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Probing fundamental physics with Extreme Mass Ratio Inspirals: a full Bayesian inference for scalar charge

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Extreme Mass Ratio Inspirals (EMRIs) are key sources for the future space-based gravitational wave detector LISA, and are considered promising probes of fundamental physics. Here, we present the first complete Bayesian analysis of EMRI signals in theories with an additional massless scalar, which could arise in an extension of General Relativity or of the Standard Model of Particle Physics. We develop a waveform model accurate at adiabatic order for equatorial eccentric orbits around spinning black holes. Using full Bayesian inference, we forecast LISA's ability to probe the presence of new fundamental fields with EMRI observations.

Introduction. In the last decade, gravitational wave (GW) astronomy has revolutionized our ability to observe the Universe, offering unique opportunities to test the nature of gravity and search for new fundamental fields in previously unexplored regimes [1–3]. Observations with gravitational-wave detectors allow us to probe the highly dynamic and strong-field regime of compact binary coalescences.

Ground-based interferometers have paved the way for searches for new physics beyond General Relativity (GR) or the Standard Model (SM) [4]. The constraining power of current facilities is limited to signal-to-noise ratio up to 30 [5], comparable mass binaries, with the most asymmetric system detected so far having a mass ratio of $q \sim 1/10$ with GW190814 [6–8]. The next generation of ground-based detectors, such as the Einstein Telescope [9] and Cosmic Explorer [10], along with space-based missions like the Laser Interferometer Space Antenna (LISA) [11], TianQin [12], and Lunar Gravitational Wave Antenna [13], are expected to observe sources across a broader mass range. Probing fundamental physics is featured prominently in science cases for future detectors [2, 3, 14–16].

LISA is expected to detect gravitational waves from Extreme Mass Ratio Inspirals: binary systems composed of stellar-mass compact objects (the secondary), inspiralling into massive black holes at the center of galaxies (the primary). Due to their small mass ratios $q \lesssim 10^{-4}$, EMRIs perform tens of thousands of cycles on highly relativistic trajectories with large inclination and orbital eccentricities within the LISA sensitivity band. The orbit complexity results in gravitational signals with multiple harmonics. The rich harmonic content and the many cycles of EMRI signals allow sub-percent parameter measurement precision, rendering EMRIs natural laboratories to test gravity and offering a unique opportunity to probe fundamental physics in unprecedented regimes [2]. These same characteristics also make the modeling and generation of EMRI waveforms particularly challenging.

A variety of studies have investigated the scientific potential of such sources to test the nature of black holes (BHs) [17–42], the propagation speed of gravity [43, 44], and the existence of new fundamental fields [45–64]. While testing gravity with EMRIs is a key science goal for LISA, calculations of waveform models beyond GR are in their infancy. The majority of works carried out so far have resorted to ad-hoc modifications of GR templates, adopting hybrid schemes based, for example, on post-Newtonian (PN) expansions [65]. Moreover, such studies have mainly focused on the potential of EMRI observations to identify deviations from the spacetime of the primary, forecasting — in some cases — constraints on such parameters from LISA observations [17, 48].

Contrary to the standard lore, which has the EMRI acting as a probe for the spacetime of the primary, Ref. [51] demonstrated that changes from the Kerr metric of the primary can be neglected at leading order in the mass ratio for a large family of theories with scalar fields non-minimally coupled to gravity. It is instead the scalar *charge* of the much lighter secondary that can leave a significant imprint on the GW emission. This is because for massless scalars, the scalar charge, if any, is inversely proportional to the square of the mass of the black hole [66–70]. The framework developed in Ref. [51] allows the construction of waveforms that are correct at the leading adiabatic order (first order in the mass ratio) and for which deviations from GR are uniquely determined by the scalar charge of the secondary. This framework has recently been framed into a consistent approach to compute post-adiabatic (second order in the mass ratio) waveform corrections to the GR baseline model [70].

This formalism has been exploited to study changes in the GW fluxes for binaries on eccentric equatorial [53] and circular inclined orbits [55, 71] for massless scalar fields, and for circular equatorial inspirals in the case of massive scalars [54]. Preliminary analyses have also assessed LISA's capability to infer the measurement precision of the scalar charge using Fisher information matrix calculations [52, 54, 72–74].

Here, we provide the first implementation of adiabatic waveforms for EMRIs with scalar fields, in eccentric equatorial orbits around spinning BHs. We perform a Bayesian analysis on all the waveform parameters to assess LISA's ability to detect the scalar charge, and, therefore, probe deviations from General Relativity and the Standard Model. Our analysis is state-of-the-art in multiple ways: it uses the most accurate waveforms at adiabatic order and includes eccentricity; the inference is based on Markov Chain Monte Carlo (MCMC) sampling; and it faithfully includes the effects of a new fundamental field at adiabatic order.

Our results show that a single EMRI is able to constrain the scalar charge of the secondary, with precision of 10% in a theory-agnostic way, i.e., independent of the origin of the scalar field. This also determines LISA's ability to probe the nature of black holes and indirectly probes the existence of additional GW polarizations. Moreover, if one selects a specific theory, a constraint of the charge can be converted into a bound on a coupling constant of this theory, which controls deviations from GR. As a characteristic example, we consider here the case of linear-Gauss–Bonnet gravity.

EMRIs and fundamental fields. In this Section, we briefly summarise the theoretical approach we use to model EMRIs with scalar fields [51–53, 55]. We refer the reader to [70] for a detailed description of the formalism and its extension within a Self-Force (SF) scheme.

We consider theories with a single massless scalar field ϕ , non-minimally coupled to the metric tensor **g**, described by the following action

$$\int d^4x \frac{\sqrt{-g}}{16\pi} \left[R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \right] + \alpha_c S_c \left[\mathbf{g}, \phi \right] + S_m \left[\mathbf{g}, \phi, \Psi \right] ,$$
(1)

where R is the Ricci scalar and g is the metric determinant. $S_{\rm m}$ describes the dynamics of the matter fields Ψ . The action $\alpha_c S_c$ describes (each of) the scalar field interactions, with the constant α_c having dimensions $(\text{mass})^n$, with n > 1. In physical units, this corresponds to interactions that are suppressed by a characteristic energy scale [69]. Varying with respect to the metric and the scalar field, we obtain the equations for the fields

$$G_{\mu\nu} = 8\pi T_{\mu\nu}^{\rm scal} + \alpha_c T_{\mu\nu}^{\rm c} + T_{\mu\nu}^{\rm m} \quad , \quad \Box \phi = T^{\rm c} + T^{\rm m} \; , \; (2)$$

where $\Box = \nabla_{\mu} \nabla^{\mu}, \ T_{\mu\nu}^{\text{scal}} = \frac{1}{16\pi} \left[\partial_{\mu} \phi \partial_{\nu} \phi - \frac{1}{2} g_{\mu\nu} (\partial \phi)^2 \right]$ and

$$T^{\rm c,m}_{\mu\nu} = -\frac{16\pi}{\sqrt{-g}} \frac{\delta S_{\rm c,m}}{\delta g^{\mu\nu}} \quad , \quad T^{\rm c,m} = -\frac{16\pi}{\sqrt{-g}} \frac{\delta S_{\rm c,m}}{\delta \phi} \quad . \tag{3}$$

Since we are considering a massless scalar, we will assume that S_c respects shift symmetry, $\phi \rightarrow \phi + \text{constant}$. However, our approach can be generalised to light scalars [54]. We focus on EMRIs in which the primary is a BH of mass M, and assume that solutions in theories controlled by (1) are continuously connected to GR solutions for $\alpha_c \rightarrow 0$. For shift-symmetric scalars, the scalar charge for black holes, if any [75–80], is fixed in terms of their mass, spin and the coupling constants of the theory [69]. Hence, M and α_c are the only meaningful physical scales of this problem. Their ratio can be expressed as

$$\zeta = \frac{\alpha_c}{M^n} = q^n \frac{\alpha_c}{\mu^n} , \qquad (4)$$

where μ is the mass of the secondary, and $q = \mu/M$. Existing bounds already require that $\alpha_c/\mu^n \leq \mathcal{O}(1)$ [81], so ζ is order q^n . One can then use q as a single bookkeeping parameter, as for the SF approach in GR.

This introduces several simplifications in the description of EMRIs ¹. Indeed, by expanding the metric and the scalar field in powers of q,

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + qh^{(1)}_{\mu\nu} + \dots , \ \phi = \phi^{(0)} + q\phi^{(1)} + \dots , \ (5)$$

it was recently shown how to derive a consistent SF formalism that includes post-adiabatic corrections to the binary dynamics [70]. In this paper, we focus on the leading EMRI dissipative contribution, which is fully determined by the linear order perturbations $h^{(1)}_{\mu\nu}$ and $\phi^{(1)}$. Equation (4) implies that: (i) the background spacetime is suitably described by the Kerr metric, with beyond GR deviations being $\mathcal{O}(q^{2n})$ corrections to $g^{(0)}_{\mu\nu}$, (ii) $\phi^{(0)}$ is constant due to the no-hair theorem [75–80] and can be set to zero by a shift, (iii) at adiabatic order, metric and scalar field perturbations induced by the secondary decouple, leading to a separate set of equations:

$$G^{\alpha\beta}[h^{(1)}_{\alpha\beta}] = 8\pi\mu \int \frac{\delta^{(4)}\left(x - y_p(\lambda)\right)}{\sqrt{-g}} \frac{\mathrm{d}y_p^{\alpha}}{\mathrm{d}\lambda} \frac{\mathrm{d}y_p^{\beta}}{\mathrm{d}\lambda} \mathrm{d}\lambda , \quad (6)$$

$$\Box \phi^{(1)} = -4\pi d\,\mu \int \frac{\delta^{(4)}\left(x - y_p(\lambda)\right)}{\sqrt{-g}} \mathrm{d}\lambda \,, \tag{7}$$

 $^{^1}$ The approach extends to less asymmetric binaries, like Intermediate Mass Ratio Inspirals, so long as ζ remains a perturbative parameter.

where $G_{\alpha\beta}$ is the Einstein tensor and $dy_p^{\mu}/d\lambda$ is the four velocity of the secondary, along its worldline.

Eqs. (6) are identical to GR, while Eq. (7) determines the scalar field evolution and depends on the scalar charge of the secondary, d, which enters as the only extra EMRI parameter. The solution for $h_{\mu\nu}^{(1)}$ and $\phi^{(1)}$ allows the energy and angular momentum fluxes to be computed for the gravitational (grav) sector, $(\dot{E}_{\text{grav}}, \dot{L}_{\text{grav}})$, and the scalar sector (scal) $(\dot{E}_{scal}, \dot{L}_{scal})$. Beyond GR modifications to the EMRI evolution are *uniquely* controlled by the latter. At this order in mass ratio, the gravitational waveform amplitudes are the same as in GR, whereas the waveform phase is affected by the extra channel of emission. In fact, if the charge is not zero, the secondary plunges faster than in GR. In this work, we consider systems on equatorial eccentric orbits [53], such that E, Ldepend on the semi-latus rectum p, on the eccentricity e, and on the primary spin a.

For a given theory, there is a mapping between d and the coupling constant(s) α_c . This allows constraints on d to be translated into bounds on such couplings. As an example, we will consider here linear Gauss Bonnet Gravity (GB) [66], for which

$$\alpha_c S_c = \frac{\alpha}{4} \int d^4 x \frac{\sqrt{-g}}{16\pi} \phi \mathcal{G} , \qquad (8)$$

where $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$ is the Gauss Bonnet invariant, $R_{\alpha\beta\mu\nu}$, $R_{\alpha\beta}$ are the Riemann and Ricci tensors, and α has dimensions of mass squared (n = 2). For this theory $\alpha \simeq 2d\mu^2$ [82]. In Appendix A, we review different normalizations of the action and obtain the relation between the scalar charge d and the coupling constant $\sqrt{\alpha}$.

Waveform modelling and data analysis setup. We develop a waveform model for EMRIs [83–98] that accounts for the scalar emission and implement it within the FastEMRIWaveform (FEW) package, which allows for fast generation of EMRI templates on Graphics Processing Units (GPUs). Since the FEW package has not yet been extended to Kerr eccentric equatorial orbits in GR, we provide the trajectory model for such orbits here for the first time, before adding the scalar field contribution.

In the first order in the mass ratio, the orbital evolution is obtained by solving the following system of ordinary differential equations:

$$\frac{dJ}{dt} = qf_J \quad , \quad \frac{d\Phi_i}{dt} = \frac{\Omega_i}{M} \qquad J = \{p, e\} \; , \qquad (9)$$

where $\Omega_{i=r,\phi}$ are the dimensionless fundamental frequencies of the Kerr spacetime, that depend on the BH spin a and on the orbital elements $\{p, e\}$ [99, 100]. Equations (9) can be integrated given the initial conditions for the semi-latus rectum p_0 , for the eccentricity e_0 , and for the phases $(\Phi_{\phi 0}, \Phi_{r0})$ (see Appendix E for a study of the ordinary differential equations' accuracy). The ordinary differential equation is integrated until we reach the separatrix plus a threshold of 0.1M. We call these plunging orbits.

The orbital-element fluxes, $f_{p,e}$, are written in terms of energy and angular momentum fluxes. For example, for the semi-latus rectum, we have $f_p = (\partial p/\partial E)\dot{E} + (\partial p/\partial L)\dot{L}$, where

$$\dot{E} = \dot{E}_{\text{grav}} + d^2 \dot{E}_{\text{scal}} \quad , \quad \dot{L} = \dot{L}_{\text{grav}} + d^2 \dot{L}_{\text{scal}} \,, \quad (10)$$

where both the gravitational and scalar fluxes are the sum of the horizon and infinity fluxes and have been computed² using the code of [101, 102] and packages of the Black Hole Perturbation Toolkit [103], using the mode summation implemented in [104]. Angular momentum and energy fluxes are computed on a 3d grid in (a, p, e), and interpolated using Chebyshev polynomials [105] (see Appendix B for details).

Once the trajectory is implemented, we can pass it to the Augmented Analytical Kludge (AAK) [106] waveform amplitude model implemented³ in FEW [107–110]. The AAK waveform is GPU-accelerated and allows the exploitation of the long-wavelength approximation of the LISA response, providing a generic model for investigating tests of GR. The typical waveform generation speed of this new time-domain model is of order 0.1 seconds.

Our trajectory is fully relativistic at adiabatic order, and therefore the measurement precision of the intrinsic parameters $\Theta_i = (\ln M, \ln \mu, a, p_0, e_0, \Phi_{\phi 0}, \Phi_{r0}, d)$ is not strongly affected by the choice of the AAK template [110]. The inclusion of post-adiabatic corrections, and the study of how such terms will affect the scalar charge detectability will be considered in a followup work [85, 111].

The AAK amplitudes and the LISA response may, however, affect the accurate reconstruction of the extrinsic parameters Θ_e defined here by the luminosity distance d_L , the polar and azimuthal sky location angles, (θ_S, ϕ_S) , and the polar and azimuthal orientation angles (θ_K, ϕ_K) that determine the orientation of the primary spin (both set of angles are expressed with respect to the Solar System barycenter reference frame [110]). We have checked using Fisher matrices and MCMC that our results are unchanged when using the full LISA response, and an early implementation of the relativistic amplitudes.

To forecast the constraints on the scalar charge with EMRI observations we sample over all the intrinsic Θ_i and extrinsic parameters Θ_e of a Kerr equatorial eccentric EMRI signal with scalar charge. We obtain the

² Note that the scalar fluxes $\dot{E}_{\rm scal}$ and $\dot{L}_{\rm scal}$ derived from the Black Hole Perturbation Toolkit and in [101] must be divided by a factor 4, to account for a different normalization of the scalar field used in the action (1).

 $^{^3}$ In this study we did not use the relativistic version of FEW since that version of the model is still under revision.



FIG. 1. Histograms of posterior samples of the scalar charge inferred by LISA observations of EMRIs with different orbital configurations of central black hole mass M and spin a, compact object mass μ , initial eccentricity e_0 and time to plunge T. The colored vertical dashed lines show the one-sided 95% credible interval of the distribution. All EMRI systems are characterized by an SNR of 50.

13-dimensional posterior distribution using the package Eryn [112], which provides a Bayesian inference tool based on Markov Chain Monte Carlo sampling. The technical details about the likelihood, priors, and sampling techniques are extensively discussed in Appendix C. This is the first appearance of a complete Bayesian parameter inference of EMRI systems in these types of orbits. Moreover, it is the first analysis to include beyond GR corrections that are accurate at adiabatic order.

Results. We consider eight different orbital configurations, specified by their component masses, the primary spin, the initial eccentricity and semi-latus rectum. We assume binaries evolve in the LISA band, plunging over a period T. This fixes the initial semi-latus rectum p_0 . For each system, the luminosity distance is fixed to give SNR= 50.

We first focus on *agnostic* forecasts of new fundamental fields, assessing LISA's ability to constrain the scalar charge. To this aim, we study the case in which the injected signal is modelled in GR, i.e., assuming d = 0, while the recovery template includes the scalar charge. This setup allows us to investigate the upper bound (or constraint) on d, which we define as given by the upper 95% credible interval of the corresponding marginalized posterior.

Fig. 1 shows histograms of the marginalized posteriors of the scalar charge for the EMRIs we considered (see Fig. 6 of Appendix C for the full posterior). Bounds on d are tighter for large eccentricity and primary spin. For fixed component masses and evolution time, doubling the eccentricity from $e_0 = 0.2$ to $e_0 = 0.4$ yields a 10% stronger bound on d (cf. systems 6 and 7 in Fig. 1). We find the same level of improvement when increasing the BH spin from a = 0.8 to a = 0.95 (systems 5 and 6). However, the precision might vary differently for lower eccentricity binaries, see for instance Fig. 9 of [73].

We also find that increasing the mass ratio provides narrower posteriors. Assuming $\mu = 10 M_{\odot}$, if we reduce the primary mass by a factor of two, we obtain a bound on d that is ~ 50% tighter (systems 4 and 6). This is primarily because a less asymmetric system plunges faster. For a fixed evolution time, such binaries have larger initial orbital separations, where the effect of scalar emission is stronger. To illustrate this point, we analyze a system with component masses $M = 10^5 M_{\odot}$ and $\mu = 5 M_{\odot}$ (system 1). This system has the largest initial semilatus rectum $p_0 \approx 16$ and, although we consider only the last half a year before the plunge for computational reasons, it yields the best 95% upper bound on d, $d_{95\%} =$ 0.015.

Fixing the intrinsic source parameters, the measurement precision improves for EMRIs evolving over a longer timescale. For a $10^6 M_{\odot} + 10 M_{\odot}$ system, doubling of Timproves the constraint on d by 60%, and increases the number of cycles from 1.1×10^5 to 1.7×10^5 , and the semi-latus rectum from $p_0 = 8.34$ to $p_0 = 10$.

We now explore the case in which *both* the injected and the recovery waveforms have a non-vanishing scalar charge. We inject a signal with d = 0.025, consistent with the upper bound from GW230529 ($d \approx 0.035$), and study constraints on the charge for a $10^5 M_{\odot} + 5 M_{\odot}$ EMRI, with the same orbital parameters as system 1 in Fig. 1. This binary provides a measurement of the charge accurate to ~ 10%, with median and 95% credible intervals of $d = 0.0244^{+0.006}_{-0.007}$. The marginalized posterior of d for this system is shown in Fig. 2.



FIG. 2. Marginalized posterior distribution of an EMRI system with scalar charge d = 0.025 and source parameters $M = 10^5 M_{\odot}, \mu = 5 M_{\odot}, a = 0.95, e_0 = 0.4, T = 2$ yrs and SNR=50. The estimated median and 95% credible interval are $0.0244^{+0.006}_{-0.007}$.

For the same system, we also explore the impact of ignoring the scalar charge and fitting the data with a GR template (see Appendix F for further details). We find that the GR waveform recovers the injected signal with $2-3\sigma$ biases in the source intrinsic parameters, i.e., its masses and spins. While such systematic errors are large compared to the size of the posterior, they can be considered small for astrophysically motivated studies.

To illustrate how a bound in d can be converted to a constraint on a specific theory, we now consider GB gravity defined by the action in Eq. (8). The forecasted constraints on the coupling constant are shown in Fig. 3. An interesting feature is that the strongest bound for $\sqrt{\alpha}$ comes from system 2, while the strongest bound for dcame from system 1. This is because the relation between d and α involves the mass of the secondary.

Selecting a specific theory also allows for a comparison between bounds from EMRIs and bounds from other systems. The analysis of the event GW230529 $\left[7,\,113,\,114\right]$ yielded $\sqrt{\alpha_{95\%}} = 1.4$ km, which is a few percent larger than the EMRI constraint obtained with systems 1, 2, and 3. Interestingly, the forecasted best constraint on $\sqrt{\alpha}$ for LVK Voyager is larger (see extremal bound from Figure 21 of [115]). The initial observational frequency of GW230529 is 20 Hz, and the total system mass is 5.1 M_{\odot} and its SNR=11.1 [7]. The dimensionless velocity of GW230529 is $v = (\pi M_{\rm tot} f)^{1/3} \approx 0.117$, whereas the same initial velocity for the lowest total mass EMRI system, $10^5 M_{\odot} + 5 M_{\odot}$, is $v = (\Omega_{\phi})^{1/3} \approx 0.23$. This means that GW230529 is in a weaker field compared to the EMRI systems we considered. In fact, we expect that if we were to consider non-plunging EMRIs with initial dimensionless velocity $v \approx 0.1$ and $p_0 \approx 60$, the EMRI constraints would further improve. We expect that intermediate-mass ratio inspirals with sufficiently small secondaries can provide even tighter constraints on fundamental fields since their observed inspiral can start in much weaker field regions.

Discussion. In this work, we produced the first readyto-use fully relativistic EMRI waveforms in theories of gravity that include a massless scalar field and perform the first Bayesian analysis of EMRI signals in this context. Our waveforms are correct at the adiabatic order and include a single extra parameter, the scalar charge of the EMRI secondary. This is the only parameter needed to capture the effect of the scalar field at this order [51].

Our study provides Bayesian methods and gravitational wave models to investigate the fundamental physics potential of LISA observations of EMRIs. These are two key objectives outlined in the LISA definition study report [16] and the fundamental physics white paper [3]. We also provide an approximate comparison of EMRI constraints on agnostic PN deviations in the phase in Appendix D, where we find that EMRIs provide three orders of magnitude improvement compared to current detectors. The methodologies and findings presented in



FIG. 3. Posterior distribution of the Gauss-Bonnet coupling mapped from the scalar charge constraints (see Fig. 1). The black dotted line in the right panel shows the bound [113] from the observation of the gravitational wave event GW230529 [7], while the black solid line shows the best forecasted bound for LVK Voyager configuration obtained in [115]. We use a different normalization with respect to [113, 115] (see Appendix A).

this paper will play a crucial role in guiding the gravitational wave fundamental physics community.

The main result of our analysis is a forecast of LISA's ability to measure or place a bound on the scalar charge of the secondary, d, in a theory-agnostic manner. This can be interpreted in multiple ways: as a way to look for new fundamental fields; as a way to probe the nature of BHs, by probing the structure of the secondary; and as an indirect test of additional gravitational wave polarizations, as the effects we probe are related to the additional scalar emission.

Bounds on d can be translated to bounds on the coupling constant(s) or a particular theory. We have demonstrated this for a specific example, that of linear-Gauss-Bonnet gravity. Focusing on a specific theory allowed us to compare our forecast with existing bounds and forecasts for the next generation of GW detectors for this particular theory. One of the caveats of this comparison is that our exploration of the parameter space is not exhaustive. As such, it is not clear if the EMRI parameters we have considered are the ones that would yield the most stringent bounds for either d or for the coupling constant of some particular theory. We plan to address this question in future work.

Our waveform model can be extended in several directions. Realistic EMRIs are expected to follow generic, inclined orbits [55]. Studies of EMRIs with new fundamental fields evolving on eccentric *and* inclined orbits are underway [116]. Inclusion of post-adiabatic corrections, along the lines of Ref. [70], is also essential to include known GR effects that enter at post-adiabatic order, such as the secondary spin [111], and to assess how new fundamental physics can affect the waveform at this order. Our approach can also be generalised to capture massive scalars [54] and more general vector/tensor fields [64].

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LS: Conceptualization, Data Curation, Formal Analysis, Investigation, Methodology, Project Administration, Resources, Software, Supervision, Validation, Visualization, Writing – Original Draft.

SB: Data Curation, Investigation, Resources, Software, Writing - Review & Editing

AM: Conceptualization, Methodology, Writing - Original Draft, Writing - Review & Editing, Supervision, Project Administration.

TPS: Conceptualization, Methodology, Writing - Original Draft, Writing - Review & Editing, Supervision, Project Administration.

NW: Data Curation, Software, Writing - Review & Editing

MvdM: Methodology, Software, Writing - Review & Editing.

AJKC: Conceptualization, Methodology, Writing - Review & Editing.

OB: Investigation, cross-Validation (MCMC & FM studies), Data curation, Writing - Review & Editing

JG: Methodology, Writing - Review & Editing, Supervision.

Appendix A: Normalization of the action

We provide here details on the mapping between constraints on the Gauss-Bonnet coupling parameter inferred in this paper and from current and future groundbased detectors.

To compare bounds on the Gauss-Bonnet coupling α we need to take into account the different normalizations considered in the literature for the actions (1) and (8). In particular, constraints derived from current GW observations [121] and by the network of future detectors [115] are obtained by assuming the following action for GB gravity:

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{\kappa} - \frac{1}{2} (\nabla \bar{\phi})^2 + \bar{\alpha}_{\rm GB} \bar{\phi} \mathcal{G} \right] , \qquad (A1)$$

where $\kappa = 16\pi$. In this paper, we consider instead

$$S = \int d^4x \frac{\sqrt{-g}}{\kappa} \left[R - \frac{1}{2} (\nabla \phi)^2 + \frac{\alpha}{4} \phi \mathcal{G} \right] , \qquad (A2)$$

If we scale $\bar{\phi} = \phi \kappa^{-1/2}$ the actions (A1) -(A2) coincide so long as $\alpha = 4\kappa^{1/2}\bar{\alpha}_{\rm GB}$. In our units the constraint available for GB gravity of [121], $\sqrt{\bar{\alpha}_{\rm GB}} \simeq 1.18$ km, translates to $\sqrt{\alpha} = 4\pi^{1/4}\sqrt{\bar{\alpha}_{\rm GB}} \simeq 6.3$ km and the constraints from GW230529 $\sqrt{\bar{\alpha}_{\rm GB}} \simeq 0.260$ km $\rightarrow \sqrt{\alpha_{\rm GB}} \simeq 1.4$ km [113]. Similarly, for the projected bound by LIGO O8, $\sqrt{\bar{\alpha}_{\rm GB}} \simeq$ 0.4km (see Fig. 21 of [115]), we obtain $\sqrt{\alpha} \simeq 2.1$ km.

The map between the scalar charge and the GB coupling has been obtained in [82], using the action

$$S_J = \int \frac{\sqrt{-g}}{\kappa} d^4 x \left[R - 2(\nabla \tilde{\phi})^2 + \tilde{\alpha} \tilde{\phi} \mathcal{G} \right] .$$
 (A3)

In this setup, at the leading order in $\tilde{\alpha}$, the scalar charge is given by $\tilde{d} = \tilde{\alpha}/(2\mu^2)$. Passing to the units we use in Eq. (A2), $\tilde{d} = d/2$ and $\tilde{\alpha} = \alpha/2$, which leaves the relation $d = d(\alpha)$ unchanged.

Appendix B: Flux interpolation

Energy and angular momentum fluxes are interpolated over a three-dimensional grid constructed using 13 Chebyshev-Gauss-Lobatto (CGL) nodes for the eccentricity $e \in [0.0, 0.5]$ and the primary spin $a \in$ [-0.99, 0.99]. Rather than using the semi-latus rectum, we find it more convenient to introduce the variable

$$u = (1+e) \left[\frac{\Omega_{\phi}(a, p, e)}{\Omega_{\phi}\left(a, \frac{p_{\text{sep}}(a, e)}{1+e}, e\right)} \right]^{2/3}, \qquad (B1)$$

where $p_{sep}(a, e)$ is the separatrix for Kerr spacetime, and negative spins correspond to retrograde orbits. We compute u on 17 CGL nodes within $u \in [0.08, 0.97]$. Therefore, we calculate scalar and gravitational fluxes on a total of $13 \times 13 \times 17 = 2873$ grid points. Before interpolation, we normalize the fluxes by their leading order contribution in a post-Newtonian expansion. We construct 4 Chebyshev interpolants for the gravitational and scalar energy and angular momentum fluxes. Assuming the fluxes are smooth functions of the variables on our interpolation domain, the accuracy of the Chebyshev interpolation should converge exponentially with the number of grid points. Consequently, we can estimate the interpolation "aliasing" error from the magnitude of the coefficients of the highest order Chebyshev polynomials in each direction. The interpolated quantities and the Chebyshev errors σ are shown in Table I.

If we write the total energy flux as:

$$\dot{E} = \frac{32}{5p^5} \left(\frac{5p^5}{32} \dot{E}_{\text{grav}} + \frac{5p^5}{32} d^2 \dot{E}_{\text{scal}} \right) \,, \qquad (B2)$$

we can estimate the size of the scalar charge that is comparable to the error in the gravitational fluxes

$$\frac{5p^5}{32}\dot{E}_{\rm scal}d^2 > \sigma_{\rm grav} \to 0.12 \, d^2 > \sigma_{\rm grav} \,, \qquad (B3)$$

where we inserted $p^4 \dot{E}_{\rm scal} = 0.3$ which is a typical value accross the grid for p = 10. This gives a relation between the scalar charge and the Chebyshev interpolation. For values of d = 0.01, we obtain a constraint on the error of the order 10^{-5} , which is one order of magnitude smaller than what we obtained with our interpolation scheme. This does not invalidate our work, as we treat our interpolated fluxes as the true fluxes and use them consistently for injection and recovery. However, this does highlight the need for denser flux grids and accurate interpolation schemes. The production of dense self-force grids with accurate interpolation methods is a current major challenge for extending the FastEMRIWaveforms package to fully generic orbits in Kerr spacetimes.

Using the Gremlin code for flux calculations evaluated at a = 0.95, p = 10.1930405906075M, e = 0.4081632653061225 we obtain a flux value for $\frac{5}{32}p^5\dot{E}_{\rm grav}$ which differs from our interpolation by absolute and relative errors of 3.8×10^{-4} , 3.6×10^{-4} , respectively. This is compatible within our estimated interpolation error.

TABLE I. Chebyshev interpolation of the energy and angular momentum fluxes and the estimates of their absolute interpolation errors.

Interpolated expression	Abs. error
$rac{5}{32}p^5\dot{E}_{ m grav}$	7.5×10^{-4}
$rac{5}{32}p^{7/2}\dot{L}_{ m grav}$	7.1×10^{-4}
$p^4 \dot{E}_{ m scal}$	2.3×10^{-4}
$p^{5/2}\dot{L}_{ m scal}$	2.2×10^{-4}

Having the fluxes in hand, we can obtain the right hand side of the Eqns. (9) and then use the FEW package to obtain the EMRI trajectory. As an example, we show in Fig. 4 the trajectories in the p - e plane for binaries with spin a = 0.95 in GR, i.e., setting the scalar charge to zero, and different EMRI masses. For reference we also show the Chebyshev grid points at the spin a =0.9562665680261776.



FIG. 4. Trajectory evolution of semi-latus rectum and eccentricity for four EMRIs with different component masses and a = 0.95. For reference, we show the location of the grid points used for interpolation in the (p, e) plane for a constant spin slice at a = 0.9562665680261776, where the largest value of p reached for this spin is approximately $p \approx 30$.

Appendix C: Data analysis setup

The posterior distributions presented in this work are obtained using MCMC sampling with the package Eryn [112, 122]. To run the MCMC, we need to specify the priors $p(\Theta)$ and the likelihood function

$$\ln p(s|\Theta) = \frac{1}{2} \langle s - h(\Theta)|s - h(\Theta) \rangle , \qquad (C1)$$

where we defined the inner product between two GW templates $h_1(t)$ and $h_2(t)$ [123]:

$$\langle h_1(t)|h_2(t)\rangle = 4 \operatorname{Re} \int_0^\infty \frac{\tilde{h}_1^*(f)\tilde{h}_2(f)}{S_n(f)} \,\mathrm{d}f \;.$$
 (C2)

The noise spectral density $S_n(f)$ of LISA is taken from [124] and assumed to be known. The tilde denotes the Fourier transform of the waveform and the symbol * denotes complex conjugation. The Fourier transform and likelihood evaluation are performed on GPUs using cupy [125]. Before passing to the frequency space, we taper h(t) with a Tukey window. The parameter that controls the magnitude of the sinusoidal lobes of the window has been fixed to alpha= 0.005 [126]. The sampling interval was adjusted for different black hole masses M to avoid aliasing.

The full parameter space of the AAK model is given in Table II. The response of the detector to the signal is described by the two data channels $h_I(t; \Theta)$ and $h_{II}(t; \Theta)$. The full log-likelihood is given by the sum of the loglikelihood for each channel. For reference, we show in Figure 5 the spectrogram of h_I for a system with parameters $M = 10^6 M_{\odot}, \mu = 10 M_{\odot}, e_0 = 0.4, d = 0.0025, p_0 =$ 8.3M, T = 2 yrs.



FIG. 5. Spectrogram of the gravitational wave signal output of the AAK waveform obtained from an EMRI system with parameters $M = 10^6 M_{\odot}, \mu = 10 M_{\odot}, e_0 = 0.4, d = 0.0025, p_0 = 8.3, T = 2 \text{ yrs.}$ The different color bands represent the different harmonics, and their color intensity represents their power.

We assume flat priors for all parameters apart from the luminosity distance, which is assumed to follow a power-law distribution with slope -2 in the range [0.01, 10.0] Gpc. A summary of the priors we consider is given in Table II. We restrict the prior in Φ_{ϕ_0} to π and not to 2π because there is an exact degeneracy every π . The prior choice of power-law with slope -2 for the luminosity distance is motivated by the fact that we found some chains getting stuck at large unphysical values of d_L . This was caused by chains getting stuck on secondary modes of the likelihood. Our choices allow for better sampling efficiency without affecting the results. For parameters that are typically constrained with high precision, i.e., (M, μ, a, p_0, e_0) , we center the priors around the true injected values Θ_{true} . In all runs, we find that the posterior support of all parameters is much tighter than the assumed prior.

Since the deviation due to the scalar emission enters the fluxes as d^2 , we sample in the parameter $\Lambda = d^2$ with a uniform prior $\mathcal{U}_{[-0.6,0.6]}$, and then select the samples with $\Lambda > 0$. Sampling in Λ , both positive and negative, allows us to obtain near-Gaussian posteriors, which can be sampled more efficiently with MCMC methods. We verified that this approach does not change the posteriors. We use two proposals to efficiently sample: the stretch move [122] and an adaptive metropolis move that jumps along the eigen-directions of the covariance ma-

TABLE II. Prior distributions on the waveform parameters used for MCMC posterior sampling.

Priors ($\delta = 0.01$)
Uniform $[\ln M^*(1-\delta), \ln M^*(1+\delta)]$
Uniform $[\ln \mu^*(1-\delta), \ln \mu^*(1+\delta)]$
Uniform $[a^*(1-\delta), 0.98]$
Uniform $[p_0^*(1-\delta), p_0^*(1+\delta)]$
Uniform $[e_0^*(1-\delta), e_0^*(1+\delta)]$
Power Law [0.01, 10.0]
Uniform $[-0.99999, 0.99999]$
Uniform $[0.0, 2\pi]$
Uniform $[-0.99999, 0.99999]$
Uniform $[0.0, 2\pi]$
Uniform $[0.0, \pi]$
Uniform $[0.0, 2\pi]$
Uniform $[-0.6, 0.6]$

trix. We sample the posteriors using 26 walkers, monitoring the integrated autocorrelation time τ as a function of the iteration. We assume that the estimator for τ is reliable when it plateaus below the line given by $N_{\rm it}/50$ for $N_{\rm it}$ the number of iterations (see this link for further details). We show in the upper corner plot of Figure 6 the full posterior distribution used for the constraints obtained in Figures 1 and 3.

To obtain a bound on this specific theory from the posteriors on $\Lambda = d^2$ and $\ln \mu$, we use

$$\sqrt{\alpha} = \sqrt{2}\mu\sqrt{d} = \sqrt{2}\,\mu\,\Lambda^{1/4}\,,\tag{C3}$$

and the determinant of the Jacobian of such transformation

$$\mathrm{d}\sqrt{\alpha} = \left|\frac{\partial\sqrt{\alpha}}{\partial\Lambda}\right| \mathrm{d}\Lambda \propto \mu \Lambda^{-3/4} \mathrm{d}\Lambda \ . \tag{C4}$$

Similarly, one can obtain the bound on the scalar charge d. In the lower corner plot of Figure 6, we can see how the posterior samples are mapped to the coupling $\sqrt{\alpha}$. The posteriors are non-Gaussian and have long tails. This demonstrates the importance of sampling in Λ instead of $\sqrt{\alpha}$. Sampling in the latter requires longer iterations to reach convergence and to resolve the tails of the "banana-shaped" distributions.

Appendix D: Mapping to agnostic bounds

To compare the potential of EMRIs to constrain deviations from General Relativity for agnostic parametrization, we provide a comparison in terms of the parametrized post-Einsteinian (ppE) formalism or Flexible Theory Independent formalism [13, 127–133]. We



FIG. 6. Posterior distribution of EMRI injections with different orbital configurations. The posteriors are centered around the injected parameters. Diagonal and off-diagonal plots provide marginalised and 2D-joint posteriors, respectively. Contour lines in off-diagonal panels identify the $1,2,3-\sigma$ Gaussian credible contours of each distribution. The upper corner plot shows the posterior distribution output of the MCMC analysis in the parameters of Table II. The lower corner shows how the list row of the upper corner plot transforms when mapping to the coupling $\sqrt{\alpha}$. In the legend, we provide the system parameters and the number of orbital cycles performed N_{cycles} .

consider the system with $10^6 M_{\odot} + 10 M_{\odot}$ solar masses, initial eccentricity $e_0 = 0.4$, and time to plunge T = 2years and run a GR inspiral. We obtain the gravitational energy fluxes \dot{E}_{grav} from the samples obtained, evaluated at the start of the inspiral. If we subtract the median value from this set of samples, and divide by the median, we obtain a set of samples representing the fractional deviation from the expected energy dissipation in GR, that are consistent with the posterior. These can be related to ppE deviations by writing

$$\Delta \dot{E} / \dot{E}_{\rm grav} = B v^{2n} \tag{D1}$$

where $v = (\pi(M + \mu)f)^{1/3}$ with f frequency of the $(l, n_{\phi}, n_r) = (2, 2, 0)$ and n the post-Newtonian (PN) order. For EMRIs we can approximate $v = \Omega_{\phi}^{1/3}$. The quantity B can be mapped to an "agnostic" deviation in the waveform phase $\delta\varphi$ at different PN orders using the formalism described in [132] (see Eqns. (9)-(11) and (19)-(28) [132] for the mapping $B \to \delta\varphi$). We provide constraints on $\delta\varphi$ obtained with this procedure as "GR mapping" in Fig. 7. Constraints are 95% upper limits obtained from the posterior on $\delta\varphi$.

We remark that this analysis is approximate for two reasons. Firstly, post-Newtonian expansions do not provide a good description of the evolution of EMRIs. Secondly, this mapping only considers deviations in the waveforms that can be described by changes in the GR parameters. While any GR deviation causing such a change would definitely not be detectable, larger deviations could also be undetectable, since they are clearly strongly correlated with changes in the EMRI parameters. In that sense, this should be considered an optimistic bound. We note that this method cannot assess the detectability of components of the deviation that change the waveforms in ways that are orthogonal to the GR waveform space, but we expect these to be subdominant.

For comparison, we also provide the mapping between the agnostic approach and the scalar charge at the -1PN order, which would correspond to the leading contribution of our full adiabatic scalar emission. In this case, the parameter B is given by [136]:

$$B = d^2 \,\Omega_{\phi}^{2/3} \dot{E}_{\rm scal} / \dot{E}_{\rm grav} \,. \tag{D2}$$

In Fig. 7, we show the constraints obtained using the GR mapping at different PN orders for the EMRI configuration we considered (blue dots). The scalar charge mapping shows degradation of one order of magnitude (orange cross) at the -1PN order. This demonstrates the importance of correlations that are not taken into account in the GR mapping.

The constraints obtained from the gravitational wave events GW170817 [134], GW230529 [114], and using the GW transient catalogs of the third observing run



FIG. 7. Comparison of the constraints on the phase deviation at different PN orders. The EMRI constraints are obtained from mapping the posterior distribution of a system with parameters $M = 10^6 M_{\odot}, \mu = 10 M_{\odot}, a = 0.95, e_0 =$ 0.4, T = 2yrs and SNR=50 into the phase deviation at different PN orders (blue dots, EMRI GR mapping). For the case of -1PN order we use the posterior distribution obtained from an EMRI embedded in a scalar field (orange cross) as done in Figure 1. We show the current constraints obtained from the gravitational wave events GW170817 (down green triangle) [134], GW230529 (up red triangle) [114], and using the GW transient catalogs of the third observing run (GWTC-3) (violet rombo) [4]. Brown pentagon markers correspond to forecasts on $\delta \varphi$ obtained for a GW230529-like binary observed by ET (see main text). The double pulsar constraints obtained from PSR J0737-3039 [135] are shown with pink star markers.

(GWTC-3) [4] are also shown, and these are a few orders of magnitude larger than the EMRI constraints. This is expected since EMRIs complete $10^4 - 10^5$ cycles during the observation, which is two to three orders of magnitude than the number of cycles typically observed in a merger seen by ground-based detectors, leading to correspondingly higher measurement precision. However, better phase constraints do not necessarily imply tighter bounds on the coupling, as this map depends on the specific theory considered. This is the case for GB gravity for which constraints improve for lighter objects, as in the case of GW230529 (see vertical line Fig. 3).

For comparison, we compute bounds on $\delta\varphi$ forecasted for a third-generation ground-based detector like the Einstein Telescope (ET). Constraints are derived through a Fisher matrix approach, analysing the inspiral phase of a binary BH system with the same properties as GW230529 (the source parameters are fixed to the median values reported in [137]). For the analysis, we consider a TaylorF2 waveform model, integrated between 3Hz and the Schwarzschild ISCO⁴. The waveform depends on 7 parameters, $(\mathcal{M}, \eta, t_c, \phi_c, \chi_s, \chi_a, \delta\varphi)$, where \mathcal{M} and η are the chirp mass and the symmetric mass ratio, (t_c, ϕ_c) the time and phase at the coalescence, $\chi_{s,a} = (\chi_1 \pm \chi_2)/2$ combinations of the individual spin components. The phase shift enters the the frequencydomain template as $\tilde{h}(f) = Ae^{i\varphi_{\text{GR}(f)}}e^{i\psi(f)}$, where

$$\varphi_{\rm GR} = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128} (\pi \mathcal{M} f)^{-5/3} \sum_{i=0}^9 \varphi_i (\pi M f)^{i/3} , \quad (D3)$$

is the GR phase (see Appendix A of [138] for the explicit form of the PN coefficients φ_i) and

$$\psi = \frac{3}{128\eta^{n/5}}\varphi_n\delta\varphi_n(\pi\mathcal{M}f)^{(n-5)/3} \quad \text{if} \quad \varphi_n \neq 0 \ , \ \ (\text{D4})$$

$$\psi = \frac{3}{128\eta^{n/5}} \delta \varphi_n (\pi \mathcal{M} f)^{(n-5)/3} \quad \text{if} \quad \varphi_n = 0 \ . \tag{D5}$$

For the Fisher analysis we average over the angles that define the source position in the sky, and consider Gaussian priors on $\chi_{1,2}$, centered around the injected values, and with unit width. Finally we assume for ET a single L-shaped detector with 15 km arm-length [139].

We also show in Fig. 7 the constraint on dipole emission, inferred from observations of the double pulsar PSR J0737–3039 [135]. Such binaries evolve in a lowdynamical regime with $v \approx 2 \times 10^{-3}$, and provide the tightest bound on the -1PN phase deviation $\delta\varphi$.

Appendix E: Accuracy of the trajectory integration

Gravitational wave observations constrain the frequency evolution of the waveform with great precision. Therefore, it is key to check that the phase evolution is not affected by systematic errors, which could influence the parameter reconstruction. We study the accuracy of the numerical integration of the ordinary differential equations (ODEs) 9. These ODEs are solved using [140] with adaptive step size gsl_odeiv2_step_rk8pd provided by [141].

Firstly, we cross-checked the phase evolution of the implementation against Mathematica on 4 test trajectories. Then, we investigated the difference of the final phase of an inspiral in GR with an ODE absolute



FIG. 8. Difference in the final phase between an EMRI evolution obtained with a non-zero scalar charge and various ODE errors (see legend), and an EMRI evolution in GR with an ODE error of 10^{-11} . For reference, we provide the number of points N taken from the ODE integrator.

error of 10^{-14} with respect to the final phase of an inspiral with a given scalar charge d and various different ODE absolute errors. We show the results in Figure 8 for a system with $M = 10^6 M_{\odot}$, $\mu = 10 M_{\odot}$, a = 0.95, $p_0 = 8.343242843079224M$, $e_0 = 0.4$ until its plunge. For small scalar charges, $d < 3 \times 10^{-6}$, the phase difference is determined by the ODE solver's noise floor. On the contrary, the value of the phase difference is independent of the ODE error for large scalar charges $d > 3 \times 10^{-6}$ and it follows the d^2 scaling expected by an expansion of the phase difference for small d. This expansion is a good approximation up to $d \approx 1$. The ODE error adopted in this work was 5×10^{-10} .

Appendix F: Systematic bias due to non-zero scalar charge

In this appendix we provide further details on the systematic bias that could potentially affect EMRI analyses due to the mismatch between a GR recovery template, and a signal with a non-zero scalar charge. This is critical as the charge can influence the binary phase evolution and bias parameter estimation. To investigate such bias we inject an EMRI signal with d = 0.025 and source parameters $M = 10^5 M_{\odot}, \mu = 5 M_{\odot}, a = 0.95, e_0 = 0.4,$ T = 2 yrs. We analyse the data with two templates: (i) a GR waveform, and (ii) a waveform model in which the scalar charge is free to vary. The posterior distributions on the EMRI parameters are shown in Fig. 9. We find 2/3 sigma systematic biases in the intrinsic parameters recovered with the GR template. Extrinsic parameters do not present biases larger than 1 sigma and are, therefore, less affected by the waveform mismodeling. The best log-likelihood point obtained with the GR template is $\ln p(s|\Theta) \approx -9$.

⁴ Constraints obtained using the maximum frequency at the Kerr ISCO do not change significantly.



FIG. 9. Posterior distribution of an EMRI system with scalar charge d = 0.025 and source parameters $M = 10^5 M_{\odot}$, $\mu = 5 M_{\odot}$, a = 0.95, $e_0 = 0.4$, T = 2 yrs and SNR=50. The scalar charge template (blue) correctly recovers the injected parameters, whereas the GR template is biased due to the non-zero scalar charge d = 0.025.

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