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An overview tool for ICRH physics in Wendelstein 7-X

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Abstract. A new overview tool for ICRH physics in Wendelstein 7-X has been developed. It calculates components of the cold-plasma dielectric tensor and infers the spatial positions of cutoffs, resonances, and evanescent layers in the plasma. It is interfaced to a data base of pre-calculated magnetic equilibria, thus allowing the user to check any of these magnetic configurations in terms of its ICRH properties in a matter of a few minutes. Information gained from this tool can inform planning of experiments or be the basis for more comprehensive numerical modelling.

1. Introduction and Motivation

The main tool used for the modelling of radio-frequency (RF) wave propagation and fast-ion generation in Wendelstein 7-X (W7-X) is the SCENIC code package [1]. As shown in Fig. 1 it combines an anisotropic equilibrium code (ANIMEC [2]), a full-wave code (LEMan [3]), and a Fokker-Planck code (VENUS-LEVIS [4]).

ANIMEC (being the anisotropic version of the familiar VMEC code [5]) provides the magnetic equilibrium to the other two codes. LEMan solves the wave-propagation problem in which the plasma is considered with its dielectric tensor in either cold, warm, or hot-plasma approximation. The hot-plasma model is used most frequently. The properties of the RF wave (most notably E_+ , E_- , and k_{\parallel}) are given to the Monte-Carlo code VENUS-LEVIS which advances the particle trajectories in time given the pre-computed magnetic field and RF-wave field. Kicks are applied to the particles whenever they cross the ICRH resonance, thus simulating heating and the building-up of an energetic fast-ion distribution function. Of course, the heating of the plasma changes its dielectric properties (and also the pressure), hence iterations between the various codes are necessary to arrive at a converged solution.

Clearly, SCENIC is a very comprehensive tool with a lot of important physics and it is the tool of choice to investigate any specific scenario in great detail. However, the model is also very expensive to run. It can easily take several weeks on a supercomputer such as MARCONI (including simulation time and queuing time) to converge a single case.

Additionally, computing time is limited and we do not want to “burn” it needlessly by simulating scenarios that are not sensible (because, for instance, resonances and evanescence layers are in less-than-ideal places).

The logical consequence is that there is clearly a need for a simplified tool to look at the *basic* physics aspects of a specific scenario. The main task of this tool should be to find the locations of the resonances and cutoffs in the cold-plasma approximation. In this way, the user can easily

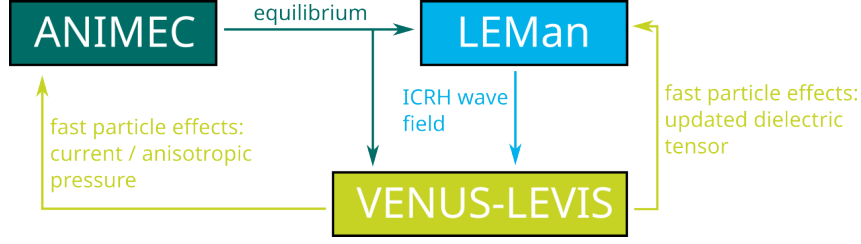


Figure 1: Schematic illustration of the SCENIC code package and the involved codes (adapted from Ref. [6]).

find and qualify scenarios that are of interest for more detailed modelling with SCENIC. The tool also allows to easily judge the experimental feasibility and relevance of certain scenarios.

In this paper, we will present the basic theory behind the tool and give some examples of its usage targeted specifically at W7-X. Appendix A and Appendix B deal with the installation and the usage of the tool.

2. Theory

The tool evaluates the plasma dielectric tensor in the cold-plasma approximation. In the notation used by Stix [7] (and for $\mathbf{B} = B_0 \hat{\mathbf{z}}$)

$$\underline{\varepsilon} \cdot \mathbf{E} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}, \quad (1)$$

where $\underline{\varepsilon}$ is the dielectric tensor and \mathbf{E} is the electric field, the components of the dielectric tensor can be expressed as

$$R = 1 - \sum_s \frac{\omega_{p,s}^2}{\omega(\omega + \Omega_s)} \quad L = 1 - \sum_s \frac{\omega_{p,s}^2}{\omega(\omega - \Omega_s)} \quad P = 1 - \sum_s \frac{\omega_{p,s}^2}{\omega^2} \quad (2)$$

and

$$S = \frac{1}{2}(R + L) \quad D = \frac{1}{2}(R - L). \quad (3)$$

Here, the plasma frequency and gyration frequency for species s are defined as

$$\omega_{p,s} = \sqrt{\frac{n_s Z_s^2 e^2}{\varepsilon_0 m_s}} \quad (4)$$

$$\Omega_s = \frac{Z_s e B}{m_s}, \quad (5)$$

respectively, where n is the density, Z denotes the charge, e is the elementary charge, ε_0 is the vacuum permittivity, and m denotes the mass. The magnetic field strength is given by B and depends (similarly as the density) on the spatial position in the plasma. Hence, the tool needs a way of importing magnetic-equilibrium and mapping data. We will discuss this below. The frequency of the ICRH antenna is denoted by ω .

In the ion-cyclotron frequency range [8] it is possible to express the components of the dielectric tensor slightly differently

$$\varepsilon_1 = 1 + \frac{\omega_{p,e}^2}{\Omega_e^2} - \sum_i \frac{\omega_{p,i}^2}{\omega^2 - \Omega_i^2} \quad \varepsilon_2 = - \sum_i \frac{\omega}{\Omega_i} \frac{\omega_{p,i}^2}{\omega^2 - \Omega_i^2}, \quad (6)$$

$$\varepsilon_{L,R} = \varepsilon_1 \pm \varepsilon_2. \quad (7)$$

Here the sums only run over the ion species as the electron contributions have already been considered separately. These expressions, as well as those by Stix, have been implemented and they yield the same results.

3. Numerical implementation

As briefly mentioned above, evaluating the components of the dielectric tensor requires knowledge of the magnetic field. As such, the newly developed ICRH overview tool is connected to the VMEC webservice [9] accessible only from within the IPP network. Thus, it is possible to use any pre-computed magnetic configuration in the data base. As is illustrated in the example input file shown below, this is done by simply specifying the string that corresponds to the desired magnetic configuration in the data base. The user can further specify an arbitrary number of species for the evaluation of the dielectric tensor. An example input file for a ^4He -(H) minority heating scenario is presented below.

```

1 # specify ID of the VMEC run from the data base (a few examples are given below)
2 # 1. using the example from http://webservices.ipp-hgw.mpg.de/docs/usecases.html
3 # (VMEC example)
4 #id = "w7x.1000_1000_1000_1000_+0500_+0500.06.088"
5 # 2. equilibria from http://svvmec1.ipp-hgw.mpg.de:8080/vmecrest/v1/geiger/w7x/
6 # (J. Geiger's calculations)
7 #id = "/w7x/0691_0686_0669_0379_+0248_+0934/01/00/"
8 # 3. equilibria from http://svvmec1.ipp-hgw.mpg.de:8080/vmecrest/v1/run/
9 # (several other users of the webservice)
10 id = "kbr_test_v2"
11
12 # number of points in first direction (R or x)
13 nR = 500
14
15 # number of points in second direction (Z or y)
16 nZ = 500
17
18 # toroidal angle in radiants (only used for cut_type = 1)
19 phi = 0.1745
20
21 # type of the cut (1: poloidal cut at specified phi / 2: cut along midplane)
22 cut_type = 2
23
24 # on-axis electron density in m^-3
25 densElecAxis = 5.0e19
26
27 # parameters for electron density profile (coefficients for normalized
28 # polynomial in s = reff^2 / a^2 = rho^2 in ascending order)
29 densElecParam = [1.0, 0.0, -0.5]
30
31 # number of species (order: elec, ions, minority, second ion species)
32 nspecies = 3
33
34 # mass of each species in multiples of the proton mass
35 mass = [5.44617021e-4, 4.0, 1.0]
36
37 # charge of each species in multiples of the elementary charge
38 charge = [-1.0, 2.0, 1.0]
39
40 # density fractions of each species (make sure the plasma is quasi-neutral)
41 dens_fractions = [1.0, 0.47, 0.06]
42

```

```

43 # antenna frequency in MHz
44 freq = 38.0
45
46 # parallel wave number in m^-1
47 kpar = 10.0
48
49 # flag for writing additional data for later plotting with gnuplot
50 gnuplot = false
51
52 # flag for specifying if a |B|-plot with resonances and cutoffs
53 # should be generated
54 onedim_res_cut = true
55
56 # a tolerance that controls the width of the lines in the |B|-plot with
57 # resonances and cutoffs
58 tol_res = 20.0
59
60 # tolerance for mapping done by webservice
61 mapping_tol = 1.0e-3

```

Listing 1: A typical input file for a ^4He -(H) minority heating scenario.

Using the values specified in the input file, the tool first checks if the plasma is quasi-neutral and, if this condition is fulfilled, proceeds to get the magnetic-field data and map the density profile to three-dimensional cylindrical (or cartesian) coordinates.

The components of the dielectric tensor and the positions of the cutoffs and resonances can be visualized in two different ways. One can choose to display the data in a poloidal cut at a specified toroidal angle or in a toroidal cut along the midplane of W7-X.

After calculation of the plasma- and gyration frequencies according to Eqs. (4)-(5), Eqs. (2) and (3) are evaluated for the chosen coordinates. Using also the definition (N being the refractive index)

$$N^2 = \frac{RL}{S} \quad (8)$$

for perpendicular wave propagation [7] enables the classification of the plasma into regions where the wave can propagate ($N^2 > 0$) and where it is evanescent ($N^2 < 0$).

Defining the parallel refractive index as

$$N_{\parallel} = \frac{ck_{\parallel}}{\omega}, \quad (9)$$

with c being the speed of light and k_{\parallel} the parallel wave vector of the RF wave launched by the ICRH antenna (an input parameter for the tool), it is possible to calculate the locations of the ion-ion-hybrid (IIH) layers, the L-cutoff, and the R-cutoff according to

$$\varepsilon_1 = N_{\parallel}^2 \quad \text{IIH-layer} \quad (10)$$

$$\varepsilon_L = N_{\parallel}^2 \quad \text{L-cutoff} \quad (11)$$

$$\varepsilon_R = N_{\parallel}^2 \quad \text{R-cutoff.} \quad (12)$$

The tool also calculates the lower and upper hybrid frequencies [7] according to

$$\frac{1}{\Omega_{\text{LH}}^2} = \frac{1}{\Omega_i^2 + \omega_{p,i}^2} + \frac{1}{|\Omega_i \Omega_e|} \quad (13)$$

$$\Omega_{\text{UH}}^2 = \Omega_e^2 + \omega_{p,e}^2 \quad (14)$$

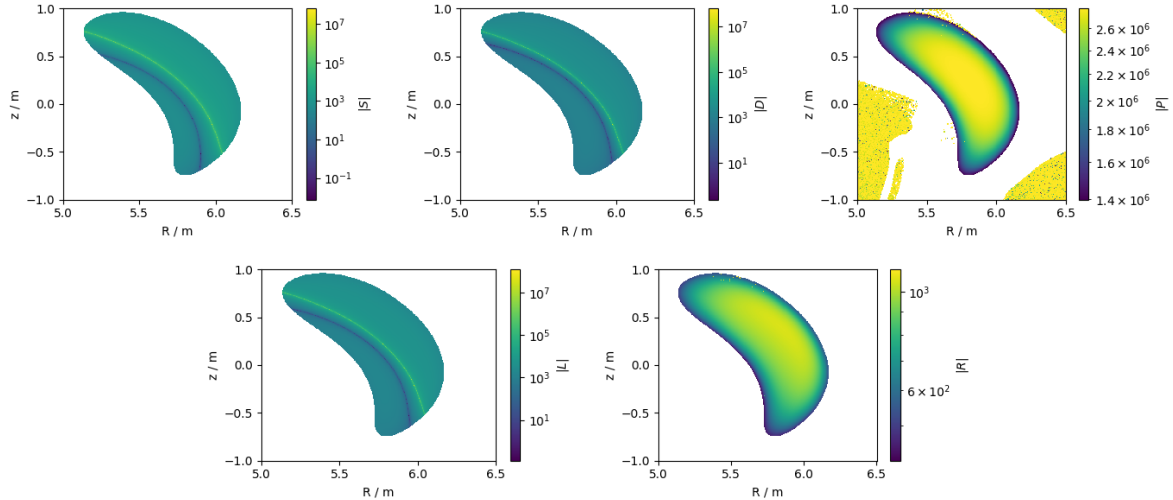


Figure 2: Components of the cold-plasma dielectric tensor in the notation of Ref. [7]. Numerical artifacts that are seen in the plot for $|P|$ (top right) are due to inaccuracies in the extrapolation routine the webservice uses to infer s -values (radial coordinate) far away from the plasma. Even if a point is far away from the plasma it may erroneously have $s < 1$ and thus have a non-negligible density assigned to it.

and calculates the resonances of the external ICRH frequency with respect to those. (Only the main ion species is considered here.)

Plots are generated for all components of the dielectric tensor and the resonances and cutoffs specified above. Dedicated plots showing the location of the (fundamental) ICRH resonance and the locations of minimum and maximum values of S and L in a $|\mathbf{B}|$ -plot are also generated for completeness. (But this can easily be adapted to show different components of $\underline{\epsilon}$.) These plots constitute the main output of the overview tool. Depending on the specified numerical resolution it only takes several seconds up to a few minutes to analyze a given case.

The tool is written in the julia programming language [10] with a few pieces of python code interfacing with the VMEC webservice.

4. Results

Below, we show the full output of the tool which is generated using the input file displayed above. Depending on the `cut_type` input variable the dielectric tensor components and the resonances and cutoffs can either be displayed in a poloidal or a toroidal cut, respectively. Both options are illustrated below.

It is also possible to plot the data along a one-dimensional line moving radially outwards at a specified toroidal angle and at $Z = 0$ (midplane). This option is even faster than the other two, but the 1D cut makes it more difficult to interpret the results due to the reduced information content. This option is not discussed further in this paper.

4.1. A poloidal cut along a specified toroidal angle (`cut_type=1`)

Figures 2-7 show the output generated by the overview tool in a poloidal cut along a specified toroidal angle of $\varphi = 0.1745$ (`cut_type=1`).

4.2. A toroidal cut along the midplane of W7-X (`cut_type=2`)

Similarly, figures 8-13 show the output generated by the overview tool for the same general inputs, but now for a toroidal cut along the midplane of W7-X (`cut_type=2`).

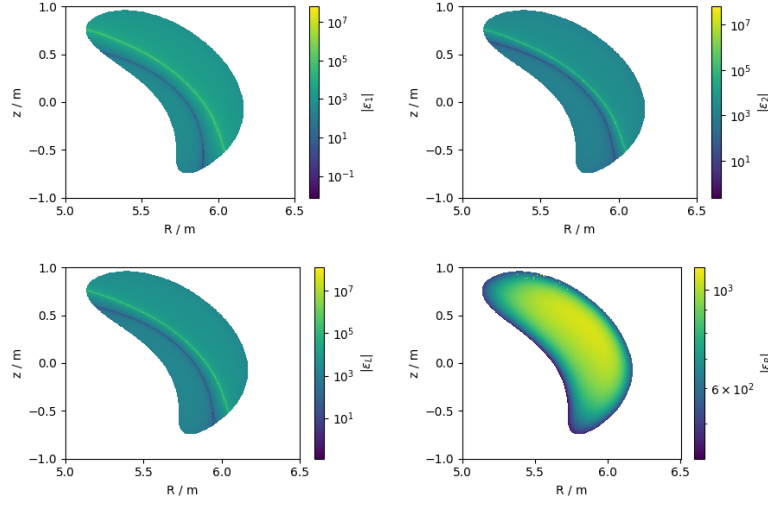


Figure 3: Components of the cold-plasma dielectric tensor in the notation of Ref. [8].

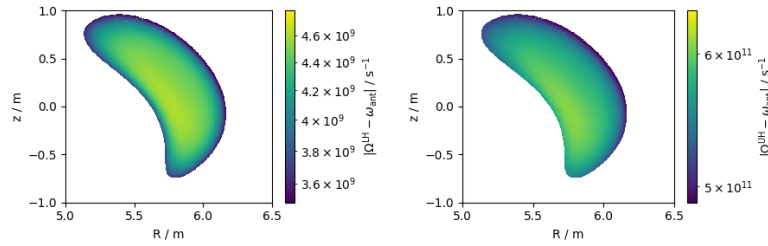


Figure 4: Resonances of the lower (left) and upper (right) hybrid frequencies with respect to the ICRH antenna frequency. In this example, no resonance is visible.

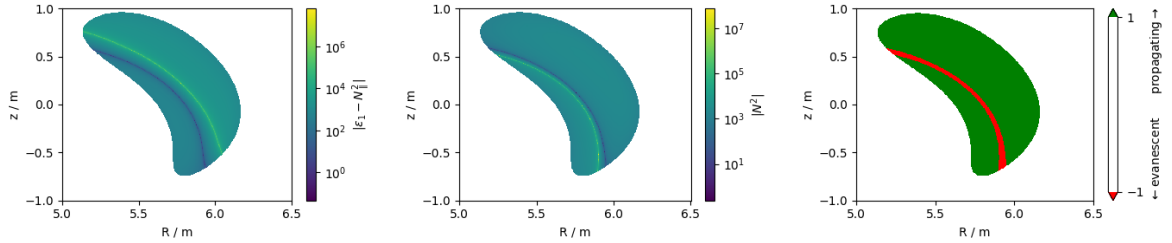


Figure 5: Ion-ion-hybrid resonance (left), the absolute value of N^2 (middle), and regions of evanescence (right).

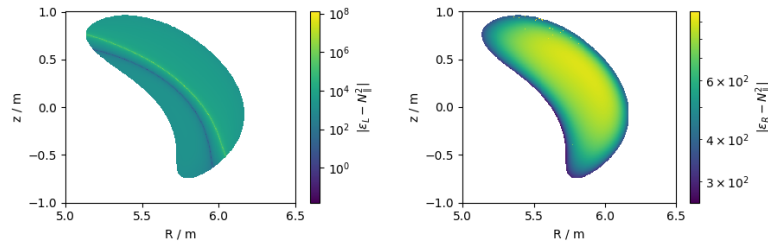


Figure 6: Cutoffs for the L-polarized (left) and R-polarized (right) components of the ICRH wave. An L-cutoff is visible, but the R-cutoff would be outside the plasma.

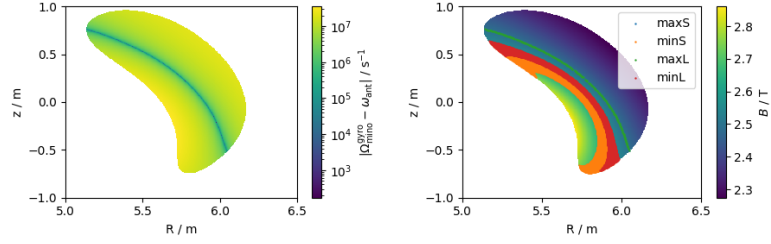


Figure 7: Location of the ICRH-resonance for the Hydrogen minority (left) and $|B|$ -plot with regions where S and L have large and small values (right).

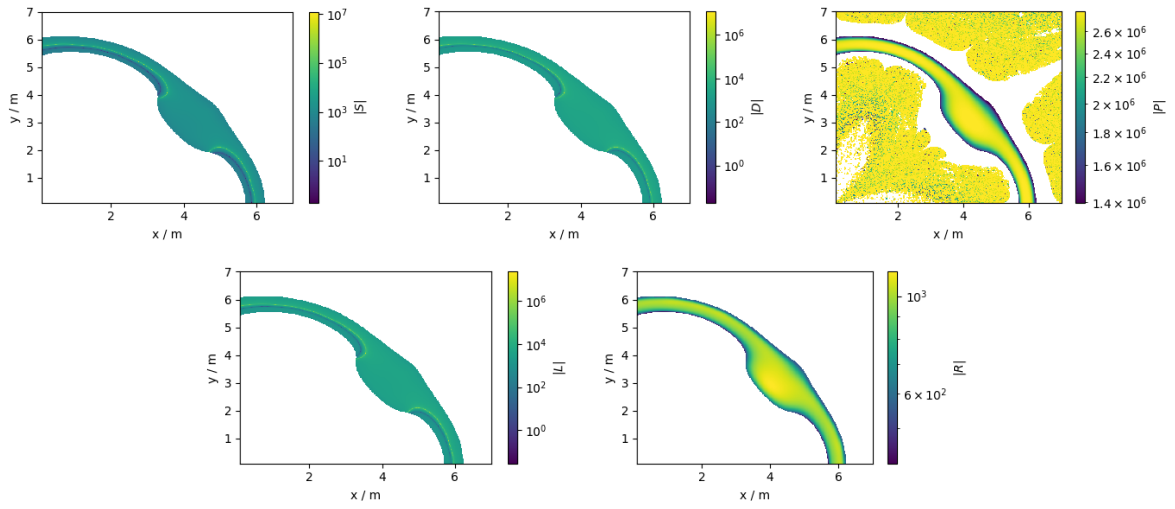


Figure 8: Components of the cold-plasma dielectric tensor in the notation of Ref. [7]. Numerical artifacts that are seen in the plot for $|P|$ (top right) are due to inaccuracies in the extrapolation routine the webservice uses to infer s -values (radial coordinate) far away from the plasma. Even if a point is far away from the plasma it may erroneously have $s < 1$ and thus have a non-negligible density assigned to it.

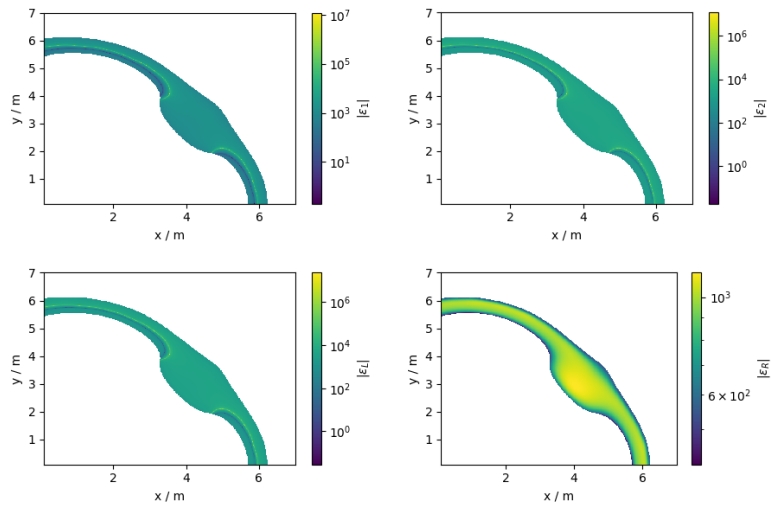


Figure 9: Components of the cold-plasma dielectric tensor in the notation of Ref. [8].

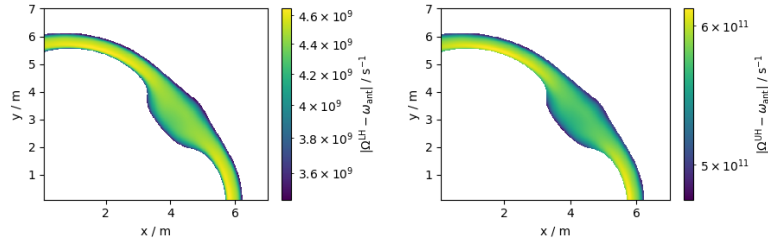


Figure 10: Resonances of the lower (left) and upper (right) hybrid frequencies with respect to the ICRH antenna frequency. In this example, no resonance is visible.

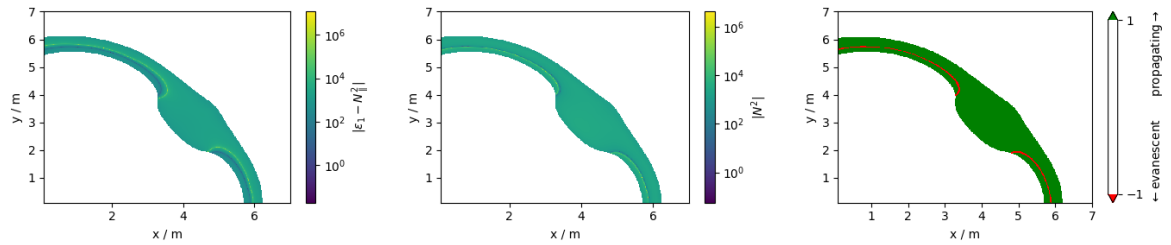


Figure 11: Ion-ion-hybrid resonance (left), the absolute value of N^2 (middle), and regions of evanescence (right).

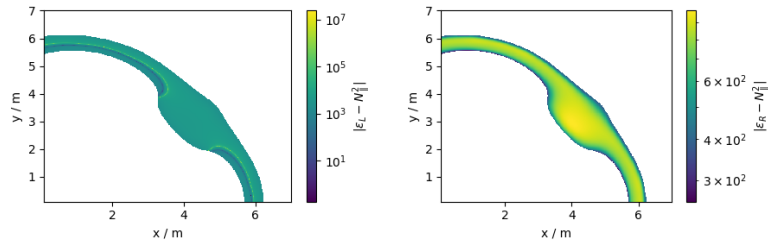


Figure 12: Cutoffs for the L-polarized (left) and R-polarized (right) components of the ICRH wave. An L-cutoff is visible, but the R-cutoff would be outside the plasma.

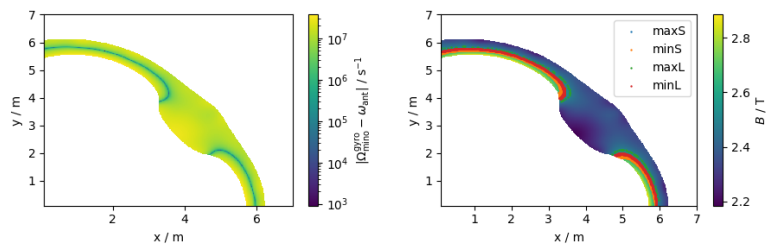


Figure 13: Location of the ICRH-resonance for the Hydrogen minority (left) and $|B|$ -plot with regions where S and L have large and small values (right).

4.3. Comparing locations of cutoffs, resonances, and evanescence layers

Nicely visible from Figs. 5 and 11 is the location of the IHH-layer in the plasma. Together with the L-cutoff (see Figs. 6 and 12) it forms a mode conversion layer that acts as a barrier for the propagation of the fast wave launched from the ICRH antenna [8]. This layer of evanescence is shown in Figs. 5 and 11 as the red area on the right-hand-side plot. On the other hand, right at the L-cutoff the wave only has a E_+ -component and is thus perfectly polarized to heat the (minority) ions. Comparing Figs. 5 and 7 (or 11 and 13 for the midplane cut) it is apparent that for this specific scenario the ICRH resonance of the H-minority ions is hit before the evanescence layer, because the wave is launched from the outboard side. The scenario is well-designed in this respect. On the other hand, one sees that the location of the ICRH resonance does not line-up very well with the location of the L-cutoff where the preferential ion heating occurs. Hence, the scenario should be optimized in this respect. This can be done, for instance, by switching to a three-ion scheme [8] with precisely dialed-in concentrations for the various plasma species.

This train of thought illustrates the basic physics considerations when designing an ICRH case for simulation or for the experiment and how the newly developed overview tool helps to guide the decision making.

5. Summary

We have presented the development of a new and simplified overview tool for ICRH physics in Wendelstein 7-X. It calculates the components of the plasma dielectric tensor in the cold-plasma approximation for a specified ICRH antenna frequency. This is used as a basis for computing the spatial positions of resonances and cutoffs as well as evanescent regions in the plasma. The tool is written in the Julia programming language and is interfaced to the VMEC webservice accessible only from within the IPP network. Thus, any magnetic configuration can quickly be checked in terms of its basic ICRH properties. Depending on the numerical resolution chosen, the tool only requires a couple of seconds up to a few minutes for its computations. We envisage this tool to be ideal for quick initial checks for certain scenarios before actual experiments or much more involved numerical simulations are attempted.

Acknowledgement

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Appendix A. Installation

The tool requires the VMEC webservice in order to run. This data base of pre-computed magnetic equilibria is accessible only from within the IPP Greifswald local network. Hence, the installation of the tool is described only for machines within the network of the IPP Greifswald. Installation is possible on Linux as well as on Windows computers.

- clone the git repository to a location of your choosing by typing

```
1 git clone https://gitlab.mpcdf.mpg.de/csla/icrh-overview.git
```

in that location

- download and install latest version of julia¹
- if on Windows: install latest version of python from the Software Pool; WinPython 3.11.8.0 has worked well in the past
- if on Linux: your Linux distribution likely already comes with a python installation; if not, install it using the proper commands for your particular Linux distribution
- install the following python packages (if not already installed):

```
1 pip install osa
2 pip install numpy
3 pip install math
```

- (if you used WinPython 3.11.8.0 under Windows this is done via the WinPython Powershell Prompt that comes with your WinPython installation)
- open julia and enter the following commands:

```
1 ENV["PYTHON"] = "...path to your python (WinPython) executable..."
2 import Pkg
3 Pkg.add("PyCall")
4 Pkg.add("PyPlot")
```

- the most important dependencies should now be installed correctly

Appendix B. Using the tool

- open julia and enter the commands:

```
1 cd("...path to the script...")
2 include("ICRH_overview.jl")
```

- if the script is run for the first time, some additional julia packages will be installed automatically (might take some time)
- overview plots of dielectric tensor components, cutoffs and resonances will be generated

¹ <https://julialang.org/downloads/>