Numerical techniques for computational magneto-hydrodynamics: application to gas-plasma interactions in tokamaks

A. Bhole[1], B. Nkonga[1], S. Pamela[2], M. Hoelzl[3], G. Huijmans[4,5], JOREK Team[1] & JET Contributors*

[1] *Université Côte d'Azur & INRIA/CASTOR, CNRS/LJAD, 06108 Nice, France*

[2] *CCFE, Culham, United Kingdom of Great Britain and Northern Ireland*

[3] *Max Planck Institute for Plasma Physics, Garching b. M. and Greifswald, Germany*

[4] *CEA, IRFM, F-13108 Staint-Paul-lez-Durance, France*

[5] *Eindhoven University of Technology, Eindhoven, Netherlands*

* *See the author list of J. Mailloux et al 2022 Nucl. Fusion 62 042026*

ashish.bhole@inria.fr

Introduction

Gas-plasma interaction (GPI) is of fundamental interest regarding Massive Material Injection (MMI) technology for Disruptions Mitigation Systems (DMS) in fusion reactors. MHD dynamics of thermal quench (TQ) and current quench (CQ) occurring during mitigated disruptions is highly complex and is associated with the destabilization of core modes, including the internal kink (1/1) mode. Understanding these MHD instabilities is crucial for ITER DMS goals and hence we need predictive GPI simulation tools. We present here our recent developments for numerical simulations of GPI that we have implemented in JOREK, a non-linear MHD code for tokamak and stellarator modeling [1].

Full MHD models

We consider full MHD modeling to describe the fusion plasma, as opposed to reduced models that are frequently used in JOREK simulations. The former include fast magnetosonic waves and hence, from a computational cost point of view, are more expensive than the latter. However, full MHD modeling is essential to capture the 1/1 mode accurately in finite-β-plasmas which can play an important role in mitigated disruptions dynamics. For GPI modeling, we extend full MHD models to include neutrals and impurities transport [3]. The complete modeling can be found in [5]. We now have a family of models with single or two temperature models with neutrals or impurities transport to simulate Massive Gas Injection (MGI) or mono or multiple Shattered Pellet Injections (SPI) in fusion plasma.

Numerical Techniques

JOREK uses high-order Hermite-Bézier (HB) finite element methods (FEMs) [1, 4] to discretize the physical models. These methods do not provide any stabilization mechanism needed to simulate nonlinear, multi-scale, and convection-dominated GPI physics. To account for the effects of unresolved scales we add two numerical stabilization mechanisms: variable-multiscale (VMS) based stabilization that provides L^2 stability and shock-capturing terms for bounded variations (BV) stability [5]. These numerical stabilization methods are residual-based. Given the complexity of physical models, there can be a hierarchy of the stabilized methods [5], depending on how well the residual is approximated.

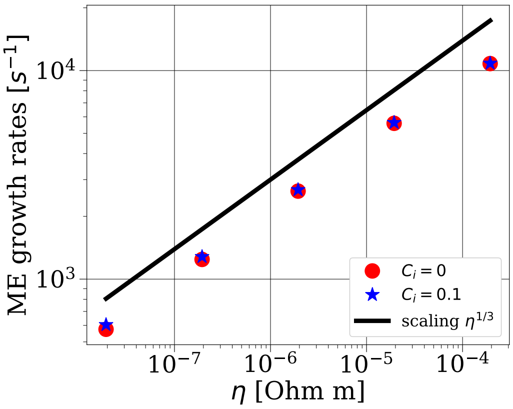

Figure 1: *Linear simulations of the internal kink modes. C_i denotes the multiplication coefficient in VMS-based stabilization.*

Figure (1) demonstrates the application of VMS-stabilized FEM in predicting growth rates of the internal kink modes. Both FEM and VMS-stabilized algorithms follow the theoretical scaling of the internal kink modes implying that the VMS-stabilized method does not alter the underlying physics.

Figure (2) demonstrates the use of VMS and shock-capturing stabilized FEM in simulating the 2D Orszag-Tang vortex. This is a standard but difficult test often performed to verify astrophysical/MHD codes. The left window shows the density field at $t = 0.2$ while the right window shows the values of the shock-capturing stabilization coefficient. We notice that the shock-capturing is active only near the discontinuities in the density field.

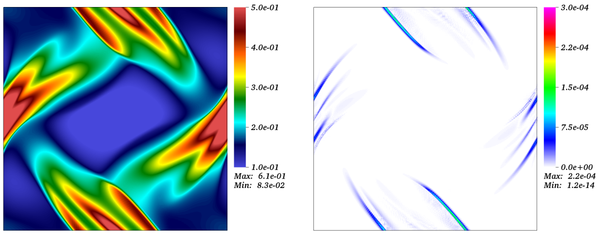

Figure 2: *2D Orszag-Tang vortex simulation using stabilized FEM. Left: density and right: locations where shock-capturing is active at $t = 0.2$.*

In [5] we have demonstrated the application of stabilized methods on a range of problems of hydrodynamics and MHD interests. We demonstrate that our stabilized algorithm converges with first-order accuracy. Sophisticated grids generated by JOREK may contain some geometrically singular points. We have developed a treatment to be applied at such points to improve their numerical properties [6]. We use all these numerical techniques for GPI simulations for MMI experiments.

Application to Gas-plasma interactions

We discuss here simulations of neon SPI in JET-like plasma. We consider that a pellet of neon gas is shattered into 100 fragments that travel in plasma with SPI velocity (V_{SPI}) m/s. We consider close to realistic values of resistivity with $\eta_0 = 4.22 \times 10^{-8}$ Ω-m and Spitzer-like temperature dependence.

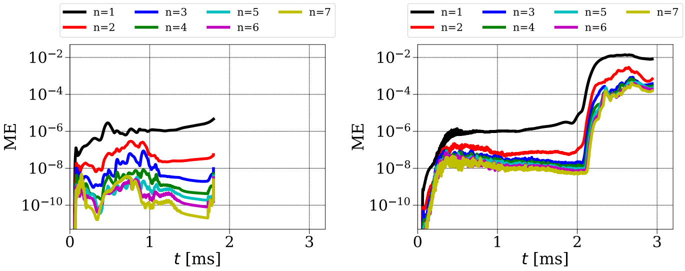

Figure 3: *Evolution of magnetic energies in the difference Fourier modes in 3D neon SPI simulations, left: without, right: with stabilized FEM*

Single-temperature full MHD model with impurities transport is considered. The plasma domain is discretized using flux-aligned grids with 7645 elements. Gears time integration method is used. An iterative GMRES solver is used with hybrid MPI and OpenMP parallelism. VMS stabilization based on the advective part of the MHD equation is added. The pressure gradient is used to detect the shock/discontinuity locations and the stabilization is added only at nearby locations. SPI configuration used is described in [5]. In 2D as well as 3D simulations we notice that numerical stabilization is crucial in capturing GPI physics. In the 3D simulations, we represent the variables using the first 7 Fourier harmonics in the toroidal direction. Figure (3) shows a comparison of magnetic energies in Fourier modes during 3D SPI simulations without and with numerical stabilization. After $t = 2$ ms the stabilized method captures the destabilization of core modes where the energies in the harmonics sharply increase. During the course of these simulations, as MHD dynamics increase, the time steps are needed to be reduced in order to remain in the numerical stability region.

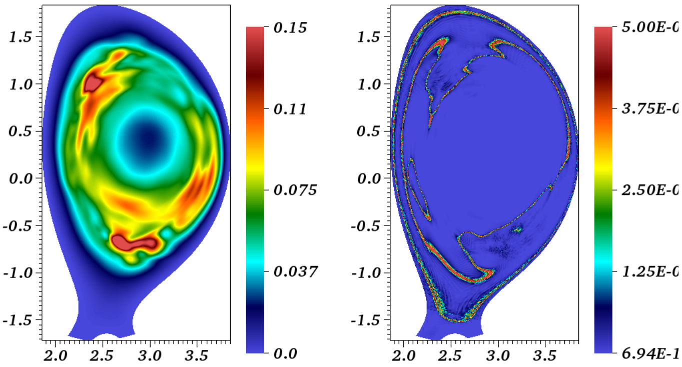

Figure 4: *Right: Impurity particle density [10^{20}] field at $t = 2.89$ ms plotted in $\phi = 0$ plane. Left: Shock-capturing stabilization coefficient denoting the locations of discontinuities in the pressure field.*

The right frame in Fig. (4) denotes the impurity density field plotted at $t = 2.89$ ms in the poloidal plane $\phi = 0$. Impurities appeared to have spread in most of the plasma domain causing discontinuities in the total pressure (ions+impurities) field. Shock-capturing strategy detects these discontinuities and adds numerical stabilization only nearby. The left frame of Fig (4) shows the values of the shock-capturing stabilization coefficient. Indeed stabilization is being added adaptively only where it is needed.

Figure (5) denotes the Poincaré plots of the magnetic field at some time instants where the color denotes values of T in keV and black dots denote the projected locations of the SPI fragments. We observe that as fragments reach the plasma core, core modes start to destabilize. The growth of 2/1 mode can be distinctly seen in the middle frame of Fig. (5). This is followed by complete ergodization of the plasma. During these dynamics, the plasma is suddenly cooled and undergoes TQ. At the end of TQ, a sign of plasma current (I_p)-spike [5] is also seen.

Summary and future scope

With full MHD modeling of the fusion plasma, we can accurately simulate 1/1 mode at finite-β-plasma and hence we can perform GPI simulations relevant to MMI with JOREK more accurately than before. The role of numerical stabilization [5] is crucial in simulating non-linear, convection-dominated, multi-scale flows with strong variations in the flow fields. Preliminary simulations presented here show the promising ability of our developments to capture highly complex TQ physics. Experimental validation is an ongoing work. The developments presented here are highly relevant to ITER DMS goals. They are also applicable to other fusion plasma problems such as Vertical Displacement Events (VDEs) and can easily be extended for stellarator applications.

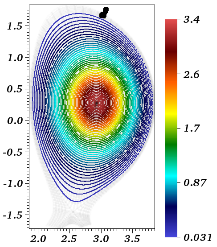

Figure 5: *Poincare plots of the magnetic field at t = 0.035, 2.25 and 2.93 ms. Black dots denote the projected locations of SPI fragments.*

References

[1] M. Hoelzl, et al., NF **61**, 065001 (2021)
[2] S. Pamela, et al., Phys. Plasmas **27**, 102510 (2020)
[3] D. Hu, et al., NF **61**, 026015 (2021)
[4] S. Pamela, et al., J. Comput. Phys. **464**, 111101 (2022)
[5] A. Bhole, et al., CAMWA **142**, 225-256 (2023)
[6] A. Bhole, et al., J. Comput. Phys. **471**, 111611 (2022)

Acknowledgements

This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them. This work has been accomplished on JFRS-1 and Marconi-Fusion HPC.