

PNAS



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2 **Supporting Information for**

3 **Linear resistivity at van Hove singularities in twisted bilayer WSe_2**

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8 **This PDF file includes:**

9 Supporting text

10 Figs. S1 to S5

11 SI References

12 Supporting Information Text

13 **Methods.** All of the flakes, including single layer WSe₂, h-BN, and few-layer graphite, used in this paper were obtained by
14 mechanical exfoliation. The overall fabrication process is the same as described in our previous work (1, 2). Briefly, we first
15 prepare a h-BN/Graphite stamp to place Hall-bar-shaped pre-patterned leads with Cr/Pt (2 nm/20 nm) evaporated on it as
16 shown in the Fig. S1. Next, the twisted bilayer WSe₂ part is fabricated using the ‘tear and pick-up’ method (3, 4). Finally, the
17 h-BN/TB-WSe₂ stamp is placed onto the pre-patterned leads. Additionally, the method used to determine the twist angles of
18 the measured devices is the same as described in our previous work (1, 2).

19 **First-principles calculation.** The electronic properties calculations for twisted bilayer WSe₂ were performed using density
20 functional theory with the Vienna ab initio software package (VASP) (5). The Perdew-Burke-Ernzerhof (PBE) exchange-
21 correlation functionals (6) were employed, along with Tkatchenko-Scheffler (TS) van der Waals corrections (7, 8), which have
22 been shown to yield results consistent with experimental observations in our previous work on WSe₂-based moiré superlattices
23 (1). An energy cutoff of 400 eV for the plane wave basis set and a Γ -centered k-mesh of $1\times 1\times 1$ were used for geometry
24 optimization and electronic structure calculations. To prevent artificial interactions between periodic slab images, a vacuum
25 thickness greater than 15 Å was applied. All atoms were fully relaxed, ensuring a residual force less than 0.01 eV Å⁻¹ per
26 atom. Considering the computational cost of superlattice calculations, while the internal atomic positions were fully optimized,
27 the lattice constant for the moiré supercell was kept fixed at a value corresponding to the optimized lattice constant of 3.30 Å
28 for a 1×1 unit cell with 2H stacking. For all band structure calculations, the spin-orbit coupling (SOC) effect was taken into
29 account due to the presence of heavy elements in the WSe₂ system.

30 **Tight-Binding calculation.** To speed up the calculations and obtain the density of states (DOS) with a high resolution in the
31 momentum space, we adopt the Tight-Binding method for the calculations of DOS. The Tight-Binding calculations are based
32 on a twisted bilayer honeycomb lattice with one orbital for each lattice site to simulate the K-valley states in WSe₂. We adopt
33 parameters fitted to DFT results to ensure the faithful description of the top valence flat band in twisted bilayer WSe₂ (see
34 Fig.S1 and Fig. 13 in Ref. (1)). The lattice constants are the same as those used in the DFT calculations and the interlayer
35 separation is set to be 6.53Å. We employed a hopping term that only depends on the separation between the two sites, similar
36 to the one used for twisted bilayer graphene (9, 10). The onsite energy difference between the two sublattices in each layer is
37 set to be 1.23eV. The displacement field is applied by setting additional onsite energy difference between the atomic sites in the
38 top and the bottom layers with the same parameters as we used in our previous work (1). We apply a Gaussian smearing of
39 2meV in the DOS calculation with the Tight-Binding method and a k-grid as large as 40x40x1 in the supercell Brillouin zone
40 to ensure the convergence of DOS in our calculations.

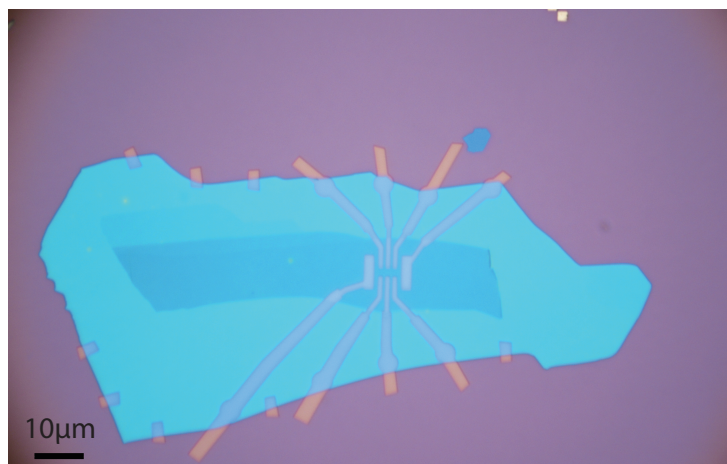
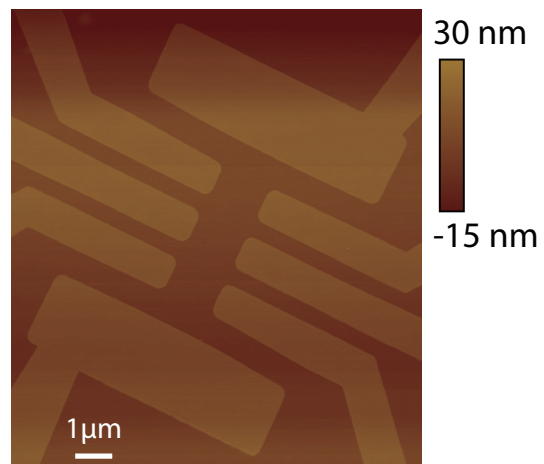
A**B**

Fig. S1. Characterization of the prepattern leads. (A) Optical image of the prepattern leads. (B) AFM image of the prepattern leads in (A)

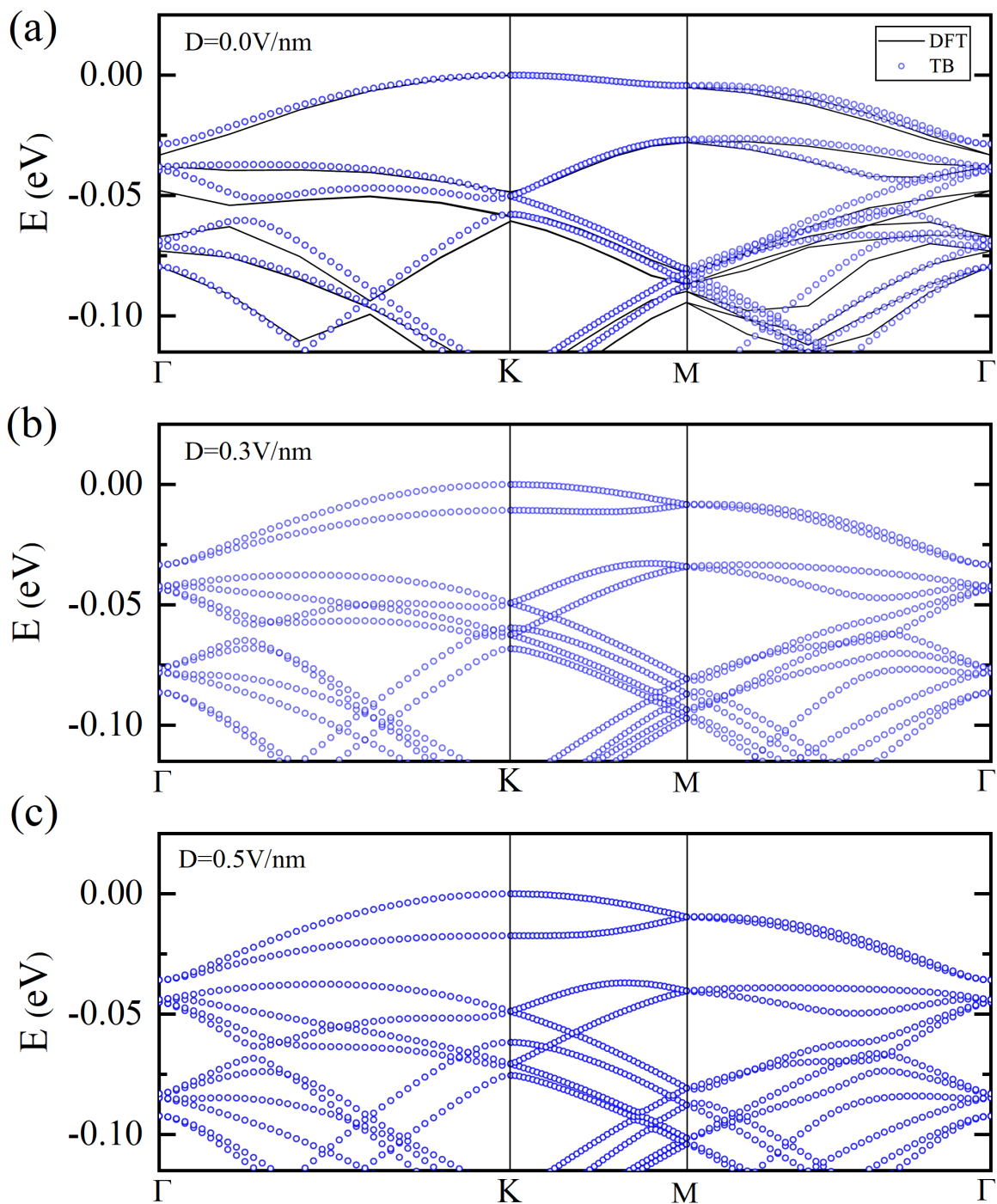


Fig. S2. Band structures of 3.15° twisted bilayer WSe_2 under different displacement fields. The results calculated by density functional theory (DFT) calculations are indicated by solid line, while those calculated by tight-binding (TB) methods are indicated by blue circles.

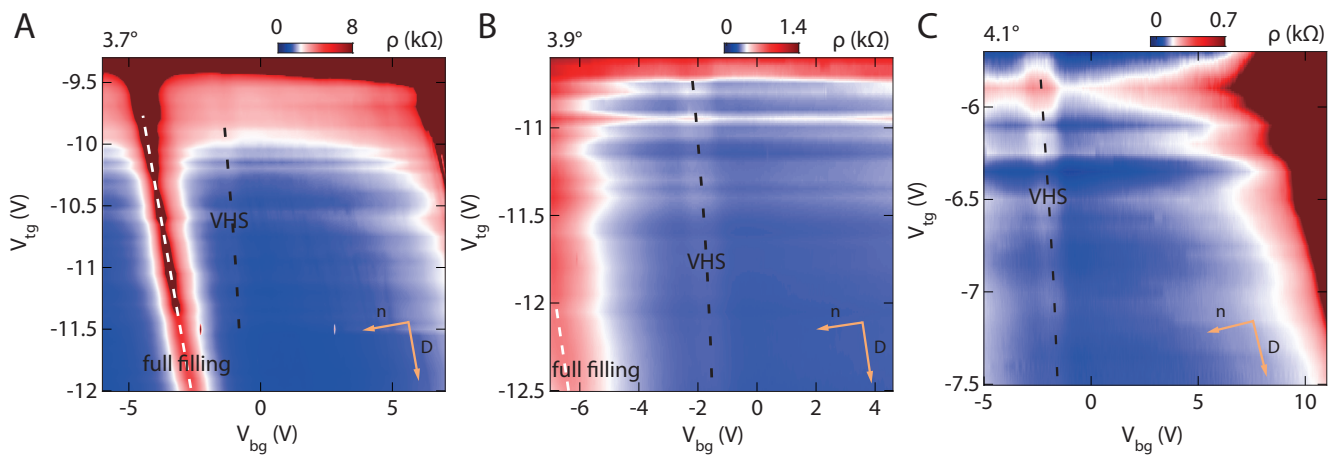


Fig. S3. Resistivity ρ as a function of top gate voltage V_{tg} and back gate voltage V_{bg} in TB-WSe₂ devices with different twisted angles under $T = 1.5$ K. The white and black dashed lines mark the full filling and VHS, respectively.

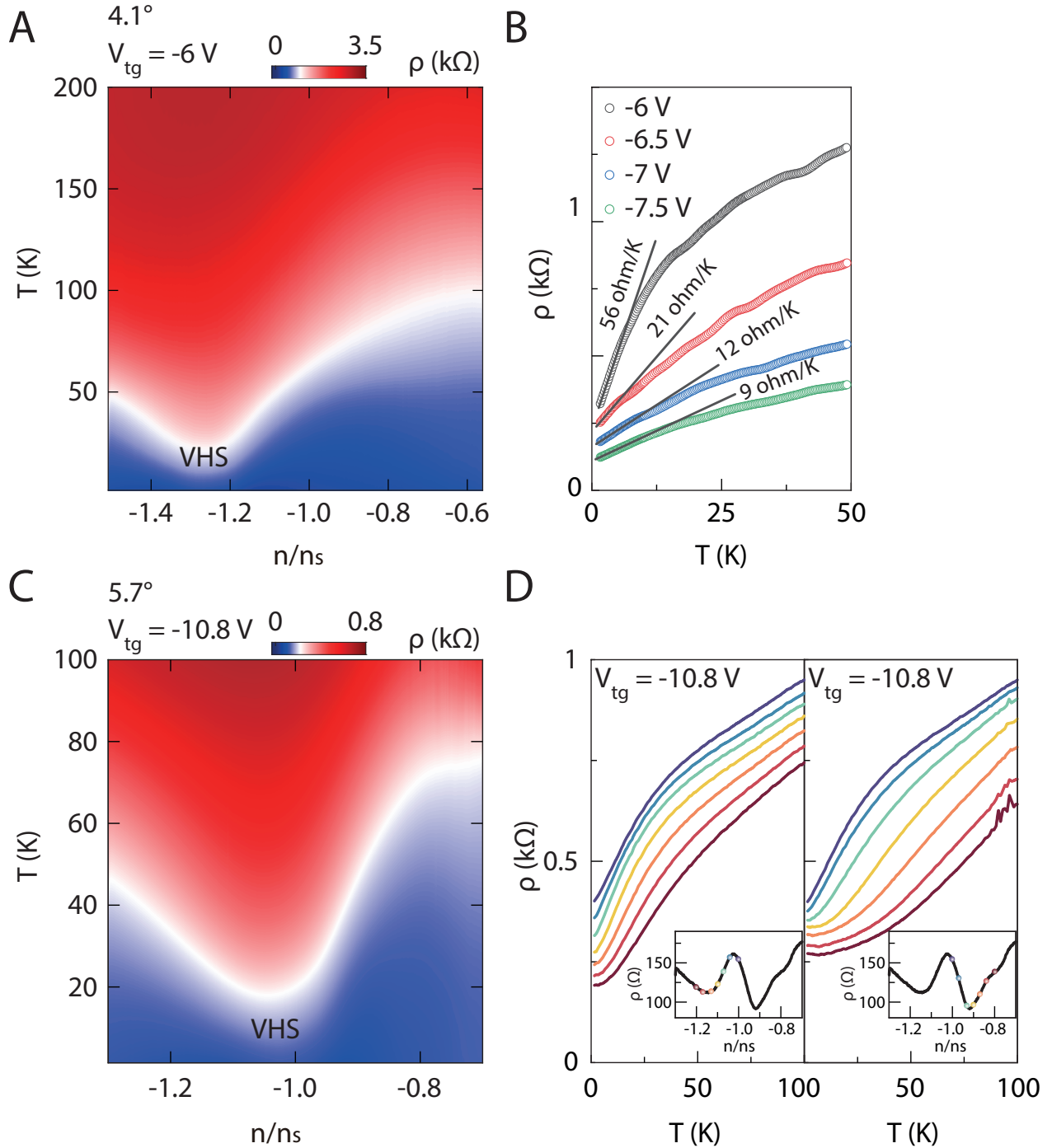


Fig. S4. Temperature dependence of the resistivity ρ in TB-WSe₂. (A) Resistivity ρ as a function of normalized carrier density n/n_s and temperature T at V_{tg} of -6 V in 4.1° TB-WSe₂ device. The resistivity of VHS shows a positive temperature dependence similar to the device in the main text. (B) ρ of the VHS plot against T at different V_{tg} values in 4.1° TB-WSe₂. These curves have offset for clarity. The black lines are fitting lines. (C) Resistivity ρ as a function of normalized carrier density n/n_s and temperature T at V_{tg} of -10.8 V in 5.7° TB-WSe₂. (D) ρ plot against T curves at V_{tg} of -10.8 V at different normalized carrier density in 5.7° TB-WSe₂. Each curve corresponds to a point in the inset figure with the same color. These curves are offset for clarity. As we approach the VHS from higher (lower) doping, the temperature dependence of ρ changes from a Fermi-liquid behavior with a T^2 relation to a T -linear behavior.

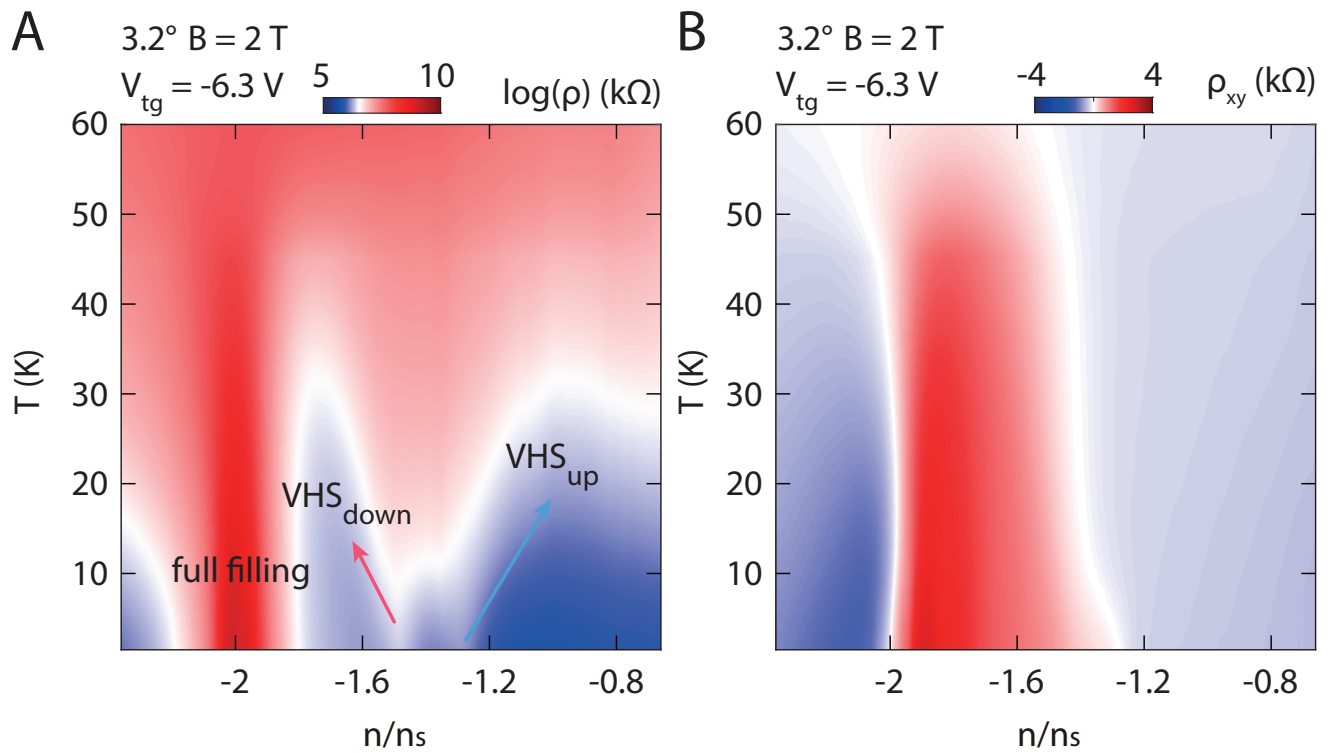


Fig. S5. Temperature dependence of the resistivity ρ and Hall resistivity ρ_{xy} in 3.2° TB-WSe₂ at V_{tg} of -6.3 V and $B = 2$ T.

41 References

- 42 1. L Wang, et al., Correlated electronic phases in twisted bilayer transition metal dichalcogenides. *Nat. materials* **19**, 861–866
43 (2020).
- 44 2. A Ghiotto, et al., Quantum criticality in twisted transition metal dichalcogenides. *Nature* **597**, 345–349 (2021).
- 45 3. L Wang, et al., One-dimensional electrical contact to a two-dimensional material. *Science* **342**, 614–617 (2013).
- 46 4. K Kim, et al., van der Waals heterostructures with high accuracy rotational alignment. *Nano Lett.* **16**, 1989–1995 (2016).
- 47 5. G Kresse, J Furthmüller, Efficient iterative schemes for ab initio total-energy calculations using a plane-wave basis set.
48 *Phys. Rev. B* **54**, 11169 (1996).
- 49 6. PE Blöchl, Projector augmented-wave method. *Phys. Rev. B* **50**, 17953 (1994).
- 50 7. A Tkatchenko, M Scheffler, Accurate molecular van der waals interactions from ground-state electron density and free-atom
51 reference data. *Phys. Rev. Lett.* **102**, 073005 (2009).
- 52 8. O Anatole von Lilienfeld, A Tkatchenko, Two-and three-body interatomic dispersion energy contributions to binding in
53 molecules and solids. *The J. chemical physics* **132** (2010).
- 54 9. G Trambly de Laissardière, D Mayou, L Magaud, Localization of dirac electrons in rotated graphene bilayers. *Nano letters*
55 **10**, 804–808 (2010).
- 56 10. GT De Laissardiere, D Mayou, L Magaud, Numerical studies of confined states in rotated bilayers of graphene. *Phys. Rev.*
57 *B* **86**, 125413 (2012).