Real Virtuality and Actual Transitions: Historical Reflections on Virtual Entities before Quantum Field Theory

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This paper studies the notion of virtuality in the Bohr-Kramers-Slater theory of 1924. We situate the virtual entities of BKS within the tradition of the correspondence principle and the radiation theory of the Bohr model. We show how, in this context, virtual oscillators emerged as classical substitute radiators and were used to describe the otherwise elusive quantum transitions. They played an effective role in the quantum theory of radiation while remaining categorically distinct and ontologically separated from the quantum world of the Bohr model. The notion of virtuality thus differs markedly from its counterpart in quantum mechanics or QFT.

1. Introduction

The virtual particles of modern physics are troubling objects, for philosophers and physicists. They do not seem to fit into the discourse on scientific realism, which is more about what is real and what is not, a dichotomy that virtual particles seem to transcend. Fitted into such an either-or analysis, the argument has gone either way, with virtual particles having being identified both as real, in some sense of the word, or as merely instrumental constructs arising in the calculational practices of quantum field theory.

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With this discussion of the ontological status of virtual entities in full bloom, recent years have seen an increased interest in the history of virtual entities in twentieth-century physics. The first prominent use of the term "virtual" is actually rather easily located and well-known. It appeared in the years right before the construction of quantum mechanics, in the last years, that is, of the old quantum theory of the Bohr model. Within this context, Niels Bohr, Hendrik Kramers, and John C. Slater attempted to develop a full-fledged theory of electromagnetic radiation in 1924. Their theory, which is commonly known as BKS, gained prominence for an ultra-short period of time, before it was spectacularly falsified by experimental evidence.

Its quick demise aside, BKS introduced and centrally relied on the concepts of "virtual oscillators" and a "virtual radiation field." It thus provides a natural starting point for the history of virtuality in quantum physics. On closer inspection, however, there is something rather odd about this lineage. The virtual particles in QED, or virtual states in non-relativistic quantum mechanics for that matter, are firmly grounded within the formalism of quantum theory.[1] They emerge as quantum objects (regardless of whether they are considered as real, instrumental or illustrative) and live in the same theoretical space as the non-virtual objects of the theory. The relation between the virtual oscillators and virtual radiation field of BKS and the stationary states of the Bohr model was different. The two, as Bohr put it somewhat mystifyingly, were only conjugated with each other.

We will try to work out more clearly what this conjugation between the virtual entities and the stationary states entailed and thus study the role played by the virtual entities of BKS theory. Previous attempts at interpreting this concept by historians of quantum physics have been brief and ultimately dismissive. The virtual appeared as little more than a label or metaphor without substance; at best, virtual entities were contrasted in different ways with their "more real" counterparts.[2] As we see it, the notion of virtuality in BKS can be further clarified. In this paper, we will present an analysis that (a) reveals a very specific and in fact adequate use of the term "virtuality" in BKS and (b) shows that this notion emerged as a direct reaction to a central tension in Bohr's quantum theory of radiation. This

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[1] The introduction of the concepts of virtual particle and virtual state in modern physics, as well as their relation to BKS theory, is discussed in detail in Martinez' contribution to this volume.

[2] See Hendry (1981, p. 198), Darrigol (1992, p. 245), and Duncan and Janssen (2007, p. 603) for mere-label-without-substance interpretations. For comparisons between virtual and real entities see Dresden (1987, p. 172) and Darrigol (1992, p. 220). Dresden, in particular, identified the salient features of virtuality in BKS. We add to his assessment by tracing the origins of the concept in the correspondence tradition, thereby further clarifying the meaning of virtuality.

improved understanding of virtuality in BKS, it would appear to us, provides the necessary foundation for discussions of the connection between virtuality in the old quantum theory and the virtual particles of QFT.

In order to understand the role of virtual entities in BKS theory, we need to situate that theory within its proper historical context and in particular within the theoretical practices from which it emerged. These theoretical research practices normally go by the name of the "old quantum theory," long considered a misnomer given the heterogeneity of its methods and concepts. We will therefore begin with a short overview of Bohr's atomic model of 1913 and the ways in which it described the radiation process as a discrete transition between stationary states (section 2). We will then argue that the virtual oscillators emerged from the notion of a "substitute radiator," introduced by Kramers in 1919 in the context of Bohr's correspondence principle as an attempt to flesh out (and treat quantitatively) the discrete and featureless quantum radiation process (section 3). BKS, we claim, built on these substitute radiators and developed them into virtual oscillators in order to provide a detailed dynamical picture of the radiation process and of energy transfer, without running afoul of the less and less negotiable need for fundamental discontinuity (section 4). After this study of the methodological role of the virtual entities in historical practice, we will conclude with reflections on the notion of virtuality that was at play in BKS (section 5).

2. Bohr's Blackbox: Radiation and the State-Transition Model in the Old Quantum Theory

Bohr's planetary model of the atom was introduced in 1913 (Bohr 1913).[3] It quickly consolidated and expanded to become the core of the old quantum theory, which thereby moved its focus from thermodynamics (as in particular in Planck's law) to atomic spectroscopy.[4]

Already in the trilogy of papers that introduced it, the Bohr atom was based on two non-classical assumptions: the stationary states and the energy-frequency relation. The first stated that the atom could be in a number of stationary states with definite energies. In these states, electrons move around the nucleus only on select orbits, singled out by a quantum

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[3] For the genesis of the Bohr model, see Heilbron and Kuhn (1969), Jammer (1966), and Darrigol (1992). In this section, we will provide a rough-and-ready summary of the Bohr atom and its concept of radiation, as it is necessary to understand the roles played by the later substitute radiators and virtual oscillators. A reader familiar with the history of the old quantum theory should feel free to merely skim this section.

[4] As shown by Alexi Assmus (Assmus 1990, 1992a, b), the Bohr model was not the only impetus for this shift of focus. As physicists applied thermodynamical quantum approaches to specific heats, they increasingly had to engage with the specifics of the quantized energy levels of molecules.

condition, and do not emit radiation. The second assumption stated that radiation is emitted and absorbed when the system makes a transition between two such stationary states, n_1 and n_2. The frequency ν of the radiation is determined by the relation:

$$\nu = \frac{1}{h}(E_{n_2} - E_{n_1})$$

Gradually accepted over the course of the 1910s, these two basic assumptions came to provide a bare-bones description of quantum systems, which we refer to as the state-transition model.[5]

Applying the state-transition model could mean calculating the spectra of atoms via the energy-frequency relation. In turn, it could also mean using the energy-frequency relation to translate observed spectra back into energy differences, in order to obtain essential input for atomic model building. The main empirical goal was to produce a classification of series spectra as well as their fine structure and to express this classification in terms of quantum numbers. The archetypical result of this approach was the Rydberg formula, which determined the frequencies of the hydrogen spectrum by two integers identified as the quantum numbers of different stationary states of the hydrogen atom:

$$\nu = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

As the calculation of spectral frequencies was thus based on one single equation, it was the determination of the energy levels that took up most of the calculational labor and was the central focus of the first extensions of the Bohr model after its inception in 1913.

The state-transition model was a decisive break with the past, introducing a rift both on a conceptual level and on the level of practice. First of all, it implied a strict compartmentalization between the states, which were radiation-free, and the radiative transitions. The two were studied in relative isolation from each other. The motion of the electron in a stationary state was determined using the formalism of classical point mechanics, yielding the energy levels.[6] In contrast, the frequency of the emitted radiation was determined from the energy levels through the mathematically

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[5] For an extended discussion, see Blum and Jähnert (2022).

[6] This opened up a large field of study, which attracted experts in celestial analytical mechanics, such as Sommerfeld, Epstein, Schwarzschild, and later on Max Born, Wolfgang Pauli, and Werner Heisenberg. They spent considerable time and effort in the late 1910s and early 1920s to develop sophisticated mechanical models and employed the calculation tools of perturbation theory in order to determine energy levels or the ionization potentials of atoms and molecules. There are many excellent historical studies of this mode of theorizing (see in particular Darrigol 1992; Eckert 2013; Duncan and Janssen 2019).

elementary energy-frequency relation, with no reference to the actual dynamics inside the atom.

The transition process not only did not care about the details of the inner-atomic dynamics; it also made no reference to the dynamics of the radiation field. Instead, the radiation field appeared as an amorphous external reservoir, a sink or a source for the energy differences involved in quantum transitions. There was no dynamical interaction in a field-theoretical sense between the system and the surrounding radiation field. The quantum transition was, in other words, a lonely and isolated event, disconnected from both the dynamics of the atom and of the field.

While this picture had clear advantages when dealing with the states (which could be constructed and studied without having to consider radiative energy loss), it was highly problematic when it came to the description of the radiation process. Prior to 1918, the state-transition model simply had nothing to offer here except for Bohr's frequency condition and thus merely provided a minimalistic black-box description. It ignored radiation properties such as intensity and polarization and provided no guidance on how to incorporate these properties.

Going beyond this description was difficult for two interconnected reasons. On the one hand, there was the aforementioned disconnect between transitions and stationary states. On the other hand, the very definition of the radiation-free states meant that in the Bohr model one could no longer apply classical electrodynamics, where the accelerated electrons in the atom would act as sources in Maxwell's equations. In this situation, any deeper dynamical laws governing the production of radiation remained completely unspecified and were relegated to a still unknown theory of the interaction between light and matter.

Bohr was keenly aware of this dire situation early on. For him, it was clear that the break between the state-transition model and classical radiation theory ran deep and would be irrevocable. Nonetheless, he tried to establish a novel relation between motion and radiation within quantum physics in formal analogy to classical radiation theory. This eventually led to the formulation of the correspondence principle, summarized by Bohr in the following manner in 1923:[7]

> [T]he possibility of the occurrence of a transition, accompanied by radiation, between two stationary states of a multiply periodic

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[7] Bohr's correspondence principle is often misunderstood as a statement about the limit where quantum theory somehow merges into classical theory. More nuanced histories of the old quantum theory discarded these classical-limiting-case interpretations and showed that the correspondence principle was a statement about the relation between motion and radiation within quantum theory (see in particular Darrigol 1992; Tanona 2002; Jähnert 2019, and the literature cited in these works).

> system, whose quantum numbers are respectively $n'_1 \ldots n'_u$ and $n''_1 \ldots n''_u$ [is] considered as conditioned by the presence of certain harmonic components [in the Fourier series] for the electric moment of the atom, for which the frequencies $\tau_1\omega_1 + \ldots + \tau_u\omega_u$ are given by the following equation:
>
> $$\tau_1 = n'_1 - n''_1, \ldots, \tau_u = n'_u - n''_u$$
>
> We, therefore, call these the "corresponding" harmonic components in the motion, and the substance of the above statement we designate as the "Correspondence Principle" for multiple periodic systems. (Bohr 1923, p. 142)[8]

Leaving the actual mechanism governing the transitions unspecified, Bohr's principle stated that there was a direct, one-to-one correspondence between the Fourier modes of the electronic motion and the transitions of the system. This correspondence applied only to spontaneous transitions;[9] it determined whether transitions were allowed or not and implied most importantly that pure harmonic motions, with only the fundamental frequency, only led to transitions to adjacent states, i.e., to a strict selection rule. Anharmonic motions, by contrast, include all overtones and thus allowed for transitions to all states, without selection rules.

In this reading, the possibility of a transition was a property of the atomic state itself, irrespective of the surrounding electromagnetic field. While the correspondence principle thus sought to re-establish contact between the transitions and the inner mechanical workings of the atom,

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[8] (Bohr 1923, p. 142): "[Die] Möglichkeit des Auftretens eines von Strahlung begleiteten Übergangs zwischen zwei stationären Zuständen eines mehrfach periodischen Systems, deren Quantenzahlen bzw. gleich $n'_1 \ldots n'_u$ und $n''_1 \ldots n''_u$ sind, als bedingt ansehen von der Gegenwart derjenigen harmonischen Schwingungskomponente in dem durch (2) gegeben Ausdruck für das elektrische Moment des Atoms, für deren Schwingungszahl $\tau_1\omega_1 + \ldots + \tau_u\omega_u$ die Gleichungen gelten:

$$\tau_1 = n'_1 - n''_1, \ldots, \tau_u = n'_u - n''_u$$

Diese nennen wir deshalb die 'korrespondierende' Schwingungskomponente in der Bewegung, und den Inhalt der obigen Aussage bezeichnen wir als das 'Korrespondenzprinzip' für mehrfach periodische Systeme."

[9] This is highlighted by Bohr's selective reception of Einstein's radiation theory. He only considered what Einstein had called "spontaneous transitions," which occurred irrespective of the radiation field; the induced transitions, i.e., absorption and stimulated emission of radiation, were left aside, precisely because they would require more detailed assumptions on the dynamic interaction between matter and radiation (see Einstein 1916a, b).

the quantum systems remained dynamically isolated from the external electromagnetic field, leaving the core of the state-transition model intact.

3. The Emergence of the *Ersatzstrahler*

By 1918, Bohr had established the core idea of the correspondence principle and presented it in his treatise "On the Quantum Theory of Line Spectra" (Bohr 1918). It postulated that the connection between motion and the radiation emitted in transitions would persist in quantum theory: each observed transition was to be associated with a nonzero Fourier component of the system's motion.

In the following, Bohr went on to generalize and extend this new correspondence relation and turned it into a relation between the numerical value of a Fourier coefficient and the intensity of the corresponding spectral line. To this end, he took up the statistical description of the state-transition model that Einstein had used in his 1916/17 derivation of Planck's law. Bohr argued that the intensities of spectral lines were given by the probability that an atom in a given state would perform the corresponding transition in unit time.

The central open question was how to quantify the relation between the Fourier coefficients and the intensities/transition probabilities. Any further elaboration had to deal with a central conceptual ambiguity: the principle postulated a connection between radiation and motion; but was this the motion in the initial state, the final state or some combination of the two? We refer to this as the "initial-final-state problem."[10] Its resolution was hardly obvious: while in the classical limit, the initial state was the one doing the radiating, the frequency condition employed a combination of initial and final states.

Bohr largely avoided this problem and shied away from an explicit mathematical formulation of the correspondence principle except for the case of disallowed transitions and the limit of large quantum numbers. In these two cases, the relation between Fourier coefficients and intensities/transition probabilities was the same as in classical theory. But for the spectroscopically relevant small quantum numbers, Bohr's foundational 1918 treatise offered no quantitative relation whatsoever and instead stated that "we cannot without a detailed theory of the mechanism of transition obtain exact expressions of [the transition] probabilities" (Bohr 1918, p. 16). While such a detailed theory was still a long way off, other physicists did not follow Bohr in his assessment that a

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[10] The term is coined in Jähnert (2019), from which we draw most of the following discussion.

quantitative relation would have to wait that long. They believed that there were more immediate ways to apply the correspondence principle and identified the initial-final-state problem as the main operational as well as conceptual obstacle. For example, in a letter to Bohr on 6 June 1918, Peter Debye argued in this way and even explored possible solutions:

> In particular your *Ansatz* for the calculation of the intensities is evidently of major importance! Personally, I feel a little dissatisfied, when I see, that you are connecting the intensity to the Fourier coefficient of a single orbit. It seems to me, that if a system goes over from one orbit [...] to another orbit [...] the Fourier coefficients will be different for the first orbit, say C, and for the second, say C'. Wouldn't it be closer to the spirit of your considerations to measure the probability of the transition by $C + C'/2$ or perhaps $\sqrt{CC'}$?[11]

Debye thus expected—in analogy to the frequency condition—a combination of the electron's initial- and final-state Fourier series to appear in a mathematical formulation of the correspondence principle. This indeed became the approach taken by Bohr's assistant Hendrik A. Kramers in his PhD thesis. Here, Kramers proposed explicit expressions for the relation between the Fourier coefficients of the underlying electronic motion and the transition probabilities.

In contrast to Debye, Kramers did not attempt to determine the intensity in direct analogy to the frequency condition but painted a far more complex picture of the transition process. Instead of using some combination of initial and final states, he argued that one had to consider the "multitude of mechanically possible states of the system lying 'between' the initial state and the final state" (Kramers 1919, p. 327). Performing an integration from the initial to the final value of the quantum numbers involved in the transition, one could then arrive at the radiation frequency in an alternative way: indeed, the radiation frequency of the discontinuous quantum transition appeared as the mean value of the orbital frequencies in the intermediate states.

In the next step, Kramers switched to his actual target and extended this method to the transition probabilities. Transition probabilities, so the main idea held, would have to be calculated in a similar manner by averaging over the respective Fourier coefficients. Fleshing out this idea was not straightforward, however. While the orbital-mechanical and the radiation frequencies could simply be identified—both being frequencies,

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[11] Debye to Bohr 6 June 1918 in Nielsen (1976, p. 607).

Fourier coefficients and transition probabilities were two distinct physical quantities with different units. This made a direct identification impossible. In order to connect them, Kramers introduced a new figure of thought, which, as we will see in following, became the foundation of the correspondence approach up to BKS:

> [It appears possible to] obtain an expression for the probability in question by comparing the emitted radiation with the intensity of radiation emitted on ordinary electrodynamics by an electron performing a simple harmonic motion. (Kramers 1919, pp. 329–30)

The averaged Fourier coefficient was thus to be connected with the transition probability in a somewhat indirect manner: it was taken to be the amplitude of "an electron performing a simple harmonic motion" which emitted radiation according to "ordinary electrodynamics." This classical radiation process was then to be formally compared to the actual, quantum-theoretical one.

This formal comparison was at the basis of Kramers' approach. It implied that one could obtain the quantum-theoretical transition probability by taking the classical intensity and dividing by the energy emitted in a single quantum transition, $h\nu$, obtaining an expression for the transition probability. The classical intensity, in turn, was easily calculated: identifying the oscillating variable x with the electron's distance from the nucleus in the Bohr model, one had an oscillating dipole—at least in the case of hydrogen, where the nucleus has the same charge as the electron, only with an opposite sign. Dipole radiation has an intensity proportional to $\nu^4 C^2$, where ν and C are the frequency and the amplitude respectively. In this case, ν was the averaged frequency (and thus the radiation frequency) and C was the averaged Fourier coefficient associated with the transition. The end result was the numerical relation between the averaged Fourier coefficient and the transition probability envisioned by the correspondence principle.

The actual averaging process used by Kramers need not concern us here. It was merely tentative and would be varied widely over the course of the following years, the initial-final-state problem still being far from resolved. Far more stable was the new theoretical entity that Kramers had introduced here, the classically radiating electron, which soon came to be known within the quantum community as the *Ersatzstrahler* (substitute radiator). This entity was not intended as a physical specification of the radiation process, which was still assumed to be more or less instantaneous. Neither Kramers, nor those who took up his ideas on calculating transition probabilities, insinuated that the electron in the atom actually performed the harmonic motion associated with the substitute oscillator.

This assessment, as we will show in the following, carries over at least in part to BKS theory and its conception of virtuality, and it is thus worth highlighting that Kramers's argument left the state-transition model intact; the classical *Ersatzstrahler* was merely introduced for the purposes of a formal comparison and lived in a theoretical space, which was ontologically separated from the space of the state-transition model. In contrast to BKS's virtual oscillators, this theoretical space was very limited. It belonged only to a single transition, and the radiation emitted by the substitute radiator merely served the purpose of determining the intensity of the associated spectral line. This conception of the substitute radiator mirrored the Bohr atom: both were isolated from the surrounding radiation field and emitted their energy into an unspecified external reservoir. For the Bohr atom, this isolation was a matter of necessity: there was simply no theoretical framework for describing its interaction with an outside world populated with dynamical, propagating radiation. For the *Ersatzstrahler*, there was no such in-principle restriction: it was a classical object and could be well at home in a world of classical interactions between matter and radiation, such as absorption, stimulated emission, and dispersion. It was this feature that would make the substitute oscillator a central element in the further extension of the correspondence principle, all the way to BKS theory.

4. From Substitution to Virtuality

In the summer of 1923, the *Ersatzstrahler* was alive and well. It was used by a host of historical actors, from which we only mention a few relevant examples: Frank C. Hoyt explored Kramers' averaging procedure in his work on spectral intensities in Copenhagen under the auspices of Bohr and Kramers. In Breslau, Fritz Reiche, Rudolf Ladenburg and Willy Thomas calculated transition probabilities using Kramers' approach in the context of their quantum theory of dispersion and absorption (Hoyt 1923a, b; Ladenburg and Reiche 1923; Thomas 1925).[12] Last but not least, the young American post-doc John C. Slater in Cambridge studied the emission process using Fourier coefficients constructed from "a suitable mean between the amplitudes of the corresponding motions in the initial and final states" as part of a new dual theory of light, which would eventually give rise to BKS theory.[13] These works gradually went beyond the limited picture of a single, isolated emission process and thereby introduced some variations into the *Ersatzstrahler* concept.[14] In December

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[12] For an extensive discussion of the applications of the correspondence principle, in particular of the role of Kramers' concept, see Jähnert (2019).

[13] See Slater's dossier as referenced in footnote 17.

1923 and January 1924, however, these different bits started to coalesce, leading to a new formulation of the *Ersatzstrahler* in BKS theory.

This development is usually described following Slater's trajectory in 1923, which he put forward in his later recollections.[15] During his stay in Cambridge (until December 1923), the story goes, Slater developed a theory sketch, which aimed to connect emission and absorption processes in distant atoms and thus to combine them into a unified, global picture.[16] In this global picture, quantum systems performing a transition emitted an actual light quantum, "a lump of energy," which propagated through space and was eventually absorbed by a distant atom. To describe the motion of the light quantum in accordance with wave phenomena like interference, Slater further introduced an energy-less classical radiation field, which determined the probability for such a quantum to be emitted in a given direction and at a given time.[17]

Having thus proposed a dual theory of light, which was in line with but probably independent from de Broglie's "*onde* fictive" or Einstein's "*Gespensterfeld*," Slater set out to ground his approach in the correspondence principle.[18] He assumed that the classical field was produced by a classical source, the oscillations of which were determined through a Kramersian averaging procedure. While Kramers had been content with calculating the radiation emitted by such a source, Slater now studied the

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14. These variations were perceived as minor, sometimes awkward adaptations rather than major reconceptualization by the historical actors, for an extended discussion see Jähnert (2019, chap. 8).

15. The development of BKS has been studied extensively (e.g., by Jammer 1966; Hendry 1981; Konno 1983; Dresden 1987; Darrigol 1992; Duncan and Janssen 2007; Blum and Jähnert 2022). The following relies heavily on these reconstructions. Our interpretation of virtuality in BKS extends the analysis of BKS by highlighting certain aspects which remained implicit in these reconstructions.

16. In a sense, this was a continuation of Einstein's work of 1916/17. Einstein had combined both emission and absorption processes in his *statistical* description of radiation interacting with quantized matter. In contrast, Slater's theory aimed at a detailed *dynamical* description of the connection between individual emission and absorption processes.

17. This reconstruction is based on a manuscript by Slater dated 1 and 4 November 1923, and the quotes are taken from there. We have only seen a typed transcript of this manuscript (the page numbers are those of this transcript), prepared by Slater himself for a dossier he put together in 1968 as an addendum to the interview with him, conducted by Thomas Kuhn in 1963 in the context of the SHQP (Sources for the History of Quantum Physics) project https://cstms.berkeley.edu/research/ahqp/. This dossier is held by the Niels Bohr Library and Archives at the American Institute of Physics in College Park, MD. According to Konno, this transcript is identical to the original manuscript, held by the American Philosophical Society in Philadelphia (Konno 1983, n. 6).

18. The similarities (and subtle differences) between de Broglie, Einstein, and Slater are well established. The independence has been discussed, e.g., by Konno (1983).

propagation and spatial distribution of the classical field of the *Ersatzstrahler*. Using the tools of classical electrodynamics (in particular the Poynting vector S), he determined the classical energy distribution and then finally assumed that it provided the probability distribution for the emission of individual light quanta.

From the perspective of the *Ersatzstrahler*, it was easy to see this approach as a direct continuation and a considerable extension of Kramers's initial approach into a global, unified picture. At the same time, Slater's theory tweaked the *Ersatzstrahler* and its relation with the state-transition model in minute ways. The process of a quantum system emitting or absorbing radiation was now spelled out as a temporal sequence of events, probabilistically caused by the classical radiation of the *Ersatzstrahler*. The *Ersatzstrahler* and its radiation were thus conceived as spatio-temporal entities, which were much more directly related to the events taking place in the actual space of atomic transitions than before.[19] For Slater, this implied that the temporal relation between the state-transition model, the *Ersatzstrahler* and its radiation needed to be specified. He assumed that the *Ersatzstrahler* radiated continuously while the system was in a stationary state. The radiation then triggered transitions in distant atoms at a later time.

This picture was decidedly different from the original *Ersatzstrahler*, for which the temporal relation between the two radiation processes had never been consequential, leading to the tacit association of the *Ersatzstrahler* with the transition process. As such, Slater's shifting of the *Ersatzstrahler* from the transitions to the states was no small thing. Rather, as Bohr highlighted, it presented a key innovation, which led him to endorse the approach.[20]

When Slater presented his work upon his arrival in Copenhagen in December 1923—so the story of BKS continues—Kramers and Bohr adopted his ideas rather quickly but modified them in a crucial way. For them, the crucial features of Slater's theory were the concept of an energyless radiation field and his modification of the *Ersatzstrahler* approach. Slater's substitute radiators, which Bohr and Kramers now dubbed "virtual oscillators," were coupled to a virtual radiation field and interacted with each other in a classical manner.[21] They could thus be used to model the

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[19] As we will see below, there is a crucial difference between Slater's original version and BKS in this point. In Slater's theory, the classical radiation field and the space through which the light quanta travelled was the same: actual space. In BKS, virtual radiation lived in a virtual space and was disjoint from the space of the quantum systems. The causal connection, however, persisted.

[20] Bohr to Slater, 10 January 1925 (BSC 16.2; Bohr's Scientific Correspondence, Niels Bohr Institute, University of Copenhagen).

interaction between light and matter and thereby introduced a global (or spatiotemporal) picture for this process: disturbances in the virtual radiation field, produced by a virtual oscillator at one place, propagated through space and could be absorbed by the virtual oscillators of other atoms, the amplitude of the induced oscillations corresponding to the probability of an induced transition. At the same time, the virtual radiation from different sources would interfere in the virtual space thereby allowing for an integration of classical wave optics.

Within this comprehensive picture, the light quantum appeared as a feature, which was redundant at best.[22] Kramers and Bohr argued that the integration of an energyless field into the *Ersatzstrahler* tradition was all that was necessary and that light quanta could be removed from the theory entirely. Eventually, Slater came to agree with this position and remarked in a letter to his parents on 6 January 1924:[23]

> That theory is about the way it was. Part of it they believe, and part they don't. But I have been thinking about it, and have come to the conclusion that the part they believe is the only part that really leads to any results anyway, and the rest is more a matter of taste than anything else, and you have about the same thing whether you keep it or not.

Last but not least, Kramers and Bohr changed another key aspect of Slater's theory and the *Ersatzstrahler* tradition. While Slater had bought into the standard assumption that the *Ersatzstrahler* would be constructed on the basis of Kramers's averaging procedure, Bohr and Kramers developed BKS as a purely qualitative theory, yet to await mathematical formulation. This feature of BKS did not arise in reaction to Slater's work but rather from another discussion unfolding prior to Slater's arrival. Here, Bohr's and Kramers's thoughts had revolved around the new quantum theory of dispersion, which emerged with the work of Rudolf Ladenburg and Fritz Reiche. In this context, calculations of transition probabilities from Kramers's *Ersatzstrahler*, among other things, raised serious doubts about

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[21] The reader should note that there is a different interpretation in the secondary literature which states that BKS in fact did not specify the way in which virtual oscillators interacted with the field. See Blum and Jähnert (2022, n. 76) for our response to this interpretation.

[22] One major argument for the redundancy of light quanta was the asymmetry between the probabilistic mechanism of emission and the deterministic mechanism of absorption in Slater's theory, which could not be reconciled with the symmetry of emission and absorption generally assumed and vouched for by spectroscopy (for a more detailed discussion of this point see Blum and Jähnert 2022, p. 137).

[23] The letter was transcribed in Slater's dossier, see footnote 17.

the feasibility of the original averaging procedure and eventually led Kramers to abandon the initial approach.[24] Lacking a quantitative alternative, the virtual oscillators were merely "conjugated with the motion of the atom" in an unspecified or, as Heisenberg later put it, "highly symbolic" manner (Bohr et al. 1924, p. 794; Heisenberg 1925, p. 617).

In this new qualitative theory, the virtual oscillators and virtual radiation extended the *Ersatzstrahler* tradition into a compact and general description of the interaction between atoms and radiation. In the introduction to the BKS paper, Bohr and Kramers highlighted this continuity and presented the new picture as little more than the next logical extension of the correspondence approach operating within the framework of classical electrodynamics:

> The correspondence principle has led to comparing the reaction of an atom on a field of radiation with the reaction on such a field which, according to the classical theory of electrodynamics, should be expected from a set of "virtual" harmonic oscillators with frequencies equal to those determined by the equation (1) [frequency condition] for the various possible transitions between stationary states. (Bohr et al. 1924, pp. 789–90)

Since BKS aimed to provide a comprehensive account of radiation phenomena, Bohr felt that it might not be adequate to talk about quantum systems performing a transition (emission and absorption) but rather about atoms reacting to a radiation field (encompassing not only emission and absorption but also dispersion and scattering of radiation). This did not imply, however, that the essence of the correspondence approach had changed. As Bohr highlighted in a subsequent paper, the virtual oscillators still relied on the formal comparison between an actual quantum process and its subjunctive classical counterpart. This comparison was now merely being extended to different types of radiation processes:

> According to the correspondence principle, the possibility [of a transition] is connected with the existence of a certain corresponding harmonic component in the electric moment of the atom, which is determined by the motion of the atomic particles. While it is the very essence of quantum theory that the reaction of atoms to a radiation field is not connected in a simple manner with the motion in the stationary state, this reaction, with regard to both scattering and emission, is still *compared formally* with a number of virtual oscillators,

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[24] See Jähnert 2019, chap. 6 for a detailed analysis of the discussion between Reiche and Kramers.

> whose frequencies are directly connected to the energy differences of the possible transitions. (Bohr 1924, p. 1115, emphasis ours)

The virtual oscillators of BKS thus still provided a classical substitute model for the transition process, a direct continuation of the approach pioneered in Kramers's dissertation. As Bohr explicitly stated, the new approach "[did] not involve any change in the connection between the structure of the atom and the frequency, intensity, and polarization of the spectral lines"(Bohr et al. 1924, p. 791). In adding the virtual radiation to the *Ersatzstrahler* picture, BKS thus replaced neither the state-transition model nor the correspondence principle.

5. The Concept of Virtuality in BKS

As we can see from the preceding historical analysis, the notion of virtuality in BKS differed substantially from the notion of virtuality in particle physics. Virtual oscillators did not present quantum objects, which could be seen as living in the actual world. The virtual oscillators of BKS emerged within the context of the correspondence principle and the idea of the classical *Ersatzstrahler*. Within this tradition, the virtual world of BKS remained categorically separate from the world of quantum systems and their transitions between different stationary states. The separation between the real or the actual and the virtual was clear cut, not fuzzy as it would later be in QFT (quantum mechanics) where the distinction between virtual and real particles is a matter of degree.

In BKS, as we have seen, this separation between the virtual and the actual was bridged on a formal level rather than through ontological connection. Yet the formal connection between virtual oscillators and the state-transition model went beyond the purely calculational approach of Kramers's dissertation, where the *Ersatzstrahler* was merely a methodological stand-in for actual transitions. Extending the approach from isolated emission processes to problems of energy exchange and spatiotemporal ordering and correlation of events, as we have seen, the original methodological as-if of the *Ersatzstrahler* was transformed into the notion of virtuality in BKS. Here, the formal comparison became a formal coordination—or, in Bohr's words, conjugation—of the virtual and the actual world, a pre-established harmony of sorts.

This formal coordination was not straightforward and in certain cases required stretching the imagination, as is most pronounced in the case of Compton scattering. BKS explained the by then well-established frequency shift observed in Compton's experiment as a Doppler shift of virtual radiation. The virtual oscillator of the target system was to receive momentum from the impinging virtual radiation; the virtual oscillator, now moving with a specific velocity, then emitted Doppler-shifted radiation. To produce the observed frequency shift, BKS required a tailor-made velocity for the virtual oscillators. This gave a consistent description in the virtual world, but the mapping of this picture onto the actual world was problematic: the virtual oscillators had to move with a velocity different from that of the actual electrons, which were being knocked out of the atom in Compton's experiment. As a result, the virtual radiator and the actual radiating system were in different places.[25]

The central characteristics of the virtual in BKS come to the fore in this extreme warping of the mapping from the virtual onto the actual world. The theory required considerable leeway in fixing the motion of the virtual oscillators and in mapping the virtual world to the actual world of atomic transitions. This leeway was hardly to be had if virtual oscillators and actual quantum systems occupied the same world. Rather it required a symbolic relation between them, along the lines of the "formal comparison" of the *Ersatzstrahler* approach.

Bohr, Kramers, and Slater clearly acknowledged that they were stretching the boundaries of acceptable theorizing by introducing "a feature strikingly unfamiliar to the classical conceptions." For the sake of a better argument, they concluded with a check on the future:

> In view of the fundamental departures from the classical space-time description, involved in the very idea of virtual oscillators, it seems at the present state of science hardly justifiable to reject a formal interpretation as that under consideration as inadequate. (Bohr et al. 1924, p. 799)

While the separation of the virtual from the actual world is most pronounced in the case of Compton scattering, this is just the tip of the iceberg. The separation plays a central role throughout BKS. It also shows up in the attempted coupling between distant atoms—the heart of the theory—leading to the theory's most (in-)famous prediction. As Bohr, Kramers and Slater pointed out, actual absorption processes were only dependent on the possibility of transition occurring (i.e., of another atom being in an excited state and emitting virtual radiation of the right frequency), not on the actual occurrence of that transition:

> [W]e assume that an induced transition in an atom is not directly caused by a transition in a distant atom for which the energy

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[25] This result has long been identified as one of the most problematic aspect of BKS (see Darrigol 1992, p. 223; Duncan and Janssen 2007, pp. 602–03; Blum and Jähnert 2022, pp. 138–9).

> difference between the initial and the final stationary state is the same.
>
> On the contrary, an atom which has contributed to the induction of a certain transition in a distant atom through the virtual radiation field conjugated with the virtual harmonic oscillator corresponding with one of the possible transitions to other stationary states, may nevertheless itself ultimately perform another of these transitions. (Bohr et al. 1924, p. 792)

This meant that energy would still be conserved statistically but not for individual processes. A direct upshot of this picture was that, in the Compton effect for example, there would be no correlation between the recoiling electron (the emission process) and the observation of the secondary X-ray (the absorption process). In other words, causality was violated in individual quantum scattering processes.

Here too, Bohr, Kramers, and Slater were well aware that this mere statistical conservation of energy presented a strikingly novel feature, not present in classical radiation theory. In contrast to the awkwardness of mapping events in virtual space onto actual space, however, they were much more comfortable with this type of radical departure from classical physics, almost praising it as a virtue of the new theory and awaiting its empirical testing.[26]

In light of the clear awareness of the pitfalls of the approach, the question arises whether the virtual oscillators and radiation fields were just hypothetical entities, with unusual—if not to say awkward—properties. As we see it, there is more to it. In BKS, virtual radiation was designed to produce an effect, namely, to induce transitions and thus establish communication between distant atoms. At the same time, radiation was not allowed to stand in a continuous dynamical exchange of energy with those

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[26] As is well-known, this empirical test rather quickly led to the demise of BKS, when Bothe and Geiger showed that the statistical correlations predicted by BKS were not sufficient to explain experimental results. The Bothe-Geiger experiments were widely viewed as immediate disproof of the theory and many saw the light quantum vindicated as Bohr's alternative of dumping energy and momentum conservation had failed. This vindication is often taken as the final word on the subject of radiation in the old quantum theory. Yet, the failure of BKS did not lead directly to a successful theory of radiation in terms of light quanta. It rather exacerbated the dilemma involved in constructing a theory of light and matter interaction. For the historical actors, the main lesson of BKS was the demand that a new theory of light-matter interaction would have to lead to correlations for individual processes that ensured strict validity of energy and momentum conservation. This "problem of coupling" (between distant atoms, through radiation) emerged as the main conceptual problem for future dynamical theories of light-matter interaction (see Blum and Jähnert 2022).

atoms, so as not to endanger the stability of the stationary states. Granting that the transport of energy, at least since Maxwell, was an essential feature of a wave field, the virtualization undertaken in BKS theory fits surprisingly well with the notion of the virtual formulated by Charles S. Pierce (Peirce 1902).[27] Far more than a fancy name for a hypothetical entity, it succinctly described the separation of (quantum) effect from (classical) essence and thus how the virtual entities differed from their non-virtual counterparts.

Going even further, we can observe that this virtuality was not just an accidental feature. Rather, it was determined by the role that virtual entities played in BKS theory. This role was to avoid a central and manifest tension within Bohr's radiation theory of states and transitions. As we have seen, BKS's main idea was the introduction of a coupling mechanism between distant atoms in order to break the isolation of quantum systems from their surroundings. Yet, the virtualization of radiation effectively shifted all dynamical interactions *out of* the actual quantum world. Transitions still took place within *isolated* quantum systems, and transition processes in distant atoms remained independent. There was strictly speaking no dynamical coupling between systems in the actual space of the state-transition model, only statistical correlations. Rather than unifying the individual quantum systems and the surrounding field, the virtualization of BKS thus solidified, even reified, the isolation of the single quantum systems. To make it quite explicit, the point here is not that this strategy was misguided given the empirical failure of BKS. Even as a potentially adequate solution, we see the virtualization of BKS first and foremost as a symptom of an unresolved conceptual tension.

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[27] While there is, to our knowledge, no direct evidence that Bohr took his cue from Pierce, his general fluency in pragmatic philosophy and general sensitivity to terminology is well documented (see Jammer 1966; Faye and Folse 1994). Whether influenced by Pierce or not, Bohr highlighted the importance of the term in his correspondence with Pauli (see Bohr to Pauli 16 February 1924 in Hermann et al. 1979, p. 146). Peirce's definition of the virtual is discussed in detail in Steinle's contribution to this volume.

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