Supporting Information: The relevance of degenerate states in chiral polaritonics

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FIG. S1. Magnitude of the different transition matrix elements in atomic units with an applied magnetic field ($B_0 = 0.235$ T): A) Electronic dipole moment with respect to x or y components. B) The z component of the magnetic dipole moment. C) The xx or yy components of the quadrupole tensor. D) the xy component of the quadrupole tensor, between the ground state (gs) and the states M_{\pm} , which correspond to the degenerate first excited state of the uncoupled system. The mainly contributing elements of each matrix representation are highlighted in red.



FIG. S2. Linear electronic dipole absorption spectra, when tuning the A) chiral cavity (-) or B) cavity (+) to the first electronic excitation of the quantum ring. An external magnetic field is applied along the z-axis ($\hbar \omega = 1.21 \text{ meV}$), with different coupling strengths λ . The broadening of every peak has been set ad-hoc by a Gaussian function.



FIG. S3. Angular momentum of the ground state (black), lower (blue) and upper (green) polaritons, as well as the weakly-coupled state M_{\pm} (red) obtained in a A) cavity (-) and B) cavity (+), as a function of the cavity frequency, with $\lambda = 5.0 \times 10^{-6}$ for $B_0 = 0.235$ T.



FIG. S4. Magnetic-dipole transition (m_z) matrix obtained for a coupling strength $\lambda = 5 \times 10^{-6}$, with a cavity frequency of 1.21 meV and a constant magnetic field $B_0 = 0.235$ T. In A) the cavity corresponds to the (-) and in B) to the (+) polarization. The dominant contributions are highlighted in red. In C) we show the magnetic-dipole moment transition between LP and UP, for different cavity frequencies and polarizations, with the same λ and magnetic field. In the three cases, the magnetic-dipole moment is expressed in atomic units.



FIG. S5. Matrices for the xx (or yy) and xy components of the quadropolar tensor, obtained for the first four states, when $\lambda = 5 \times 10^{-6}$ and $B_0 = 0.235$ T, with a cavity A) (-) and B) (+) configuration. The main contributions are highlighted in red. All the magnitudes are in atomic units.