi = 0.15$; 3) integral of the PSZS on $P\phi$ and E_K; 4) 2D cut at at fixed $\mu = 0$, during the early nonlinear phase of an electromagnetic simulation for the NLED-AUG case, using a pitch dependent slowing-down distribution function for the energetic particles. Resolution: $(N_{P\phi} = 21, N_\mu = 25, N_{Ek} = 51)$, linear B-splines.

This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them. The simulations discussed in this work were performed on the MARCONI FUSION HPC system.

[1] Lauber P 2013 *Physics Reports* **533** 33–68 ISSN 0370-1573

[2] Chen L and Zonca F 2016 *Rev. Mod. Phys.* **88** 015008

[3] Falessi M V and Zonca F 2019 *Physics of Plasmas* **26** 022305 (*Preprint* `https://doi.org/10.1063/1.5063874`)

[4] Zonca F, Chen L, Briguglio S, Fogaccia G, Vlad G and Wang X 2015 *New Journal of Physics* **17** 013052

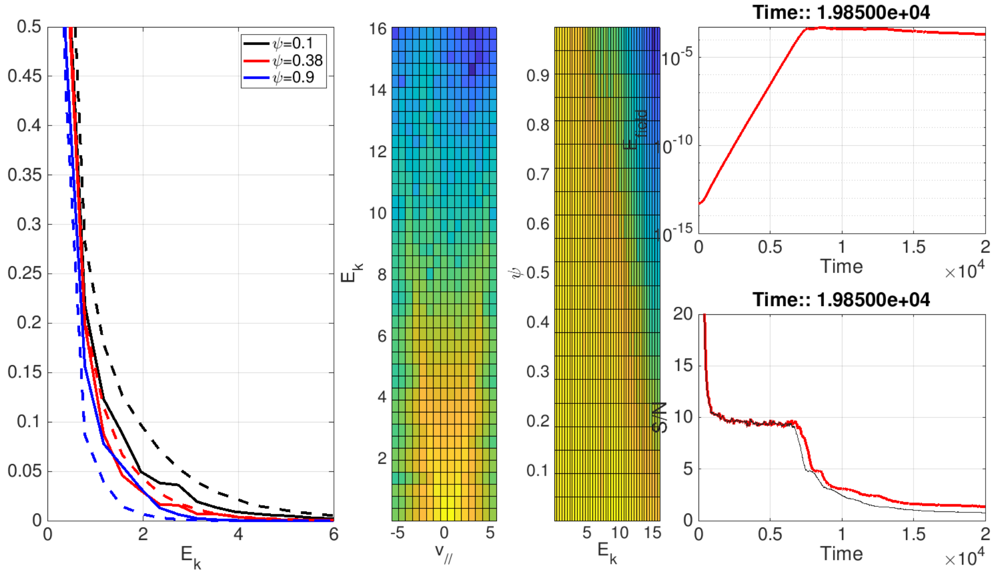

Figure 2: From left to right: 1) 1D cut at at fixed $v_{||} = 0$ for different values of ψ at the beginning (dashed) and end (solid) of the simulation, the expected temperature profile relaxation is reproduced; 2) 2D cut at at fixed $\psi = 0.38$, position of the maximum of the temperature gardient, at $t \simeq 2.e4$; 3) 2D cut at at fixed $v_{||} = 0.$, at $t \simeq 2.e4$; 4) Top: time evolution of the field energy E_{field}; Botton: time evolution of the signal-to-noise ratio, S/N (the thin black line corresponds to a standard ORB5 simulation). S/N and E_{field} follow the definition of in Ref. [16]. Cyclone base case, from Ref. [15], with adiabatic electrons and using a numerically reconstructed control-variate. Resolution: ($N_\psi = 21, N_{v_{||}} = 25, N_{Ek} = 51$), linear B-splines.

[5] Eurofusion ENR ATEP Project, (2021) `https://wiki.euro-fusion.org/wiki/Project_No10`

[6] Falessi M V and Zonca F 2018 *Physics of Plasmas* **25** 032306 (*Preprint* `https://doi.org/10.1063/1.5018175`)

[7] Zonca F, Chen L, Falessi M V and Qiu Z 2021 *Journal of Physics: Conference Series* **1785** 012005

[8] Lanti E, Ohana N, Tronko N, Hayward-Schneider T, Bottino A, McMillan B F, Mishchenko A, Scheinberg A, Biancalani A, Angelino P, Brunner S, Dominski J, Donnel P, Gheller C, Hatzky R, Jocksch A, Jolliet S, Lu Z X, Collar J P M, Novikau I, Sonnendruecker E, Vernay T and Villard L 2020 *COMPUTER PHYSICS COMMUNICATIONS* **251** ISSN 0010-4655

[9] Bottino A, Falessi M, Hayward-Schneider T, Biancalani A, Briguglio S, Hatzky R, Lauber P, Mishchenko A, Poli E, Rettino B, Vannini F, Wang X and Zonca F 2022 *Journal of Physics Conference Series* **2397** 012019

[10] Wang X, Briguglio S, Di Troia C, Falessi M V, Fogaccia G, Fusco V, Vlad G and Zonca F 2022 *Physics of Plasmas* **29** 032512 (*Preprint* `https://doi.org/10.1063/5.0080785`)

[11] Lauber P *et al.* 2018 *27th IAEA Fusion Energy Conference (Proc. 27th IAEA FEC)* (https://conferences.iaea.org/event/151/contributions/6094/)

[12] Rettino B, Hayward-Schneider T, Biancalani A, Bottino A, Lauber P, Chavdarovski I, Weiland M, Vannini F and Jenko F 2022 *Nuclear Fusion* **62** 076027

[13] Vannini F, Biancalani A, Bottino A, Hayward-Schneider T, Lauber P, Mishchenko A, Poli E, Rettino B, Vlad G and Wang X 2022 *Nuclear Fusion* URL `http://iopscience.iop.org/article/10.1088/1741-4326/ac8b1e`

[14] Rettino B, Hayward-Schneider T, Biancalani A, Bottino A, Lauber P, Weiland M, Vannini F and Jenko F 2023 *submitted to Nuclear Fusion*

[15] Goerler T, Tronko N, Hornsby W A, Bottino A, Kleiber R, Norscini C, Grandgirard V, Jenko F and Sonnendruecker E 2016 *Physics of Plasmas* **23** 072503 URL `https://doi.org/10.1063/1.4954915`

[16] Bottino A and Sonnendrücker E 2015 *J. Plasma Phys.* **81** 435810501 URL `https://doi.org/10.1017/s0022377815000574`