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Are Zeno's Paradoxes of Motion Fallacies? Evidence from the Hebrew Aristotelian Logical Tradition

I was unable to attend in person the conference on fallacies in Lille in May 2021, at which the papers in this volume were presented. In order for me to have come to Lille, I would have had to come half way to Lille first. But in order to do that, I would have had to come half way to half way to Lille. And in order to do that, I would have had to come half way to half way to half way to Lille and so on *ad infinitum*. My absence in person from the conference can thus be explained by appeal to Zeno's problem of dividing motion into halves, outlined, e.g., in Aristotle's *Physica* 239b9-14. This problem can be understood to prove that all motion over divisible space is impossible, since any divisible space can be divided into infinitely many halves. If motion is impossible, then my absence from Lille is easily explained[1]. This excuse is not unlike the argument we find at Aristotle's *De Sophisticis Elenchis* 172a8-9 that 'one could deny that walking about after a meal is rather good, because of Zeno's argument'[2]. Such a claim, says Aristotle, is 'not doctorly' (οὐκ ἰατρικός), thereby giving us the impression that the contentious quarreler (ὁ ἐριστικὸς) here is recommending against exercising after dinner on the grounds that motion is impossible. While I will not deny having viewed this claim with some sympathy after some of my larger dinners, on the whole I would have to agree with Aristotle that it is too universal (κοινὸς) for the argument in question[3]. The same can be said of my excuse for not coming to Lille: denying all motion to

1. Zoom provided a modern-day solution to Zeno's paradox, allowing the conference to take place amid the Covid-19 outbreak. I thank the organizers for putting together a wonderful and stimulating conference even in the face of great obstacles.

2. τις μὴ φαίη βέλτιον εἶναι ἀπὸ δείπνου περιπατεῖν διὰ τὸν Ζήνωνος λόγον. Aristotle's Greek text is taken from W.D. Ross 1958 edition. Unless otherwise noted, all translations are my own.

3. The Greek here does not, in fact, contain the comparative, 'too', but has only κοινὸς γάρ, "for it is universal". My addition of 'too' is based on the implication of the statement's context that the suitable argument here would be more particular. We shall see below that Averroes gives much more detailed criteria for precisely what kind of particular argument should be made in this context.

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explain my absence would work, but it involves making an excessively universal claim. While excessive universality is improper under certain circumstances, it is hardly fallacious reasoning in itself. What makes it improper is not that it is contrary to reason, but that it does not fit in the context of the contentious argument.

Aristotle's proper arguments against Zeno's paradoxes are, in fact, to be found at *Physica* VI. His mentions of the paradoxes in *De Sophisticis Elenchis* and *Topica* do not explain why Zeno's reasoning is not correct, but apparently refer the reader to the *Physica* for the full treatment. Medieval and Renaissance Hebrew treatments of Zeno's arguments in commentaries and super-commentaries on Aristotle, however, argue for the fallaciousness of Zeno's arguments in the context of the *Topica* and *De Sophisticis Elenchis* without directly referring the reader to the *Physics* in those works. This is because the authors of the Hebrew works view Zeno's arguments as properly dialectic and accordingly treat their refutation in works dedicated to dialectic. This shift in the context of the arguments is present in the primary sources for the study of the *Topica* and *De Sophisticis Elenchis* in Hebrew: Al-Fārābī's *Art of Dialectic* and *Sophistical Refutations* and Averroes' Middle Commentaries on Aristotle's *Topica* and *De Sophisticis Elenchis*. Aristotle's works, *Topica* and *De Sophisticis Elenchis*, have to this day never been translated into Hebrew. I shall argue that Al-Fārābī and Averroes respond to Zeno in detail in these commentaries on dialectic and sophistic because the paradoxes of infinity associated with Zeno were used by the Muslim *Kalām* as important parts of the arguments for atomism. This will also explain why the four paradoxes of motion are more or less assimilated into the Paradox of the Stadium, which Al-Fārābī calls "The Question of the Halves"[4]. The Hebrew treatments of Zeno are, as we shall see, heavily indebted to Al-Fārābī and Averroes.

In what follows, we shall first turn to Aristotle's presentation of Zeno in the *Physica*, *Topica* and *De Sophisticis Elenchis*, before examining how Al-Fārābī and Averroes differ in their approach from Aristotle. We shall focus on the Hebrew translations of these works and then turn to the Hebrew commentary tradition and how it viewed these works in the absence of serious engagement with *Kalām* atomism among Hebrew thinkers in Southern Europe in the 12th-16th centuries.

1. Aristotle

Aristotle briefly discusses the paradoxes once in the *Topica* and twice in the *De Sophisticis Elenchis*[5]. We have already seen that in one of these places (*De Sophisticis Elenchis* 172a8-9), Aristotle does not argue the fallaciousness of the claim that mo-

4. For an overview of Zeno's paradoxes, and their continued importance for philosophy and mathematics to this very day, see Huggett 2019.

5. See *Topica*, 160b6-10 (below), *De Sophisticis Elenchis* 172a8-9 (mentioned above) and 179b20-

tion is impossible, but only that the argument in one specific case is too universal for its context. In the short mention of Zeno at *Topica* 160b, Aristotle discusses raising objections (ἔχων ἔνστασιν) and making counterarguments (ἀντεπιχειρεῖν) to universal propositions. Upholding universal propositions without objection is, according to Aristotle, to act peevishly (δυσκολαίνειν), but one would seem to be even more peevish to reject universal propositions without raising objections and counterarguments. Aristotle continues:

> "Yet this is not enough. We have many arguments that are contrary to accepted opinions and which are difficult to solve, like Zeno's argument that motion is impossible or that the stadium cannot be traversed. But it is not the case that we should not accept the [arguments] opposite to these on this account"[6].

That is, the addressee of Aristotle's *Topica* is not expected to be able to solve Zeno's argument, but he does not have to accept the impossibility of motion. Arguments of this kind, which Aristotle calls difficult (χαλεπός), are an apparent counterexample to the peevishness normally incurred by one who cannot answer objections or bring counter-arguments. Clearly, Aristotle would not have us admit Zeno's argument against motion, but showing its fallaciousness is beyond the scope of the dialectical argumentation discussed in the *Topica*. That Zeno's arguments are contrary to accepted opinions may also play a part in Aristotle's not giving full arguments in the *Topica*. Indeed, the audience of the *Topica* would appear to be expected to include both that motion exists and that the stadium can be traversed among their accepted opinions (δόξαι). In this case, there is no need to refute views contrary (ἐναντίοι) to them in the *Topica*.

Aristotle's second mention of Zeno's paradox of motion in the *De Sophisticis Elenchis* (at 182b26) is similar in that it too offers no obvious logical solution to the paradox. Instead, it would seem to imply that the reader and the interlocutor with sophists would not be able to expose the false deduction made by Zeno or his followers and accordingly would not have a true refutation of Zeno's argument against motion. Aristotle says:

> "There is nothing to prevent the same argument from having a number of flaws; but it is not the exposition of any flaw that constitutes a solution; for it is possible for a man to prove that a false conclusion has been deduced, but not to prove on what it depends, e.g. in the case of Zeno's argument to prove that motion is impossible. So that even if anyone were to try to establish that this is impossible, he still is

24. Aristotle also refers to Zeno at *De Sophisticis Elenchis*, 182b26, but this concerns the paradox of being, viz. Zeno's claim that all being is one.

6. *Topica*, 160b6-10 (Pickard-Cambridge 1984a's translation): καίτοι οὐδὲ τοῦθ' ἱκανόν· πολλοὺς γὰρ λόγους ἔχομεν ἐναντίους ταῖς δόξαις, οὓς χαλεπὸν λύειν, καθάπερ τὸν Ζήνωνος ὅτι οὐκ ἐνδέχεται κινεῖσθαι οὐδὲ τὸ στάδιον διελθεῖν, ἀλλ' οὐ διὰ τοῦτο τἀντικείμενα τούτοις οὐ θετέον.

> mistaken, even if he has deduced it ten thousand times over. For this is no solution; for a solution is an exposition of a false deduction, showing on what its falsity depends. If then he has not made a deduction, whether he is trying to establish a true proposition or a false one, to point this out is a solution"[7].

This is not to say that Zeno's arguments were not fallacious, but only that the reader of the *De Sophisticis Elenchis* is not expected to be able to show on what their falsity depends.

Indeed, in his main discussion of Zeno's paradoxes of motion in *Physica* VI Aristotle explicitly says both ὁ Ζήνωνος λόγος ψεῦδος λαμβάνει, 'Zeno's argument takes up something false' (233a21-22) and Ζήνων δὲ παραλογίζεται· 'Zeno argues fallaciously' (239b5). Aristotle explains Zeno fallacious reasoning in both chapters 2 and 9 of *Physica* VI, but it is the first account that addresses in greatest detail what later came to be called 'the Question of the Halves'. Aristotle's explanation is:

> "It is impossible for a thing to pass over or severally to come in contact with infinite things in a finite time. For there are two senses in which length and time and generally anything continuous are called 'infinite': they are called so either in respect of divisibility or in respect of their extremities. So while a thing in a finite time cannot come in contact with things quantitatively infinite, it can come in contact with things infinite in respect of divisibility: for in this sense the time itself is also infinite: and so we find that the time occupied by the passage over the infinite is not a finite but an infinite time, and the contact with the infinites is made by means of moments not finite but infinite in number"[8].

This explanation relies on a distinction between different types of continuity: continuity in divisibility, i.e. divisibility into things always divisible[9], and continuity in extremities, i.e. when the extremities touch and are one[10]. It would, indeed, take an infinite amount of time to pass over an infinite set of things whose extremities touch and are one, which Aristotle here calls 'quantitatively infinite'.

7. *De Sophisticis Elenchis*, 179b17-26 (Pickard-Cambridge 1984b's translation). οὐδὲν δὲ κωλύει τὸν αὐτὸν λόγον πλείους μοχθηρίας ἔχειν, ἀλλ' οὐχ ἡ πάσης μοχθηρίας ἐμφάνισις λύσις ἐστίν· ἐγγωρεῖ γὰρ ὅτι μὲν ψεῦδος συλλελόγισται δεῖξαί τινα, παρ' ὃ δὲ μὴ δεῖξαι, οἷον τὸν Ζήνωνος λόγον, ὅτι οὐκ ἔστι κινηθῆναι. ὥστε καὶ εἴ τις ἐπιχειρεῖ συνάγειν ὡς δυνατόν, ἁμαρτάνει, κἂν [εἰ] μυριάκις ᾖ συλλελογισμένος· οὐ γάρ ἐστιν αὕτη λύσις· ἦν γὰρ ἡ λύσις ἐμφάνισις ψευδοῦς συλλογισμοῦ παρ' ὃ ψευδής. εἰ οὖν μὴ συλλελόγισται, †εἰ καὶ ἀληθὲς ἢ ψεῦδος† ἐπιχειρεῖ συνάγειν, ἡ ἐκείνου δήλωσις λύσις ἐστίν.

8. *Physica*, 233a21-31 (Hardie / Gaye 1984's translation): τὸ μὴ ἐνδέχεσθαι τὰ ἄπειρα διελθεῖν ἢ ἅψασθαι τῶν ἀπείρων καθ' ἕκαστον ἐν πεπερασμένῳ χρόνῳ. διχῶς γὰρ λέγεται καὶ τὸ μῆκος καὶ ὁ χρόνος ἄπειρον, καὶ ὅλως πᾶν τὸ συνεχές, ἤτοι κατὰ διαίρεσιν ἢ τοῖς ἐσχάτοις. τῶν μὲν οὖν κατὰ τὸ ποσὸν ἀπείρων οὐκ ἐνδέχεται ἅψασθαι ἐν πεπερασμένῳ χρόνῳ, τῶν δὲ κατὰ διαίρεσιν ἐνδέχεται· καὶ γὰρ αὐτὸς ὁ χρόνος οὕτως ἄπειρος. ὥστε ἐν τῷ ἀπείρῳ καὶ οὐκ ἐν τῷ πεπερασμένῳ συμβαίνει διιέναι τὸ ἄπειρον, καὶ ἅπτεσθαι τῶν ἀπείρων τοῖς ἀπείροις, οὐ τοῖς πεπερασμένοις.

9. See, e.g., *Physica* 232b24-25: λέγω δὲ συνεχὲς τὸ διαιρετὸν εἰς αἰεὶ διαιρετά. Cf. 231b15.

10. See, e.g., *Physica* 227a10: λέγω δ' εἶναι συνεχὲς ὅταν ταὐτὸ γένηται καὶ ἓν τὸ ἑκατέρου πέρας οἷς ἅπτονται, καὶ ὥσπερ σημαίνει τοὔνομα, συνέχηται.

Yet, says Aristotle, that which is infinitely divisible does not require an infinite amount of time to pass over. This is probably a consequence of Aristotle's earlier claim that infinite divisibility exists as a potential that cannot be actualized all at once[11]. Time, too, Aristotle points out, is infinitely continuous in divisibility and its infinite divisions which exist potentially could be taken to correspond to the potentially existing infinite divisions of space[12]. Zeno's fallacy, then, is of a kind discussed repeatedly in the *Topica* and *De Sophisticis Elenchis*: when a term, in this case 'infinite' or 'infinitely continuous', is said in many ways, it can be misunderstood according to the wrong meaning. Zeno understands it one way, when in fact he should have understood it in another.

Still, Aristotle does not address Zeno's paradoxes of the stadium, of the halves, and of motion in general in any detail in dialectic or sophistical refutations, but rather in physics. Undoubtedly, Aristotle does this because he does not want to include extensive discussions of infinity, continuity, or divisibility in the *Topica* and *De Sophisticis Elenchis*. Indeed, when it comes to equivocal terms, Aristotle prefers to discuss terms like 'sharp' (ὀξύς), whose meaning in music is easily distinguished from its meaning in describing objects[13]. The background required to disambiguate the terms of *Physics* VI takes much longer to explain. Indeed, Aristotle may also be using his refutation of Zeno as a pedagogical opening to encourage his readers to turn to the *Physics* in order to grasp those complicated concepts. It is also possible that Aristotle does not think that the readers of the *Topica* and *De Sophisticis Elenchis* and perhaps regular Greeks in general include the denial of motion or the other issues Zeno raised among their δόξαι; accordingly, there is no great need to refute these claims in the dialectical works[14].

Nevertheless, in the Arabic Aristotelian tradition and consequently in the Hebrew Aristotelian tradition, Zeno's paradoxes were, in fact, given serious treatment in commentaries and super-commentaries on Aristotle's *Topica* and *De Sophisticis Elenchis*. In what follows, I shall trace in outline how Zeno's paradoxes came to be seen as part of dialectic and sophistic through the commentaries of Al-Fārābī and Averroes and then Hebrew commentaries on those works. This move often led to simplifying the paradoxes or assimilating them into what is often known as the paradox of the stadium. Further, I shall argue that for Al-Fārābī and Averroes, this change served to remove dialectical considerations, especially those of Kalamic atomists, from physics. Jewish commentators later followed this trend, thereby perpetuating the distinction between Aristotelian science and religious dialectical argumentation.

11. See *Physica* 206a14-b3. See, e.g., Bostock 1972 and Hintikka 1966.

12. For a recent comprehensive and clear account of these issues see Sattler 2020, pp. 277-334.

13. See, e.g., *Topica* 106a12-14.

14. Sattler 2020, pp. 124-175 argues that even Zeno himself did not believe that motion is impossible, but only that it is beyond human knowledge. Indeed, she argues, Zeno joins other Eleatics in maintaining that physics cannot properly be known by man.

2. Al-Fārābī

Al-Fārābī discusses Zeno's paradox(es) of motion twice in his commentary on Aristotle's *Topica*, the *Book of Dialectic* (*Kitāb al-jadal*)[15] and once in his commentary on the *De Sophisticis Elenchis*, the *Book of Refuting the Misleading* (*Kitāb al-ʾamkana al-mughalaṭa*)[16]. The first discussion in the *Book of Dialectic* occurs within an account of the benefits of dialectic for philosophy (paragraph 20) and the second discussion echoes the conclusions of the first (paragraph 88). For Al-Fārābī, the first benefit (نافع) of dialectic for the study of philosophy is primarily in testing traditionally received opinions, i.e. those opinions one encounters first and which are inculcated through education or acculturation (أدب) and habituation (عود); these opinions would apparently include religious views. Such tests, says Al-Fārābī, are not possible without 'opposition' (عناد), which in turn is not possible without the art of dialectic (صناعة الجدل). Indeed, Al-Fārābī dedicates a large part of his *Kitāb al-jadal* to describing how in debates questioner and respondent put forward various opposing views and arguments whose resolution brings them closer to practicing philosophy. Still, rather than bring an example of a test of traditionally received opinions, Al-Fārābī brings the example of Zeno's paradox, which shows, he says, that dialectical arguments can even bring people to become skeptical (حمل ... على الاسترابة) about the sensibles (بالمحسوسات). Indeed, Al-Fārābī attributes knowledge of three types of propositions to a beginner who has not yet studied philosophy: widely held opinions (الآراء المشهورة), traditionally received opinions (الآراء المقبولة), and sensible opinions (الآراء المحسوسة). The implication is that if dialectical arguments such as Zeno's paradox can make one skeptical of sensible opinions (e.g. that motion exists), then they can also make one skeptical of widely-held opinions and traditionally received opinions. The resolution of such dialectical difficulties is accordingly the first task of dialectic[17].

15. An edition and French translation of Al-Fārābī's *Book of Dialectic* is in D. Mallet's unpublished 1992 doctoral thesis (Mallet 1992). A recent English translation of the work can be found in Di Pasquale 2019. Di Pasquale's translation lists the page and paragraph numbers of Mallet's edition. For the anonymous, probably twelfth century, Hebrew translation of Book I of Al-Fārābī's *Book of Dialectic* see Y. Halper and G. Weber's 2022 edition, which also follows the paragraphing of Mallet's edition. Citations from Al-Farabi's commentary will accordingly be to paragraphs of Mallet's edition.

16. An edition of Al-Fārābī's *Kitāb al-ʾamkana al-mughalaṭa* is in R. Al-ʿAjam's 1987 edition of Abū Naṣr Al-Fārābī's *Al-Manṭiq ʿinda al- Fārābī* (vol. ii, p. 131-164). An anonymous medieval Hebrew translation of this text, probably from the thirteenth century, is extant in at least six manuscripts: mss. München, Bayerische Staatsbibliothek, hebr. 110, ff. 219r-223v and hebr. 244, ff. 213r-222r; ms. Wien, Österreichische Nationalbibliothek, hebr. 53, ff. 27v-40v (second pagination); ms. Paris, Bibliothèque Nationale, héb. 929, ff. 227v-241r; ms. Parma, Biblioteca Palatina, Parm. 2761, ff. 118v-131r; ms. Jena, Universitätsbibliothek, Rec. adj. f. 10, ff. 45v-53r.

17. Abū Naṣr Al-Fārābī, *Kitāb al-jadal*, p. 52: ويحمله ذلك على أن يمتحن الآراء المقبولة التي كان لقيها أوّلاً وأدّب بها وعوّدها، حتّى أنّه ربّما حمل كثيرا من الناس في كثير من الأوقات على الاسترابة بالمحسوسات وامتحانها كما عرض لبرمنيدس ولزيئن (= ولزنين) إلى أن قالا في الحركة إنّها غير موجودة وإنّ الكثرة غير موجودة وإنّ الموجود واحد ورأوا أن يتبعوا ما توجبه المشهورات التي كانت هي المعقولات عندهم وأن يستريبوا بالمحسوسات إذ كانت المعقولات أخصّ بالإنسان من المحسوسات.

In the *Book of Refuting the Misleading*, Al-Fārābī says the following:

> "Zeno's doubts about motion … include the question of the halves: it is known that one who transverses some distance has crossed half of that distance prior to crossing the entire distance and has crossed half of that half prior to crossing the entire half. Now, when a body is infinitely divisible into halves, it necessarily follows that the one in motion crossed an infinite distance in an infinite amount of time. But this is false. Now this necessarily follows because a distance is infinite in one of two ways: in length or in division. Accordingly, it is not possible for one to cross a distance that is infinite in length in a time that is finite in length. Nor is it possible for one to cross a distance that is finite in length in a time that is infinite in length. Nor is it possible for one to cross a distance that is infinite in division in a time that is infinite in division. Or vice versa. And since he took a distance that is infinite in division and a time that is finite in length he was led to error and imagined there to be an infinite time because of the infinite distance"[18].

This argument, which focuses on the distinction between infinite in division and infinite in length, is clearly based on Aristotle's *Physics* 233a21-31, which I quoted in full above. Al-Fārābī's treatment, though, avoids the concept of continuity altogether. Instead, it concentrates on elucidating cases of crossing a distance[19] in time, based on whether the distance or time is finite, infinite in division, or infinite in length. The latter is apparently equivalent to Aristotle's 'quantitatively infinite'. Al-Fārābī singles out three cases which are impossible, but maintains that it *is* possible to cross a distance that is infinite in division in a time that is finite in length, even though that time is infinite in division. Zeno apparently did not see this because he was misled by the ambiguity of the infinite and so conflated infinite in division and infinite in length. That is, Zeno fell pray to the fallacy of ambiguity and thus judged something impossible that was in fact completely possible.

18. Abū Naṣr al-Fārābī, *Al-Manṭiq ʿinda al- Fārābī*, vol. ii, p. 148: امهنم … ةكرحلا يف نينز كوكشو لبق فصنلا كلذ فصنو عطق ءانو امهعطقو نأ لبق ةقفاسملا كلت فصنو عطق ءنأ رهاظف ام ةقفاسم عطق اذإ لقنتملا نأ وهو .فاصنإلا ةلأسم كلذو ,مانتم نامز يف ةيهانتم ريغ ةقفاسم عطق كرحتملا نوكي نأ مزل ةيهانتم ريغ أفاصنأ مسقني مسجلا نأك اذإو .امهفصنو ءامتم عطقي نأ نكمي لا كرحتملاو نامزلا كلذكو .ةمسقلا يف امإو لوطلا يف امإ نيتهج ىدحإ ةيهانتم ريغ نوكت ةقفاسملا نأ لبق نم مزل امنإو لاحم لاو ,لوطلا يف مانتم ريغ نامز يف لوطلا يف ةيهانتم ةقفاسم عطقي نأ ,لوطلا يف مانتم نامز يف لوطلا يف ةيهانتم ريغ ةقفاسم عطقي أيهانتم نامزلا ذخأو ةمسقلاب ةيهانتم ريغ ةقفاسملا ذخأ امإو ,سكعلاب كلذكو .ةمسقلاب مانتم نامز يف ةمسقلاب ةيهانتم ريغ ةقفاسم عطقي نأ لقنتملا rather than Al-ʿAjam's ةقفاسملا يهانتو لا ةهجو نم نامزلا يهانتو نأ ةمهوأو طلاغ لوطلا يف. I am reading in accordance with the three manuscripts in Al-ʿAjam's footnote 5 and the Hebrew translation. The Hebrew translation, as found in Jena, Universitätsbibliothek, 10, f. 49^r has: וספקות זנין בתנועה … מהם שאלת החציים והוא שהמעתיק כשילך מהלך מן המהלכים בידוע שהלך חצי המהלך ההוא קודם שילך כל המהלך והלך חצי חציה קודם שהלך כל חציה. וכאשר יהיה הגוף מתחלק לחציים שאין לו תכלית יתחייב שיהיה המתנועע הלך מהלך שאין לו תכלית בזמן שיש לו תכלית. וזה שקר. וזה יתחייב מפני שהמהלך יהיה באין תכלית באחד משני צדדים באורך או בחלוקה. וכן הזמן והמתנועע אי אפשר שילך מהלך שאין לו תכלית בארך בזמן שיש לו תכלית בארך. ולא שילך מהלך שיש לו תכלית בארך בזמן שאין לו תכלית בארך. ולא שילך מהלך בלא תכלית בחלוקה נלקח הזמן בתכלית באורך הטעה ודמה שהתכלית הזמן מצד לא תכלית המהלך.

19. Or, perhaps, 'interval'. The Arabic ةقفاسم is used to translate the Greek διάστασις at *Physics* 202b17-18. See *Glossarium Græco-Arabicum*, ةقفاسم. Διάστασις refers to an 'interval' or an 'extension', at *Topics* 142b5 (see also Plato, *Timaeus,* 36a). Still, the term, ةقفاسم, does not here refer to an interval of time and so I have preferred to translate it 'distance'.

In sum, Al-Fārābī's discussions of Zeno leads us to two conclusions. One is that Zeno's paradox is actually a sophistical fallacy. It is accordingly properly placed in the *Sophistical Refutations* which he sees as a continuation of dialectic. Second, refutations of this kind are somehow connected to testing received opinions, which include religious opinions. That is, Al-Fārābī apparently views Zeno's doubts, i.e. Zeno's paradoxes as connected to received opinions and possibly more generally to well-known views, assuming that the received opinions as presented in Al-Fārābī's *Book of Dialectic* are included among the well-known views that are the primary subject of the book. Aristotle, however, did not connect Zeno's paradox to received opinions, and if he saw the paradox as connected to widely held views, he did not, apparently, consider such a relation to be relevant for the audience of the *Topica*. In contrast, Al-Fārābī addresses Zeno's paradoxes of motion, even if he subsumes them under "the question of the halves", in the *Book of Dialectic*, apparently considering the readers of that work to connect this question to received opinions and perhaps even to widely held views more generally.

But who were the readers of Al-Fārābī's *Book of Dialectic* and the *Book of Refuting the Misleading* who would consider Zeno's paradoxes relevant for their received opinions? To my mind, it seems most likely that Al-Fārābī has in mind believers in Kalām atomism, particularly those influenced by Basrian Muʿtazila. Abū al-Hudhayl who came to be recognized as the "most influential early Muʿtazilī theologian" developed a theory of atoms and their accidents which he used to explain God's acts of creation[20]. In his view, Zeno's paradox of the halves proved the existence of indivisible units, *viz.* atoms, such that any locomotion traverses a necessarily finite number of those units[21]. Not all Muʿtazilites accepted this solution; indeed, Abū al-Hudhayl's nephew, the 9th century al-Naẓẓām rejected atomism arguing that Zeno's paradox of the halves could be solved by accepting the possibility of taking a leap, *ṭafra*, over infinitely many units of space[22].

That these views were not fringe, but mainstream is emphasized by their presence in the *Book of Beliefs and Opinions* of Saʿadia Al-Fayyumi, a contemporary of Al-Fārābī and head (*gaʾon*) of the Jewish academy at Pumbedita. Among his arguments that the world is created Saʿadia includes an argument against the infinity of time. According to Saʿadia, if time were infinite it could never be traversed. Saʿadia argues for this by considering[23] each unit of time (*al-ʾān*) to be a

20. Mourad 2018.

21. The attribution of this argument to the eighth-ninth centuries Abū al-Hudhayl is made by the eleventh century Muʿtazilī theologian Ibn Mattawayh. Still, there is no reason to assume it is not genuine or to question the importance of Zeno's paradoxes for Muʿtazilī atomism before Al-Fārābī. For a translation and analysis of Ibn Mattawayh's account of Abū al-Hudhayl's argument, see Dhanani 1994, pp. 160-161.

22. See Dhanani 1994 and Dhanani 2004. Still, it is not entirely clear that al-Naẓẓām thought that there were *infinitely* many parts between all distances. See Pines 1997, pp. 14-15, n. 37.

23. Note that Saʿadia's word for 'considering' or 'establishing' is from the root w-ḍ-ʿ, the same

point (*al-nuqṭa*) and reducing the argument to that of Zeno's paradox of the stadium. Saʿadia does not name Zeno, but rather mentions "one of the unbelievers (*al-mulḥidūn*)" who met with and debated one of "those who affirm the unity of God (*al-muwaḥidūn*)". The unbeliever mentions the paradox of traversing an infinite number of parts. We do not hear the response of the monotheist, but Saʿadia offers a number of solutions to the paradox: the first is atomism, i.e. there is but a finite number of parts to every distance or length of time; the second is the theory of the leap (*al-ṭafra*) over infinite parts; the third is that there is a infinite number of parts of time corresponding to an infinite number of parts of space[24]; and the fourth solution, which Saʿadia identifies as his own, depends on the distinction between potential and actual infinity that we find in Aristotle's *Physics*, though it receives no attribution here[25]. Saʿadia's prominent use of these arguments in the first chapter of his *magnum opus* suggests that they were well known outside of Basra in Al-Fārābī's time even among non-Muslims[26]. Further, Saʿadia's inclusion of atomism as a monotheistic solution to Zeno's unbelief tells us that monotheists, both Muslims and Jews, took Zeno's paradox seriously and that arguments against it in favor of atomism and the leap theory were considered part of mainstream religiously acceptable views. Finally, note that Saʿadia's invention of the dialogue between the infidel and the monotheist is wholly unnecessary to the argument; in its context, indeed, it would seem to have no other purpose than to bring this discussion into a context of dialectic and debate.

Al-Fārābī's treatment of Zeno's paradox in his commentaries on the *Topica* and *De Sophisticis Elenchis* is likely to be a response to the kind of dialectical discussions of Zeno that Saʿadia encountered. Like Saʿadia, Al-Fārābī prefers a solution rooted in Aristotle's *Physica*; but unlike Saʿadia, Al-Fārābī does not admit atomism as an acceptable view on theological grounds. Indeed, it might be that countering this view is at the heart of his critique of accepted views in the *Book of Dialectic*. If so, then Al-Fārābī treats Zeno's paradoxes in greater detail in the dialectical works in order to counter their uses by those he perceives as contemporary dialecticians and sophists, including especially atomists. For Aristotle, I argued above, Zeno's paradoxes raise interesting theoretical questions without causing serious doubts about basic physical principles. For Al-Fārābī, the paradoxes have indeed gained an audience that takes them as serious critiques and accordingly developed an entire non-Aristotelian scientific system to treat them. One cannot

root used to translate Aristotle's τόπος throughout the *Topica*.

24. This solution is stated very briefly in the vaguest possible terms and Saʿadia does not point out that it would allow the world to be eternal.

25. See Saʿadja b. Jûsuf al-Fajjûmî, *Kitâb al-Amânât waʾl-Iʿtiqâdât*, §36. English translation in Rosenblatt 1948, pp. 44-45.

26. Note also that Avicenna mentions the paradoxes of Zeno, attributing them to both ancient thinkers and modern, implying that they continued to be well known and much discussed in his day and in his circles. See Avicenna, *Al-Ṭabīʿīyāt, al-samāʿ al-ṭabīʿī*, 276 (trans. in McGinnis 2009, p. 276).

merely address these paradoxes in the physics since they deny the very possibility of physics as Aristotle understood it. Far better to address them in more detail and point out their fallaciousness in earlier logical works studied before one comes to study science proper.

3. Averroes

Averroes may be best known in the West for his return to Aristotle and for following Aristotle's text closely in translation. Yet he also follows Al-Fārābī in treating Zeno's paradox of motion as part of dialectic and the sophistical refutations, rather than primarily in physics (though Averroes does discuss it there too[27]). Toward the opening of Averroes' *Middle Commentary* on Aristotle's *Topica* we find a list of three benefits (منافع) of dialectic: becoming accustomed to arguments, arguing with the general public, and in grasping well-known premises necessary for the theoretical sciences (العلوم النظرية, הידיעות העיוניות). Averroes divides this third benefit into five parts (وجوه, פנים).

> "The fourth is [those principles] through which the sophistry of those who mislead people with regard to the principles of the sciences is rejected. This is like what Aristotle did in the first treatise of the *Physica* with those who denied plurality and the existence of motion"[28].

Here Averroes clearly locates Zeno's paradox, even including its treatment in the *Physica*, in dialectic and the refutation of sophistry, which he seems to consider a single enterprise.

When he treats Aristotle's short discussion of Zeno's paradox at *Topics* 160b6-10 in his *Middle Commentary*, Averroes clarifies at some length how precisely it fits in to dialectic. He says:

27. See Averroes, *Epitome in Physicorum Libros*, ed. Puig, p. 52 (Spanish translation in Puig 1987, p. 153 – see also Puig's discussion on p. 38). The Arabic of the Middle and Long Commentaries on the *Physica* is not extant. For a discussion of Zeno's paradoxes of motion as they appear in the Michael Scot's Latin translation of the *Long Commentary* on the *Physica* see Puig Montada 2018. For a discussion of how this paradox appears in an anonymous medieval Hebrew translation of the *Long Commentary* see Glasner 2001. On the Hebrew translations of Averroes' three commentaries on Aristotle's *Physica*, see Glasner 2011, pp. 183-184.

28. Averroes, *Middle Commentary on Aristotle's Topics*, ed. Butterworth / Haridi, pp. 33-34: اودحج نيذلا عم عامسلا نم ىلولأا ةلاقملا يف وطسرأ لعفب امك – هولعلا ءىدابم يف ىناطسفوسلا ةطلاغم ىقلتت امهب نأ عبارلاو ةكرحلا دوجو ترثكلا. Qalonimos ben Qalonimos's fourteenth century Hebrew translation of this line is: והרביעי שבהם ידחו הטעאות המטעי' בהתחלות החכמות כמו שעשה אריסטו במאמר הראשון מספר השמע עם אשר כפרו ברבוי ומציאות התנועה. All citations from and references to the Hebrew translation of Averroes *Middle Commentary* on Aristotle's *Topica* are from the critical edition currently being prepared by my doctoral student, Arye Rainer as part of his dissertation. Since the focus of this article is on the Hebrew tradition, I cite the Hebrew as well as the Arabic.

> "One who brings a proposition that contradicts a universal premise that is supported by induction ought not bring a proposition that gives a universal refutation which is a refutation of the contrary. Rather he should bring a proposition that gives a partial refutation which is a refutation of the contradictory. For refuting the premises which proceed through universality is peevish. Thus could one be required to deny the sensibles. For example, if we saw fit to explain that every animal is moving through an induction that proceeds through animals who sense by that thing which is moving, it would not be fitting to respond with something like Zeno's argument from which it follows that nothing is moving. This argument is that in which he said that every moving thing will cross half a distance before crossing the entire distance, and half of that half before crossing the half, and half of the half of that half, and so on *ad infinitum*. So it is absurd that someone will cross an entire distance in finite time. Accordingly, nothing is moving. Arguments like this reject the sensibles, but with such arguments it is difficult to accept their contradictory. But were it not for this [contradictory], it would be impossible that sensibles would occur. However, the master of this science ought to be warned about these [arguments], though they are primarily contained in sophistic. This injunction is one that a respondent ought to employ with premises like these"[29].

This comment agrees with Aristotle at *Topica* 160b6-10 that Zeno's paradox of the stadium is difficult to contradict, but nevertheless should not be accepted. Yet, Averroes' version differs in six key respects. In the first place, Averroes has included here a concise statement of the stadium paradox which is nearly identical to his presentation of the same paradox in the *Middle Commentary* on *Physica* 239b9-14[30]. As we saw, Aristotle's *Topica* 160b6-10, in contrast, merely alludes to the problem by name. In the second, Averroes understands the whole discussion here to refer to the problem of excessive universality, a problem we en-

29. Averroes, *Middle Commentary on Aristotle's Topics*, ed. Butterworth / Haridi, pp. 228-922: لوقبي تأيلب – دضلابلطبلإا وهو – ةيلكلابلطبيلوقيي تأيلا نأ ءارقتسلابلا ةتيثملا ةيلكلا ةمدقملا ضقانيلوقبي تأ اذإ يغبنيو لئاثم – تاسوسحملا راكنإ هذع مزليذإ عينش ةيلكلابلهنأش اذه ي تلا تاسمدقملا لطبإ نإف .ضيقنلابلطبلإا وهو – اينزج لاطبإ هم لهلطبي سايقلثمب كلذ بيجملا يقلتيف كرحتت لهنأ اهرمأ نم سحي تاناويح ءارقتسلابلا نييبن أ دارأ اذإ كلذ لبق فصنلا فصنو لهعيمج عطقي نأ لبق ةفاسملا فصنع طقي ةفاف كرحتم لكه يفإ لاق ي ذلا وهو .كرحتم ءيش لاو هذع مزلالا نينز لاحم كلذو ةيهانتم ريغ لماظعأ مانتم نامز يف لهلك ةفاسملا عطقي وهو ةيهان ريغ يلإ كلذكو فصنلا لبق فصنلا فصنو فصنلا ضرلاعبن أ نكمي مل كلذ لاولو لهضقنب بعصيب تاسايقلا اذه عم ي هن كلا تاسوسحملا عفدت تاسايقلا هذه لئاثمأ نإف .كرحتي ءيش لاو نذإف لئاثمأ يف لهلثتمين أ ب بيجملا ي غبنيي تلا ةيصولا ي هذه ي لوأ ي ناطسفوسلاب ي هو اهرذحي ن أ ي غبنيي ةعانصلا هذه نأ لإا .تاسوسحملا لهذ .تامدقتملا هذه. Qalonimos' translation (para. 336) has: וראוי כאשר יביא מאמר יסתור ההקדמה הכללית המקיימת בחפוש שלא יביא מאמר שיבטלה בכללות והוא הבטול להפך אבל יביא מאמר שיבטלה בטול חלקי, והוא הבטול בסותר. כי בטול ההקדמות אשר זה דרכם בכללות מרוחק אחר שיחוייב ממנו הרחקת המוחש. דמיון זה כאשר ראינו שנבאר בחפוש בשכל חי מתנועע בשנחפש בעלי חיים שיורגש מעניינם שהם מתנועעים, הנה אין ראוי למשיב זה, בכמו היקש זנין המחוייב ממנו שהוא אין דבר מתנועע. והוא אשר אמר בו כל מתנועע הנה הוא יחתוך חצי המהלך קודם שיחתוך כלו וחצי החצי קודם החצי וחצי חצי החצי קודם חצי החצי, וכזה אל בלתי תכלית, והוא יחתוך המהלך כלו בזמן בעל תכלית וזה שקר. הנה אם כן אין דבר מתנועע. כי כמו אלו ההקשים ידחו המוחשים אבל הם עם זה הקשים יקשה סתירתם. ולולי זה אי איפשר שיחלוק בהם המוחשים, אלא שבעל זאת המלאכה שיזהר מהם. והם בהטעאה יותר ראשונים. הנה זאת היא הצוואה אשר ראוי למשיב שיקיים אותה בדמיוני אלו ההקדמות

30. See, e.g., ms. Paris, Bibliothèque Nationale, hebr. 934, ff. 109v-110r: ספק זנון ... וספקות זה האיש המפורסמים בבטול התנועה ארבע ... והספק השני הוא אשר יודע בחצאים. וזה שהוא יאמר שהמתנועע האחד כאשר חתך המהלך כלו יחוייב שיחתוך חציו קודם שיחתוך המהלך כולו וחצי החצי קודם החצי וזה אל בלתי תכלית הנה יחוייב שאם היתה התנועה נמצאת שיתנועע המתנועע מהלך בלתי ב"ת. Averroes identifies this as commenting on Treatise VI, chapter 11; our editions of the *Physica* locate this passage in Book VI, chapter 9.

countered at Aristotle's *De Sophisticis Elenchis* 172a8-9 and in my bad joke at the opening of this article. Third, in Aristotle's *Topica*, the accusation of peevishness (δυσκολαίνειν) was leveled at those who do not raise objections or counter-arguments; for Averroes the corresponding Arabic term, شنيع,[31] refers to the excessively universal character of the argument. Fourth, for Aristotle, as we saw, Zeno's argument went against the δόξαι; for Averroes, Zeno's argument is due to mistaken, excessively far reaching inductions. Fifth, a directed response, treating the relevant part of the discussion alone is, according to Averroes, the proper strategic response to excessive universality. Sixth and finally, for Averroes, this kind of argumentation is properly part of sophistics, i.e. sophistical refutation, while for Aristotle, its proper place was in physics.

What do these changes and additions tell us about how Averroes saw the importance of Zeno's paradox? That he does not mention that Zeno's paradox runs counter to generally accepted opinions (presumably, המפורסמות, المشهورات) here suggests that he does not consider Zeno's views to be obviously contrary to widely held views. This may be a sign that he, like Al-Fārābī before him, recognized that Zeno's paradox of the halves was important for grounding atomism and the like among the Kalām and that, moreover, such theories had attained a fairly widespread acceptance in Averrroes' day[32].

In Averroes' view, then, such Kalām atomism is apparently argued by excessively universal arguments. For Averroes, this approach is peevish and is governed by mistaken inductions. Rather than looking at all animals and inferring the existence of some thing (*ʾamr*) that makes them moving and which is connected to sense-perception, they make excessively universal arguments about the impossibility of moving at all. Atomism can be proposed as an answer to these arguments about the apparent impossibility of motion, but it will not be connected to sense-perception since atoms cannot be seen. That is, I am suggesting that Averroes' account here is designed to reject any attempt by followers of Abū al-Hudhayl to use atomism and the denial of infinite divisibility as a solution to Zeno's

31. According to *Glossarium Graeco-Arabicum*, this word more frequently translates ἄτοπος or ἀτοπία. Qalonimos here uses מרוחק, which works, but is not used exclusively with this meaning.

32. On Averroes' critique of atomism and especially the notion that atomism negates the possibility of causation, see Kogan 1985, pp. 71-164, esp. p. 91-97. In his *Guide of the Perplexed*, Moses Maimonides, a contemporary of Averroes, lists atomism as the first of the 12 scientific premises common to the Mutakallimūn (*Dalālat-al-ḥā'irīn*, 105^{v} – transl. Pines 1963, p. 195). Maimonides addresses some problems that could be seen as related to Zeno, such as the problem of the arrow, on (ff. 106r-107v – pp. 197-198). Among the difficulties Maimonides raises is that half distances are not always possible since a given distance may have an odd number of atoms. In such a case, dividing the line into a finite number of atoms would not be sufficient to explain how a distance can be divided in to half (see p. 198). Note that Maimonides says only that this problem is related to the Kalām proofs for the first three premises he lists, but not which one. He clearly expects his readers to be familiar with this argument and other arguments mentioned here and their use in extracting what he terms the "premises of the Mutakallimūn".

paradox. Such atomism denies what has been sensed and poses something that has no basis in sensation.

Moreover, according to Averroes, the Zenoist/atomist seeks to refute the universal premise without addressing the particulars. In the example Averroes gives, someone notices that many animals move and inductively infers that all animals move. The proper refutation, says Averroes, is to bring a counterexample showing the contradictory, namely an animal that does not move. Such a refutation would result in rejecting the proposition that all animals move. What the Zenoist or Kalām atomist does, however, is to oppose the entire universal proposition, "all animals move", with its contrary: "no animal moves". In order to accept this, one would have to reject the particular premises s/he had previously accepted, namely that some particular animals move. To do this, one would have to reject what his/her senses perceived and so reject sensation itself as a valid means to attain truth.

Averroes' point, then, is methodological; Zeno's paradox of the halves is mentioned in the context of discussing how the proper way to refute an induction is by bringing a contradictory example, not a contrary to the universal proposition. Unlike Al-Fārābī, he is not interested in explaining infinity across distance and time here, but in explaining how to make and refute inductions. Since induction is discussed in Aristotle's *Topica*, this argument is properly a part of dialectic and accordingly it is discussed at some length in Averroes' *Middle Commentary* on the *Topica*. Yet, since the Zenoist's attempt to refute the induction is an error in argument, namely to bring a contrary where a contradictory is needed, its full discussion is most properly in sophistic. It is, thus, clear why Averroes felt the need to go into more detail than Aristotle did in discussing Zeno's paradox of the halves in the context of the logical *Organon*.

4. The Hebrew Tradition

The commentaries of Al-Fārābī and Averroes on dialectic and sophistical refutation would have been the primary sources for Hebrew readers interested in Aristotle's *Topica* and *De Sophisticis Elenchis*. Since neither Aristotle's *Topica* nor his *De Sophisticis Elenchis* has ever been translated into Hebrew, medieval Hebrew readers would have only had access to translations of Al-Fārābī's and Averroes' commentaries. Some few may have had access to Latin texts of Aristotle's works, but Jews did not begin study Aristotle in Latin in earnest until the sixteenth century when they began to attend the University of Padua. Even then, manuscript evidence suggests a preference for Hebrew texts until the seventeenth century.

Al-Fārābī's extensive commentary on Aristotle's *Topics*, the *Book of Dialectic*, was one of the earliest philosophical translations made into Hebrew, probably appearing in the twelfth century. The translation is unfinished and includes only the

first treatise of the book. Moreover, the choice of terms would appear to have been unfinished; numerous Arabic terms have been translated with a view to homophony, but these terms do not appear consistently. My suspicion is that the translator used homophonic terms as placeholders in a pioneering translation with the intention of replacing them with better terms. The translator replaced some but not all of these terms, before abandoning his translation, leaving the work short and with a great deal of inconsistency. It is, unfortunately, cut off right before the first mention of Zeno's paradox. Yet, the more detailed account of Zeno's problem of the halves is preserved quite clearly, as we saw, in Al-Fārābī's commentary on *De Sophisticis Elenchis*, the *Book of Refuting the Misleading*. This commentary is in the same series as the Hebrew translation and uses a very similar set of Hebrew terms. The translator of *Refuting the Misleading* remains anonymous, but the work is clearly intended to be read alongside *Kitāb al-jadal* and various *opuscula* of Al-Fārābī, which were translated into Hebrew around the same time.

Judging from the dispersion of the manuscripts, these works were read in Europe fairly continuously until well into the sixteenth century. There is even a short gloss commentary on Al-Fārābī's two works, *Dialectic* and *Refuting the Misleading*, preserved in marginal notes in two manuscripts. Unfortunately, there are no glosses where Al-Fārābī discusses Zeno's paradox of the halves in the *Refuting the Misleading*[33].

One reader of Al-Fārābī's *Book of Dialectic*, though not necessarily in its Hebrew translation, was Shem Tov Falaquera (ca. 1225-1295)[34]. In his *Epistle of the Debate*, Falaquera's Scholar responds to a Pietist's assertion that his faith cannot be questioned with the following remarks:

> "If you knew the difficult questions that [arise] by way of dialectics you would not say this. Have you not heard that among the ancient philosophers there were those who brought proof for the refutation of motion even though it is a thing perceived by the senses (this is the problem known as the problem of the halves)... The fallacy of these proofs that are dialectical cannot be recognized and cannot be refuted except by him who knows the science of demonstration and he alone can recognize the lie in them. Now, if concerning these things that are sensed, they [those practiced in dialectics] pose great difficulties for man and lead him astray, then how much more so is this the case concerning tradition?"[35]

33. The marginal notes are preserved in ms. Wien, Österreichische Nationalbibliothek, hebr. 53, ff. 20r-40v (second pagination) and ms. Paris, Bibliothèque Nationale de France, hebr. 928, ff. 24v-32v. Of these two manuscripts only the first contains the Hebrew translation of Al-Fārābī's *Refuting the Misleading* (ff. 27v-40v). In the case of the *Dialectic*, the marginal commentary has slight divergences in the two manuscripts that suggest it was copied from at least one other manuscript. Accordingly, there is reason to suspect that marginal Hebrew commentary on *Refuting the Misleading* is not unique to the Vienna manuscript.

34. See Halper 2021.

35. Shem Tov Falaquera, *Epistle of the Debate*, pp. 61-62 (transl. Harvey 1987, pp. 24-25).

While Falaquera does not tell us the solution to Zeno's paradox, he does locate it in the science of dialectic. That is, one should encounter such fallacies in dialectic (and probably also the sophistical refutations, which are a part of dialectic). The full refutation, he says, requires demonstration, i.e. science itself, but the question is dialectical in character. Moreover, like Al-Fārābī, he connects this paradox to testing tradition, even if this connection is somewhat tenuous, and notes that if dialectic can cause one to question the senses, it can cause one to question tradition as well.

Joseph Ibn Kaspi (1280 - c. 1345) not only read Al-Fārābī, but included a summary of *Refuting the Misleading* in his main logical text, *ṣeror hakesef*[36]. This work reproduces in slightly simplified form Al-Fārābī's response to Zeno's paradox in his *Refuting the Misleading*, a response which was highly indebted to Aristotle, *Physica* 233a21-31, as we saw above. Ibn Kaspi, however, includes this simplified argument in the section on fallacies of meaning rather than in the section on fallacies of linguistic expression[37]. Yet, this change is connected with Kaspi's understanding of homonymous terms and his emphasis on the ambiguity of meaning in cases like these, rather than on ambiguity of the linguistic term. In this case, he emphasizes that Zeno's argument confuses two different judgments (*dinim*) about two different properties (*tekhunot*), *viz.* infinity and finiteness about length and division. This ought to lead the Zenoists to two separate premises, but in fact they take them to be one. Still, Kaspi's account remains quite close to Al-Fārābī's and so can be understood to include a fairly extensive discussion of Zeno's paradox in the part of logic that treats sophistic.

In the second decade of the fourteenth century Qalonimos ben Qalonimos of Arles translated Averroes' *Middle Commentaries* on the *Topics* and *Sophistical Refutations* in their entirety into Hebrew. These translations were made in the style of the Ibn Tibbon family and so could be more easily read alongside the other translations of Averroes' Short, Middle and Long Commentaries on the Logical Organon[38]. Indeed, within 10 years of Qalonimos' translations of these works, Levi Gersonides (1288-1344) wrote extensive commentaries on both of them.

Unfortunately, the two manuscripts containing Gersonides' complete commentaries on the *Topics* and *Sophistical Refutations* are not in good shape and I have not yet been able to decipher them[39]. Yet, part of Gersonides' commentary

36. For an edition of this work, see Rosenberg 1984. The discussion of Zeno's paradox is on p. 288. While I shall point to some idiosyncrasies of Ibn Kaspi's Hebrew, it adheres quite closely to that of the anonymous translation found above in note 17.

37. See Charles H. Manekin's article in this volume (Manekin 2023).

38. On the edition of the *Topics* commentary, see note 28 above. An edition of the *Sophistics* commentary is still a desideratum.

39. See ms. Torino, Biblioteca nazionale universitaria, A I 14 and ms. Oxford, Bodleian Library, Mich. 64. The Turin manuscript was damaged in the fire of 1904 and the Oxford manuscript is very faint.

on the *Topics* survives in legible form in München, Bayerische Staatsbibliotek, Heb. MS 26, f. 319r-350v[40]. This manuscript is in an Italian hand and dated 1551. The text of the super-commentary is not complete and another scribe has continued the text (from 351r to 403r) with Qalonimos' translation of Averroes' *Middle Commentary* on the *Topics*. This section of the manuscript also contains numerous illustrations and is, I believe, the only illustrated text of Averroes' *Middle Commentary* on the *Topics* and *Sophistical Refutations* in any language.

Gersonides' super-commentary does not comment on every sentence, but selects sentences it finds interesting and then attaches its remarks to those. Yet it does not clearly indicate which sentences are quotations and which are the commentary, and a reader looking at this super-commentary alone would find the text somewhat mystifying. That is, the super-commentary assumes its readers to have Qalonimos' translation of Averroes' *Middle Commentary* on the *Topics* in front of them and to compare the two texts continuously.

One sentence the super-commentary focuses on is Averroes' statement that one of the benefits of dialectic is learning to refute those who "deny the existence of motion". Gersonides does not connect this statement to Zeno's paradox, but instead notes that the existence of motion and moving things are "among the principles that the physicist assumes" when he does physics. Still, he notes, this is not trivial since "the nature of moving things is different" among different moving things. Thus, "earth moves downwards, while fire moves upwards". Gersonides tells us, "Aristotle explains there [i.e. in the *Physica*] using dialectical methods that their [i.e. Zeno's and the like's] argument is of the utmost absurdity"[41]. That is, Gersonides actually reverses the pattern we have seen until now. He suggests that the proper place to address Zeno's paradoxes of motion is in the physics. Still, he recognizes that the method employed there is not demonstrative, but dialectical. In this respect, Gersonides follows Averroes, though it is possible that he was aware of Latin traditions that treated Zeno's paradoxes in the *Physica* while only alluding to them in the *Topica* and *De Sophisticis Elenchis*.

Yet, Gersonides still gives Zeno a prominent place in his discussion of how syllogisms with well-known premises are usually neither always true nor always false in his super-commentary on Averroes' *Middle Commentary* on the *Topica*. Gersonides writes:

> "You ought to know that it will not happen that that which is well-known to the general public will be false in every respect. However, it can happen among theoret-

40. At the 2022 XXVI[th] Annual SIEPM Colloquium on "Dialectic in the Middle Ages: Between Dialectic and the Foundation of Science" at Bar Ilan University, Manekin discussed this manuscript in detail, estimating that about 2/5 of Gersonides' *Commentary* on the *Topics* was extant therein.

41. Ms. München, Bayerische Staatsbibliotek, heb. 26, f. 321r-v: הכחישו התנועה ... הם התחלות בעל החכמה ... שהארץ התנועע אל המטה והאש אל המעלה ... וארסטו' ביאר להם שם בדרכים הטבעית ... ולזה יהיה טבע הנמצאות מתחלף הנצוחיים שמאמרם הוא בתכלית הבטול.

> ical scientists that they will reject sensibles because of an absurd opinion that arises as true in their thought. Zeno and many other ancients, for example, did this when they denied many of the sensible things"[42].

Gersonides, accordingly, joins Al-Fārābī in considering Zeno's view to be among the widely held views, if only among those well known to theoreticians. He may have the Kalām in mind, but he may also have had in mind any number of other later developments physics, medicine or even magic that could lead people who consider themselves theoreticians to reject what their own eyes behold. We may suppose, though, that since these well-known opinions are those of theoreticians, they are better addressed in physics than in dialectic and Gersonides accordingly elaborates them only there.

5. Conclusion

Aristotle saw the *Physica* as the most appropriate place to treat Zeno's paradoxes of motion. Still, he alluded to the paradoxes, or at least some of them, in the *Topica* and *De Sophisticis Elenchis*. These allusions, however, were quite general and implied that fuller treatments were to be found elsewhere. Yet for Al-Fārābī, the proper place to address Zeno's paradoxes was precisely in commentaries on the *Topica* and *De Sophisticis Elenchis*. This is because he saw those paradoxes as relating to widely-held opinions, presumably those held and promoted by Kalām theologians. Averroes, however, treated Zeno's paradoxes in his commentaries on Aristotle's *Physica*, but also went into them in fairly extensive detail in the his Middle Commentaries on the *Topica* and *De Sophisticis Elenchis*. While Al-Fārābī focused on the physical theory that would refute Zeno's paradoxes, Averroes's *Middle Commentary* on the *Topica* focused on the logical argumentation for making and refuting inductions. Jewish students of the *Topica and De Sophisticis Elenchis* inherited from the Muslim predecessors the notion that Zeno's paradoxes are dialectical fallacies and so should be treated in the context of the *Topica and De Sophisticis Elenchis*. Still, Gersonides seems to have made some effort to direct his readers away from examining Zeno's paradoxes in the context of the *Topica*, and to return them to the *Physica*.

42. Ms. München, Bayerische Staatsbibliotek, heb. 26, ff. 319v-320r: וראוי שתדע שהמפורסם אל ההמון לא יקרה שיהיה כוזב בכל, אבל יקרה במפורסם אצל | בעל מלאכה מהמלאכות העיוניות שידחה המוחש מפני דעת בטל עלה במחשבתו שהוא צודק כמו שעשה זנין וזלותו רבים מהקודמים בהכחישם רבים מהדברים המוחשים.

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Abstract: Following the Arabic tradition, medieval Hebrew commentaries on Aristotle's *Topica* and *De Sophisitics Elenchis* understood Zeno's paradoxes of motion as dialectical fallacies related to widely-held opinions or incorrect inductive arguments. Following Al-Fārābī and Averroes, Hebrew Aristotelian commentators include discussions of Zeno's paradoxes of motion in their commentaries on *Topica* and *De Sophisitics Elenchis*. Aristotle's own discussions of Zeno's paradoxes in those works, however, merely allude to the difficulties without presenting solutions. Indeed, they point elsewhere, most likely to the *Physica* where Aristotle provides a detailed account of those paradoxes and their solutions. The shift in emphasis in the discussions of Zeno's paradoxes of motion as dialectical paradoxes in the *Topica* and *De Sophisticis Elenchis* in the works of Al-Fārābī and Averroes was likely due to the importance of those paradoxes for *Kalām* atomism. Hebrew commentators inherited this approach, even though they did not operate in the context of the *Kalām*.

Keywords: Logic; Aristotle; Zeno; Al-Farabi; Averroes; Dialectic.

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