Multi-Frequency ICRF Simulations of NBI-Heated Plasmas with TORIC-SSFPQL package

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1. Introduction

Addressing the physics of burning plasmas and validating the present numerical codes have motivated in the recent years experiments to study the influence of fast ions (FI) on the nature and strength of MHD instabilities, Alfvén activity and microturbulence in the plasma core. Neutral beam injection (NBI) is one of the possible FI sources in present devices, together with ion acceleration by radio-frequency (RF) waves in the ion-cyclotron range of frequencies (ICRF). The FI energy of the former is limited to about the injection energy (presently up to about few hundreds of keV), whereas in the case of ICRF the FI energy is mainly limited by their confinement. Because of the finite-Larmor-radius (FLR) nature of the ICRF acceleration at the cyclotron harmonics, NBI-FIs can be selectively further energised by ICRF to increase their population (both in energy and density), and thus their effects on the plasma. This synergy can be boosted by simultaneously launching waves with frequencies corresponding to different IC harmonics inside the plasma. These experiments have been recently started on ASDEX Upgrade (AUG) tokamak by operating, for instance, at low magnetic field, ≈ 1.3 T, and setting the frequencies of the two pairs of antennas at 30 MHz and 41.8 MHz [1]. At ≈ 1.3 T/30 MHz the $3^{\rm rd}$ IC resonance of Deuterium (D) is at mid-radius on the LFS, while all the IC resonances of Hydrogen (H) minority are outside the plasma. At ≈ 1.3 T/41.8 MHz, D has its $3^{\rm rd}$ IC resonance on the HFS and its $4^{\rm th}$ IC resonance in the core; H has its $2^{\rm nd}$ IC resonance in the core as well, but it absorbs very little RF power because of the low concentration and low background temperatures. The aim of using both frequencies (30 and 41.8 MHz with 1.3 T) is to accelerate NBI-D at $3^{\rm rd}$ and $4^{\rm th}$ harmonics in the core and at the $3^{\rm rd}$ on the HFS.

These AUG discharges motivated the extension of the full-wave TORIC code [2] coupled [3] with the steady-state Fokker-Planck quasilinear solver SSFPQL [4] in order to simulate multi-frequency ICRF scenarios. In Section 2, we discuss the implementation of the multi-frequency quasilinear operator, with emphasis on the numerical challenges and necessary optimisations. Including FIs effects makes necessary to account for the radially broadening of the absorbed RF power profiles by finite-orbit-width (FOW) effects. In Section 3, we discuss a heuristic model for these effects implemented in TORIC-SSFPQL. Finally, we outline the future steps.

2. Optimized Quasilinear Operator

In the kinetic equation of the heated species, ICRF is described by the quasilinear operator,

$$\begin{aligned} \left(\frac{\partial F}{\partial t}\right)_{\mathrm{RF}} &= \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left(v_\perp\, D_{\mathrm{RF}}(v_\perp)\,\frac{\partial F}{\partial v_\perp}\right), \\ D_{\mathrm{RF}}(v_\perp) &= D_0 \sum_N \sum_{n,m} P_{N;m,n}\, |J_{N-1}(\chi_\perp v_\perp) + \lambda_p\, J_{N+1}(\chi_\perp v_\perp)|^2 , \end{aligned} \tag{1}$$

where: $v_\perp = (v/v_{\mathrm{thi}})\sqrt{1-\xi^2}$; $\xi = v_\parallel/v$; $\chi_\perp = k_\perp v_{\mathrm{thi}}/\Omega_{\mathrm{ci}}$, with $\Omega_{\mathrm{ci}} = Z_i eB/m_i c$ the ion cyclotron (IC) angular frequency and $v_{\mathrm{thi}} = \sqrt{2T_i/\mathrm{m}_i}$ the thermal speed; $\lambda_p = (E_-/E_+)_{\mathrm{res}}$ the wave polarization at the resonance in terms of circular components of the wave electric field, and $J_N(x)$ the Bessel functions, with N the IC harmonic (the resonance condition is $\omega = N\Omega_{\mathrm{ci}} + k_\parallel v_\parallel$). The sum over the poloidal (m) and toroidal (n) wavenumbers follows from the spectral representation of the RF fields along the periodic coordinates. The wavevector components, $k_\perp$ and $k_\parallel$, and λ_p depend on both m and n. The sum over the harmonic index N accounts for the possible coexistence of multiple IC resonances inside the plasma,i.e. high-harmonic and/or multiple frequencies scenarios. $P_{N;m,n}$ is proportional to the fraction of RF power absorbed by the considered species from TORIC (the index of ion species has been omitted). The coefficient D_0 is determined iteratively by requiring that the quasilinear heating rate is close enough to the absorbed ICRF power predicted by TORIC, P_{RF},

$$\left(\int P_{\mathrm{RF}}\mathrm{d}V\right)^{-1} \int \left| P_{\mathrm{RF}} - \mathrm{m}\int \left[v_\perp D_{\mathrm{RF}}(v_\perp)\left(-\frac{\partial F}{\partial v_\perp}\right)\right] v_\perp \mathrm{d}v_\perp \right| \mathrm{d}V < \varepsilon \tag{2}$$

where P_{RF} is the local RF absorbed power density calculated by TORIC and the second term is the quasilinear heating rate. The threshold ε is specified by the user. $P_{N;m,n}$ entails the resonance kernel of the wave-particle interaction [5].

In SSFPQL the solution is expanded in a truncated series of Legendre polynomials in ξ, whose convergence is ensured by using a multiplication theorem for Bessel functions. The complete expressions of the coefficient D_{RF} in the Legendre representation are given in [4, 6].

To enable SSFPQL to deal with IC heating at more than one frequency simultaneously several non trivial changes have to be made. For the fast (compressional) wave the wave polarization, λ_p, and $\chi_\perp$ depend weakly on n, and even more weakly on m. Thus, in the standard version of SSFPQL for each n the sum over m is simplified with a single contribution evaluated at $m = 0$. This assumption had to be relaxed to allow for multi-harmonic scenarios: full consistency between the value of λ_p and the fields calculated by TORIC has been found necessary to avoid numerical issues which spoil the convergence of consistency iteration between the two codes. In addition, the interface between the two code had to be substantially rewritten to make SSFPQL read and elaborate the data from two or more runs of TORIC for different frequencies. This task is in progress, and has proven more complicated than initially planned.

3. Heuristic Model of the Finite-Orbit-Width Effects in TORIC-SSFPQL

In the standard version of SSFPQL it is assumed that the resonant particles have negligible radial drifts along their orbits. This assumption is particularly restrictive for FIs, whose contribution becomes important at high-harmonic resonances because of the energy dependence of $v_\perp$ in the argument of J_N in (1). In heating problems, FOW effects enter mainly in two ways: 1) radial excursions can make fast ions resonate with the same wave component in separate radial positions [7]; 2) the RF power absorbed by the resonant particles is in principle *available* to all the magnetic surfaces intercepted by the particle orbits. Accounting for these FOW effects in SSPFQL goes beyond the capability of its numerical scheme. However, a way to partially mimic the second FOW effect is to average *a posteriori* the RF power calculated by TORIC over an appropriate set of particle orbits. First of all, the guiding-center following code, GCENTER [7], of SSFPQL is used to calculate the radial position $\rho(\tau|v_i,\xi_j,\rho_k)$ along the orbit, initialized with the (v_i,ξ_j,ρ_k) values of a representative sample of resonant particles. τ is the time along the trajectory. The weight for orbit averaging is the time spend by the resonant particles on each magnetic surface, ρ_r, intercepted by their orbits,

$$\mathscr{W}_{i,j,k;r} = \frac{1}{N_\tau}\sum_{\tau=1}^{N_\tau}\left(1-\frac{\left|\rho(\tau|v_i,\xi_j,\rho_k)-\rho_r\right|}{\Delta\rho}\right)\cdot\begin{cases}1 & |\rho_r-\rho_k|<\Delta\rho\\ 0 & \text{otherwise}\end{cases} \tag{3}$$

$(\sum_{r=1}^{N_\rho}\mathscr{W}_{i,j,k;r}=1)$. Additionally, not all the orbits have the same weight in the orbit average of the RF absorbed power calculated by TORIC. In fact, the orbits that contribute the most are those responsible for the highest RF power absorption. A natural way to account for this is to multiply the weights (3) with the corresponding $v_\perp$ flux induced by RF (2^{nd} term in the integrand of (2)), so that the orbit average becomes

$$\bar{P}_{N;m,n}(\rho_r) = \sum_{k=1}^{N_\rho} P_{N;m,n}(\rho_k)\sum_{i=1}^{N_v}\sum_{j=1}^{N_\xi}\left(\left(|J_{N-1}(\chi_\perp v_\perp)+\lambda_p\, J_{N+1}(\chi_\perp v_\perp)|^2\right)_{m,n} v_\perp\left(-\frac{\partial F}{\partial v_\perp}\right)\right)_{v_i,\xi_j}\mathscr{W}_{i,j,k;r} \tag{4}$$

with the weights normalized to unit sum. In (4), F is the distribution function used in TORIC (the Maxwellian at the beginning of the loop, then the last SSFPQL solution).

There is some degree of arbitrariness in choosing these orbits, except that all trapped particles that do not reach the resonance are manifestly irrelevant for our purpose. The weights need to be evaluated only once before the first SSFPQL call. Figure 1 shows an example of an AUG like case with 1.3T/30MHz with the 3^{rd} D IC resonance at $\rho_{\text{pol}} \approx 0.2$ on the LFS for ≈ 1.3 MW of ICRF absorbed power. About 24% of ICRF power is directly absorbed by D, and the rest is absorbed by electrons. Because of the low RF power density, the loop converges in only three iterations.

The red and blue lines show the profiles of direct D absorption without and with NBI, respectively. The black dashed line is the absorption at the very first step of the loop, when Maxwellians are used, whereas the coloured lines refer to the last step of the loop. Dashed and solid lines are the profiles without and with orbit average, respectively. The latter are the profiles used in calculating the quasilinear operator in SSFPQL. Without NBI there is not a large difference between Maxwellians and F at convergence (black vs red dashed lines). The presence of fast NBI-D broadens the the effective IC resonance, and thus the deposition profiles. The broadening is further substantially increased when FOW effect are included, as it is evident by comparing dashed with solid lines. As a consequence, the peak of absorption around $\rho_{\mathrm{pol}} \approx 0.2$ is substantially decreased in favour of RF power deposition in the plasma periphery.

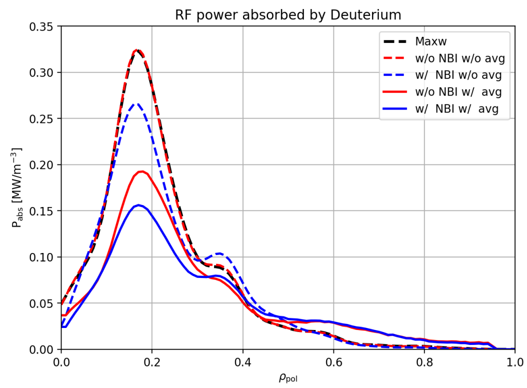

Figure 1: *Direct deuterium RF-power absorption with and without NBI and orbit average in the case of AUG-like discharge with 1.3T/30MHz, i.e. the 3rd Deuterium IC resonance is at $\rho_{\mathrm{pol}} \approx 0.2$.*

4. Conclusions

The interface between TORIC and SSFPQL has been substantially rewritten to allow SSFPQL to elaborate data from two or more runs of TORIC with different frequencies while ensuring consistency between λ_p in SSFPQL and the fields calculated by TORIC. Rewriting of this interface is still in progress, despite the initially optimistic plan. After debugging, TORIC-SSFPQL with the new interface will be validated against the experiments done on AUG with multi-frequencies ICRF [1]. On the other hand, the module for the FOW effects on TORIC absorbed power profiles was adapted from a previous version used in the past for the forward modeling of CXRS diagnostic in the presence of ICRF minority heating [8].

References

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